

**REAL INVERTED PENDULUM CONTROL
USING LQR CONTROLLER**

MARIANI BINTI MOHAMAD

UNIVERSITI TEKNIKAL MALAYSIA MELAKA

REAL INVERTED PENDULUM CONTROL USING
LQR CONTROLLER

MARIANI BINTI MOHAMAD

This report is submitted in partial fulfillment of the requirements for the award of
Bachelor of Electronic Engineering (Industrial Electronics) With Honours

Faculty of Electronic and Computer Engineering
Universiti Teknikal Malaysia Melaka

April 2009



UNIVERSITI TEKNIKAL MALAYSIA MELAKA
FAKULTI KEJURUTERAAN ELEKTRONIK DAN KEJURUTERAAN KOMPUTER

BORANG PENGESAHAN STATUS LAPORAN
PROJEK SARJANA MUDA II

Tajuk Projek : Real Inverted Pendulum Control using LQR Controller

Sesi Pengajian : 2008/ 2009

Saya **MARIANI BINTI MOHAMAD** mengaku membenarkan Laporan Projek Sarjana Muda ini disimpan di Perpustakaan dengan syarat-syarat kegunaan seperti berikut:

1. Laporan adalah hakmilik Universiti Teknikal Malaysia Melaka.
2. Perpustakaan dibenarkan membuat salinan untuk tujuan pengajian sahaja.
3. Perpustakaan dibenarkan membuat salinan laporan ini sebagai bahan pertukaran antara institusi pengajian tinggi.
4. Sila tandakan (\checkmark) :

SULIT*

(Mengandungi maklumat yang berdarjah keselamatan atau kepentingan Malaysia seperti yang termaktub di dalam AKTA RAHSIA RASMI 1972)

TERHAD*

(Mengandungi maklumat terhad yang telah ditentukan oleh organisasi/badan di mana penyelidikan dijalankan)

TIDAK TERHAD

Disahkan oleh:

(TANDATANGAN PENULIS)

(COP DAN TANDATANGAN PENYELIA)

Alamat Tetap: F-801, Kampung Bukit Aceh,
08400 Merbok, Kedah.

Tarikh: 30 April 2009

Tarikh: 30 April 2009

“I hereby declare that this report is the result of my own work except for quotes as cited
in the references.”

Signature :
Author : MARIANI BINTI MOHAMAD
Date : 30 APRIL 2009

“I hereby declare that I have read this report and in my opinion this report is sufficient in terms of the same scope and quality for the award of Bachelor of Electronic Engineering (Industrial Electronics) With Honours.”

Signature :
Supervisor’s Name : MOHD SHAKIR BIN MD. SAAT
Date : 30 APRIL 2009

To my beloved parents and family, who had strongly encouraged and supported me in the journey of learning. Their love of learning has been an inspiration.

ACKNOWLEDGEMENT

Alhamdulillah, a lot of praise to Allah the Almighty for His blessing for allowing me and has helped me in so many ways to perform during the final project. With His blessing, I have successfully finished my final year project along with this thesis.

First of all, I would like to thank my parent and also the whole family for their loves, prays and supports during my study at UTeM. I would like to pin point my gratitude and appreciation to my project supervisor, Mr. Mohd Shakir Md. Saat, who has helped me a lot through all the rough time and has shared his knowledge and experiences with me during this project.

Secondly, I would like to thank all my friends in the team, Safwan, Zulfadzli, Awang and Zaini for their support and skill exchange. Without their direct involvement, it would be difficult for me to complete all the task and project assigned successfully. I had a fun and pleasant moments working with these four fellows.

Besides that, my thanks also go to each and every person who has been very friendly and helpful to me towards the completion of this project. I was surrounded with great peoples which made my study at UTeM the most memorable moment that I will cherish forever.

ABSTRACT

This project is about controlling of an inverted pendulum system using linear quadratic regulator (LQR). Inverted pendulum system is a highly nonlinear and unstable system. Basically, an inverted pendulum system consists of a cart which can move horizontally along its track, a dc motor to drive the cart and also an inverted pendulum, mounted on the cart with a constant rotating pivot. This system demonstrates some basic concepts in control being nonlinear, non-minimum phase and multivariable behaviors. Due to these behaviors, the system requires a continuous correction mechanism for the pendulum to stay at upright position. The objective of this project is to design LQR controller to control the unstable system for both cart's position and pendulum's angle. LQR is developed as an optimal and effective controller that will be used to place the cart at desired position, at the same time to keep the pendulum straight upwards. This project also includes the interfacing procedures using real-time windows target application in Simulink. Both the system model and LQR controller are implemented in real time, connected to the real mechanism. A single data acquisition (DAQ) card is used as a medium to convert the digital signal from Simulink into analog signal.

ABSTRAK

Projek ini mengisarkan tentang kawalan sebuah sistem bandul songsang dengan menggunakan *Linear Quadratic Regulator*. Sistem bandul songsang ini merupakan sebuah sistem yang tidak linear dan tidak stabil. Pada dasarnya, sebuah sistem bandul songsang terdiri daripada sebuah kereta sorong yang boleh bergerak secara mendatar di atas landasannya, sebuah motor dc yang digunakan untuk menggerakkan kereta sorong tersebut dan juga sebuah bandul terbalik yang diletakkan pada kereta sorong dengan satu paksi putaran yang tetap. Sistem ini menunjukkan konsep asas tentang sifat-sifat tidak linear, fasa tidak minimum dan kepelbagaian perubahan. Disebabkan oleh sifat-sifat ini, sistem bandul songsang ini memerlukan satu mekanisme pembetulan yang berterusan untuk bandul tersebut terus berada pada posisi ke atas. Objektif projek ini adalah untuk membina LQR untuk mengawal sistem yang tidak stabil untuk kedua-dua sudut bandul dan posisi kereta sorong. LQR telah dikenali sebagai sebuah pengawal yang optimum dan berkesan, akan digunakan untuk meletakkan kereta sorong pada kedudukan yang dikehendaki, dan pada masa sama untuk mengekalkan bandul ke atas. Projek ini turut melibatkan proses menghubungkan di antara *software* (Simulink) dan *hardware* (bandul songsang) dengan menggunakan aplikasi *real-time windows target* pada *Simulink*. Satu kad data perolehan (DAQ) akan digunakan sebagai medium perantaraan untuk menukarkan isyarat digital kepada isyarat analog.

CONTENTS

CHAPTER	TITLE	PAGE
	PROJECT TITLE	i
	STATUS VERIFICATION FORM	ii
	DECLARATION	iii
	SUPERVISOR APPROVAL	iv
	DEDICATION	v
	ACKNOWLEDGEMENT	vi
	ABSTRACT	vii
	ABSTRAK	viii
	CONTENTS	ix
	LIST OF TABLES	xii
	LIST OF FIGURES	xiii
	LIST OF ABBREVIATIONS	xv
I.	INTRODUCTION	
	1.1 Overview	1
	1.2 Objectives	3
	1.3 Problem Statement	3
	1.4 Scope of Work	4
	1.5 Project Significant	4
	1.6 Research Methodology	5

II. BACKGROUND RESEARCH

2.1	Control System	7
2.1.1	Open-Loop and Closed-Loop Control Systems	8
2.1.2	Inverted pendulum System	9
2.2	Literature Review of Inverted Pendulum System	10

III. MATHEMATICAL MODEL OF THE SYSTEM

3.1	Introduction	12
3.2	The Mathematical Model of the System	13
3.2.1	Nonlinear State Equations	15
3.2.2	Linearization	16
3.2.3	Disturbance Input	17
3.3	Controllability	19
3.4	Observability	20

IV. LQR CONTROLLER DESIGN

4.1	Linear Quadratic Regulator (LQR)	21
4.2	LQR Controller Design	24
4.2.1	Controller Design Without Integral Control	24
4.2.2	Controller Design With Integral Control	26
4.2.3	Controller Design With Disturbance	28

V.	INTERFACING THE HARDWARE WITH REAL-TIME WINDOWS TARGET APPLICATION	
5.1	Real-Time Windows Target (RTWT)	29
5.2	Data Acquisition Card – Advantech PCI 1711	33
5.3	Interfacing Procedures	34
VI.	RESULT	
6.1	Result	38
6.1.1	Unstable System	38
6.1.2	Controller Design without Integral Control	39
6.1.3	Controller Design with Integral Control	40
6.1.4	Controller Design with Disturbance	41
6.1.5	Interfacing	46
VII.	DISCUSSION	
7.1	Closed-Loop System	52
7.2	Real-Time Interfacing	54
VIII.	CONCLUSION AND RECOMMENDATION	
8.1	Conclusion	55
8.2	Recommendation	56
	REFERENCES	57
	APPENDICES	59

LIST OF TABLES

NO	TITLE	PAGE
3.1	Physical Data of the Inverted Pendulum System	13
4.1	Algorithm for LQR Design	23
5.1	I/O Connector Signal Description Used in Interfacing	34
6.1	Reference Voltages for Cart's Position	47
6.2	Reference Voltages for Pendulum's Angle	48
7.1	Summary of Simulation Result	53
7.2	Summary of Interfacing Result	54

LIST OF FIGURES

NO	TITLE	PAGE
1.1	Inverted Pendulum System	2
1.2	Block Diagram of the Overall System	2
1.3	Flow Chart for the Whole Project	6
2.1	Block Diagrams of Open-Loop System	8
2.2	Block Diagrams of Closed-Loop System	9
3.1	Inverted Pendulum	13
3.2	Free Body Diagram of the Inverted Pendulum System	14
4.1	Plant for Non-integral Control System	25
4.2	Plant of the LQR Controller Design	26
4.3	Plant for Integral Control System	27
5.1	Development Process of Software Component	32
5.2	Connections between Hardware and Wiring Board	34
5.3	Plant to check the sensor output voltage	35
5.4	Final Plant of Inverted Pendulum System for Interfacing	36
6.1	Unstable Step Responses for Open Loop Plant	38
6.2	Step Responses without Integral Control	39
6.3	Step Responses with Integral Control	40
6.4	Step Responses with Disturbance Added to the System	41
6.5	Step Responses for 1.0N Disturbance	42
6.6	Step Responses for 1.3N Disturbance	43
6.7	Step Responses for 1.5N Disturbance	44

6.8	Step Responses for 1.6N Disturbance	45
6.9	Graph plotted for cart's position voltage	46
6.10	Graph plotted for pendulum's angle voltage	47
6.11	Graph Plotted for Desired Point of Cart 2.67V	49
6.12	Graph Plotted for Desired Point of Cart 3.48V	50
6.13	Graph Plotted for Desired Point of Cart 1.84V	51

LIST OF ABBREVIATIONS

ANG	-	Analog
ARE	-	Algebraic Riccati Equation
A/D	-	Analog/Digital
COM	-	Common
CW/CCW	-	Clockwise/Counter Clockwise
DAQ	-	Data Acquisition Card
DC	-	Direct Current
DIN	-	Deutsches Institut fur Normung
D/A	-	Digital/Analog
I/O	-	Input/Output
LQR	-	Linear Quadratic Regulator
PC	-	Personal Computer
PCI	-	Peripheral Component Interact
PID	-	Proportional-Integral-Derivative
RTWT	-	Real-Time Windows Target
SCSI	-	Small Computer System Interface

CHAPTER I

INTRODUCTION

1.1 Overview

A physical pendulum has two equilibrium positions if the pivot point is held at a constant position. The vertically down position is a stable equilibrium while the vertically up or inverted equilibrium is clearly unstable. If the pendulum is released from any position other than precisely straight up, then the pendulum will fall and oscillate about the vertically down position with decreasing amplitude until it comes to rest.

This project deals with the nonlinear system of an inverted pendulum system. An inverted pendulum system is a system that demonstrates some basic concepts in control being nonlinear, multivariable and non-minimum phase. Basically, the inverted pendulum system consists of three main components. Firstly is a cart, which can move horizontally along its track, a motor that is used to move the cart and also an inverted pendulum mounted on the cart with a constant rotating pivot. The control objectives are to maintain the pendulum straight upwards and at the same time to place the cart at desired position.

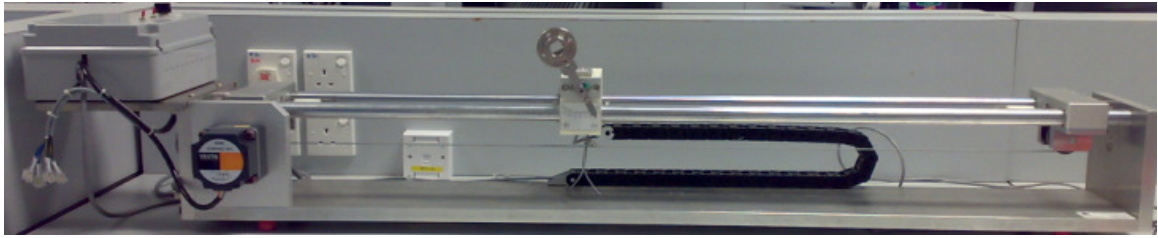


Figure 1.1 Inverted Pendulum System

In this project, a linear-quadratic regulator (LQR) is introduced as an optimal and effective controller that will be used to control the unstable inverted pendulum system. The main objective of optimal control is to determine control signals that will cause a process to satisfy some physical constraints and at the same time maximize or minimize a chosen performance criterion. In this project, the formulation of optimal control needs a mathematical description of the process to be controlled and a specification of the performance index.

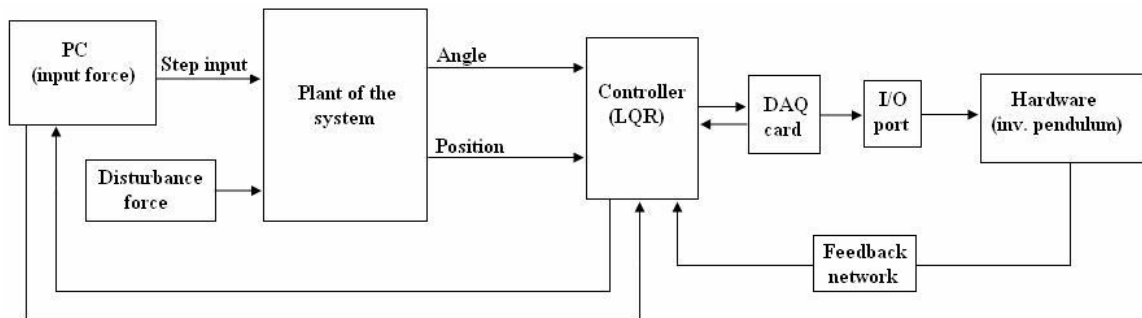


Figure 1.2 Block Diagram of the Overall System

Figure 1.2 shows the block diagrams which represent the inverted pendulum system. The block diagrams can be divided into two components, the software and hardware components. Software component consists of a plant of the system. In this case the system has two inputs – the force applied to the cart and the possible disturbance force from the surrounding environment, and two outputs – the cart's position and the pendulum's angle. The outputs are fed to a controller. During simulation, the controller will compare the actual value with the desired value, do correction if necessary and feed back the correct value to the system.

In the right hand side, the hardware component consists of the mechanical system of inverted pendulum, together with DAQ card and I/O port for interfacing purpose. The hardware responses through the signal received from the software. At the same time, it connects to the controller for corrective task.

1.2 Objectives

The objectives of this research are as follows:

- i. To synthesis the mathematical model of the inverted pendulum system.
- ii. To design linear-quadratic regulator (LQR) controller using Matlab/Simulink.
- iii. To interface between the software (Simulink) and hardware (Inverted Pendulum).

1.3 Problem Statements

- i. An inverted pendulum system is a challenging system from the engineering point of view. This is because of the nonlinearities, non minimum phase and multivariable behaviors.
- ii. Due to these behaviors, the system requires a continuous correction mechanism for the pendulum to stay at upright position.
- iii. The problems that include in this project are to control the unstable system of inverted pendulum for both the cart's position and the pendulum's angle.

1.4 Scope of Works

The scopes of work for this project are:

- i. Design the LQR controller using Simulink to control the system.
- ii. Interfacing between Simulink (LQR) and hardware (inverted pendulum) using real-time windows target application to control the position's cart and pendulum's angle.

1.5 Project Significant

Inverted pendulum is a classic example of an unstable system, which is often used to illustrate various types of control strategies. Since it demonstrates some basic concepts in control system, this system is significantly suitable in providing a platform for teaching and hands-on practice in Control Engineering course. Control concepts such as the stability and the controllability of a control system, can all be shown visually through the inverted pendulum system.

Besides, the success of this project can be applied on laboratory usage in UTeM and subsequently enhanced the understanding of MATLAB/Simulink application among students. This can be helpful to both parties, lecturers and students.

With the success of this project, thus the same method can be applied to control the other complex system that having the same problem with inverted pendulum system. Moreover, this project can be as the starting stage in designing other controllers such as proportional-integral-derivative (PID), fuzzy logic, etc.

1.6 Research Methodology

Generally, first task involved to accomplish this project is to do a literature review regarding the inverted pendulum system. Next task is to analyze the mathematical model of the system. From the analysis, a nonlinear final state space equation is obtained. However, the LQR controller design is based on a linear state equation. Therefore, the nonlinear equation is linearized about the origin.

Based on the linear state equation, the controller is designed using Simulink. The design is simulated through Matlab. In the design, gain K plays important role because it is an exact value that will bring the system from unstable system to stable system. Consequently, if the simulation result does not meet the design requirements, a new value of gain K is reset and the simulation process is done again until a satisfaction result is obtained.

Figure 1.3 below shows the flow chart which represents the methodology for the whole project. In the flow chart also shows the option that the system will go through for each action taken.

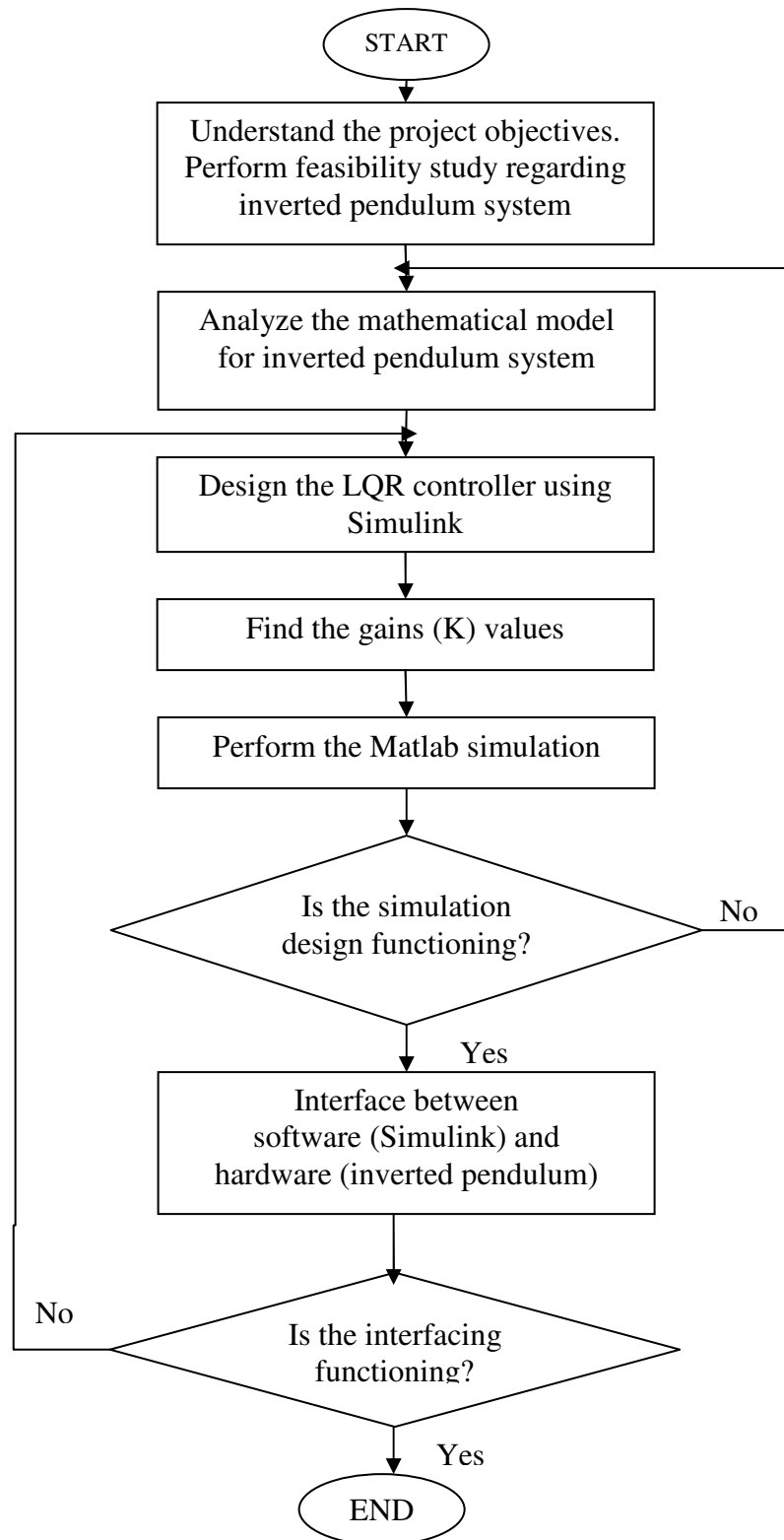


Figure 1.3 Flow Chart for the Whole Project

CHAPTER II

BACKGROUND RESEARCH

2.1 Introduction

The literature review undertaken as a part of the inverted pendulum project was focused on understanding the background and application of inverted pendulum systems, mathematical modeling, control, and other successful projects of a similar nature.

2.2 Background

The inverted pendulum is a classic example of a non-linear control topic and one studied frequently with reference to design, implementation and development of control for nonlinear systems. It appears in undergraduate control text books such as K.Ogata (1978) where it is used as an example of how to mathematically describe physical systems.

2.3 Present Applications

Inverted pendulum is currently used as teaching aids and research experiments. Quanser (2004), a supplier of educational and research based equipment produce modular systems which can be configured as single or double inverted pendulum. Their range offers both a rotary and a linear version. Many researchers have also built their own inverted pendulum systems (Astrom and Furuta, 1996) to suit their investigations.

2.4 System Modeling Methodology

Chinichian (1990) design and analyze a controller for balancing one pendulum with two degrees of freedom, “spatial inverted pendulum”. The pendulum, with two degrees of freedom, has a three dimensional motion, and it will be more analogous to the design of a controller for attitude control during launching a rocket. A full state-variable feedback controller design for a state-space linear model of a three dimensional inverted cart/pendulum system is presented. This design was based on pole-placement technique. Alternative solutions to the simple pole-placement technique were also proposed to exploit non-uniqueness of the feed-back gains for a certain closed-loop pole locations and the closed-loop system response was simulated on a digital computer.

The rapid increase of the aged population in countries like Japan has prompted researchers to develop robotic wheelchairs to assist the infirm to move around (Takahashi, 2000). The control system for an inverted pendulum is applied when the wheelchair maneuvers a small step or road curbs.

Rich Chi Oii (2003) discusses the processes developed and considerations involved in balancing a two-wheeled autonomous robot based on the inverted pendulum model. The experimental examines the suitability and evaluates the performance of a Linear Quadratic Regulator (LQR) and a Pole-placement controller in balancing the system. The LQR controller uses several weighting matrix to obtain the appropriate