A Sensorless Speed Identifier using Adaptive Digital Filtering Technique

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This report is submitted in partial fulfillment of the requirements for the award of Bachelor of Electronic Engineering (Telecommunication Electronics) With Honours

> Faculty of Electronic and Computer Engineering Universiti Teknikal Malaysia Melaka

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This work is dedicated to my family, lecturer and also to all my friends.

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# ABSTRACT

It has been known that the speed of an induction motor can be determined from its rotor slot harmonic (RSH)component, leading to the development of several analog RSH-based sensorless speed estimation techniques. A common problem encountered by these methods is that the speed estimation extraction process is complicated by the small level of the RSH signal relative to the fundamental signal and the presence of other harmonics. Furthermore, the frequency of the RSH changes as the load changes. To overcome the shortcomings of the analog techniques ,a few all digital techniques are proposed. In this project, one of these digital techniques which is based on an adaptive filters proposed to be modeled in MATLAB. The student is first exposed to a fixed frequency digital filter design before moving on to design the adaptive digital filter based on a given algorithm. The designed filter is then to be tested using a simulated induction motor signal. It is expected that the filter can extract the fundamental and the RSH components from the signal and subsequently the components are used to identify the speed of the motor.

# ABSTRAK

Telah diketahui bahawa kelajuan motor induksi dapat ditentukan dari bahagian slot rotor harmonik (RSH),ini telah mengarah ke pembangunan beberapa anggaran kelajuan RSH berasaskan teknik analog kurang sensor.Satu masalah umum ditemui dalam kaedah ini adalah proses ekstraksi dalam anggaran kelajuan adalah rumit oleh peringkat kecil dari isyarat RSH relatif terhadap isyarat fundamental dan kehadiran lain harmonik.Di samping itu, perubahan frekuensi RSH berkadaran dengan perubahan beban.Untuk mengatasi kekurangan teknik analog, beberapa teknik digital telah dicadangkan. Dalam projek ini,. Salah satu daripada teknik digital yang didasarkan pada penapis adaptif dicadangkan menjadi model dalam MATLAB. Pelajar mula dengan mereka frekuensi tetap penapis digital sebelum mereka penapis digital adaptif berdasarkan algoritma yang diberikan. Penapis yang direka kemudian akan diuji dengan menggunakan isyarat simulasi motor induksi. Diharapkan bahawa penapis dapat mengekstrak fundamental dan bahagian-bahagian RSH dari isyarat dan kemudian bahagian-bahagian itu digunakan untuk mengenalpasti kelajuan motor.

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# LIST OF ABBREVIATION

AC	-	Alternating Current
AOM	-	Air over Motor
DC	-	Direct Current
DOL	-	Direct On Line
DSP	-	Digital Signal Processor
FFT	-	Fast Fourier Transform
FIR	-	Finite Impulse Response
FLC	-	Full Load Current
FLT	-	Full Load Torque
FRLS	-	Fast Recursive Least Square
IIR	-	Infinite Impulse Response
LMS	-	Least-Mean-Square
LRT	-	Locked Rotor Torque
LRC	-	Locked Rotor Current
MATLAB	-	Matrix Laboratory
MRAS	-	Model Reference Adaptive system
PF	-	Power Factor Correction
RML	-	Recursive Maximum likelihood
RPM	-	Round per Minute
RSH	-	Rotor Slot Harmonic
TEFC	-	Totally Enclosed Forced air Cooled

# LIST OF APPENDIX

# NO TITLE

### PAGE

# A Functional MATLAB Coding

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### **CHAPTER I**

### **INTRODUCTION**

### 1.1 Project Introduction

It has been known that the speed of an induction motor can be determined from its rotor slot harmonic (RSH) component, leading to the development of several analog RSH-based sensorless speed estimation techniques. A common problem encountered by these methods is that the speed information extraction process is complicated by the small level of the RSH signal relative to the fundamental signal and the presence of other harmonics. Furthermore, the frequency of the RSH changes as the load changes. To overcome the shortcomings of the analog techniques, a few all digital techniques are proposed.

In this project, one of these digital techniques which is based on an adaptive filters is proposed to be modeled in MATLAB. The student is exposed to a fixed frequency digital filter design before moving on to design the adaptive digital filter based on a given algorithm. The designed filter is then to be tested using a simulated induction motor signal. It is expected that the filter can extract the fundamental and the RSH components from the signal and subsequently are used to identify the speed of the motor.

#### **1.2 Problem Statements**

The performances of a speed transducer for control of an electric drive are the accuracy of the speed estimate, the speed range of operation and the dynamic response. Speed information of an induction motor is encoded in its stator currents and voltages by the rotor slots. For many years, several attempts RSH-based sensorless speed estimation, using analog and digital techniques have been reported. However, extracting the speed information is complicated by the small level of the RSH signal relative to the fundamental and the presence of other harmonics arising both from the inverter and the machine itself. Also, the machine slows down under load, the RSH amplitude increase and its frequency reduces, so it moves toward the similarly sized harmonics of the fundamental, making identification even more difficult. As many analog filter have been design, still the performance of each techniques was limited in terms of accuracy, linearity, resolution, speed range, or speed of response. To solve the problem, digital techniques based on the use of a continuously updated frequency spectrum computed via the fast Fourier transform was proposed. This provided good accuracy and linearity over a very wide range of speed and load condition. However, the FFT-based approach have limitation due to compromise between the required frequency resolution to allow speed detection and the required frequency resolution to allow speed detection and the response time to changes of speed which deteriorates with the resulting long data record.

#### 1.3 Objective

In this project, a technique based on adaptive filter is proposed to be implemented in MATLAB in order to solve the problem faced by FFT-based approach.

- a) To design the filter using a given algorithm.
- b) To implement the filter in MATLAB in order to extract the fundamental and RSH components, consequently to determine the speed of the motor.
- c) To produce a functional MATLAB program code.

Prject work divided into three stage

- 1) Introduction to digital filter and adaptive filter technique.
  - Gain knowledge to understand the concept of digital filter as well as adaptive filter.
  - Implementation of recursive maximum likelihood algorithm in designing the adaptive filter.
- 2) Implementation of the desiged filter in MATLAB.
  - FIR and IIR fixed frequency filter design.
  - Adaptive digital filter design.
- 3) Calculation of motor speed.

### **1.5 Thesis Structure**

# **Chapter 1 Introduction**

General description on the project idea, clarification on the scope of the project, reviews of problem statement which introduces this project and thus the objectives of doing this project.

#### **Chapter 2 Literature Review**

This chapter includes the study on the induction motor theory, FIR filter, IIR filter, adaptive filter and reviews the related previous methods. Review on induction motor theory is to make us understand how the induction motor work and what parameter will influence the speed of the induction motor while review on the filter make us have an idea about the type of filter to be used according to the characteristic and whether it is suitable or not for this project.

# **Chapter 3 Methodology**

This chapter shows the project planning. The project is divided into seven steps and each step is being described. The methodology is a kind of time table to make us clear on the project progress and what should be done on the time.

#### **Chapter 4 Result and Discussion**

This chapter shows presentation of the obtained result and discussion is made based on the result. The result is analyzed and then identify whether the project is a success or not.

# **Chapter 5 Conclusion and Suggestion**

This chapter describes an overall view on the project as well as suggestion or recommendation to upgrade the project or make improvement on it.

# **CHAPTER II**

#### LITERATURE REVIEW

### **2.1 Induction Motor Control Theory** [1]

An induction motor (or asynchronous motor or squirrel-cage motor) is a type of alternating current motor where power is supplied to the rotor by means of electromagnetic induction.

An electric motor converts electrical power to mechanical power in its rotor (rotating part). There are several ways to supply power to the rotor. In a DC motor this power is supplied to the armature directly from a DC source, while in an induction motor this power is induced in the rotating device. An induction motor is sometimes called a rotating transformer because the stator (stationary part) is essentially the primary side of the transformer and the rotor is the secondary side. Unlike the normal transformer which changes the current by using time varying flux, induction motor uses rotating magnetic field to transform the voltage. The primary side's currents evokes a magnetic field which interacts with the secondary side's emf to produce a resultant torque, henceforth serving the purpose of producing mechanical Induction widely used, especially poly energy. motors are phase induction motors, which are frequently used in industrial drives.

Induction motors are now the preferred choice for industrial motors due to their rugged construction, absence of brushes and thanks to modern power electronics where the ability to control the speed of the motor have.

#### 2.1.1 Induction Motor Design [1]

#### **2.1.1.1 Stator Design** [1]

The stator is the outer body of the motor which houses the driven windings on an iron core. In a single speed three phase motor design, the standard stator has three windings, while a single phase motor typically has two windings. The stator core is made up of a stack of round pre-punched laminations pressed into a frame which may be made of aluminium or cast iron. The laminations are basically round with a round hole inside through which the rotor is positioned. The inner surface of the stator is made up of a number of deep slots or grooves right around the stator. It is into these slots that the windings are positioned. The arrangement of the windings or coils within the stator determines the number of poles that the motor has.

A standard bar magnet has two poles, generally known as North and South. Likewise, an electromagnet also has a North and a South pole. As the induction motor stator is essentially like one or more electromagnets depending on the stator windings, it also has poles in multiples of two etc. the winding configuration, slot configuration and lamination steel all has an effect on the performance of the motor. The voltage rating of the motor is determined by the number of turns on the stator and the power rating of the motor is determined by the losses which comprise copper loss and iron loss, and the ability of the motor to dissipate the heat generated by these losses. The stator design determines the rated speed of the motor and most of the full load, full speed characteristics.

#### **2.1.1.2 Rotor Design** [1]

The rotor comprises a cylinder made up of round laminations pressed onto the motor shaft, and a number of short-circuited windings. The rotor windings are made up of rotor bars passed through the rotor, from one end to the other, around the surface of the rotor. The bars protrude beyond the rotor and are connected together by a shorting ring at each end. The bars are usually made of aluminium or copper, but sometimes made of brass. The position relative to the surface of the rotor, shape, cross sectional area and material of the bars determine the rotor characteristics. Essentially, the rotor windings exhibit inductance and resistance, and these characteristics can effectively be dependent on the frequency of the current flowing in the rotor. A bar with a large cross sectional area will exhibit a low resistance, while a bar of a small cross sectional area will exhibit a high resistance. Likewise a copper bar will have a low resistance compared to a brass bar of equal proportions. Positioning the bar deeper into the rotor, increases the amount of iron around the bar, and consequently increases the inductance exhibited by the rotor. The impedance of the bar is made up of both resistance and inductance, and so two bars of equal dimensions will exhibit different AC impedance depending on their position relative to the surface of the rotor. A thin bar which is inserted radialy into the rotor, with one edge near the surface of the rotor and the other edge towards the shaft, will effectively change in resistance as the frequency of the current changes. This is because the AC impedance of the outer portion of the bar is lower than the inner impedance at high frequencies lifting the effective impedance of the bar relative to the impedance of the bar at low frequencies where the impedance of both edges of the bar will be lower and almost equal. The rotor design determines the starting characteristics.

#### **2.1.2 Equivalent Circuit** [1]

The induction motor can be treated essentially as a transformer for analysis. The induction motor has stator leakage reactance, stator copper loss elements as series components, and iron loss and magnetising inductance as shunt elements. The rotor circuit likewise has rotor leakage reactance, rotor copper (aluminium) loss and shaft power as series elements. The transformer in the centre of the equivalent circuit can be eliminated by adjusting the values of the rotor components in accordance with the effective turns ratio of the transformer. From the equivalent circuit and a basic knowledge of the operation of the induction motor, it can be seen that the magnetising current component and the iron loss of the motor are voltage dependant, and not load dependant. Additionally, the full voltage starting current of a particular motor is voltage and speed dependant, but not load dependant.

#### 2.1.3 Starting Characteristics [1]

In order to perform useful work, the induction motor must be started from rest and both the motor and load accelerated up to full speed. Typically, this is done by relying on the high slip characteristics of the motor and enabling it to provide the acceleration torque. Induction motors at rest, appear just like a short circuited transformer, and if connected to the full supply voltage, draw a very high current known as the "Locked Rotor Current". They also produce torque which is known as the "Locked Rotor Torque". The Locked Rotor Torque (LRT) and the Locked Rotor Current (LRC) are a function of the terminal voltage to the motor, and the motor design. As the motor accelerates, both the torque and the current will tend to alter with rotor speed if the voltage is maintained constant. The starting current of a motor, with a fixed voltage, will drop very slowly as the motor accelerates and will only begin to fall significantly when the motor has reached at least 80% full speed. The actual curves for induction motors can vary considerably between designs, but the general trend is for a high current until the motor has almost reached full speed. The LRC of a motor can range from 500% Full Load Current (FLC) to as high as 1400% FLC. Typically, good motors fall in the range of 550% to 750%FLC. The starting torque of an induction motor starting with a fixed voltage, will drop a little to the minimum torque known as the pull up torque as the motor accelerates, and then rise to a maximum torque known as the breakdown or pull out torque at almost full speed and then drop to zero at synchronous speed. The curve of start torque against rotor speed is dependant on the terminal voltage and the motor/rotor design. The LRT of an induction motor can vary from as low as 60% Full Load Torque (FLT) to as high as 350% FLT. The pull-up torque can be as

low as 40% FLT and the breakdown torque can be as high as 350% FLT. Typical LRTs for medium to large motors are in the order of 120% FLT to 280% FLT. The power factor of the motor at start is typically 0.1 - 0.25, rising to a maximum as the motor accelerates, and then falling again as the motor approaches full speed. A motor which exhibits a high starting current, i.e. 850% will generally produce a low starting torque, whereas a motor which exhibits a low starting current will usually produce a high starting torque. This is the reverse of what is generally expected. The induction motor operates due to the torque developed by the interaction of the stator field and the rotor field. Both of these fields are due to currents which have resistive or in phase components and reactive or out of phase components. The torque developed is dependant on the interaction of the in phase components and consequently is related to the I<sup>2</sup>R of the rotor. A low rotor resistance will result in the current being controlled by the inductive component of the circuit, yielding a high out of phase current and a low torque. Figures for the locked rotor current and locked rotor torque are almost always quoted in motor data, and certainly are readily available for induction motors. Some manufactures have been known to include this information on the motor name plate. One additional parameter which would be of tremendous use in data sheets for those who are engineering motor starting applications, is the starting efficiency of the motor. By the starting efficiency of the motor, it is refer to the ability of the motor to convert amps into newton meters. This is a concept not generally recognised within the trade, but one which is extremely useful when comparing induction motors. The easiest means of developing a meaningful figure of merit, is to take the locked rotor torque of the motor (as a percentage of the full load torque) and divide it by the locked rotor current of the motor (as a percentage of the full load current). i.e.

Stating efficiency = 
$$\frac{\text{Locked Rotor Torque}}{\text{Locked Rotor Current}}$$
 (2.1)

If the terminal voltage to the motor is reduced while it is starting, the current drawn by the motor will be reduced proportionally. The torque developed by the motor is proportional to the current squared, and so a reduction in starting voltage will result in a reduction in starting current and a greater reduction in starting torque.