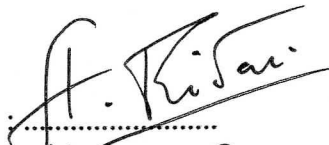


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*\*Line which is irrelevant*

**ROLLING AND YAWING AERODYNAMIC PERFORMANCE OF AN AIRCRAFT  
MODEL**

**LIM SWEE MENG**

**This report is presented in  
Partial fulfilment of the requirements for the  
Degree of Bachelor of Mechanical Engineering (Thermal Fluid)**

**Faculty of Mechanical Engineering  
Universiti Teknikal Malaysia Melaka**

**MAY 2009**

**DECLARATION**

“I hereby, declare this thesis is result of my own research except as cited in the references”

Signature : .....

Author's Name : Lim Swee Meng

Date : 9/10/2008

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## ABSTRAK

Kajian ini menumpu kepada penyelidikan kesan permukaan kawalan (*rudder* dan *aileron*s) bagi sebuah model kapal terbang terhadap pekali *side force*,  $C_Y$ , pekali momen *yawing*,  $C_n$ , dan pekali momen *rolling*,  $C_l$ , dengan menggunakan terowong angin subsonik yang tertutup. Kesan terhadap model dengan mengubah sudut gelincir, sudut *rudder* dan sudut *aileron*s akan diuji dengan menggunakan terowong angin subsonik adalah untuk mendapatkan nilai pekali *side force*,  $C_Y$ , pekali momen *yawing*,  $C_n$ , dan pekali momen *rolling*,  $C_l$ . Seperti yang dijangkakan, perubahan sudut *rudder* adalah berkesan dalam mengawal pekali moment *yawing*. Perubahan sudut *rudder* adalah berkadar secara tidak langsung dengan pekali momen *yawing* apabila perubahan sudut *rudder* meningkat pekali momen *yawing* akan berkurang. Keputusan eksperimen menunjukkan bahawa perubahan sudut *rudder* adalah berkesan dari  $-5^\circ$  sehingga  $-10^\circ$  untuk mengawal pekali momen *yawing*. Selain daripada itu, keputusan eksperimen juga menunjukkan bahawa perubahan sudut *aileron*s adalah berkesan dalam mengawal pekali momen *rolling* dan pekali momen *yawing*. Kesan langsung bagi perubahan sudut *aileron*s adalah dalam mengawal pekali momen *rolling*. Perubahan sudut *aileron*s berkadar secara langsung dengan pekali momen *rolling*. Apabila perubahan sudut *aileron*s meningkat pekali momen *rolling* juga akan meningkat. Keputusan eksperimen menunjukkan bahawa perubahan sudut *aileron*s adalah berkesan dari  $-5^\circ$  sehingga  $-15^\circ$  dalam mengawal pekali momen *rolling*.

## ABSTRACT

The present investigation primarily studies the effect of control surfaces on the coefficient of side force,  $C_Y$ , yawing moment,  $C_n$ , and rolling moment,  $C_l$ , of an airplane model using a close-loop subsonic wind tunnel. Effect of variation for the sideslip angle, angle of rudder and angle of ailerons will be tested in the wind tunnel in order to get the coefficient of side force,  $C_Y$ , yawing moment,  $C_n$ , and rolling moment,  $C_l$ . As expected, the rudder deflection is effective in controlling the yaw moment coefficient. The rudder deflection is inversely proportional to the yawing moment coefficient where as the rudder deflection increases the yawing moment coefficient will decrease. From the results the effective angle of rudder deflection is from  $-5^\circ$  till  $-10^\circ$  in controlling the yawing moment coefficient. Besides, from the results the ailerons deflection is effective in controlling the rolling moment coefficient and yawing moment coefficient. The direct effect of ailerons deflection is on the rolling moment coefficient. The ailerons deflection is directly proportional to the rolling moment coefficient where as the angle of ailerons deflection increases the rolling moment coefficient will increase. From the results the effective angle of ailerons deflection is from  $-5^\circ$  till  $-15^\circ$  in controlling the rolling moment coefficient.

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## LIST OF SYMBOLS

Concise list of symbols in order of appearance:

UAV	=	Unmanned Aerial Vehicles
$C_y$	=	Coefficient of Side Force
$C_l$	=	Coefficient of Rolling Moment
$C_N$	=	Coefficient of Yawing Moment
$\beta$	=	Sideslip Angle, °
$\varphi$	=	Roll Angle, °
$C_l$	=	Lift Coefficient
$F_d$	=	Drag Force, N
$\rho$	=	Density of the Fluid, kg/m <sup>3</sup>
$v$	=	Velocity of The Object Relative to The Fluid, m/s
$A$	=	Reference Area, m <sup>2</sup>
$C_d$	=	Drag Coefficient
$T$	=	Torque, Nm
$F$	=	Force, N
$L$	=	Length, m
$N$	=	Moment, Nm
$N_{wb}$	=	Moment About Wing Body, Nm
$l_{vt}$	=	Vertical Tail Length, m
$D_{vt}$	=	Drag of Vertical Tail, N
$L_{vt}$	=	Lift of Vertical Tail, N
$\alpha_{vt}$	=	Angle of Attack of Vertical Tail, °
$C_{L_{vt}}$	=	Lift Coefficient of Vertical Tail
$S_{vt}$	=	Vertical Tail Area, m <sup>2</sup>
$a_{vt}$	=	Vertical Tail Lift Curve Slope

$a_r$	=	Rudder Lift Curve Slope
$\delta_r$	=	Angle of Rudder, °
$C_{N_{wb}}$	=	Coefficient of Yawing Moment of Wing Body
$C_{N\beta}$	=	Coefficient of Yawing Moment of Sideslip Angle
$C_{N\delta_r}$	=	Coefficient of Yawing Moment of Rudder Angle
$C_{N\beta_{wb}}$	=	Coefficient of Yawing Moment of Sideslip Angle At Wing Body
p	=	Propeller
$L_{roll}$	=	Lift Due to Roll, N
$C_{l\beta}$	=	Coefficient of Rolling Moment of Sideslip Angle
$C_{ldr}$	=	Coefficient of Rolling Moment of angle of rudder
$C_{lda}$	=	Coefficient of Rolling Moment of angle of ailerons
$C_{n\beta}$	=	Coefficient of yawing Moment of Sideslip Angle
$C_{ndr}$	=	Coefficient of yawing Moment of angle of rudder
$C_{nda}$	=	Coefficient of Rolling Moment of Sideslip Angle
$C_{Y\beta}$	=	Coefficient of side force of Sideslip Angle
$C_{Ydr}$	=	Coefficient of side force of angle of rudder
$C_{Yda}$	=	Coefficient of side force of angle of ailerons
$v$	=	Sideslip Velocity, m/s
$\Gamma$	=	Dihedral Angle, °
c.g	=	Centre of Gravity
$\mu$	=	Fluid Viscosity, kg/ms
$c$	=	Speed of Sound
[ n ]	=	Refer to References n (n = 1,2,3...)

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## CHAPTER 1

### INTRODUCTION

#### 1.1 Background Study

Unmanned Aerial Vehicles (UAVs) are typically rotorcraft or airplanes and are used in many applications, including surveillance, terrain mapping, military combat and environmental monitoring. In UAV the performances of aircraft in lateral and directional direction is very important. The UAV will depart from one point and arrive another point automatically by setting the coordinate of depart and arrive. We notice that there are many problems in aircraft control due to its performances. Therefore I will do a research on the performances of aircraft in lateral and directional direction to determine the variation of side force coefficient, rolling moment coefficient and yawing moment coefficient function of slip angle and roll angle. Analysis and discussion of stability derivative will be done by using the experimental results get from the wind tunnel. Through the research we will find out the behavior of UAV and control surfaces which affected the flight behavior. The law that affected the UAV is forces, directions and moments. The control surfaces can control the direction of force and moment by using rudder, elevator and aileron. The control surfaces that included in my research are rudder and ailerons where the rudder is use to control the yawing while aileron is use to control the rolling.

## 1.2 Problem Statement

Flight behavior and performance of an aircraft is very important because of with poor flight behavior and performances an aircraft accident may occur. Control surfaces are the components that used to control the flight behavior and performance of an aircraft. Therefore an analysis and study on the effect of control surfaces to the flight behavior and performances of an aircraft is carried out. In the context of this project, some open ended question include:-

- What are the corresponding rolling moment coefficient, side force coefficient and yawing moment coefficient for a certain degree of ailerons and rudder deflection?
- What is the relationship between the side forces, rolling moment, yawing moment and control law of an aircraft?

## 1.3 Objectives

- To determine the variation of side force, rolling moment and yawing moment ( $C_Y$ ,  $C_l$  and  $C_N$ ) function of sideslip angle ( $\beta$ ), angle of attack ( $\alpha$ ), roll angle ( $\phi$ ), angle of rudder ( $\delta_r$ ) and angle of ailerons ( $\delta_a$ ).
- To study the effect of control surfaces to the flight motion of the UAV model.

## 1.4 Scope

The research will be focus on the data analysis and determination of side force coefficient, rolling moment coefficient and yawing moment coefficient. An airplane model will be tested at wind tunnel in order to determine lateral and directional derivatives. Before start to test literature research will be done, and the parameter will be set to test at wind tunnel.

## CHAPTER 2

### LITERATURE REVIEW

#### 2.1 Basic Aerodynamics of an Aircraft

The word Aerodynamics comes from two Greek words that is aëros which concerning the air, and dynamis, which means force. Aerodynamics is the study of forces and the resulting motion of objects through the air. Since we live in a three dimensional world, it is necessary to control the attitude or orientation of a flying aircraft in all three dimensions. In flight, any aircraft will rotate about its center of gravity, a point which is the average location of the mass of the aircraft. We can define a three dimensional coordinate system through the center of gravity with each axis of this coordinate system perpendicular to the other two axes. We can then define the orientation of the aircraft by the amount of rotation of the parts of the aircraft along these principal axes. [1]

The yaw axis is defined to be perpendicular to the plane of the wings with its origin at the center of gravity and directed towards the bottom of the aircraft. A yaw motion is a movement of the nose of the aircraft from side to side. The pitch axis is perpendicular to the yaw axis and is parallel to the plane of the wings with its origin at the center of gravity and directed towards the right wing tip. A pitch motion is an up or down movement of the nose of the aircraft. The roll axis is perpendicular to the other two axes with its origin at the center of gravity, and is directed towards the nose of the aircraft. A rolling motion is an up and down movement of the wing tips of the aircraft. [1]

In flight, the control surfaces of an aircraft produce aerodynamic forces. These forces are applied at the center of pressure of the control surfaces which are some distance from the aircraft centre of gravity and produce torques or moments about the principal axes. The torques causes the aircraft to rotate. The elevators produce a pitching moment, the rudder produces a yawing moment, and the ailerons produce a rolling moment. The ability to vary the amount of the force and the moment allows the pilot to maneuver or to trim the aircraft. [1]

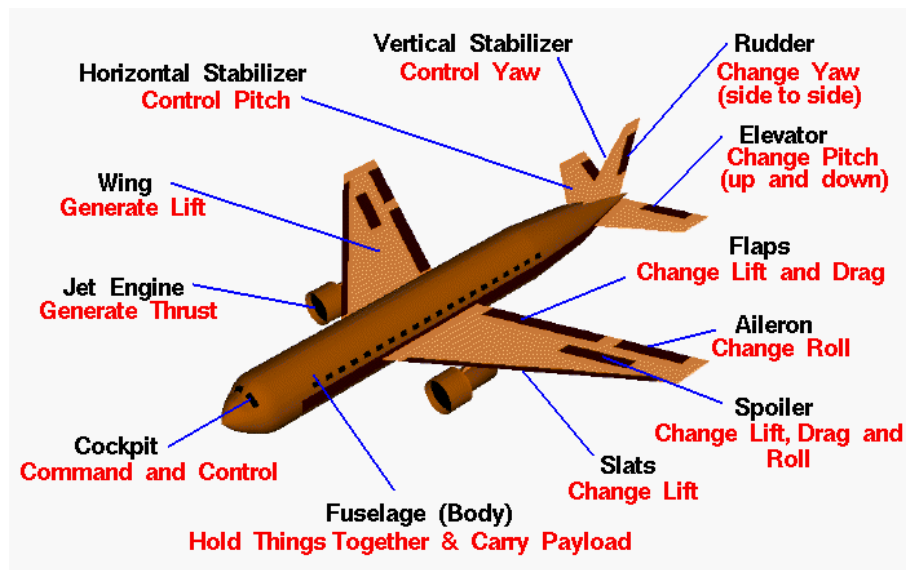


Figure 2.1: Aircraft model components

(Source: Tom Benson (2008))

## 2.2 Aerodynamics Studies for effect of control surfaces on An Aircraft

A classic airplane has three basic controls: ailerons, elevator, and rudder. They are designed to change and control the moments about the roll, pitch, and yaw axes. These control surfaces are flaplike surfaces that can be deflected back and forth at the command of the pilot. [1]

The ailerons are located at the trailing edge of the wing. It was used to generate a rolling motion for an aircraft. Ailerons are small hinged sections on the outboard portion of a wing. Ailerons usually work in opposition: as the right aileron is deflected upward, the left is deflected downward, and vice versa. The ailerons are used to bank the aircraft; to cause one wing tip to move up and the other wing tip to move down. The banking creates an unbalanced side force component of the large wing lift force which causes the aircraft's flight path to curve. Airplanes turn because of banking created by the ailerons, not because of a rudder input. [1]

The ailerons work by changing the effective shape of the airfoil of the outer portion of the wing. Changing the angle of deflection at the rear of an airfoil will change the amount of lift generated by the foil. With greater downward deflection, the lift will increase in the upward direction. The aileron on the right wing is deflected up. Therefore, the lift on the left wing is increased, while the lift on the right wing is decreased. For both wings, the lift force of the wing section through the aileron is applied at the aerodynamic center of the section which is some distance from the aircraft center of gravity. This creates a torque

$$T = F * L$$

about the center of gravity. If the forces and distances are equal there is no net torque on the aircraft. But if the forces are unequal, there is a net torque and the aircraft rotates about its center of gravity. The resulting motion will roll the aircraft to the right. If the pilot reverses the aileron deflections where the right aileron down and the left aileron up so the aircraft will roll in the opposite direction. [1]

At the rear of the fuselage of most aircraft one finds a vertical stabilizer and a rudder. The stabilizer is a fixed wing section whose job is to provide stability for the aircraft, to keep it flying straight. The vertical stabilizer prevents side-to-side, or yawing, motion of the aircraft nose. The rudder is the small moving section at the rear of the stabilizer that is attached to the fixed sections by hinges. Because the rudder moves, it varies the amount of force generated by the tail surface and is used to generate and control the yawing motion of the aircraft. [2]

The rudder is used to control the position of the nose of the aircraft. It is not used to turn the aircraft in flight. Aircraft turns are caused by banking the aircraft to one side using either aileron. The banking creates an unbalanced side force component of the large wing lift force which causes the aircraft's flight path to curve. The rudder input insures that the aircraft is properly aligned to the curved flight path during the maneuver. Otherwise, the aircraft would encounter additional drag or even a possible adverse yaw condition in which, due to increased drag from the control surfaces, the nose would move farther off the flight path. [2]

The rudder works by changing the effective shape of the airfoil of the vertical stabilizer. As described on the shape effects slide, changing the angle of deflection at the rear of an airfoil will change the amount of lift generated by the foil. With increased deflection, the lift will increase in the opposite direction. The rudder and vertical stabilizer are mounted so that they will produce forces from side to side, not up and down. The side force is applied through the center of pressure of the vertical stabilizer which is some distance from the aircraft center of gravity. This creates a torque

$$T = F * L$$

on the aircraft and the aircraft rotates about its center of gravity. With greater rudder deflection to the left from the back of the aircraft, the force increases to the right. If the pilot reverses the rudder deflection to the right, the aircraft will yaw in the opposite direction. [2]

### 2.3 Effect of side force, rolling moment and yawing moment ( $C_N$ , $C_r$ and $C_y$ ) function of slip angle ( $\beta$ ) and roll angle ( $\phi$ ) on Control Law

An airplane produces both yawing and rolling moments due to the sideslip angle. This interaction between the roll and the yaw produces the coupled motion. In the sideslip condition, the airplane's longitudinal axis remains parallel to the original flight path, but the airplane no longer flies straight along its original track. Now, the horizontal component of lift forces the airplane to move sideways toward the low wing.

For the directional stability many of the basic ideas involving longitudinal stability also apply to directional stability. In the usual equilibrium condition, an airplane flies so that the yaw angle is zero. To have static directional stability, a positive yawing moment should be generated if the airplane is disturbed to a negative yaw angle or alternatively by convention, a positive sideslip angle  $\beta$  and a negative yawing moment generated for a negative sideslip angle excursion. If the airplane holds its disturbed position, it has neutral directional stability. If the tendency is to increase the disturbed position, further away from equilibrium, the airplane is directionally unstable.

#### 2.3.1 Yaw Moment

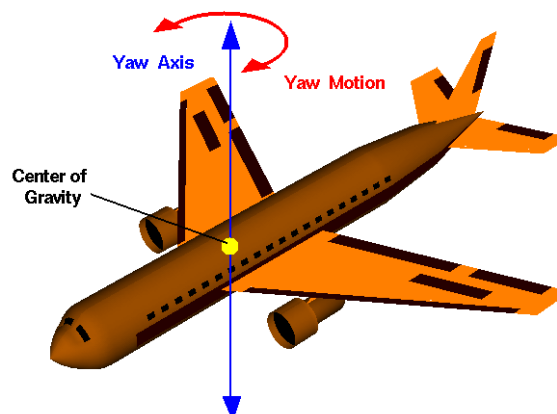


Figure 2.2: Yaw axis and yaw motion

(Source: Tom Benson (2008))

For the yaw moment is the moment about the  $z_{\text{body}}$  axis and is positive if it moves the nose of the plane to the right. The big contributor to the yaw moment is the vertical tail. We can write the yaw moment equation in a similar manner to the way we wrote the pitch-moment equation by considering the contributions from the wing-body combination and from the vertical tail. If we take moments about the center of gravity we have: [3]

$$N = N_{wb} - L_{vt} l_{vt} \cos \alpha_{vt} - D_{vt} l_{vt} \sin \alpha_{vt} \quad (1)$$

where  $l_{vt}$  is the vertical tail length, the distance from the cg to the aerodynamic center of the vertical tail  $L_{vt}$ ,  $D_w$  are the lift and drag of the vertical tail, and  $\alpha_{vt}$  is the angle of attack of the vertical tail measured positive so as to create a positive side force. If we make the usual assumptions such as that  $\alpha_{vt}$  is a small angle, and that  $D_{vt} \ll L_{vt}$ , we can reduce Eq. (1) to the form: [3]

$$N = N_{wb} - L_{vt} l_{vt} \quad (2)$$

$$N = N_{wb} - C_{L_{vt}} \frac{1}{2} \rho V^2 S_{vt} l_{vt} \quad (3)$$

where  $S_{vt}$  is the vertical tail area. Dividing by  $\frac{1}{2} \rho V^2 S b$  we obtain the yaw-moment equation in coefficient form: [3]

$$\begin{aligned} C_N &= C_{N_{wb}} - C_{L_{vt}} \eta_v \left[ \frac{S_{vt} l_{vt}}{S b} \right] \\ &= C_{N_{wb}} - C_{L_{vt}} \eta_{vt} V_{vt}^{-} \end{aligned} \quad (4)$$

If we let the vertical tail lift coefficient depend on a vertical tail lift curve slope and a rudder deflection we can write it as: [3]

$$C_{L_{vt}} = a_{vt} \alpha_{vt} + a_r \delta_r \quad (5)$$

where  $a_{vt}$  and  $a_r$  are the vertical tail lift curve slope and rudder “lift curve slope” respectively.