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FINITE DIFFERENCE SIMULATION OF A FLEXIBLE RECTANGULAR
PLATE STRUCTURE

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"I hereby, declare this thesis is the result of my own research except as cited in the references"

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ABSTRACT

Finite difference simulation of a flexible rectangular plate structure is an investigation which presented about the dynamic characterization of a two dimensional flexible plate structure. To control the vibration of the plate efficiently, it is required to obtain an accurate model of the rectangular plate structures. An accurate model will lead to the realization of satisfactory control. Therefore, a thin, flat rectangular aluminum plate, with all edges clamped, is considered. Simulation algorithm which characterizing the dynamic behavior of the plates is developed through a discretisation of the governing partial different equation formulation of the plate by using finite difference method and implemented within Matlab. It will allow application and sensing of a disturbance signal at any mesh point on the plate. Such a provision is desirable for the design and development of active vibration control technique. The performance of the developed algorithm in characterizing the dynamic behavior of the system is assed in comparison to previous result reported by various methods. The algorithm validation will be presented in both, time and frequency domain. That the measured parameter associated with the first three resonance modes of the system compare with the previous result which has been reported. Thus, the simulation algorithm developed and validated forms a suitable test and verification platform in subsequent investigations for development of active vibration control strategies for flexible rectangular plate structures.

ABSTRAK

Simulasi perbezaan terbatas bagi struktur fleksibel kepingan segiempat tepat adalah kajian yang mempamerkan tentang ciri-ciri dinamik bagi dua dimensi struktur kepingan fleksibel. Untuk mengawal kecekapan getaran kepingan, adalah amat penting mendapatkan model yang tepat bagi struktur segiempat tepat. Model yang tepat akan memimpin kepada kesedaran untuk memperolehi kawalan yang memuaskan. Oleh itu, satu kepingan aluminium yang nipis dan rata dengan semua sisi diapit digunakan. Simulasi algoritma yang mencirikan sifat dinamik bagi kepingan dibangunkan melalui pengasingan persamaan kawalan perbezaan sebahagian bagi plat dengan menggunakan kaedah perbezaan terbatas dan seterusnya dilaksanakan dengan Matlab. Ini akan mengizinkan aplikasi dan mengesan gangguan isyarat pada mana-mana titik jaringan pada kepingan tersebut. Proses penyediaan seperti ini diperlukan untuk merekabentuk dan memajukan teknik kawalan getaran aktif. Perkembangan dalam membangunkan algoritma bagi tujuan mencirikan sifat dinamik sistem disahkan melalui perbandingan dengan hasil keputusan kajian sebelum ini yang menggunakan pelbagai kaedah lain. Pengesahan algoritma akan dipamerkan dalam dua keadaan, masa dan frekuensi kawasan. Sebaliknya, ukuran parameter yang dikaitkan dengan tiga gema mod yang pertama bagi sistem akan dibandingkan dengan hasil dari kajian lepas yang telah dilaporkan.

TABLE OF CONTENTS

CHAPTER	TOPIC	PAGE
1	INTRODUCTION	1
	1.1 Overview	1
	1.2 Problem statement	6
	1.3 Scope of research	7
	1.4 Outline of research	8
	1.5 Gant chart	9
2	LITEARTURES REVIEW	10
	2.1 Flexible plate structure	10
	2.2 FD method	12
	2.2.1 FD method is different from other method	14
3	METHODOLOGY	16
	3.1 Overview	16
	3.2 Classical dynamic equation of plate	16
	3.2.1 Origin of plate equation	18
	3.2.2 Kinematics equation	19
	3.2.3 Constitutive equation	21
	3.2.4 Resultants equation	23
	3.2.5 Equilibrium equation	25
	3.2.6 Combination of four origin plate equation	27

3.3	The dynamic equation of motion	29
3.4	Discretisation of the Plate	31
3.5	Initial condition	37
3.6	Boundary condition	37
3.7	Algorithm discretisation	40
3.8	Algorithm stability	45
4	RESULTS AND DISCUSSION	49
4.1	Algorithm implementation	49
4.2	Algorithm validation	57
5	CONCLUSION	60
	REFERENCES	61

LIST OF TABLES

NUMBER OF TABLE	TITLE	PAGE
1.1	Central difference approximations	33
4.1	Parameter of the rectangular plate	49
4.2	Modes of vibration of the rectangular plate with various width to length, a/b ratios	58
4.3	Average of percentage error for all first three modes	59

LIST OF FIGURES

NUMBER OF FIGURE	TITLE	PAGE
1.1	Thin flat plate applications in industries	1
1.2	Flexible plate structure in engineering system	
3.1	A flexible rectangular plate structure	19
3.2	Four origin of plate equation	27
3.3	Finite difference discretization	31
3.4	Biharmonic operator in central difference form	36
3.5	Finite difference discretisation along distance coordinates	36
3.6	Boundary condition for rectangular plate plane view	38
3.7	Description of the nodal points	40
4.1	Finite difference simulated of the plate with ratio a/b is 0.9	52
4.2	Finite difference simulated of the plate with ratio a/b is 0.5	54
4.3	Finite difference simulated of the plate with ratio a/b is 0.2	56
4.4	Percentage of error for the first three modes	59

LIST OF SYMBOLS

SYMBOL	DEFINITION
w	Deflection (m)
M_x, M_y	Bending moments
M_{xy}	Twisting moment
Q_y, Q_x	Vertical shear force
$q(x, y)$	Transverse external force (kg / m^2)
D	Flexural rigidity
b	Length of the plate (m)
a	Width of the plate (m)
h	Thickness of the plate (m)
E	Young's Modulus
Δx	Change in x direction
Δy	Change in y direction
Δt	Change in time, sampling time (s)
n	Number of section along the length of the plate
m	Number of section along the width of the plate

GREEK LETTER	DEFINITION
ω	Frequency (rad / s)
λ	Frequency parameter
ρ	The mass density per unit area
ν	Poisson ration
α_x, α_y	Spatial frequencies on the x and y directions

SUBSCRIPT**DEFINITION**

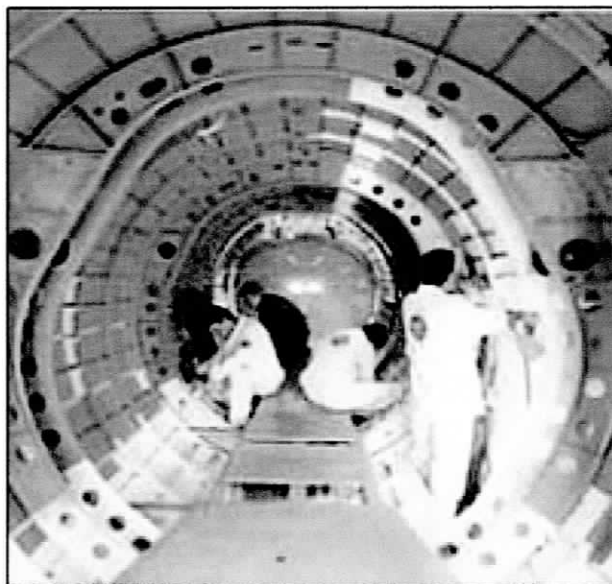
x	Axis in x direction
y	Axis in y direction
z	Axis in z direction

CHAPTER 1

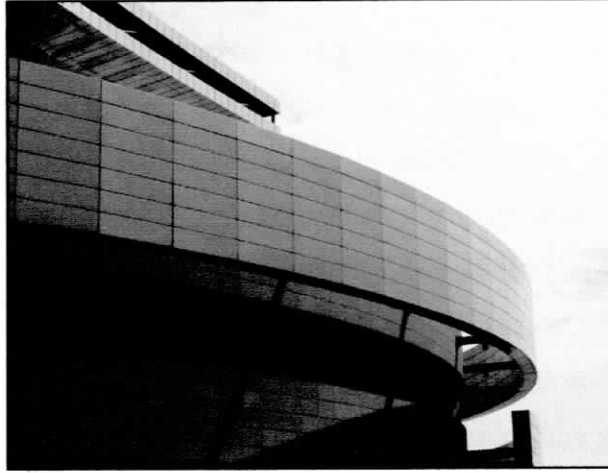
INTRODUCTION

1.1 Overview

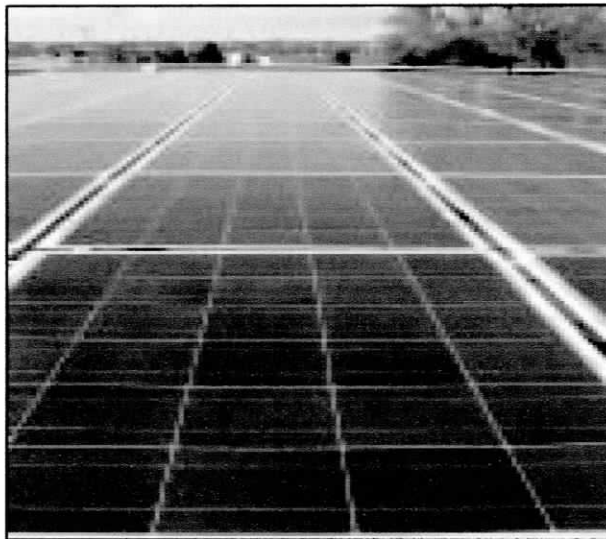
The dynamic behavior of thin isotropic rectangular plates is a subject that has received considerable attention in recent years because of its technical importance. In addition, its important application in industry. Thin flat plates are found widely used in industry. Examples are found in bridge deck, solar panels, aerospace (fuselage and airplane body skin), wall building and ship building as shown in Figure 1.1 (Mat Darus and Tokhi, 2001).



(a) Fuselage (In aerospace)



(b) Glass wall building

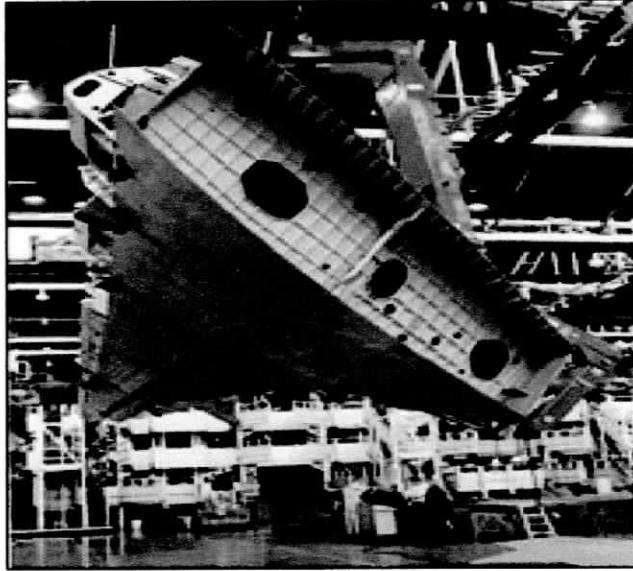


(c) Solar panel

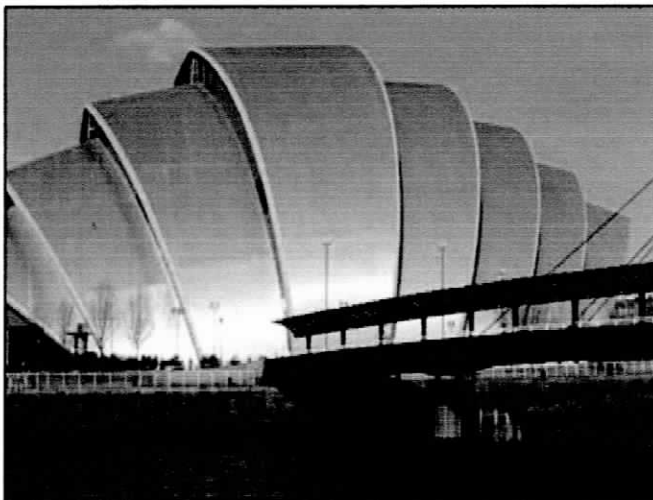
Figure 1.1: Thin flat plate applications in industries

Hossain, 1996 is noted that flexible plate structures are utilized in a wide range of engineering systems as shown in Figure 1.2. For example, these in civil engineering application include skyscrapers and bridges, in aerospace structures include propellers, aircraft fuselage and wings, satellite solar panels and helicopter blades and in electromechanical systems include turbo generator shaft, engines, gas turbines rotors and electric transformer cores (Tokhi and Hashim, 2005).

Leitch, 1992 also have reported that flexible structure systems are known to exhibit an inherent property of vibration when subjected to disturbance forces, leading to component and/or structural damage. Therefore, the purpose of vibration control in flexible structures is to dampen the response of the structure to external excitation. In all cases there are the alternatives of passive or active control solutions. Active vibration control consists of artificially generating canceling sources to destructively interfere with the unwanted source and thus result in a reduction in the level of vibration at desired locations (Tokhi and Hashim, 2005).



(a) Wing ribs (From an A340 Airbus wing box are made from aluminum plate produced by British Aluminum Plate (BAP), a division of The Luxfer Group)



(b) Conference Centre, Glasgow (From amount of aluminum plates. Including, windows, doors, facades and followed by roofs and walls)

Figure 1.2: Flexible plate structure in engineering system

It is important initially to recognize the flexible nature of the plate and construct a mathematical model for the system. In order to control the vibration of a plate efficiently, it is required to obtain an accurate model of the plate structure. An accurate model will result satisfactory and good control. Such a model can be constructed using a partial differential equation (PDE) formulation of the dynamic of the flexible plate. A commonly used approach for solving the PDE, representing the dynamics of the plate, is to utilize a representation of the PDE, obtained through a simplification process, by finite set of ordinary differential equations (Tokhi and Hashim, 2005).

The FE method has been chosen to solve partial different equation (PDE). The FD method has been previously utilized to describe infinite structure plates, Wang and Lai (2000). FE method is its ability to handle complex geometries (and boundaries) with relative ease. While FD method in its basic form is restricted to handle rectangular shapes and simple alterations thereof, the handling of geometries in FE method is theoretically straightforward. FD method is suitable for uniform structures, such as the plate system.

1.2 Problem Statement

The aim of this project is to develop a simulation algorithm by characterizing the dynamic behavior of the rectangular plate through FD method and to compare the result obtained with various other methods from previous reported results.

In this investigation, FD method was chosen to develop a simulation algorithm characterizing the dynamic behaviour of the rectangular plate, because FD method is more appropriate and more suitable in real-time applications than other method. According to this method the governing differential equation is replaced by a set of simultaneous algebraic equation. Matlab software can be used as the programming language to find the solution to these algebraic equations. The investigation is accomplished by varying ratio of the length to width of the rectangular plate. The dynamic behaviour characterization of the system in performance of the developed algorithm is assessed in comparison with previously reported results by using various other methods.

1.3 Scope of Research

The scope of this research is to:

- 1) Investigate the dynamic behaviour of the rectangular plate
- 2) Understand a simulation algorithm of rectangular plate through a discretisation of the governing partial differential equation formulation of the plate dynamics by reviewing past research and literatures.
- 3) Develop simulation algorithm for rectangular plate structure.
- 4) Implement the algorithm within Matlab environment.

1.4 Outline of Research

The research outlines are as follows:

a) Literature review

Describe the previous research and concept of flexible plate structure, FD method.

b) Methodology

Thin flexible plate is studied and learned the derivation of classical equation of plate and dynamic of equation. Solved the uniform plate structure, actually rectangular plate by using FD method and will be apply in Matlab environment.

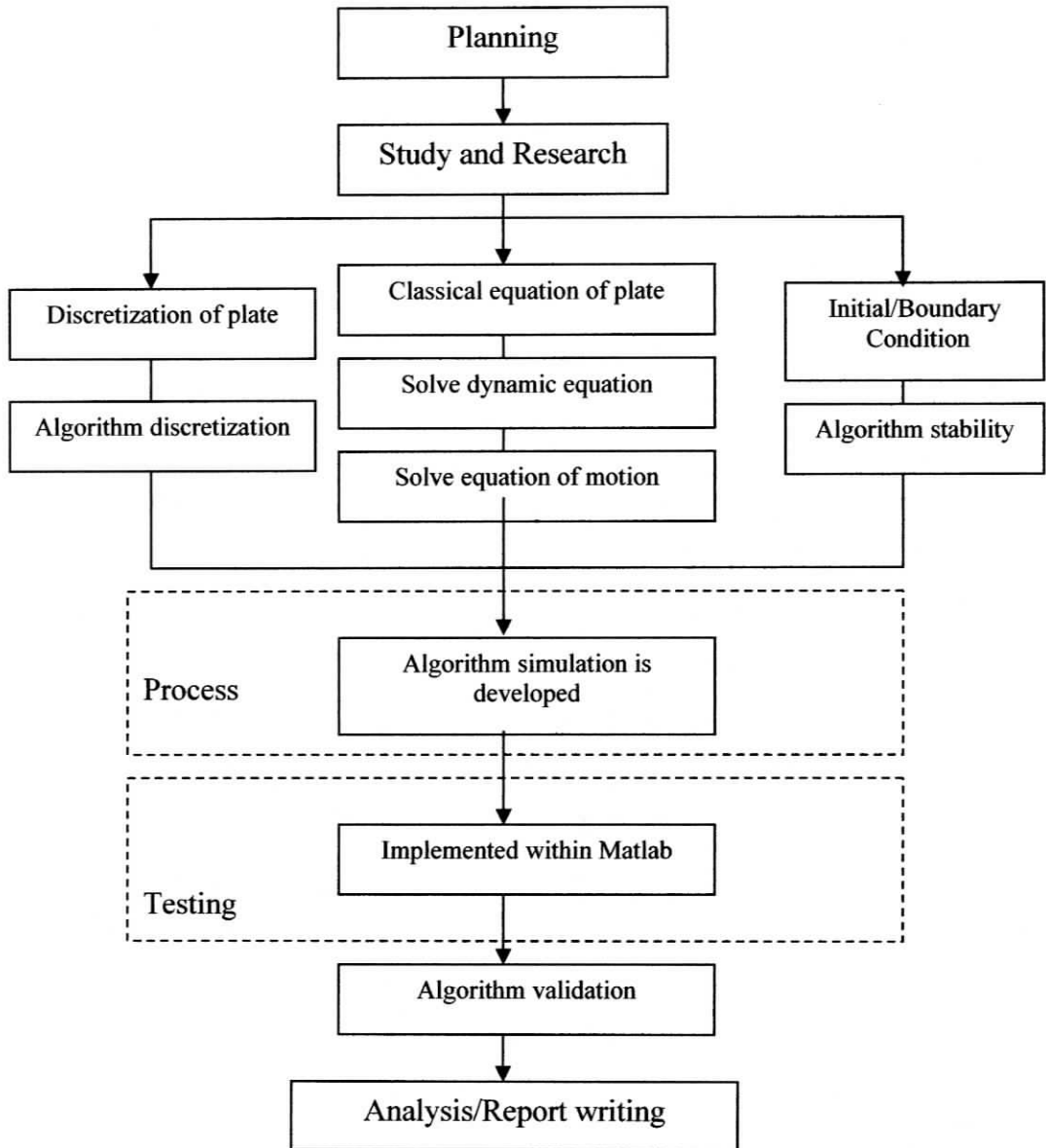
c) Analysis result

The result from the Matlab will be compared with previously reported results by using various other methods. Chapter 2 and 4 should be review for further detail.

d) Discussion

Results obtained from this research work were compiled and discussed in Chapter 4. Conclusions for this study were presented in Chapter 5.

1.5 Gant Chart



CHAPTER 2

LITERATURE REVIEW

2.1 Flexible Plate Structure

Plate structures are elements of practical importance in many engineering applications. Study of the natural modes, frequencies and the dynamic behavior of flexible plates is a subject that has received considerable attention due to its technical importance, for the last decade. In addition to being a problem of academic interest, many applications of thin flat plates are found in industry. Such as ship building (ship hull), vehicle body, machine part, storage tank, table tops, street manhole covers, side panel, bulkheads and tank bottom. Various approaches have been previously been developed for modeling flexible plate system. A symmetric and positive definite boundary element (BE) formulation for lateral vibration of plates was introduced by Davi and Milazzo (1997). Numerical results using this method have been in very good agreement with those obtained using other techniques with the accuracy of the BE model was demonstrated.

The finite element (FE) method has been previously utilized to describe infinite structure plates, Wang and Lai, 2000. It is noted, an advantage of the FE method is that it allows irregularities in the structure to be accounted for. However, the computational complexity and consequent software coding involved in both the BE formulation and the FE method constitutive major disadvantages of these techniques, especially in real-time application.

Thin plates are initially flat structural members bounded by two parallel planes, called faces, and a cylindrical surface, called an edge or boundary. The generators of the cylindrical surface are perpendicular to the plane faces. The distance between the plane faces is called the thickness (h) of the plate. It will be assumed that the plate thickness is small compared with other characteristic dimensions of the faces (length, width, diameter, etc). The load-carrying action of a plate is similar, to a certain extent, to that of beams or cables; thus, plates can be approximated by a grid work of an infinite number of beams or by a network of an infinite number of cables, depending on the flexural rigidity of the structures. This two-dimensional structural action of plates results in lighter structure and therefore offers numerous economic advantages. The plate, being originally flat, develops shear forces, bending and twisting moment to resist transverse loads. Because the load are generally carried in both plates is considerably stiffer than a beam of comparable span and thickness. So, thin plates combine light weight and form efficiency with high load-carrying capacity, economy and technological effectiveness and have general behavior of plate such as the material is homogenous, isentropic and linear elastic; that is, it follows Hooke's law, the plate is initially flat, the middle surface of the plate remains unstrained during bending and the constant thickness of plate, h , is small compared to its other dimensions; that is, the smallest lateral dimension of the plate is at least 10 times larger than its thickness (Vantsel and Krauthammer, 2001).

2.2 FD Method

In applications involving uniform structures, such as the plate system considered here, the FD method is found to be more appropriate, and the relatively reduced amount of computation involved in the FD method makes the technique more suitable in real-time applications. Thus, the FD method is used to obtain an efficient numerical method of solving the PDE formulation of the plate dynamics by developing a finite-dimensional simulation of the flexible plate through discretisation of the PDE in both the time and space coordinates. The simulated system is constructed to provide a suitable platform for subsequent implementation and verification of various controller designs (Mat Darus and Tokhi, 2003).

Thomas and Finney, 1988 have reported FD methods allow us to approximate the effect of a differential operator on a function so that we can easily see these effects. This may complement results derived analytically or sometimes may be the only practical way of deriving a result. Numerical methods used to approximate functions or differential operators do not guarantee the "correct" results. It is important to check results against known analytical results whenever possible watch your convergence very carefully and get graphical displays of each stage of your computation.