

**EXAMINATION TIMETABLING USING GRAPH COLORING
TECHNIQUE**

PRIYADARISINI A/P SUBRAMANIAM

UNIVERSITI TEKNIKAL MALAYSIA MELAKA

BORANG PENGESAHAN STATUS TESIS*

JUDUL: EXAMINATION TIMETABLING USING GRAPH COLORING
TECHNIQUE

SESI PENGAJIAN: 2010 / 2011

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EXAMINATION TIMTABLING USING GRAPH COLORING TECHNIQUE

PRIYADARISINI A/P SUBRAMANIAM

The report is submitted in partial fulfilment of the requirements for the Bachelor
of Computer Science (Artificial Intelligence)

FACULTY OF INFORMATION AND COMMUNICATION TECHNOLOGY
UNIVERSITI TEKNIKAL MALAYSIA MELAKA
2011

DECLARATION

I hereby declare that this project report entitled
EXAMINATION TIMETABLING SYSTEM

is written by me and is my own effort and that no part has been plagiarized without
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STUDENT : _____ Date: 5 JULY 2011
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SUPERVISOR: _____ Date: _____
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DEDICATION

To my beloved parents, Mr. Subramaniam Appalanayadu and Mrs. Mariamma Achanaidu for their unconditional love and full support for my thesis.

For my Supervisor Dr. Abd. Samad Hasan Basari for making it all worthwhile.

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ABSTRACT

Timetabling at large covers different types of problems which have their own unique characteristics. In education, the three most common academic timetabling problems are lecture timetable, lab or tutorial timetable and exam timetable. Exam timetable is essential but difficult to be done manually due to the complexity of the problem such as dual academic calendar, increasing student enrolments and limitations of resources. Thus, the main objective of this project is to implement a system with a graph coloring technique. The target user for the system is the registrar of the faculty. The programming languages used to develop the system are Visual Basic programming language and MySQL database. The results show that the system has generated better quality of timetable compared to the existing method.

ABSTRAK

Penjadualan secara keseluruhannya merupakan suatu masalah yang merangkumi pelbagai jenis dengan ciri-ciri yang unik. Terdapat tiga jenis penjadualan yang biasa dalam bidang pendidikan iaitu jadual waktu kuliah, jadual waktu makmal atau tutorial dan jadual waktu peperiksaan. Jadual peperiksaan sukar dibuat secara manual disebabkan beberapa faktor seperti calendar akademik yang bertindan dengan kursus lain. Maka, objektif utama projek ini ialah untuk melaksanakan satu sistem dengan menggunakan teknik pewarnaan graf. Pengguna sasaran untuk sistem ini adalah pendaftar akademik fakulti. Bahasa pengaturcaraan yang digunakan untuk menghasilkan sistem ini ialah *Visual Basic* dan pangkalan data MySQL.. Jadual yang dihasilkan daripada sistem didapati lebih berkualiti berbanding dengan jadual yang dibuat menggunakan kaedah sedia ada.

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CHAPTER I

INTRODUCTION

1.1 Introduction

This chapter aims to describe the project background, problem statement, objectives, scopes, project significance and expected output of the project.

1.2 Project Background

Timetabling is a common example of a scheduling problem but it is an important application in graph coloring. Many problems in timetabling involve various pair wise restriction on the items being scheduled, that is, there exist restrictions on which items can be scheduled to take place simultaneously. The major goal of this project is the implementation of timetabling using graph coloring approach which solving university examination timetabling problems using techniques of graph coloring that incorporates the satisfaction of both “essential” timetabling conditions (i.e., conditions or constraints that must be satisfied in order to produce a legal or feasible timetable) as well as suggested “preferential” timetabling conditions (i.e., additional conditions or soft constraints that need not necessarily be satisfied to produce a legal or legitimate timetable, but if satisfied may very well produce a more “acceptable” timetable for students and/or faculty members).The method is applied in creating a window- based exam scheduling system.

1.3 Problem Statement

The problems that exist currently are exam timetables are done manually by the faculty of Information technology and Communication (FTMK) academic registrar. This manual process considers the hard constraints and the soft constraints. For example, if the duration for the exam is seven days, the registrar will make sure the entire exam involve will be spread out within that duration with checking the student constraints. With manual process, there are no standards for solution qualities that measure either the exam timetable is feasible or not. Examination timetables should have a standard solution quality so that if the person in charge resigns or change, the new person in-charge has a benchmark to measure the quality of exam timetable.

This project focuses on semester 1 2010/2011 session examination scheduling for faculty (FTMK) students only. Exam timetable for this semester has been done manually and the proposed system will be used the same data to generate the timetable.

1.4 Objective

The objective of this project is:

1. To analyze the examination timetable in order to understand the current processes and problems involved during preparing the timetable.
2. To develop an examination timetable system using graph coloring approach.
3. To generate better quality examination timetable for the use of faculty (FTMK) academic registrar.

1.5 Scope

Several scope and barriers in this project are:

- The system will be limited only for examination timetable for faculty (FTMK).

- Only students and lecturers from faculty (FTMK) involved as candidates or variables in the invigilation timetable.
- There is only one user which is academic registrar who can control, monitor and initiate the system.

1.6 Project Significance

As mentioned in the previous section, the target users for the proposed system are academic registrars. By developing this system, the academic registrar can control and monitor student examination timetable smoothly thus produce a good timetable which satisfies all the constraints. Besides, it also will benefit student's as it satisfy some of the soft constraint (e.g. students not need to be under pressure for two different subject in the same day) and students will have positive view on their university that the management fulfil the students' needs and wants which are some of the soft constraint for an examination timetabling.

1.7 Expected Output

The expected output of the project is to understand how the graph coloring approach is used to group the subject based on color and one color is represent one exams. In addition, the window-based system is developing from manual type activities into a computerized system. This computerized system is user friendly whereby any type of user capable to handle the system without any confusion or mystification. The output of this system will be more consistent.

1.8 Conclusion

An examination timetable is considered as good quality if all the soft constraints under consideration are minimized. This system is considered as a good project to be developed where it brings more benefit for the academician who use this system and make their task become manageable and the time tables produced are acceptable by students itself. The next chapter will discuss about literature review and project methodology whereby the overview of the previous researches will be considered. Besides, the technique used, project methodology and project requirements are also discussed.

CHAPTER II

LITERATURE REVIEW AND PROJECT METHODOLOGY

2.1 Introduction

A literature review is a body of text that aims to review the critical points of current knowledge on a particular topic. Literature review also shows the research that related to the project's topic. Thus, another meaning of literature review is a critical look of existing research that significant to the work that we are carrying out. Literature review is important because it shows the target of the project.

This chapter contains all the research that has been done on the previous and existing timetabling methods on the internet including reviews on the features, capabilities and so on. All the weaknesses on the current or existing system were identified in order for this project to overcome those complications, thus the strength of the existing system were identified and studied for it can be implemented in the upcoming system.

The methodology consists of several phases as a guideline that are to be achieved. The project requirements such as software and hardware for the project development have been identified. The project milestone from the start until the delivery phases are also briefed and listed in this chapter. The milestone and Gantt chart are important as a guideline to ensure the project is completed according to the schedule and plan.

2.2 Fact and Findings

In this section, all the information related to the scheduling is found by surfing the Internet and go to the library. Literature review is done and findings are come out after read through all the information.

2.2.1 Domain

This section describes and discusses the technique used in producing a timetable. Timetabling is an application of a scheduling problem which is also an application of graph coloring. It is a NP-complete problem that is the timetable system produced can only be used by certain users, as each user has its own needs and requirements that differ specifically.

2.2.2 Existing System

The current system for final examination that has been used by the faculty (FTMK) is manually operated. Users will attach name of lecturers in FTMK before considering about the constraints. Normally, users will be depending on the constraints to set up the time tabling system. The current system need the users to be detail with every variable, constraints, possible risks and other things since the process of creating the timetable is a manual process.

2.2.2.1 Related domain and approaches used

2.2.2.1.1 Previous Researches

In the article “The Mathematics of map coloring”, which was published in a 1971, its author, Coxeter(1971), mentioned that in nearly every instance when a map of the United States is colored to distinguish neighbour states, at most five or six colors are used. What is the minimum number of colors that can be used to color the states in the United States if every two states share a common border is required to be colored differently? Two states that share only a common point, however, such as Utah and New Mexico, are permitted to be colored the same. Since Nevada and Utah are neighbouring states, that is, they share a common boundary, they must be assigned different colors. In fact, Nevada has a ring of five neighbour states, namely, Utah, Idaho, Oregon, California and Arizona. Therefore, each of these five states must be assigned a color different from that used for Nevada. On the other hand, three colors are needed to color five states bordering Nevada, So four colors are needed in all to color these six states. Indeed, all states in the United States can be colored with four colors.

The paper by Razak, Ibrahim and Hussin(2010) “Bipartite Graph Edge Coloring Approach to Course Timetabling” presents research finding on implementing a bipartite graph edge coloring approach in solving a course timetabling problem. The results are analysed by comparing the total penalties of violation on a set of predefined soft constrain between the current manual timetable and the timetable from developed prototype. The research showed that the bipartite graph edge coloring approach in this case study was able to minimize the number of penalties when violating the soft constraints, thus producing better quality timetable compared to current timetable. Future research will include testing the algorithm with larger scope of data sets from different institutions.

In 1993, Burke, Elliman and Weare(1993) introduced plans for a university timetabling system based on graph coloring and constraint manipulation. Graph coloring and room allocation heuristic algorithms were described along with an

illustration of how the two can be combined to provide the basis of a system for timetabling. The authors also discussed the handling of several common timetabling features within the system, primarily with regards to examination timetabling. Some particular features are specifically exclusive to examination timetabling (e.g., the constraint that an exam with more students should come at the beginning of the exam period), while other features could be translated and applied to a system for course timetabling (e.g. the constraint that an exam must or must not occur in a specific time slot).

Timothy (1971) paper “University Timetabling via Graph Coloring: An Alternative Approach” have presented an alternative approach to university course timetabling via graph coloring. The traditional graph coloring models involve graphs in which a vertex represents a course to be scheduled; an edge represents a pair of courses that conflict and the color of a vertex represents the time period. In addition, they also have determined a course’s room assignment to determine its time slot and avoided the need for a separate algorithm to assign courses to classrooms after coloring but also did with traditional graph coloring.

2.2.2.1.2 Scheduling

Scheduling involves the arrangement, coordination and planning of the utilization of resources to achieve an objective. Timetabling is an application of a scheduling problem. A timetable is presented for events to take place and it does not necessarily imply the allocation of resources. However, in reality it is important to know whether the resources available are sufficient or not for the given event to take place at particular time so that they do not get in conflict with each other.

Timetabling application used in various forms such as educational timetabling, hospital scheduling, sports timetabling, transportation scheduling and etc. In education, the three most common academic timetabling applications are school timetable, university timetable and examination timetable.

2.2.2.2 Microsoft Visual Basic

Visual Basic, created by Microsoft for building stand-alone Windows-based programs, Visual Basic was originally created to make it easier to write programs for the Windows computer operating system. The basis of Visual Basic is an earlier programming language called BASIC that was invented by Dartmouth College professors John Kemeny and Thomas Kurtz. Developers can use it for quickly building GUI applications.

Visual Basic was one of the first systems that made it practical to write programs for the Windows operating system. This was possible because VB included software tools to automatically create the detailed programming required by Windows. These software tools not only create Windows programs, they also take full advantage of the graphical way that Windows works by letting programmers "draw" their systems with a mouse on the computer. This is why it's called "Visual" Basic.

Visual Basic also provides unique and complete software architecture. "Architecture" is the way computer programs, such as Windows and VB programs, work together. One of the major reasons why Visual Basic has been so successful is that it includes everything that is necessary to write programs for Windows.

2.2.3 Technique

2.2.3.1 Graph Coloring

In graph theory, Graph coloring is an assignment of labels traditionally called "colors" to an elements of a graph to certain constraints. It has two types' f coloring which are vertex coloring and edge coloring. Vertex coloring is the way of coloring the vertices of a graph such that no two adjacent vertices share the same color while edge coloring assigns a color to each edge so that no two adjacent edges share the same color. In more details, a graph G with n vertices and m edges consists of a vertex set $V(G) = \{v_1, v_2, \dots, v_n\}$ and an edge set $E(G) = \{e_1, e_2, \dots, e_m\}$. Each edge of