

ANALYSIS AND MODELLING OF PROJECTILE MOTION

WITH

AIR RESISTANCE

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This report is writing to fulfill the part of conditions the conferral of

Degree of Mechanical Engineering (Structure and Material)

Faculty of Mechanical Engineering

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MAY 2010

I declare that this thesis entitled  
“Analysis and Modelling Of Projectile Motion with Air Resistance”  
is the result of my own research except as cited in the references.

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Date : 24<sup>th</sup> MAY 2010

Special dedicated to my beloved Mak and Abah

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## ABSTRACT

Projectile motion is a common motion and it is use in many applications in our daily life. However, for study purpose at school, college or university, it is assumed that air-resistance effects are negligibly small or just assume it does not exist. In this research, an experiment of projectile motion with air resistance will be conducted. The objective is to study the effect of air resistance towards projectile motion. For that, an experiment of projectile motion will be conducted complete with air resistance. This research will study not only the effect of air resistance, but the effect of mass of ball use, distance and velocity of the projectile as well. In order to make sure this research will achieve its objective, a few methods were used. First of all, a flow chart needs to be constructed in order to make sure the work flow of this research will move on smoothly. To get the data need for this research, a laboratory experiment will be conducted. The data obtained then will be collected and then the data will be analyzed to get the result. The experiment then will be converting to mathematical model in order to find the distance travel, speed, and all the data those are related. At the end of the study, the effect of the air resistance towards projectile motion will be verified. It is shown that with air resistance, the distance travelled by the projectile motion will be decreased. The distance travel also will be affected by the mass of specimen used for the projectile motion and the speeds of the specimen during undergo the projectile motion.

## ABSTRAK

Gerakan projektil adalah satu gerakan yang biasa dan banyak digunakan di dalam aplikasi dalam kehidupan seharian kita. Walau bagaimanapun, untuk tujuan pembelajaran di sekolah, kolej mahu pun universiti, kesan rintangan angin dianggap terlalu kecil atau dianggap tiada. Di dalam kajian ini, satu ujikaji tentang gerakan projektil akan dijalankan. Tujuannya adalah untuk mengkaji kesan rintangan angin terhadap gerakan projektil. Untuk itu, satu ujikaji gerakan projektil akan dijalankan lengkap dengan rintangan angin. Kajian ini tidak sahaja mengkaji kesan rintangan angin tetapi juga kesan jisim bola yang digunakan, jarak projektil bola dan juga kelajuan projektil. Untuk memastikan kajian ini mencapai objektif, beberapa langkah telah digunakan. Pertama sekali, carta alir akan dilukis untuk memastikan perjalanan kajian ini akan berjalan lancar. Untuk mendapatkan data yang diperlukan bagi kajian ini, satu ujikaji makmal perlu dijalankan. Data-data yang diperolehi akan dikumpul dan dianalisa untuk mendapatkan keputusannya. Ujikaji ini kemudiannya akan ditukar kepada model matematik untuk mencari dan mendapatkan jarak luncuran, kelajuan dan data-data yang berkaitan. Pada penghujung kajian ini, kesan rintangan angin terhadap gerakan projektil akan diperolehi. Kajian menunjukkan bahawa dengan adanya rintangan angin, jarak projektil akan berkurangan. Selain itu, jarak projektil juga dipengaruhi oleh berat spesimen yang digunakan untuk gerakan projektil dan juga kelajuan spesimen ketika mengalami gerakan projektil.



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**LIST OF SYMBOLS**

F	=	Force, $N$
a	=	Acceleration, $m/s^2$
g	=	Gravity Acceleration, $m/s^2$
$F_D$	=	Drag Force, N
A	=	Cross sectional area, $m^2$
$\rho$	=	Density, $kg/m^3$
C	=	Numerical drag coefficient
W.A.R	=	Without Air Resistance
A.R	=	Air Resistance
I	=	Mass Moment of Inertia, $kgm^2$
$\alpha$	=	Angular acceleration, $m/s^2$



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## CHAPTER 1

### INTRODUCTION

#### 1.1 Background

Projectile motion is a common motion in our life. A soccer ball being kicked, a baseball being thrown or athletes long jumping are examples of simple projectile motion. Even fireworks and water fountains are also examples of projectile motion. Projectile motion is a special case of two dimensional motions with constant acceleration. Here, force due to gravity moderates linear motion of an object thrown at certain angle to the vertical direction. The resulting acceleration is a constant, which is always directed in vertically downward direction.

The projectile motion emphasizes one important aspect of constant acceleration that even constant acceleration, which is essentially unidirectional, is capable to produce two dimensional motions. The basic reason is that force and initial velocity of the object are not along the same direction. The linear motion of the projected object is continuously worked upon by the gravity, which results in the change of both magnitude

and direction of the velocity. A change in direction of the velocity ensures that motion is not one dimensional.

The change in magnitude and direction of the velocity is beautifully managed so that time rate of change in velocity is always directed in vertical downward direction i.e. in the direction of gravity. This aspect is shown qualitatively for the motion in Figure 1.1 as velocity change successively at the end of every second from  $V_1$  to  $V_2$  to  $V_3$  (respectively from the left side of Figure 1.1) and so on by exactly a vector, whose magnitude is equal to acceleration due to gravity “g”.

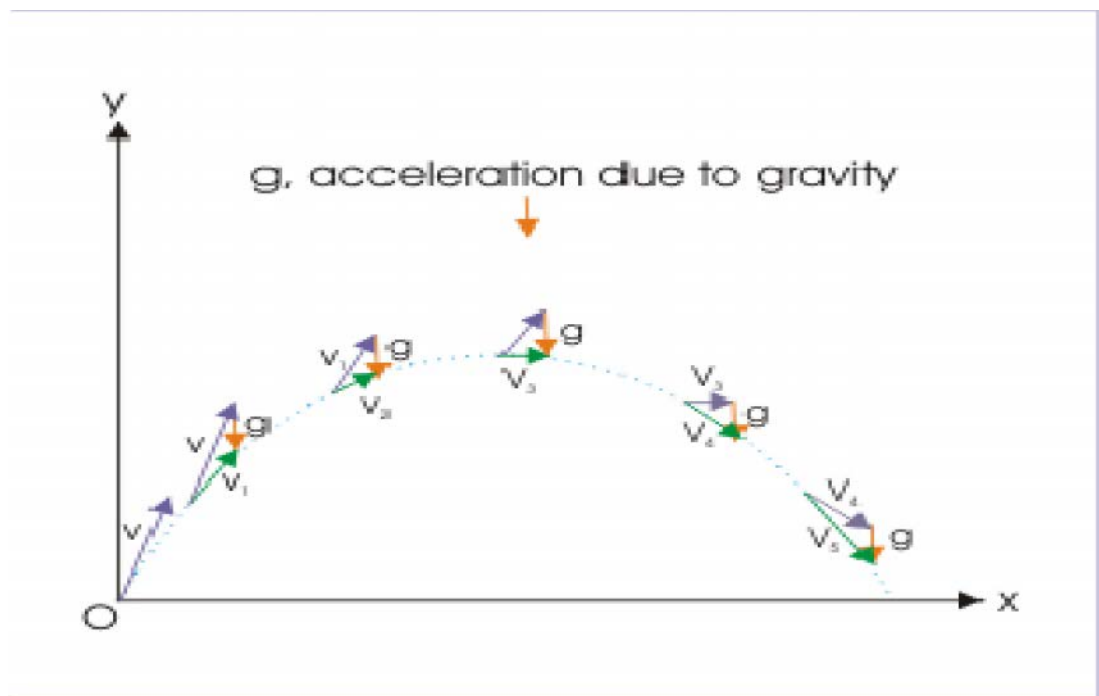


Figure 1.1 Velocity of the projectile changes by acceleration vector in unit time

## 1.2 Objectives

An objective is important in order to determine the target of a research and analysis. Below are the main objectives of this research:

- i. To setup a laboratory experiment complete with air resistance assembly.
- ii. To perform an experiment of projectile motion with variable air resistance speed.
- iii. To collect data from objective (ii) and analyze.
- iv. To validate the whole experiment with theoretical foundations.

## 1.3 Scope

Scope is the subject that is needed to be focused when performing a research. The main scopes of this research are:

- i. To perform the experiment in a laboratory-scaled environment.
- ii. To perform the experiment to a maximum of 3 air resistant speeds.
- iii. To limit mathematical modeling to fundamental projectile and fluid principle.

## 1.4 Problem Statement

In this study of projectile motion, it is assumed that air-resistance effects are negligibly small or just assume it does not exist. But in our life, air resistance (often called *air drag*, or simply *drag*) has a major effect on the motion of many objects,

including tennis balls, bicycle riders, and airplanes. It will affect the speed of projectile motion whether slower or faster depending on the direction and force of the air. It also will affect the distance of the projectile too. This research is all about the effect of air resistance to projectile motion particularly and the effect of different air resistance speed toward the direction and distance of the projectile motion. This research also will study about the effect of mass of the object to the projectile motion. The data obtained then will be analyzed and the experiment will be compared to mathematical model. In order to perform this experiment, it will be conducted in a laboratory-scaled environment which will use three (3) air resistance speeds and limit the mathematical modeling to fundamental projectile and fluid principles.

## CHAPTER 2

### LITERATURE REVIEW

#### 2.1 Introduction

Projectile motion exists commonly in our everyday lives and is particularly evident in the motion or flight of objects which are projected from a particular height. The key to working with projectile motion is recognizing that when an object with mass flies through the air, its motion is a combination of vertical and horizontal movements. Although the horizontal velocity of the object remains constant throughout the flight, its vertical velocity accelerates or decelerates due to gravity. Projectiles in two dimensions follow a parabolic trajectory like the one shown in Figure 2.1.

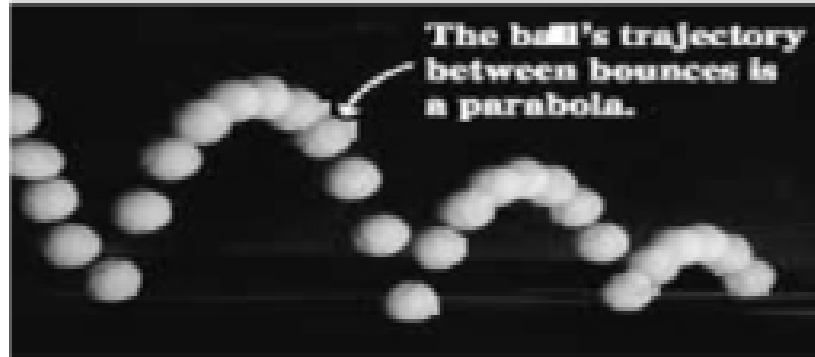


Figure 2.1: The parabolic trajectory of a bouncing ball

(Raymond A. Serway.2001)

Projectile motion is one of the major applications in kinematic particle analysis and also applied the Newton's Law which is one of particle dynamics.

## 2.2 Kinematics Of Particles

A particle is considered to be a small physical entity which may be represented by a point with an associated mass. Many objects such as projectiles, automobiles, space vehicles and even heavenly bodies are commonly regarded as being particles even though they are hardly small or minute. Hence, for such representations to produce useful results, the object, be it a ball, a car, an airplane must be small relative to its surroundings. In any event the modeling of an object as a particle, and then as a point with an associated mass, becomes increasingly accurate the smaller the object is.

The kinematics of a particle is the study of a motion of a particle without regard to the forces causing the motion. In this context, the mass of the particle is irrelevant. Therefore, a kinematical analysis of particles may be reduced to a mathematical study of the movement of points (Huston, 2001).

The principal kinematic quantities of interest for particles are: position, velocity, and acceleration (Huston, 2001). Position is simply the location of the particle (or point) in a reference frame, or relative to other particles. The position of a particle is usually represented by a vector – called position vector. Velocity is the time rate of change of position and acceleration is the time rate of change of velocity. Velocity and acceleration are thus also vector quantities.

### **2.3 Newton's Law**

In the projectile motion with air resistance, the first and second of Newton's Law are applied. First Newton's Law said an object at rest tends to stay at rest and an object in motion tends to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced force. As the ball moves in the air after it was launched, a gravity force will disturb it by pulling it downward. Otherwise, the air resistance will make the ball speed faster or slower and can affect the distance of the ball travel.

Second Newton's Law states that the acceleration of a particle is proportional to the force applied to the particle and inversely proportional to the mass of the particle. Its