

**CONTROL OF A REAL INVERTED PENDULUM  
SYSTEMS USING  
STATEFEEDBACK CONTROLLER**

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CONTROL OF A REAL INVERTED PENDULUM SYSTEMS USING  
STATEFEEDBACK CONTROLLER

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This report is submitted in partial fulfillment of the requirements for the award of  
Bachelor of Electronic Engineering (Industrial Electronics) With Honours

Faculty of Electronic and Computer Engineering  
Universiti Teknikal Malaysia Melaka

April 2009



**UNIVERSITI TEKNIKAL MALAYSIA MELAKA**  
FAKULTI KEJURUTERAAN ELEKTRONIK DAN KEJURUTERAAN KOMPUTER

**BORANG PENGESAHAN STATUS LAPORAN  
PROJEK SARJANA MUDA II**

**Tajuk Projek** : Control Of A Real Inverted Pendulum Systems Using  
Statefeedback Controller  
**Sesi Pengajian** : 2008/2009

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To my beloved parents

## **ACKNOWLEDGEMENT**

I have learnt some great knowledge's that I could not forget in my life.

A lot of praise to Allah destined me to study here.

This work is a synergistic product of many good minds. It began since the first day I got this final project. I am grateful for the inspiration and friendly culture of peoples here. I would like to thanks to all of peoples here especially to my supervisor and my team work.

For the development and production of this final report I feel deep sense of gratitude:

- To my parents for their loves, prays and encouragements.
- To my lecturers for their guides and advices.
- To my supervisor, Mr. Mohd Shakir for his carefulness, brilliant ideas and flexibility.
- To my team work, Zaini, Awang, Zulfadhli, Mariani for their full support, wisdom thinking and enjoy moments working with.

## ABSTRACT

This thesis presents the design and implementation of a complete control system for stabilizing control of an inverted pendulum. The inverted pendulum is a classical control system problem because of its nonlinear characteristics and unstable behavior. Control of inverted pendulum is a Control Engineering project based on the flight simulation of rocket or missile during the initial stages of flights. The inverted pendulum (which is simulating the rocket, here) is mounted on a moving cart. A DC motor is controlling the translation motion of the cart, through a belt/pulley mechanism. The aim of this project is to stabilize the Inverted Pendulum such that the position of the carriage on the track is controlled quickly and accurately so that the pendulum is always at its upright during such movements. The dynamic model of the whole system consists of two separate sub-models, namely the non-linear model and a linear model of the inverted pendulum. The control method is based on state feedback pole placement design techniques using the linearized model of the inverted pendulum. The Real Time Windows Target (RTWT) application is used as a host target that enables to connect Simulink models and execute in real time meanwhile the Data Acquisition Card (DAQ card) is used to generate data from computer to the real hardware.



## ABSTRAK

Tesis ini membentangkan reka bentuk dan pelaksanaan satu sistem kawalan lengkap untuk menstabilkan kawalan satu bandul terbalik. Bandul terbalik adalah satu masalah sistem kawalan yang klasik disebabkan oleh ciri-cirinya yang tak linear dan mempunyai tingkah laku tidak stabil. Kawalan bagi bandul terbalik adalah satu projek Kejuruteraan Kawalan berdasarkan simulasi penerbangan roket atau peluru berpandu semasa peringkat awal penerbangan. Bandul terbalik (yang dianggap sebagai roket, di sini) dipasang di atas sebuah kereta atau troli. Sejenis motor akan mengawal gerakan translasi kereta, melalui satu tali sawat / mekanisme takal. Tujuan projek ini adalah bagi menstabilkan Bandul Terbalik seperti bahawa kedudukan itu bagi gerabak pada trek adalah terkawal dengan cepat dan dengan tepat supaya bandul sentiasa ada pada tegaknya semasa sesuatu pergerakan. Model dinamik keseluruhan sistem ini mengandungi dua berasingan model, iaitu model tak linear dan satu model linear bagi bandul terbalik. Kaedah kawalan adalah diasaskan melalui pengawal pampasan dahulu penempatan titik menggunakan model yang dilinearkan bagi bandul terbalik. Aplikasi Real Time Windows Target (RTWT) digunakan seperti satu tuan rumah sasaran yang membolehkan untuk model Simulink berkait dan dijalankan dalam masa nyata sementara itu Data Acquisition Card (Kad DAQ) bertujuan menjana data daripada komputer untuk perkakasan yang sebenar.

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## LIST OF ABBREVIATIONS

A/D	-	Analog/Digital
ANG	-	Analog
COM	-	Common
CPU	-	Central Processing Unit
CW/CCW	-	Clockwise/Counter Clockwise
D/A	-	Digital/Analog
DAQ	-	Data Acquisition
DC	-	Direct Current
DIN	-	Deutsches Institut für Normung
I/O	-	Input/Output
PC	-	Personal Computer
PCI	-	Peripheral Component Interact
RTWT	-	Real Time Windows Target
SCSI	-	Small Computer System Interface
SFC	-	Statefeedback Controller
VR	-	Variable Resistor

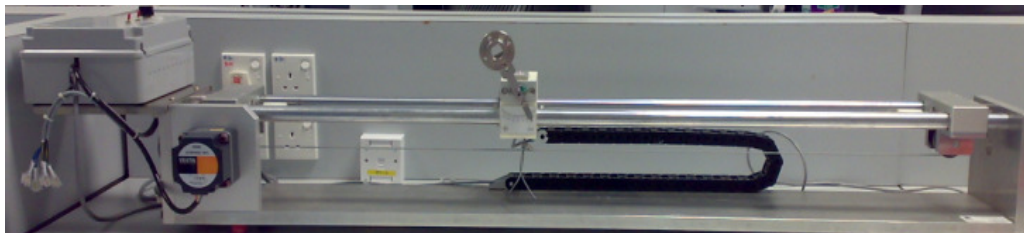


# CHAPTER I

## INTRODUCTION

### 1.1 Overview

Inverted pendulum system is a nonlinear unstable system, an ideal experiment platform for teaching control theories and conducting various control experiments. Many abstract control concepts, such as the stability and the controllability of a control system, can all be shown visually through the inverted pendulum system. In addition to educational purposes, an inverted pendulum is also a research area for many researchers of modern control theories. Through the continuous research on new ways of controlling inverted pendulum, researchers have developed new control methods, and apply them to the high tech areas such as aeronautical engineering and robotics.

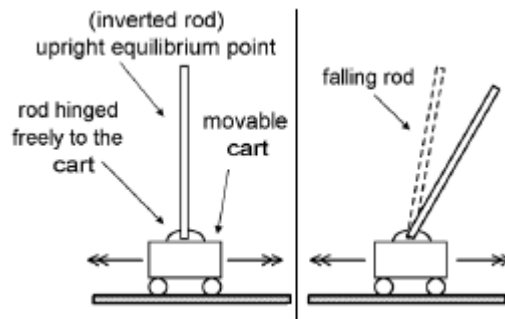


**Figure 1.1** Model of Inverted Pendulum

The inverted pendulum is an intriguing subject from the control point of view due to their intrinsic nonlinearity. The problem is to balance a pole on a mobile platform that can move in only two directions, to the left or to the right. This control problem is fundamentally the same as those involved in rocket or missile propulsion.

Being an under-actuated mechanical system and inherently open loop unstable with highly non-linear dynamics, the inverted pendulum system is a perfect test-bed for the design of a wide range of classical and contemporary control techniques. Its applications range widely from robotics to space rocket guidance systems. Originally, these systems were used to illustrate ideas in linear control theory such as the control of linear unstable systems. Their inherent non-linear nature helped them to maintain their usefulness along the years and they are now used to illustrate several ideas emerging in the field of modern non-linear control.

A single rod inverted pendulum consists of a freely pivoted rod, mounted on a motor driven cart. With the rod exactly centered above the motionless cart, there are no sidelong resultant forces on the rod and it remains balanced as shown in Figure 1.2 (left side). In principle it can stay this way indefinitely, but in practice it never does. Any disturbance that shifts the rod away from equilibrium, gives rise to forces that push the rod farther from this equilibrium point, implying that the upright equilibrium point is inherently unstable. This is called the pendant position. This equilibrium point is stable as opposed to the upright equilibrium point.



**Figure 1.2** Equilibrium points of Inverted Pendulum

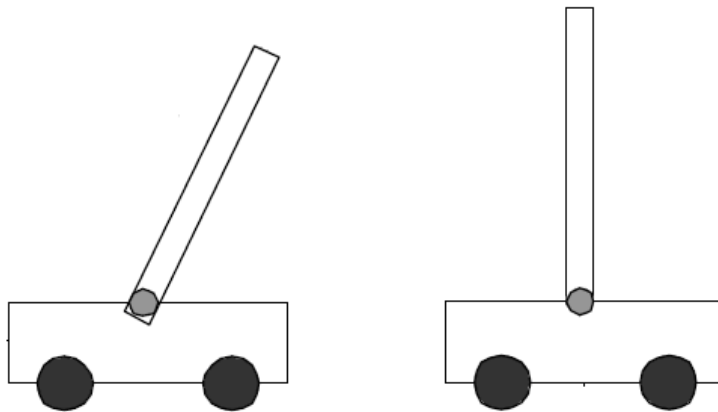
The considerations in the thesis are based on the real inverted pendulum trainer situated in the university. A single inverted pendulum is mounted on a moving cart. A DC motor controls the translation motion of the cart, through a belt mechanism. The motor is driven by DC electronic system, which also contains controller circuit. The inverted pendulum as an object of control is inherently unstable and nonlinear system. In order to balance the pendulum in the inverted position the pivot must be continuously and quickly moved to correct the falling pendulum.

## 1.2 Objectives

- a. To synthesis the mathematical model of the Inverted Pendulum based on the modeling of inverted pendulum by K.Ogata (1978).
- b. To design a statefeedback pole placement control techniques to control the cart's position and the rod's angle of the Inverted Pendulum.
- c. To carry out the simulation works for the controllers in MATLAB before applied to the real system.
- d. To interface between the software and hardware using Data Acquisition Card based on Real Windows Target Application.

### 1.3 Problem Statement

It is virtually impossible to balance a pendulum in the inverted position without applying some external force to the system. The cart balanced Inverted Pendulum system, shown below, allows this control force to be applied to the pendulum cart.



**Figure 1.3** The cart balanced Inverted Pendulum system

This Inverted Pendulum provides the control force to the cart by means of a DC motor through a belt drive system. The outputs from the system can be carriage position, carriage velocity, pendulum angle and pendulum angular velocity. The pendulum angle is fed back to an Analog Controller which controls the DC motor, ensuring consistent and continuous traction.

The aim of this study is to stabilize the pendulum such that the position of the cart on the track is controlled quickly and accurately and that the pendulum is always maintained tightly in its inverted position during such movements.

The problem involves a cart, able to move backwards and forwards, and a pendulum, hinged to the cart at the bottom of its length such that the pendulum can move in the same plane as the cart, shown below. That is, the pendulum mounted on the cart is free to fall along the cart's axis of motion. The system is to be controlled so that the pendulum remains balanced and upright, and is resistant to a step disturbance.

This problem involves a simple coupled system. If the pendulum starts off-centre, it will begin to fall. The pendulum is coupled to the cart, and the cart will start to move in the opposite direction, just as moving the cart would cause the pendulum to become off centre. To stabilize the system, such to keep the pendulum in upright position, a feedback control system must be used.

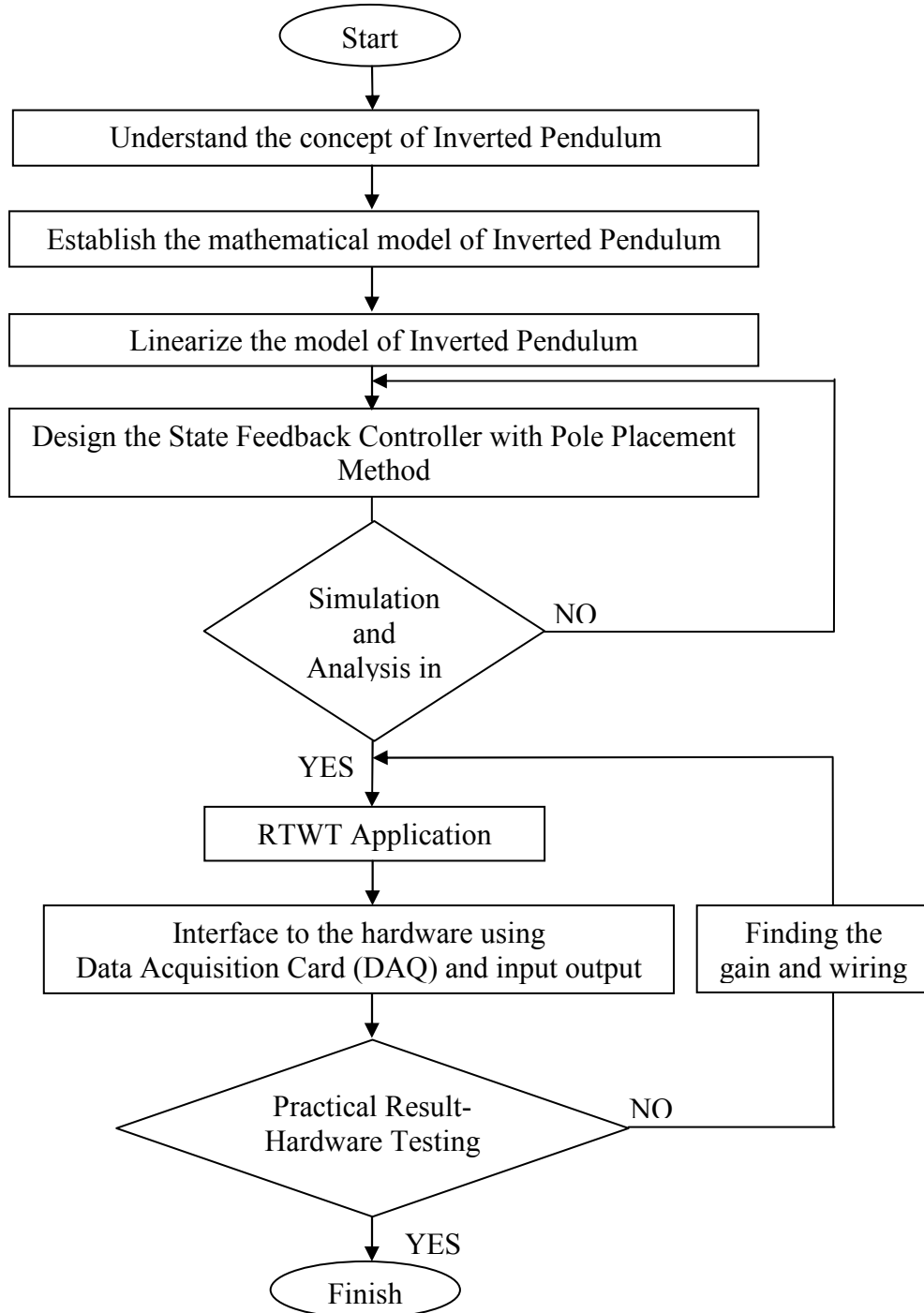
#### **1.4 Scope of Works**

The scope of work involves the whole process that will be cover in this project. The scope of work is simply can be described as below:

- a) Synthesis the mathematical model of Inverted Pendulum modeled by K.Ogata (1978)
- b) Design the state feedback controller based on the pole placement method controller to control the Inverted Pendulum.
- c) Implement the design of the controller in MATLAB software.
- d) Interfacing between the software and the hardware using the Data Acquisition Card (DAQ) based on with Real Time Windows Target Application (RTWT).
- e) Experimental results and discussion.

## 1.5 Research Methodology

Figure 1.3 shows the idea and approach in order to achieve the objective of this project.



**Figure 1.4** Flow chart of research methodology

## **CHAPTER II**

### **LITERATURE REVIEW**

#### **2.1 Introduction**

The literature review undertaken as a part of the inverted pendulum project was focused on understanding the background and application of inverted pendulum systems, mathematical modeling, control, and other successful projects of a similar nature.

#### **2.2 Background**

The inverted pendulum is a classic example of a non-linear control topic and one studied frequently with reference to design, implementation and development of control for non-linear systems. It appears in undergraduate control text books such as K.Ogata (1978) where it is used as an example of how to mathematically describe physical systems.

### **2.3 Present Applications**

Inverted pendulum is currently used as teaching aids and research experiments. Quanser (2004), a supplier of educational and research based equipment produce modular systems which can be configured as single or double inverted pendulum. Their range offers both a rotary and a linear version. Many researchers have also built their own inverted pendulum systems (Åström and Furuta, 1996) to suit their investigations.

### **2.4 System Modeling Methodology**

Chinichian (1990) design and analyze a controller for balancing one pendulum with two degrees of freedom, “spatial inverted pendulum”. The pendulum, with two degrees of freedom, has a three dimensional motion, and it will be more analogous to the design of a controller for attitude control during launching a rocket. A full state-variable feedback controller design for a state-space linear model of a three dimensional inverted cart/pendulum system is presented. This design was based on pole-placement technique. Alternative solutions to the simple pole-placement technique were also proposed to exploit non-uniqueness of the feed-back gains for a certain closed-loop pole locations and the closed-loop system response was simulated on a digital computer.

The rapid increase of the aged population in countries like Japan has prompted researchers to develop robotic wheelchairs to assist the infirm to move around (Takahashi, 2000). The control system for an inverted pendulum is applied when the wheelchair maneuvers a small step or road curbs.

Rich Chi Oii (2003) discusses the processes developed and considerations involved in balancing a two-wheeled autonomous robot based on the inverted pendulum