SELECTION OF DIFFERENCE EQUATION MINIMUM MODEL SPECIFICATION BY CORRELATION ANALYSIS

MOHAMMAD HAFIZ BIN ABDELLAH

UNIVERSITI TEKNIKAL MALAYSIA MELAKA

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MOHAMMAD HAFIZ BIN ABDELLAH

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SUPERVISOR DECLARATION

'I hereby declare that I have checked this project report and in my opinion this project is satisfactory in terms of scope and quality for the award of the degree of Bachelor of Mechanical Engineering (Structure and Material)'

Signature	:
Name of Supervisor	: DR. MD. FAHMI BIN ABD SAMAD @ MAHMOOD
Position	: Senior Lecturer
Date	: 24 May 2010

"I hereby declare this project report is my own work base on my scope except the quotations and summaries which have been duly acknowledged"

Signature :

Name : MOHAMMAD HAFIZ BIN ABDELLAH

Date : 24 May 2010

Special to my beloved Mak & Ayah…

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ABSTRACT

This project would give more understanding about least square method, model structure selection, difference equation model and correlation test. Through the study, the significance of correlation analysis in selection of difference equation minimum model specification is understood and the knowledge of variable correlation analysis is utilized in selection of difference equation minimum model specification. The scopes of the project are limited to implement the study to five simulated linear and nonlinear difference equation model. The mathematical models that apply in this research are ARX and NARX model. In system identification both the determination of model structure and model validation are important steps. Autocorrelation test and cross-correlations are two of the correlation analyses used. The results were obtained through programming using Matlab software and then compared in order to identify and select the difference equation minimum model specification by correlation analysis. It is shown that the application and interpretation of these tests to simulated data were able to identify a simpler if not the same, model specification as the original specification.

ABSTRAK

Projek ini memberi pemahaman tentang kaedah *least square*, pemilihan struktur model, model persamaan pembezaan dan analisis korelasi. Melalui kajian dan pembelajaran, kepentingan analisis korelasi dalam pemilihan struktur model difahami dan pengetahuan tentang analisis korelasi pembolehubah dikembangkan dalam pemilihan spesifikasi model minimum persamaan pembezaan. Skop projek terhad kepada mempraktikkan kajian kepada sebilangan model persamaan pembezaan lelurus dan bukan lelurus yang disimulasikan. Model matematik yang diaplikasikan dalam kajian ini adalah model ARX dan NARX. Dalam sistem identifikasi, kedua-dua pemilihan struktur model dan pengesahan model merupakan langkah-langkah penting. Ujian autokorelasi dan korelasi bersilang adalah dua analisis korelasi pembolehubah yang digunakan. Keputusan diperolehi melalui pengaturcaraan menggunakan perisian Matlab dan kemudian dibandingkan untuk mengenalpasti dan memilih spesifikasi model minimum persamaan pembezaan melalui analisis korelasi. Keputusan menunjukkan aplikasi ujian-ujian korelasi terhadap data yg disimulasikan dapat menghasilkan spesifikasi model yang mudah jika tidak sama dengan spesifikasi asal.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS

pee	=	Autocorrelation value
pue	=	Cross-correlation value
n_y	=	Output lag
n_u	=	Input lag
l	=	Degree of nonlinearity
ε	=	Equation error
θ	=	Parameter vector
λ^2	=	Variance
t	=	Time
f	=	Mathematical function
$\xi(t)$	=	Residuals
α	=	Significance level
$u(t)$	=	Input
$y(t)$	=	Output
H_o	=	Hypothesis
y_o	=	Stationery output

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Background of Project

Often when engineers analyze a system to be controlled or optimized, they use a mathematical model. In analysis, engineers can build a descriptive model of the system as a hypothesis of how the system could work, or try to estimate how an unforeseeable event could affect the system. Similarly, in control of a system, engineers can try out different control approaches in simulations. A mathematical model usually describes a system by a set of variables and a set of equations that establish relationships between the variables. The values of the variables can be practically anything; real or integer numbers, Boolean values or strings, for example. The variables represent some properties of the system, for example, measured system outputs often in the form of signals, timing data, counters, and event occurrence. The actual model is the set of functions that describe the relations between the different variables. Mathematical models can take many forms, including but not limited to differential equations, dynamical systems, statistical models, or game theoretic models. These and other types of models can overlap, with a given model involving a variety of abstract structures. The selection of difference equation minimum model specification by correlation analysis is stated and discussed.

1.2 Problem Statement

Mathematical models are usually composed of variables, which are abstractions of quantities of interest in the described systems, and operators that act on these variables can be algebraic operators, functions and differential operators. If all the operators in a mathematical model present linearity, the resulting mathematical model is defined as linear. A model is considered to be nonlinear otherwise. The question of linearity and nonlinearity is dependent on context, and linear models may have nonlinear expressions in them. For example, in a statistical linear model, it is assumed that a relationship is linear in the parameters, but it may be nonlinear in the predictor variables. Similarly, a differential equation is said to be linear if it can be written with linear differential operators, but it can still have nonlinear expressions in it. In a mathematical programming model, if the objective functions and constraints are represented entirely by linear equations, then the model is regarded as a linear model. If one or more of the objective functions or constraints are represented with a nonlinear equation, then the model is known as a nonlinear model. In the present study, correlation functions were used to detect the presence of unmodelled linear and non-linear terms in the residuals. Once a model of the process has been estimated, whether linear or non-linear, model validity tests are applied to detect if there are any unmodelled terms in the residuals that, if omitted from the model, will induced biased estimates.

1.3 Objective

The objectives of the projects are:

i. To understand the significance of correlation analysis in model structure selection.
ii. To utilize the knowledge of variable correlation analysis in selection of difference equation minimum model specification.

1.4 Scopes

The scopes of the project are:

i. To implement the study to a number of simulated linear difference equation model.
ii. To expand the finding to nonlinear difference equation model.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

In this chapter the model structure selection of system identification, difference equation model and correlation analysis are discussed. All this will give more understanding about the significance of correlation analysis in model structure selection those are used in this research. This chapter also includes the variable correlation analysis in selection of difference equation minimum model specification. To start an analysis, all the definitions must be known.

2.2 System Identification

System identification is one of the two possible ways of obtaining mathematical models for a certain object either a signal or a system by processing experimental data collected in an operation regime that is thought to be close to that in which the model is going to be used (Söderström and Stoica, 2002). The other possible way, called mathematical modelling, consists of making use of the laws those are known to govern the behaviour of the system to obtain a mathematical model for it. The main drawback of mathematical modelling is that it cannot be applied to complicated systems either because the resulting model would be too complex and difficult to use or simply because the laws that govern such systems are unknown. System identification, on the other hand can be applied to virtually any system and typically it yields relatively simple models that can well describe the system's behaviour in a certain operation regime.

2.2.1 The Least Squares Method

The Least Square method is applicable to dynamic models of the form

$$y(t) + a_1 y(t-1) + \cdots a_{na} y(t-na) = b_1 u(t-1) + \cdots + b_{nb} u(t-nb) + \varepsilon(t). \qquad (2.1)$$

Here ε (t) denotes an equation error, which can describe disturbances or unmodeled dynamics. Using the backward shift operator q^{-1}, so that $q^{-1}u(t) = u(t-1)$, one can write (2.1) in abbreviated form

$$A(q^{-1})y(t) = B(q^{-1})u(t) + \varepsilon(t)$$

(Söderström and Stoica, 2002) (2.2)

with

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na} \qquad (2.3)$$

$$B(q^{-1}) = b_1 q^{-1} + \ldots + b_{nb} q^{-nb}.$$

The model (2.2) can be equivalently expressed as the linear regression mode1

$$y(t) = \Psi^T(t)\theta + \varepsilon(t), \qquad (2.4)$$

where

$\psi^{T}(t) = (- y (t - 1) \ldots \; - y (t - na) \;\; u (t - 1) \ldots u (t - nb))$

$\theta \quad = (a_1 \; \ldots \; a_{na} \;\; b_1 \ldots b_{nb})^{T}.$ (2.5)

Assume that the data *u(1), y(1) . . . . u(N), y(N)* are available. The LS estimate, $\hat{\theta}$ of the parameter vector θ is defined as the minimizing argument of the sum of squared equation errors

$$V_N(\theta) = \left[\frac{1}{N} \sum_{t=1}^{N} \varepsilon^2(t) \right] = \frac{1}{N} \sum_{t=1}^{N} y^2(t)$$

$$- \left[\frac{2}{N} \sum_{t=1}^{N} y(t)\psi^T(t) \right] \theta + \theta^T \left[\frac{1}{N} \sum_{t=1}^{N} \psi(t)\psi^T(t) \right] \theta. \quad (2.6)$$

By setting the gradient of $V_N(\theta)$ to zero we get the normal equations

$$\left[\frac{1}{N} \sum_{t=1}^{N} \varphi(t)\varphi^T(t) \right] \hat{\theta} = \left[\frac{1}{N} \sum_{t=1}^{N} \varphi(t) y(t) \right] \quad (2.7)$$

Assuming the matrix in (2.7) to be non-singular (which it "normally" is), we thus have

$$\hat{\theta} = \left[\frac{1}{N} \sum_{t=1}^{N} \varphi(t)\varphi^T(t) \right]^{-1} \left[\frac{1}{N} \sum_{t=1}^{N} \varphi(t) y(t) \right] \quad (2.8)$$

Shown below is an example of Least Square method. Consider a class of systems given by

$$y(t) + a_0 y(t-1) = b_0 u(t-1) + e(t) + c_0 e(t-1) \quad (2.9)$$

where e(t) is a sequence of independent and identically distributed random variables with mean zero and variance λ^2. Such a sequence is called white noise. Two different sets of parameter values are used, namely,

$$S_1 : a_0 = -0.8 \;\; b_0 = 1.0 \;\; c_0 = 0.0 \;\; \lambda = 1.0$$
$$S_2 : a_0 = -0.8 \;\; b_0 = 1.0 \; c_0 = -0.8 \;\; \lambda = 1.0 \qquad (2.10)$$

The system S_1 can thus be represented as

$$S_1 : y(t) - 0.8y(t-1) = 1.0u(t-1) + e(t) \qquad (2.11)$$

whereas S_2 can be written as

$$\begin{aligned} S_2 : y(t) &= 0.8x(t-1) + 1u(t-1) + e(t) \\ y(t) - 0.8y(t-1) &= 1u(t-1) + e(t) - 0.8e(t-1) \\ 0.8y(t-1) &= 0.8x(t-1) + 0.8e(t-1) \\ x(t) - 0.8x(t-1) &= 1.0u(t-1) \\ y(t) &= x(t) + e(t) \end{aligned} \qquad (2.12)$$

The white noise *e(t)* hence enters in different ways in the two systems. For the system S_1 it appears as an equation disturbance, whereas for S_2 it is added to the output. Note that for S_2 the signal *x(t)* can be interpreted as the deterministic or noise-free output.

Shown below is why the LS estimates to be consistent for system S_1 but not for system S_2. Consider a linear system of a general order and write it as

$$A_0(q^{-1})y(t) = B_0(q^{-1})u(t) + v(t) \qquad (2.13)$$

or, equivalently,

$$y(t) = \Psi^T(t)\theta_0 + v(t). \tag{2.14}$$

Here θ_0 is called the true parameter vector. Assume that v(t) is a stationary stochastic process that is independent of the input signal.

The estimation error becomes

$$\hat{\theta} - \theta_0 = \left[\frac{1}{N}\sum_{t=1}^{N}\Psi(t)\,\Psi^T(t)\right]^{-1} = \left[\frac{1}{N}\sum_{t=1}^{N}\Psi(t)\,y(t) - \left\{\frac{1}{N}\sum_{t=1}^{N}\Psi(t)\,\Psi^T(t)\right\}\theta_0\right]$$

$$= \left[\frac{1}{N}\sum_{t=1}^{N}\Psi(t)\,\Psi^T(t)\right]^{-1}\left[\frac{1}{N}\sum_{t=1}^{N}\Psi(t)\,v(t)\right] \tag{2.15}$$

Under weak conditions, the sums in (2.15) tend to the corresponding expected values as the number of data points, N, tends to infinity. Hence $\hat{\theta}$ is consistent (That is, $\hat{\theta}$ tends to θ_0 as N tends to infinity) if (and essentially only if)

$$E[\Psi(t)\,\Psi^T(t)] \text{ is non-singular} \tag{2.16}$$

$$E[\Psi(t)v(t)] = 0, \tag{2.17}$$

where E denotes the expectation operator. Condition (2.16) is satisfied in most cases. However, there are, a few exceptions:

- The input has a spectral density that is nonzero at less than nb frequencies (*u(t)* is not persistently exciting of order n_b).
- The data are completely noise free (*v(t)* $\equiv$ 0) and the model order is chosen too high (which implies that $A_0(q^{-1})$ and $B_0(q^{-1})$ have common factors).
- The input *u(t)* is generated by a linear low-order feedback from the output.