

**MATHEMATICAL MODELLING OF NEILL MAPPING FUNCTION FOR
GLOBAL POSITION SYSTEM (GPS) TROPOSPHERIC DELAY**

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SUPERVISOR DECLARATION

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STUDENT DECLARATION

“I declare that this report entitle “*Mathematical Modelling of Neill Mapping Function for Global Position System (GPS) Troposperic Delay*” is the result of my own research except as cited in the references. The report has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

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DEDICATION

*Dedicated to my beloved father and mother,
my siblings, lecturers and to all my friends thanks
for their love and sacrifice*

ACKNOWLEDGEMENT

Alhamdulillah thank to ALLAH S.W.T for his blessed at last I finished this Final Year Project. First of all, I would like to take this opportunity to express my gratitude to my supervisor; Dr. Hamzah Bin Sakidin for encouragement, support, critics and helps. Without his guidance and interest, this project will not be a success.

My sincere appreciation also extends to all my fellow friends for their assistance and motivation at the various occasions. Their views and tips are very useful indeed. Last but not least, thank you to all people who in one way or another contribute to the success of this project.

I am also grateful to all my family members.

Thank you.

ABSTRACT

The purpose of this project is to modify a mathematical model on a mapping function models to improve the calculation of the GPS tropospheric delay. Mapping function scale factor can be used to amplify the zenith tropospheric delay to form total tropospheric delay. Mapping functions for the hydrostatic and the wet part are usually separated and mostly the tropospheric delay models use mapping functions in the form of continued fractions which is quite tedious in calculation. Mapping function values can be obtained after 26 mathematical operations for Neill Mapping Function (NMF). There is a need to modify the mapping function models to allow faster calculation and also better understanding of the models. The mapping functions for NMF models for hydrostatic and wet components are given in a form of continued fraction, whereby the elevation angle is the variable. NMF is selected due to its ability to measure the mapping function down to 3° elevation angle.

ABSTRAK

Tujuan projek ini adalah untuk mengubah suai pemodelan matematik pada model fungsi pemetaan untuk memperbaiki pengiraan kelewatan tropospheric GPS. Kelewatan kemuncaknya tropospheric boleh dikuatkan oleh faktor pekali dipanggil fungsi pemetaan untuk membentuk jumlah kelewatan tropospheric. Fungsi pemetaan untuk bahagian hidrostatik dan basah biasanya dipisahkan. kebanyakan daripada model kelewatan tropospheric menggunakan fungsi pemetaan dalam bentuk pecahan berterusan yang agak membosankan dalam pengiraan. Nilai-nilai fungsi pemetaan boleh diperolehi selepas menyelesaikan 26 operasi matematik bagi Fungsi Pemetaan Neill (NMF). Terdapat keperluan untuk mengubah suai model fungsi pemetaan untuk membolehkan pengiraan yang lebih cepat dan juga pemahaman yang lebih baik daripada model sebelumnya. Fungsi-fungsi pemetaan untuk model NMF untuk komponen hidrostatik dan basah diberi dalam bentuk pecahan yang berterusan, di mana sudut ketinggian berubah-ubah. NMF dipilih kerana kemampuan untuk mengukur fungsi pemetaan sehingga sudut ketinggian dapat mengurangkan kepada 3° .

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CHAPTER 1

INTRODUCTION

1.1 Project Background

Tropospheric delay is a delay experienced by the GPS signal while propagating through the electrically neutral atmosphere. As a coefficient of zenith hydrostatic delay and also zenith non hydrostatic delay, the mapping function can be used to increase or reduce the tropospheric delay. So, mapping function scale factor can be used to amplify the zenith tropospheric delay to form total tropospheric delay. The original mapping function model formula is more complex and the established tropospheric delay models use mapping functions in the form of continued fractions which is quite tedious in calculation. The mapping function scale factor should be unity at 90 degree [3]. Based on that, this project will be focused on Neill Mapping Function (NMF). NMF is selected due to its ability to measure the mapping function down to 3° elevation angle [3]. The original NMF model will be modified to reduce the number of operation and give better understanding of the model.

1.2 Problem Statement

The original mapping function model formula is more complex. As we know, the developed tropospheric delay models use mapping functions in the form of continued fractions which is quite tedious in calculation. There are 26 mathematical operations for Neill Mapping Function (NMF) which are very long to be calculated and difficult to understand. More time is needed to calculate the mapping function models to get the result. Neill Mapping Function, NMF as given in equation below [3]:

For hydrostatic component:

$$NMF_h(\varepsilon) = \frac{1 + \frac{a}{1 + \frac{b}{1 + c}}}{\sin\varepsilon + \frac{a}{\sin\varepsilon + \frac{b}{\sin\varepsilon + c}}} + \left[\frac{1}{\sin\varepsilon} - \left(\frac{1 + \frac{a_{ht}}{1 + \frac{b_{ht}}{1 + c_{ht}}}}{\sin\varepsilon + \frac{a_{ht}}{\sin\varepsilon + \frac{b_{ht}}{\sin\varepsilon + c_{ht}}}} \right) \right] \quad (1)$$

and for wet component:

$$NMF_w(\varepsilon) = \frac{1 + \frac{a_{wet}}{1 + \frac{b_{wet}}{1 + c_{wet}}}}{\sin\varepsilon + \frac{a_{wet}}{\sin\varepsilon + \frac{b_{wet}}{\sin\varepsilon + c_{wet}}}} \quad (2)$$

Where:

ε - Elevation angle (degree)

$NMF_h(\varepsilon)$ – Hydrostatic mapping function

$NMF_w(\varepsilon)$ – Wet mapping function

H - Station height above the sea level (km)

1.3 Objective

There are three main objectives in this project which are:

- i. To modify original model of the Neill mapping function.
- ii. To calculate the modified mapping function model.
- iii. To compare the original NMF with the modified NMF result in term of the number of operation of the models.

1.4 Scope of project

In many study, there are some limits and constrain of the project that makes the project achievable. This project will be focused on the Neill mapping function models for both hydrostatics and non hydrostatics components. The software used to solve the problem is Maple 13 or excel software and lastly the elevation angle used between 3° to 90° only.

1.5 Thesis outline

This thesis consists of five chapters. It begins with introductory chapter. This chapter will discuss about background, problem statement, objective and scope of this project.

Chapter two will discuss the literature review about Neill Mapping Function (NMF) model and also related work from the previous modeling. In this chapter, it will also discuss about the comparison between the original and the modified Neill's mapping function models.

Chapter three will explain about methodology of this project. It is consisting software to find the modified NMF model by using maple13 and excel.

Chapter four will present the result as a project finding. In this chapter also, it will discuss about the analysis and discussion of the project. In the discussion part, it will explain how the original NMF models to be modified.

The last chapter will be discussed about the conclusion. In this part, it will summarize the research and the process of modification of Neill mapping function model and also recommendation of future work.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The purpose of a literature review is to convey the knowledge and ideas that have been established on this topic and also finding the strengths and weaknesses. Literature review has been conducted prior to undertaking this project to obtain the information on the technology available and technologies that used by the other researchers on the same topic around the world. This chapter provides the summary of literature reviews on key topics related Neill mapping function model.

2.2 Tropospheric Delay

The sum of several sources of error, such as orbit error, satellite clock error, multipath error, receiver noise error, selective availability, ephemeris error and also atmospheric error will determine the accuracy of Global Positioning System (GPS) measurement. Tropospheric delay is delay experienced by the GPS signal in propagating through the electrically neutral atmosphere. Usually, this delay will be calculated in the zenith direction and is referred to as a zenith tropospheric delay.

The delay consists of a zenith hydrostatic delay and zenith wet delay. Zenith hydrostatic delay can be modeled accurately using surface barometric measurements while zenith wet delay cannot be modeled from surface barometric measurements and depends on atmospheric water vapor. As a coefficient of zenith hydrostatic delay and also zenith non hydrostatic delay, the mapping function can be used to increase or reduce the tropospheric delay.

The mapping function depends on the elevation angles. The value of mapping function scale factor equal to 1 (unity) is at 90 degree of elevation angle. So, this value will give minimum value for the tropospheric delay (TD) as given below [7]:

$$TD = ZHD \cdot m_h(\varepsilon) + ZWD \cdot m_w(\varepsilon) \quad (3)$$

where:

ZHD is zenith hydrostatic delay (m)

ZWD is zenith wet delay (m)

$m_h(\varepsilon)$ is the hydrostatic mapping function (-)

$m_w(\varepsilon)$ is the wet mapping function (-)

2.3 Neill Mapping Function Model

In this research, Neill Mapping Function model (NMF) are selected to be modified in order to reduce the computing time by reducing the percentage of number of operations.

In 1996, Arthur Neill already derive the mapping functions, are the most widely used, and are known to be the most accurate and easily-implemented functions. The new mapping function (NMF) is based on temporal changes and geographic location rather than on surface meteorological parameters. All previously available mapping functions have been limited in their accuracy by the dependence on surface temperature, which causes three dilemmas. There is more variability in temperature in the atmospheric boundary layer, from the Earth's surface up to 2000 m are the reason all these.

Variations smaller than that calculated from the mapping function is due to daily changes in surface temperature. Furthermore, upper atmosphere changes are normally lower than seasonal changes in surface temperature (but the computed mapping function yields

artificially large seasonal variations). Then, the computed mapping function for warm winter days may not significantly differ from function for cold summer days. For example, difference in lapse rates and heights of the troposphere will cause the actual mapping functions are quite different than computed values.

Temperature and relative humidity profiles, which are in some sense averages over broadly varying geographical regions can produce the new mapping functions. Niell compared NMF and ray traces calculated from radiosonde data spanning about one year or more covering a wide range of latitude and various heights above sea level [2]. The validity and applicability of the mapping function NMF can be identified by way of comparison.

We can see through the least-square fit of four different latitude data sets, Niell showed that the temporal variation of the hydrostatic mapping function is sinusoidal within the scatter of the data.

Table 2.1: Coefficients of the hydrostatic NMF mapping function [3]

Coefficients	Latitude ϕ_i				
	15°	30°	45°	60°	75°
Average					
<i>a</i>	1.2769934e-3	1.2683230e-3	1.2465397e-3	1.2196049e-3	1.2045996e-3
<i>b</i>	2.9153695e-3	2.9152299e-3	2.9288445e-3	2.9022565e-3	2.9024912e-3
<i>c</i>	62.610505e-3	62.837393e-3	63.721774e-3	63.824265e-3	64.258455e-3
Amplitude					
<i>a</i>	0.0	1.2709626e-5	2.6523662e-5	3.4000452e-5	4.1202191e-5
<i>b</i>	0.0	2.1414979e-5	3.0160779e-5	7.2562722e-5	11.723375e-5
<i>c</i>	0.0	9.0128400e-5	4.3497037e-5	84.795348e-5	170.37206e-5
Height correction					
<i>a_{ht}</i>	2.53e-5				
<i>b_{ht}</i>	5.49e-3				
<i>c_{ht}</i>	1.14e-3				

For the hydrostatic NMF mapping function, the parameter a at tabular latitude φ_i at time t from January 0.0 (in UT days) is given below [3] :

$$a(\varphi_i, t) = a_{avg}(\varphi_i) + a_{amp}(\varphi_i) \cos\left(\frac{t-T_0}{365.25} 2\pi\right) \quad (4)$$

where T_0 is the adopted phase, DOY (day of year) 28. We can obtain the value of $a(\varphi, t)$ by using the linear interpolation between the nearest $a(\varphi_i, t)$. For parameters b and c , a the same procedure was followed.

Table 2.2: Coefficients of the wet NMF mapping function [3]

Coefficients	Latitude φ_i				
	15°	30°	45°	60°	75°
a_w	5.8021897e-4	5.6794847e-4	5.8118019e-4	5.9727542e-4	6.1641693e-4
b_w	1.4275268e-3	1.5138625e-3	1.4572752e-3	1.5007428e-3	1.7599082e-3
c_w	4.3472961e-2	1.5138625e-3	4.3908931e-2	4.4626982e-2	5.4736038e-2

The wet NMF mapping function not dependence to temporal. Therefore, requirement needed for each parameter is interpolation in latitude. Height correction associated with the NMF is given below [3]:

$$\Delta m(E) = \frac{dm(E)}{dh} H \quad (5)$$

$$\frac{dm(E)}{dh} = \frac{1}{\sin(E)} - f(E, a_{ht} b_{ht} c_{ht}) \quad (6)$$

where $f(E, a_{ht} b_{ht} c_{ht})$ is a three-term continued fraction and the parameters $(E, a_{ht} b_{ht} c_{ht})$ as given in Table 2.1 were determined by a least-squares fit to the height correction at nine elevation angles, and H is the station height above sea level (use data from Table 2.1 to find the value for wet component).

Before this, the large number of mapping functions was analyzed use data from 50 stations distributed worldwide (32,467 benchmark values). The models that meet the high standards of modern space geodetic data analysis are Ifadis, Lanyi, MTT, and NMF. The models Lanyi, MTT, and NMF yield identical mean biases and the best total error performance at the elevation angle above 15 degrees Ifadis and NMF are superior at lower elevation angles.

The delay in the direction of an observation is related to the zenith delay by a mapping function to developed expression for calculating the ratios, which is modeled with sufficient accuracy for elevations down to 3° using a three term continued fraction in $\sin \varepsilon$ elevation, ε , given by Neill Mapping Function, NMF as equation (1) for hydrostatic component and equation (2) for wet component.

2.4 Modification and simplification

The modification and simplification of NMF have been carried out as given below [2]:

2.4.1 Hydrostatics Neill Mapping Function, $NMF_h(\varepsilon)$

The same shape of graph for the original NMF is found by using regression method. The original NMF (ε) models is named as Y, while Y1, Y2, and Y3 are name for simplified models. All four mapping function models give a graph of hyperbolic shape. However, there is a slightly difference between the Y model and the simplified models. The simplified model is such as below [2]:

$$Y1 = AX^B \quad (7)$$

where

Y1 : simplified $NMF_h(\varepsilon)$

A, B : constant

X : elevation angle (independent variable)

2.4.1.1 Sum of Error Calculation For NMF_h (ε)

Table 2.3: Sum of error for NMF_h, Y and simplified models (Y1, Y2, Y3) [2]

X	Y = NMF(h)	Y1 = 33.748* X ^(-0.8144)	Y2 = 33.748* X ^(-0.782)	Y3 = 39.042* X ^(-0.8144)	(Y -Y1) ²	(Y -Y2) ²	(Y -Y3) ²
2	18.581	19.191	19.626	22.201	0.372	1.093	13.104
5	10.151	9.099	9.586	10.527	1.106	0.319	0.141
10	5.556	5.174	5.575	5.986	0.145	0.000	0.185
15	3.802	3.719	4.060	4.303	0.007	0.067	0.251
20	2.898	2.942	3.242	3.404	0.002	0.119	0.256
25	2.353	2.453	2.723	2.838	0.010	0.137	0.235
30	1.993	2.115	2.361	2.447	0.015	0.136	0.206
35	1.739	1.865	2.093	2.158	0.016	0.125	0.175
40	1.553	1.673	1.886	1.936	0.014	0.111	0.146
45	1.413	1.520	1.720	1.759	0.012	0.094	0.120
50	1.304	1.395	1.584	1.614	0.008	0.078	0.096
55	1.220	1.291	1.470	1.493	0.005	0.062	0.075
60	1.154	1.203	1.373	1.391	0.002	0.048	0.056
65	1.103	1.127	1.290	1.303	0.001	0.035	0.040
70	1.064	1.061	1.217	1.227	0.000	0.023	0.027
75	1.035	1.003	1.153	1.160	0.001	0.014	0.016
80	1.015	0.951	1.097	1.101	0.004	0.007	0.007
85	1.004	0.906	1.046	1.048	0.010	0.002	0.002
90	1.000	0.864	1.000	1.000	0.018	0.000	0.000
Sum of error					1.748	2.469	15.138

From the Table 2.3, Y2 = 33.748*X^(-0.782) model has been selected as the simplification mapping function model for NMF_h (ε). The Y2 model been selected due to it meet the constraint requirement, which is at 90 degree the mapping function scale factor is unity. Smaller deviation is better. So, this model also smallest sum of error (2.469) compared to the others as given in Figure 2.1 below [2]:

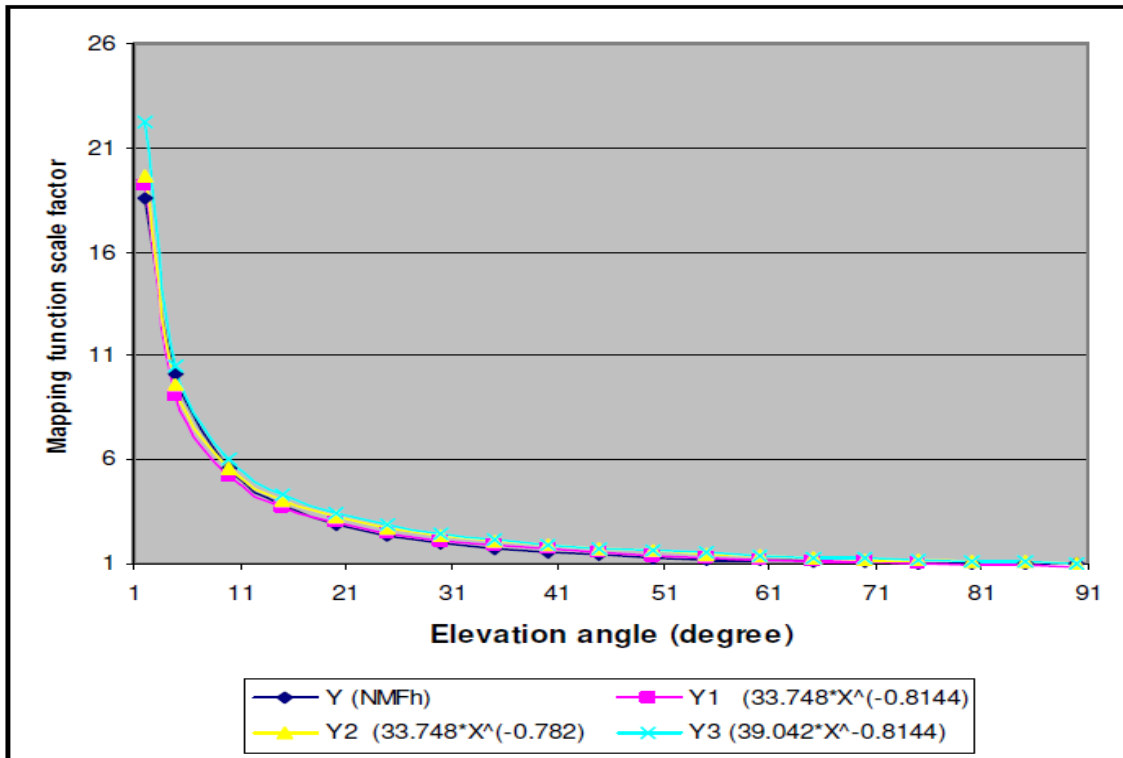


Figure 2.1: Graph of NMF_h mapping function (Y), Y1, Y2 and Y3 by regression

2.4.2 Wet Neill Mapping Function, $NMF_w(\epsilon)$

Z is the $NMF_w(\epsilon)$ mapping function model, while the simplified model named as Z1, Z2, Z3. There is a slight difference between the Z model and the three simplified model. These four models give the same shape of hyperbolic graph. The new model has been generated using regression method. We can reduce the computation time by using the simplification model. The simplified model is such as below [2]:

$$Z1 = AX^B \tag{8}$$

where

Z1: simplified $NMF_w(\epsilon)$.

A, B: constant

X: elevation angle (independent variable)

2.4.2.1 Calculation of Sum of Error For NMF_w (ε).

Table 2.4: Sum of error for NMF_w, Z and simplified models (Z1, Z2, Z3) [2]

X	Z	Z1= 38.079* $X^{(-0.8452)}$	Z2 = 38.079* $X^{(-0.8088)}$	Z3 = 44.846* $X^{(-0.8452)}$	$(Z - Z1)^2$	$(Z - Z2)^2$	$(Z - Z3)^2$
2	21.854	21.196	21.738	24.963	0.433	0.014	9.663
5	10.751	9.770	10.360	11.507	0.961	0.153	0.571
10	5.657	5.439	5.914	6.405	0.048	0.066	0.559
15	3.833	3.861	4.261	4.547	0.001	0.182	0.509
20	2.911	3.027	3.376	3.565	0.013	0.216	0.428
25	2.360	2.507	2.819	2.952	0.022	0.210	0.351
30	1.997	2.149	2.432	2.531	0.023	0.190	0.285
35	1.741	1.886	2.147	2.222	0.021	0.165	0.231
40	1.554	1.685	1.927	1.985	0.017	0.139	0.185
45	1.413	1.525	1.752	1.797	0.013	0.115	0.147
50	1.305	1.395	1.609	1.643	0.008	0.092	0.115
55	1.220	1.287	1.490	1.516	0.004	0.072	0.087
60	1.154	1.196	1.388	1.409	0.002	0.055	0.065
65	1.103	1.118	1.301	1.317	0.000	0.039	0.046
70	1.064	1.050	1.226	1.237	0.000	0.026	0.030
75	1.035	0.991	1.159	1.167	0.002	0.015	0.017
80	1.015	0.938	1.100	1.105	0.006	0.007	0.008
85	1.004	0.891	1.048	1.049	0.013	0.002	0.002
90	1.000	0.849	1.000	1.000	0.023	0.000	0.000
Sum of error					1.610	1.759	13.299

The simplified models deviate from the original model by using sum of error method. Smaller value of deviation will give better result. From the Table 2.4 above, Z2 = 38.079X^(-0.8088) model have been selected as the simplification mapping function model because it's mapping function gives unity at 90 degree elevation angle. The other reason is because due the smallest sum of error (1.759) compared to the others. Figure 2.2 below show the result between three NMF models [2]:

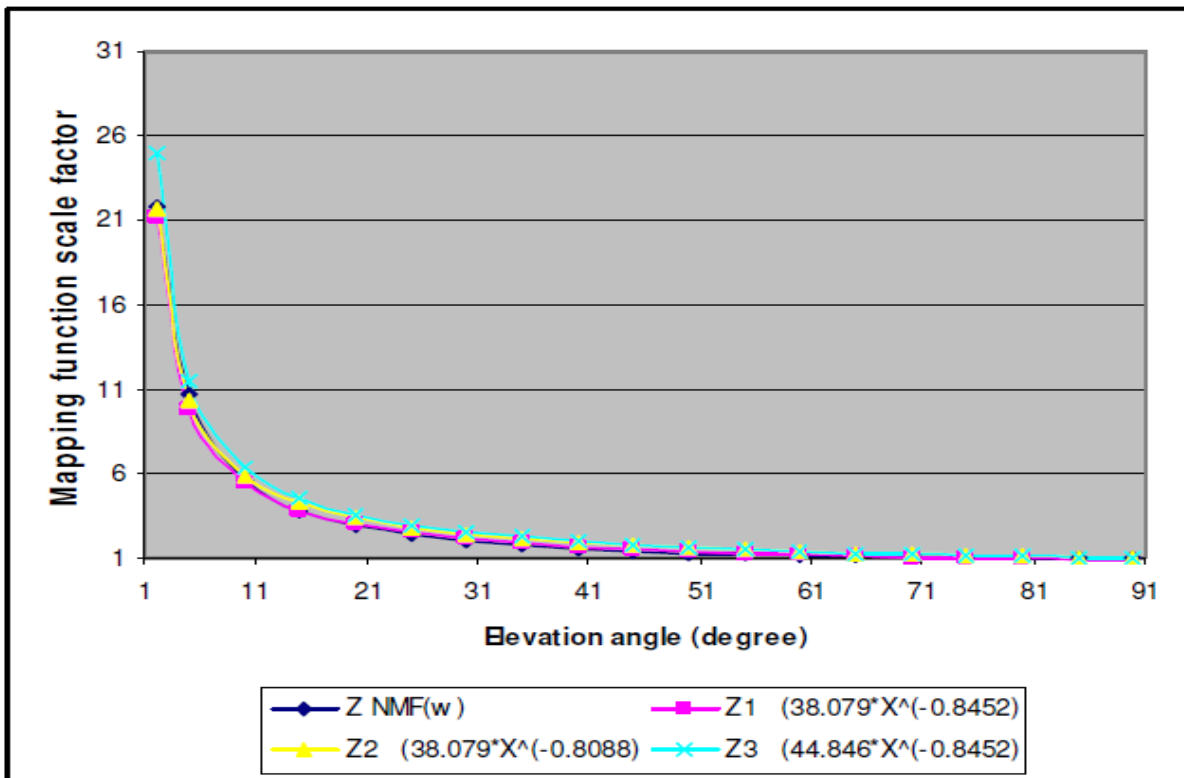


Figure 2.2: Graph of NMFw mapping function (Z), Z1, Z2 and Z3 by regression

2.5 Discussion and Conclusion

The original Neill mapping function, NMF model is given in a form of continued fraction are quite tedious. The NMF either for hydrostatic and also non hydrostatic components can be simplified to a simpler form by using regression method which have only 2 operations. The simplification of NMF can reduce the computation time and also can give simpler hyperbolic equation model. Both hydrostatic and also wet components give the similar result with the original NMF after done the simplification. The models reduction percentage can be shown in Table 2.5 [2]: