CONTROL OF A BALL BEAM:

Comparison between Lead/Lag Compensator and State Feedback Controller (SFC)

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Dedication

To my dearest father, mother, and family for their encouragement and blessing
To all of friends for their best supports and helps.

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ABSTRACT

The significance problem of the ball and beam system is that it is a simple system which is open-loop unstable. Even if the beam is restricted to be very nearly horizontal, without active feedback, it will swing to one side or the other, and the ball will roll off the end of the beam. Two types of the controllers will be synthesized in order to control the system. One is the model based controller lead/lag compensator controller and second is state feedback controller. The first stage is to develop the mathematical model of a ball and beam system based on the lagrangian equation. Then, the root locus method will be applied to get the closed loop so that the design of the State-Feedback Controller can be accomplished. The second stage is to design the State Feedback Controller to be applied to the system.The final stage is to carry out the simulation work of both controllers for comparison purpose. The simulation work is done using a MATLAB/SIMULINK platform.

Abstrak

ABSTRAK

Masalah yang ketara sistem bola dan haluan adalah sebuah sistem yang tidak stabil yakni gelung suap balik terbuka. Sekalipun alur dihadkan untuk berada pada keadaan yang sangat mengufuk tanpa suap balik yang aktif, ia akan berayun pada satu sisi atau pada sisi yang lain. Bola tersebut akan berputar melepasi had alur yang disediakan. Dua jenis pengawal akan disintesis dalam tertib untuk mengawal sistem tersebut. Pertama adalah Pengawal Pampasan Dahului/Ekori dengan menggunakan kaedah asas lokus. Kedua adalah Pengawal Suap Balik Keadaan . Tahap pertama adalah untuk membangunkan model matematik bagi sistem bola dan alur berasaskan persamaan lagrangian . kemudian kaedah asas lokus akan diaplikasikan untuk mendapatkan sistem gelung suap balik tertutup supaya pembinaan Pengawal Pampasan Dahului/Ekori dapat disemournakan. Tahap kedu adalah untuk membina pengawal suap balik keadaan untuk diaplikasikan kepada sistem. Tahap terakhir adalah untuk menghasilkan kerja simulasi bagi kedua-dua pengawal bagi tujuan perbandingan. Kerja simulasi dihasilkan menggunakan perisian MATLAB/SIMULINK.

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CHAPTER 1

INTRODUCTION

1.1 Overview

The project deals with the nonlinear system of the ball and beam. The ball and beam system consists of a long beam which can be tilted by a servo or electric motor together with a ball rolling back and forth on top of the beam. The lead-lag compensator and State Feedback Controller (SFC) controller will be intended in order to control the system.

Figure 1 shown a ball is placed on a beam, where it is allowed to roll with 1 degree of freedom along the length of the beam. A lever arm is attached to the beam at one end and a servo gear at the other. As the servo gear turns by an angle theta (disturbance occur), the lever changes the angle of the beam by alpha. When the angle is changed from the vertical position, gravity causes the ball to roll along the beam. A controller will be designed for this system so that the ball's position can be manipulated.

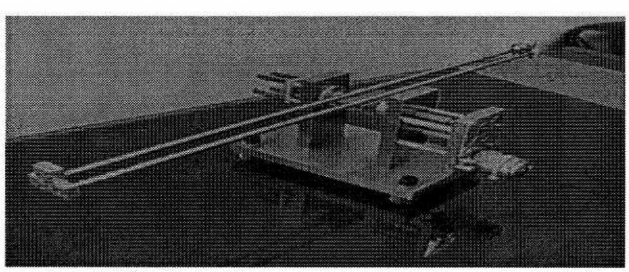

Figure 1.1: Example of Ball and Beam Prototype [1]

Both tasks will consist of literature review, mathematical modeling, and controller design and simulation progress. The programming and the simulation is using the Matlab based software.

1.2 Objective

The main objectives of the project are:

i. To design lead/lag compensator controller by using Root Locus method which measures the position of the ball and adjusts the beam accordingly.

ii. To design the state feedback controller (SFC) this measures the position of the ball and adjusts the beam accordingly.

iii. To compare the efficiency between lead/lag compensator and State Feed back Controller (SFC)

1.3 Problem Significant

i. The ball and beam system consists of a long beam which can be tilted by a servo or electric motor together with a ball rolling back and forth on top of the beam.

ii. Simple system which is open-loop unstable. Even if the beam is restricted to be very nearly horizontal, without active feedback, it will swing to one side or the other, and the ball will roll off the end of the beam.

iii. In two dimensions, the ball and beam system becomes the ball and plate system, where a ball rolls on top of a plate whose inclination can be adjusted by tilting it frontward, backwards, leftwards, or rightwards.

1.4 Scope of Work

i. A ball and beam system as described in Peter Welstead (1999).

ii. Develop mathematical model of the system

iii. Design Lead/Lag Compensator of the system

iv. Design State Feedback Controller (SFC) of the system

v. Compare the performance of these two systems (Lead/Lag Compensator and State Feedback Controller (SFC)) using simulation and animation.

1.5 Layout of Thesis

This report contains eight chapters. Chapters 2 contain literature review for details overview to this Ball and Beam system.

Chapter 3 shows the block diagram of the methodology taken in order to accomplish the task. Firstly the mathematical model of a Ball and Beam system must be derived. The mathematical model is based on Lagrangian equation.

Chapter 4 deals with the mathematical modeling of the system. The first part of this chapter contains formulation of the nonlinear model of a Ball and Beam system. The Lagrange equation is used in order to formulate the system model. In this part the linearization of a nonlinear model has also been considered. The second part of this chapter is about explanation of a transfer function. It should be noted that the plant transfer function is a double integrator. As such it is marginally stable and will provide a challenging control problem.

Chapter 5 presents the controller design using the Lead/Lag Compensator by using root locus method to find the gain. In this chapter the important part of the Lead/Lag compensator is discussed such as the definition of Lead/lag Compensator, theory of Lead/lag Compensator and how to implement the compensator.

Chapter 6 presents the controller design by using the State Feedback Controller (SFC). In this chapter the important part of the State Feedback Controller is discussed such as the definition, theory of State Feedback Controller (SFC) and the implemented of integral to eliminate the steady state error.

Chapter 7 includes result which is performed by analysis of the graph. The comparison between both of the controllers and works undertaken also included in this chapter. It contain graph from different step input in order to ease the comparison.

Chapter 8 includes discussion, conclusion and suggestion for future work. All the process or procedure to designed both controllers were concluded whether it achieve the objective or not. Recommendations for future work of this project are presented at the end of this chapter

CHAPTER 2

LITERATURE REVIEW

2.1 Literature Review

The ball on beam balancer system is one of the most enduringly popular and important laboratory models for teaching control systems engineering. The system control job is automatically regulating the position of the ball on the beam by changing the angle of the beam. The open loop of the ball and beam system is unstable. [1]

Many researches were carried out researches to control the ball and beam system. Various control strategies have been proposed by numerous researchers for controlling the ball beam such that the system is stable as well as the ball is move to the desired position. The approaches varied from the classical control to the advanced control. Lead/Lag compensator was design to control the ball and beam problem [4]. The drawback of the Lead/Lag compensator is it only can control for a Single-Input-Single-Output (SISO) system. It means that the Lead/Lag compensator only can control either for the position of the ball or angle of the beam at a one time. [4]

A ball is placed on a beam, see figure 2.1, where it is allowed to roll with 1 degree of freedom along the length of the beam. A lever arm is attached to the beam at one end and a servo gear at the other. As the servo gear turns by an angle theta, the lever changes the angle of the beam by alpha. When the angle is changed from the vertical position, gravity causes the ball to roll along the beam. A controller will be designed for this system so that the ball's position can be manipulated.

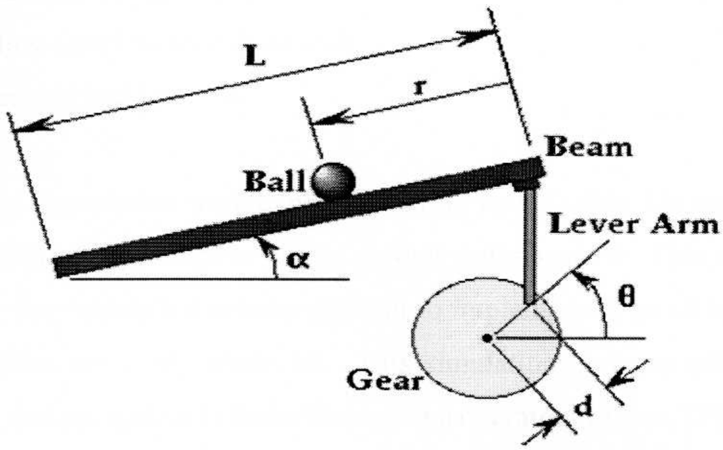

Figure 2.1: Ball and Beam Modeling

For this problem, assume that the ball rolls without slipping and friction between the beam and ball is negligible. The constants and variables for this example are defined as follows, table 2.1:

Table2.1: Ball and Beam assumption [4]

M	mass of the ball	0.11 kg
R	radius of the ball	0.015 m
d	lever arm offset	0.03 m
g	gravitational acceleration	9.8 m/s^2
L	length of the beam	1.0 m
J	ball's moment of inertia	9.99e-6 kgm^2
r	ball position coordinate	
alpha	beam angle coordinate	
theta	servo gear angle	

The design criteria for this problem are:

i. Settling time less than 3 seconds.

ii. Overshoot less than 5%.

The latest experiment were experimented by Erik Luther. He concludes the ball and beam experiment represents a simple second order system. This system is easily simulated using four states, but is more difficult to implement in real-life because not all necessary variables are easily observed. This simulation assumes all four states are easily observed and the system is linear through the operating region. [3]

The linearized system equations can also be represented in state-space form. This can be done by selecting the ball's position (r) and velocity (rdot) as the state variables and the gear angle (theta) as the input. The state-space representation is shown below:

$$\begin{bmatrix} \dot{r} \\ \ddot{r} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} r \\ \dot{r} \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{\quad}{L\left(\dfrac{J}{R^2}+m\right)} \end{bmatrix} \theta \qquad [2.1]$$

However, for state-space example it the different model will be used. The same equation for the ball still applies but instead of controlling the position through the gear angle, theta, the alpha-double dot also be controlled. This is essentially controlling the torque of the beam. Below is the representation of this system:

$$\begin{bmatrix} \dot{r} \\ \ddot{r} \\ \dot{\alpha} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \dfrac{-mg}{\left(\dfrac{J}{R^2}+m\right)} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r \\ \dot{r} \\ \alpha \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \qquad [2.2]$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r \\ \dot{r} \\ \alpha \\ \dot{\alpha} \end{bmatrix} \qquad [2.3]$$

Please take note that, for this system the gear and lever arm would not be used, instead a motor at the center of the beam will apply torque to the beam, to control the ball's position.