

**CONTROL OF 2 DIMENSIONAL INVERTED PENDULUM
USING MATLAB**

FARHANA BT ABD RAZAK

UNIVERSITI TEKNIKAL MALAYSIA MELAKA


UNIVERSITI TEKNIKAL MALAYSIA MELAKA (UTeM)
BORANG PENGESAHAN STATUS TESIS*
JUDUL: CONTROL OF 2 DIMENSIONAL INVERTED PENDULUM USING MATLAB
SESI PENGAJIAN: 2007/2008


 Saya **FARHANA BT ABD RAZAK**

mengaku membenarkan tesis (PSM/Sarjana/Doktor Falsafah) ini disimpan di Perpustakaan Universiti Teknikal Malaysia Melaka (UTeM) dengan syarat-syarat kegunaan seperti berikut:

1. Tesis adalah hak milik Universiti Teknikal Malaysia Melaka.
2. Perpustakaan Universiti Teknikal Malaysia Melaka dibenarkan membuat salinan untuk tujuan pengajian sahaja.
3. Perpustakaan dibenarkan membuat salinan tesis ini sebagai bahan pertukaran antara institusi pengajian tinggi.
4. **Sila tandakan (√)


- SULIT** (Mengandungi maklumat yang berdarjah keselamatan atau kepentingan Malaysia yang termaktub di dalam AKTA RAHSIA RASMI 1972)
- TERHAD** (Mengandungi maklumat TERHAD yang telah ditentukan oleh organisasi/badan di mana penyelidikan dijalankan)
- TIDAK TERHAD**

(TANDATANGAN PENULIS)



Alamat Tetap:
75, JALAN EMAS SURIA,
TAMAN EMAS SURIA,
08000 SUNGAI PETANI, KEDAH
TARIKH: 5 MEI 2008

Disahkan oleh:



 (TANDATANGAN PENYELIA)
SYAMIMI BINTI SHAMSUDDIN
 Pensyarah

Cop Rasmi: Fakulti Kejuruteraan Pembuatan
 Universiti Teknikal Malaysia Melaka
 Karung Berkunci 1200, Ayer Keroh
 75450 Melaka

 Tarikh: 9 MEI 2008

* Tesis dimaksudkan sebagai tesis bagi Ijazah Doktor Falsafah dan Sarjana secara penyelidikan, atau disertasi bagi pengajian secara kerja kursus dan penyelidikan, atau Laporan Projek Sarjana Muda (PSM).
 ** Jika tesis ini SULIT atau TERHAD, sila lampirkan surat daripada pihak berkuasa/organisasi berkenaan dengan menyatakan sekali sebab dan tempoh tesis ini perlu dikelaskan sebagai SULIT atau TERHAD.

DECLARATION

I hereby, declared this thesis entitled “Control of a 2 Dimensional Inverted Pendulum using Matlab” is the results of my own research
except as cited in references.

Signature :

Author's Name : Farhana Bt Abd Razak

Date :

APPROVAL

This Bachelor's Project submitted to the senate of UTeM and has been accepted as fulfilment of the requirement for the Degree of Bachelor of Manufacturing Engineering (Robotics and Automation) with Honours. The member of the supervisory committee is as follow:

.....
Mrs Syamimi bt Shamsuddin
Project Supervisor
Faculty of Manufacturing Engineering

ABSTRACT

The problem of balancing an inverted pendulum has been a benchmark problem in demonstrating various control design techniques. The principal reasons for its popularity are its nonlinear and unstable characteristics. This project presents the design and simulation of two dimensional inverted pendulum system. The design of the system is done in MATLAB environment. The selection of the controller is the core of this study. PID controller has been chosen as the generic control loop feedback to control the system. In the simulation of inverted pendulum, PID controller is used to control both the position of the cart and the pendulum and also the pendulum angle to ensure the pendulum stands in upright position when the simulation is running. The design of the system is implemented using Simulink block diagram. From the working simulation of inverted pendulum, analysis for the factors contributing in stability of the system is carried out. The analysis covers the parameters of the system, PID controller simulation, set point method and stability of the system. Full working simulation was not able to carry out due to motion equation error but analysis is done using existing data from Simulink environment. The best combination of PID controller and the lowest settling time give the best result for inverted pendulum stability.

ABSTRAK

Masalah keseimbangan bandul *inverted pendulum* telah menjadi satu masalah utama dalam mendemonstrasikan kepelbagaian teknik kawalan rekabentuk. Sebab utama masalah ini menjadi kebiasaan adalah kerana sifatnya yang tidak linear dan tidak stabil. Ianya amat bersesuaian dgn kawalan ujian disebabkan oleh ketidakstabilan dan ketidak linear yang amat tinggi. Projek ini mewakili rekabentuk dan simulasi sistem *inverted pendulum* yang mempunyai dua dimensi. Rekabentuk sistem ini telah digunakan didalam perisian MATLAB. Pemilihan kawalan amat penting di dalam proses ini. Sistem kawalan PID telah dipilih sebagai sejenis kawalan lingkaran tindak balas di mana ianya telah digunakan meluas di dalam sistem kawalan kejuruteraan . Di dalam simulasi *inverted pendulum* ini, alat kawalan PID berfungsi sebagai pengawal kedudukan tapak dan juga *pendulum*. Selain itu juga ia berfungsi untuk mengawal sudut *pendulum* itu untuk memastikan *pendulum* itu berdiri dalam keadaan yang betul ketika simulasi dijalankan. Rekabentuk sistem ini telah digunakan dengan menggunakan blok diagram *Simulink*. Daripada simulasi *inverted pendulum* yang dijalankan, analisis bagi mencari faktor yang mempengaruhi kestabilan sistem turut dilakukan. Analisis ini merangkumi parameter sistem, simulasi alat kawalan PID, kaedah *set point* dan juga kestabilan sistem. Simulasi sistem tidak dapat dijalankan disebabkan terdapat ralat dalam persamaan pergerakan, tetapi analisis masih dijalankan dengan menggunakan keputusan data yang sedia ada dalam *Simulink*. Kombinasi terbaik kawalan PID dan masa kestabilan yang terendah memberikan keputusan yang terbaik untuk pengimbangan sistem.

DEDICATION

For My beloved mother and father

ACKNOWLEDGEMENTS

Bismillahirrahmanirrahim...

Firstly, I would like to convey my gratitude towards the Al-Mighty for giving me the strength and willingness to complete Final Year Project. I am indebted to many people in this dissertation effort. First and foremost, I would like to express my sincere gratitude and appreciation to Madam Syamimi Bt Shamsuddin, my supervisor, for her advise, support and tireless effort, who guided me to this end in this area that I like most: control of a two dimensional inverted pendulum using MATLAB. I deeply appreciate for the countless hours spent by her on this dissertation since it was first started as a pilot study. Her refinement of this dissertation is invaluable.

An extended note of appreciation is in order for Mr Arfauz Bin Abd Rahman for his constructive comments and suggestions. Thanks for his effort in reviewing the pilot study of this dissertation.

Special thanks and appreciation are goes to my parent, Abd Razak Bin Ahmad and Norain Bt Md Nor for their sacrifice, encouragement, understanding and assistance in my dissertation effort. Acknowledgement of appreciation also goes to my friends for their patience and support. Without all of you, I cannot achieve what I have today. Thank you again.

Wassalam...

Farhana Bt. Abd Razak

TABLE OF CONTENTS

Abstract	i
Abstrak	ii
Dedication	iii
Acknowledgement	iv
Table of Contents	v
List of Figures	ix
List of Tables	xiii
List of Abbreviations, Symbols, Specialized Nomenclature	xiv
1. INTRODUCTION	1
1.1 Background	1
1.2 Problem Statement	2
1.3 Scope	3
1.4 Objectives	3
1.5 Brief History of Inverted Pendulum	4
1.6 Project Schedule for PSM I.....	6
2. LITERATURE REVIEW	7
2.1 Background	7
2.2 Pendulum	7
2.2.1 Definition of Pendulum	7
2.2.2 Types of Pendulum	8
2.2.2.1 Simple Pendulum.....	9
2.2.2.2 Inverted Pendulum	11
2.2.2.3 Differentiate Between Simple Pendulum and Inverted Pendulum	16
2.3 Different Study on Inverted Pendulum	17
2.3.1 Case of 3 Dimensional (3D) Inverted Pendulum.....	17

2.3.2	Case of 2 Dimensional (2D) Inverted Pendulum.....	19
2.3.3	Case of 1 Dimensional (1D) Inverted Pendulum.....	22
2.4	Controller	23
2.4.1	Definition of Controller	24
2.4.2	Type of Controller	24
2.4.2.1	Proportional Integral Derivative (PID) Controller	25
2.4.2.2	Artificial Neural Network (ANN) Controller	28
2.4.2.3	Fuzzy Logic Controller	31
2.5	Previous Research of Inverted Pendulum	33
2.5.1	2D Inverted Pendulum Using a 3-wheeled omni-directional Robot.....	33
2.5.2	Artificial Neural Network Identification and Control of the IP	36
2.6	Programming Platform	37
2.6.1	MATLAB	37
2.6.2	Simulink	39
2.6.3	Modelica	41
2.7	Control System	43
3.	METHODOLOGY	47
3.1	Introduction	47
3.2	Planning of Study.....	47
3.3	Data Collection	51
3.4	Journal	52
3.5	Project Outline	52
3.6	Flow Chart	53
3.7	Literature Review	54
3.7.1	Step in Using MATLAB Software	56
3.7.2	PID Controller	69
3.8	Project Implementation and Modification	71
3.9	Testing	73
3.10	Data Analysis	76

3.11 Writing	76
4. DESIGN OF 2D INVERTED PENDULUM SYSTEM	77
IN SIMULINK WINDOW	
4.1 Introduction	77
4.2 Design of Simulink Block Diagram	77
4.2.1 PID Controller Subsystem	78
4.2.2 2D Inverted Pendulum Subsystem	82
4.2.3 Pole Angle 3D Transformation Subsystem	92
4.2.4 Coordinates 3D Transformation Subsystem	97
4.2.5 Trajectory Graph Subsystem	100
4.2.6 Virtual Reality Inverted Pendulum Subsystem	105
4.3 Combination of Subsystems for the Inverted Pendulum System	110
5. SIMULATION, RESULT AND ANALYSIS OF	113
INVERTED PENDULUM SYSTEM	
5.1 Simulation Testing	113
5.2 Analysis of Inverted Pendulum System	116
5.2.1 Calculation of Stability for Inverted Pendulum System	118
5.2.2 Analysis of Inverted Pendulum Performance	139
5.3 Set Point Simulation	144
5.3.1 VR Sensor Simulation	145
5.3.2 Mouse Simulation	146
5.3.3 Input Signal Simulation	147
5.4 PID Controller Simulation	149
6. DISCUSSION, CONCLUSION AND SUGGESTION	154
FOR FURTHER WORKS	
6.1 Discussion	154
6.2 Conclusion	158

6.3 Suggestions for Further Works	159
7. REFERENCES	162

LIST OF FIGURES

1.1	Inverted Pendulum	5
2.1	A Simple Gravity Pendulum	9
2.2	Pendulum Clock	11
2.3	Schematic Diagram of the Inverted Pendulum	12
2.4	A schematic drawing of inverted pendulum	14
2.5	Simple pendulum	16
2.6	Inverted pendulum	16
2.7	3 Dimensional Inverted Pendulum	18
2.8	Picture of a working 2D inverted pendulum	19
2.9	A linear 2D Inverted Pendulum	20
2.10	Two degree of freedom virtual single inverted pendulum from MATLAB	21
2.11	1D inverted pendulum	22
2.12	Classical Inverted Pendulum	23
2.13	A block diagram of PID controller	26
2.14	Example model of inverted pendulum and PID controller	28
2.15	Simulink model of 1D balancing feedback system	35
2.16	The PID controller in balancing control	35
2.17	Simulink model of the linear pendulum system	40
2.18	Animation of Pendulum in Modelica	41
2.19	Model of the pendulum made in Modelica	42
2.20	Simplified description of a control system	43
2.21	Feedback Control System Block Diagram	44
2.22	Full state feedback for Inverted Pendulum	46
3.1	Flow chart for planning of study	50
3.2	MATLAB 6.5. Software	56

3.3	MATLAB Version 6.5 Window	57
3.4	MATLAB Command Window	57
3.5	Simulink Library Browser	58
3.6	Resulting untitled model window	59
3.7	Simulink block libraries for Continuous systems	60
3.8	Simulink block libraries for Sources	61
3.9	Simulink block libraries for Sink	62
3.10	MATLAB window showing how to drag Simulink library block	63
3.11	Example Assemble and Label Subsystems in Simulink	64
3.12	Untitled model window shows interconnect subsystems	65
3.13	Untitled model window showing how to get the Block Parameters for the subsystems	66
3.14	Untitled model window showing how to select Simulation parameter	67
3.15	Simulation Parameters	67
3.16	Untitled window shows Start and Stop Simulation icon	68
3.17	Virtual Reality Toolbox Viewer	73
3.18	Viewer Control Panel	73
3.19	Navigation panel	74
3.20	Example of Working Simulation in Virtual Reality Toolbox Viewer	75
4.1	Conversion from 2D to 3D Inverted Pendulum	78
4.2	PID controller block diagram	79
4.3	PID controller subsystem	79
4.4	Subsystem Mask Editor for PID Controller	80
4.5	PID controller block parameter	80
4.6	Schematic of the inverted pendulum mounted on cart	81
4.7	Detail 2D inverted pendulum	82
4.8	Subsystem of 2D inverted pendulum	83
4.9	Free body diagram of inverted pendulum	84

4.10(a)	Free body diagram (cart)	84
4.10(b)	Free body diagram (pendulum)	84
4.11	Inverted pendulum system block diagram	88
4.12(a)	Transfer function block parameter for position	89
4.12(b)	Transfer Function Block Parameter for Angle	89
4.13	Mask of inverted pendulum subsystem	90
4.14(a)	Subsystem mask editor for 2D inverted pendulum (icon)	90
4.14(b)	Subsystem mask editor for 2D inverted pendulum (parameters)	91
4.15	2D inverted pendulum block parameter	91
4.16	Pole angle 3D transformation subsystem	92
4.17	Transformation angle from 2D inverted pendulum to 3D inverted pendulum	93
4.18	Block diagram representation of inverted pendulum system	94
4.19	Pole angle 3D transformation system block diagram	95
4.20	Coordinate transformation from 2D to 3D Inverted Pendulum	95
4.21	Coordinates 3D transformation subsystem	97
4.22	Constant block parameter	98
4.23	Sketching of 3D inverted pendulum along z-axis	98
4.24	Coordinate 3D transformation system block diagram	99
4.25	Trajectory graph block diagram	100
4.26	S-function parameter for trajectory graph	100
4.27	Mask of trajectory graph subsystem	101
4.28	Subsystem mask editor for trajectory graph parameters	102
4.29	Trajectory graph block parameter	104
4.30	Trajectory graph window	105
4.31	VR Sink block	106
4.32	VR Sink block parameter	106
4.33	V-Realm Builder 2.0 window	107
4.34	3D view of inverted pendulum system in V-Realm Builder 2.0 window	108

4.35	VR Sink block parameter with drawing command	109
4.36	VR Sink block with the input labels	110
4.37	Inverted pendulum system block diagram in Simulink window	111
5.1	Result for Simulink block diagram in running mode	113
5.2	Simulation Diagnostics Viewer	114
5.3	Error block diagram	115
5.4	Closed-loop transfer function for Assumption 1	126
5.5	Closed-loop transfer function for Assumption 2	130
5.6	Closed-loop transfer function for Assumption 3	134
5.7	Closed-loop transfer function for Assumption 4	138
5.8(a)	Step response graph for assumption 1	140
5.8(b)	Step response graph for assumption 2	141
5.8(c)	Step response graph for assumption 3	141
5.8(d)	Step response graph for assumption 4	142
5.9	Set point mode for simulation of inverted pendulum	144
5.10	3-D model of inverted pendulum	145
5.11	Trajectory graph for VR sensor set point	146
5.12	Trajectory graph for mouse set point	147
5.13	PID controller parameter for existing system to control angle	150
5.14	PID controller parameter for existing system to control position	151
5.15	Simulation of inverted pendulum using Assumption 4 for PID controller	153
6.1	Setup of inverted pendulum hardware	159
6.2	Inverted pendulum robot	160

LIST OF TABLE

1.1	Project Schedule for Final Year Project	6
5.1	Parameter of inverted pendulum system	116
5.2	Summary of pole location for Assumption 1	125
5.3	Summary of pole location for Assumption 2	129
5.4	Summary of pole location for Assumption 3	133
5.5	Summary of pole location for Assumption 4	137
5.6	Summary of transfer function	139
5.7	Settling time for inverted pendulum system	143
5.8	Trajectory graph for different parameters	148
5.9	Capability of PID controller to control the position of inverted pendulum	150
5.10	Capability of PID controller to control the angle of the pendulum	151
6.1	Difference of motion equation between current study and previous study	155

LIST OF ABBREVIATIONS, SYMBOLS, SPECIALIZED NOMENCLATURE

ANN	-	Artificial Neural Network
DOF	-	Degree of Freedom
FLC	-	Fuzzy Logic Controller
IP	-	Inverted Pendulum
K_p	-	Proportional Gain
K_i	-	Integral Gain
K_d	-	Derivative Gain
MATLAB	-	Matrix Laboratory
NN	-	Neural Network
PD	-	Proportional Derivative
PI	-	Proportional Integral
PID	-	Proportional Integral Derivative
1D	-	One Dimensional
2D	-	Two Dimensional
3D	-	Three Dimensional

CHAPTER 1

INTRODUCTION

1.1 Background

Research in the field of science and technology is widening through out the world. Its aim is to produce products that are able to reduce human energy in doing work. All researches in this field also aim to investigate phenomena that occur in the engineering field. Malaysia is among the countries that is actively involved in the field of research concerned which the hope that this country can contribute to the use of sophisticated technology. One of the continuous researches that is being undertaken in the engineering field is as the control of inverted pendulum.

Extensive research on the balancing of inverted pendulum has been done through out the world using various techniques. Based on the research, scientist found that inverted pendulum is one of the well known example used to illustrate various control techniques. This system is widely applied in area such as rocket control, crane operation, and antiseismic control of buildings. The inverted pendulum is an example of an unstable non linear system.

Non-linear control is a sub-division of control engineering which deals with the control of non-linear systems. The behavior of a non-linear system cannot be described as a linear function of the state of that system or the input variables to that system. A

nonlinear system is one whose behavior cannot be expressed as a sum of the behaviors of its parts. In technical terms, the behavior of nonlinear systems is not subject to the principle of superposition. Non-Linear system is investigated with respect to the dynamics of the inverted pendulum. The inverted pendulum is an example of an unstable highly non-linear system. This problem has been a research interest of control engineers. Consequently, it has received much attention, as it is an extremely complex and challenging control problem.

The inverted pendulum system is a Control Engineering problem that is used in universities around the world. It is a suitable process to test prototype controllers due to its high non-linearity and lack of stability. The system consists of an inverted pole hinged on a cart which is free to move in the x-axis direction. The term “inverted” means opposite position and “pendulum” means a weight hung from a fixed point.

1.2 Problem Statement

The problem of balancing an inverted pendulum has been a benchmark problem in demonstrating and motivating various control design techniques. The principal reasons for its popularity are its nonlinear and unstable characteristics, uncertainty in friction terms, lack of state variable measurements, and the easy way disturbances are introduced in the process. In addition, the problem is also representative of some other well-known control problems, such as that rocket taking off, cranes with hanging loads, and the apparatus also has the attraction of inexperienced people who attempt to control using hand motion. The inverted pendulum is among the most difficult systems to control in the field of control engineering. There is a need for a control method which addresses the non-linearity of an operating system, incorporating an adaptation capability.

Due to its importance in the field of control engineering, it has been a task of choice to be assigned for students to analyze the inverted pendulum model and propose a linear compensator according to the PID control law in the MATLAB environment.

1.3 Scope

The scope of this final year project is to design a two dimensional inverted pendulum using MATLAB software. This project will focus on the design of the inverted pendulum controller using PID (proportional integral derivative) close loop system. The design also can able to control the inverted pendulum in the upright position when the working control model is achieved. PID controller needs to stabilize the angle of inverted pendulum in the simulation environment.

1.4 Objectives

The aim of this project is to design and build a working control model that can balance two dimensional inverted pendulum in an upright position by using MATLAB software. Throughout this project, the following objectives will be achieved:

- a) To conduct literature study on existing controller designs for the inverted pendulum.
- b) To design and build a control model for 2 dimensional inverted pendulum using PID controller.
- c) To build a working simulation and make analysis from the control model in MATLAB environment.

1.5 Brief History of Inverted Pendulum

The simple linear pendulum has long proved a useful model for more complicated physical systems, and its behavior in the small-amplitude limit provides a realistic yet solvable example for students in introductory classes. While the force-free, frictionless pendulum can be solved exactly for all amplitudes in terms of elliptic integrals, the solution is hardly illuminating, rarely found useful, and when damping and external driving are included, the equations of motion become intractable. With the advent of desktop computers, however, it has become possible to study in some detail the rich nonlinear dynamics of the damped, force driven pendulum and gain significant insight into its sensitivity to initial conditions for certain values of the system parameters (Chye, 1999).

In 1581, Galileo, a physicist, began studying at the University of Pisa. While at the University of Pisa, Galileo began his study of the pendulum while, according to legend, he watched a suspended lamp swing back and forth in the cathedral of Pisa. However, it was not success until 1602 that Galileo made his most notable discovery about the pendulum. The period (the time in which a pendulum swings back and forth) does not depend on the arc of the swing (the isochronism). Eventually, this discovery would lead to Galileo's further study of time intervals and the development of his idea for a pendulum clock (Helden, 2004).

Early studies of the inverted pendulum system were motivated by the need to design controllers to balance rockets during vertical take-off. At the instance of time during launch, the rocket is extremely unstable. Similar to the rocket at launch, the inverted pendulum requires a continuous correction mechanism to stay upright, since the system is unstable in open loop configuration. This problem can be compared to the rocket during launch. Here, rocket boosters have to be fired in a controlled manner, to maintain the rocket upright (Chye, 1999).

The inverted pendulum is a classic problem in dynamics and control theory and widely used as benchmark for testing control algorithms (PID controllers, neural networks and genetic algorithms). Variations on this problem include multiple links, allowing the motion of the cart to be commanded while maintaining the pendulum, and balancing the cart-pendulum system on a see-saw. The inverted pendulum is related to rocket or missile guidance, where thrust is actuated at the bottom of a tall vehicle. Another way that an inverted pendulum may be stabilized, without any feedback or control mechanism, is by oscillating the support rapidly up and down. If the oscillation is sufficiently strong (in terms of its acceleration and amplitude) then the inverted pendulum can recover from perturbations in a strikingly counterintuitive manner (Jeffers, 2001).

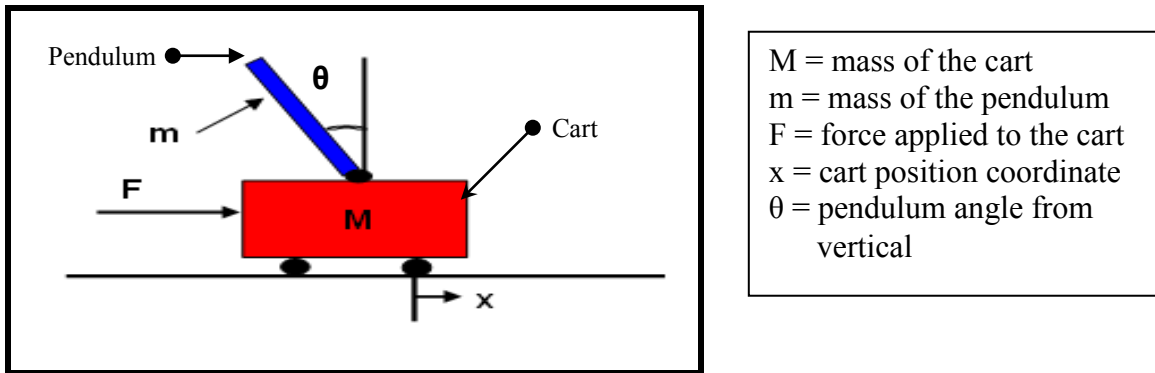


Figure 1.1: Inverted Pendulum (Callinan, 2003)

In 1898, E. Wiechert of Gottingen introduced a seismograph with a viscously damped pendulum as a sensor (Wiechert, 1899). The damping was added to lessen the effects of the pendulum eigen-oscillations. Wiechert's first seismograph was a horizontal-pendulum instrument, which recorded photographically. After a trip to Italy to study seismometers used in that country, he decided to build a mechanically-recording seismograph. For a sensor, he used an inverted pendulum stabilized by springs and frees to oscillate in any direction horizontally (Wiechert, 1904). The seismograph was completed in 1900.

Besides classroom theory and various control design methods, digital control exposure is important to the many practical aspects of implementing a digital control system. This requires a good foundation in control theory as well as knowledge in computer interfacing techniques, system modeling, instrumentation and digital signal processing. The modeling of inverted pendulum shall be created in MATLAB.

1.6 Project Schedule for Final Year Project

Table 1.1 shows the Gantt chart for the study which will be carried out through a two semester period.

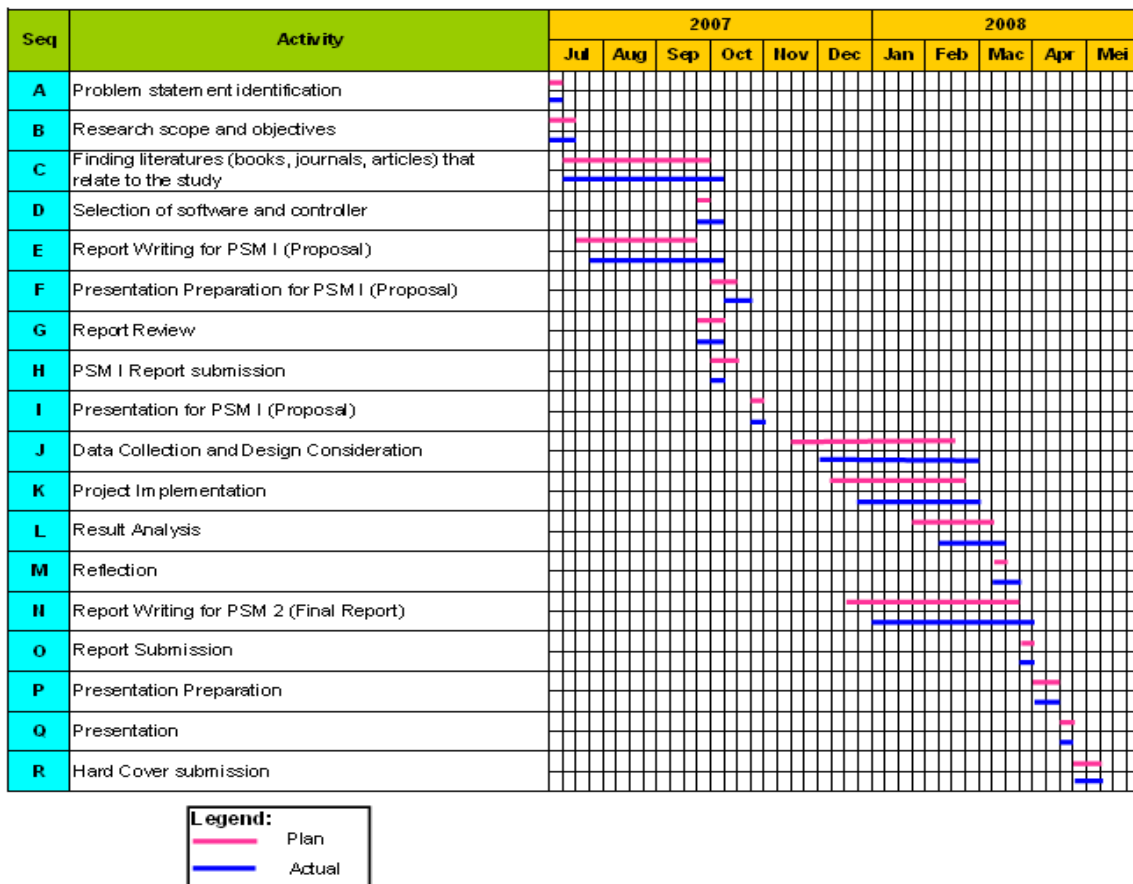


Table 1.1: Project Schedule for Final Year Project