E-LEARNING OF HEAT EQUATION

MUHAMMAD FAKHRI BIN ABDUL SAHAK

MAY 2009

"I hereby declared that I have read through this report and found that it has comply the partial fulfillment for awarding the degree of Bachelor of Electrical Engineering (Power Electronic and Drive)"

Signature :

Supervisor's Name : DR. TAY CHOO CHUAN

Date : 13TH MAY 2009

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MUHAMMAD FAKHRI BIN ABDUL SAHAK

This Report Is Submitted In Partial Fulfillment of Requirements for the Degree of Bachelor in Electrical Engineering (Power Electronic and Drive)

Faculty of Electrical Engineering
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MAY 2009

"I hereby declare that this report is a result of my own work except for the excerpts that have been cited clearly in the references."

Signature :

Name : MUHAMMAD FAKHRI BIN ABDUL SAHAK

Date : 13TH MAY 2009

"Specially dedicated to my beloved parent, brothers, sisters, lecturers and friends"

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Alhamdulillah, praise be to Allah, the Cherisher and Sustainer of world, most Gracious, most Merciful Lord.

Praise be to Allah for enabling me to completed this analysis and research for E-Learning of Heat Equation project and report for my Final Year Project.

With this opportunity, I would like to express my gratitude to the Faculty of Electrical Engineering (FKE), Universiti Teknikal Malaysia Melaka (UTeM) generally, and especially to Dr. Tay Choo Chuan as my supervisor, for his invaluable help, support and ideas to me through achieving my Final Year Project goals. His countless contributions in this project will remind forever in my heart.

Finally, I would like to honor my parent, for supporting me steadfastly and their appreciated advice through my project completion.

ABSTRACT

The purpose of this project is to develop a module of electronic learning and a new learning guide about Heat Equation. Nowadays, the improvement of technologies such as internet has used widely by users especially student to gain knowledge. Electronic learning is the perfect method of learning, instead of old method teaching in class. As we know, Heat Equation studies are an important partial differential equation which describes the distribution of heat (or variation in temperature) in a given region over time. Using Microsoft Office PowerPoint, the simulation on Heat Equation will show more specifically what is going on this topic. From the interactive learning program, users especially student will enjoy and can experience also mastered in this topic.

ABSTRAK

Tujuan projek ini adalah menghasilkan satu modul pengajaran berkomputer (e-learning) dan membuat satu analisis bersimulasi tentang Persamaan Haba. Pada masa kini, kecanggihan teknologi contohnya internet telah digunakan secara meluas oleh orang ramai terutamanya pelajar untuk menambah ilmu pengetahuan. Modul pembelajaran elektronik ini merupakan satu kaedah yang berkesan berbanding cara pembelajaran lama di dalam kelas (menggunakan papan putih misalnya). Sebagaimana yang kita ketahui , pengajian tentang Persamaan Haba adalah penting dalam menerangkan penghantaran haba (disukat dalam ^{0}C) pada sesuatu tempat melawan masa. Menggunakan simulasi Microsoft Office PowerPoint, para pengguna boleh menerokai dengan lebih mendalam tentang Persamaan Haba ini. Menerusi pembelajaran interaktif ini, para pengguna terutamanya pelajar boleh menghayati dan menguasai topic Persamaan Haba dengan mudah.

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INTRODUCTION

1.1 Project Background

The past few years have witnessed a rapid growth in the number and variety of usage in Electronic Module (e-learning). E-learning has become an advance approaching method and the most concerned path for people to acquire their expected knowledge. The uses of electronic module have been an important approach for each universities and college to contribute their knowledge both for the student also for the outsider. Many developed countries have reserved a big proportion of education funding to support their e-learning strategies to enhance the education exports.

1.2 Objectives

The purposes of this project are:

1. To developed a new learning guide about Heat Equation.
2. To developed a user friendly software (e-learning) Microsoft Office PowerPoint.
3. To help user understand the basic concept in Heat Equation.
4. To help user learn Heat Equation better than before.

1.3 Scopes

The scopes of this project are:

1. Literature study on e-learning.
2. Model of heat conduction.
3. Separable method.
4. Initial value problems.
5. Graphical illustration using Microsoft Office PowerPoint.

1.4 Problem Statement

Electronic Learning Module has become one of the benefit and popular method for the people especially student to acquire their expected knowledge. Nowadays, e-learning had a high demand among the universities because the effectiveness commonly in educations.

This project is purpose to be developed based on the problem arise related to Heat Equation. The common problem is usually the understanding on Heat Equation. Actually people especially student had a problem and difficult to understand the main point of the Heat Equation due to the tough and complexity of the equation calculation.

The other problem is users hardly to find the correct answer of the initial value problems. Nowadays, the teaching method was used make students bored to learn from book and they also can not experience and mastered the subject themselves.

So, by interactive learning on Heat Equation, it will be the good alternative for student to study with more interesting.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

What is Electronic Learning Module? When talk about e-learning, some people say it is only can run via internet. Actually, e-learning allows users to learn anywhere and usually at any time, as long as users have properly configured with computer or cell phones.

2.2 Previous e-learning

To make the interactive electronic learning module, a few literature review have been made regarding to make flash more interesting. Actually, from the UTeM portal, the several research have been made regarding to flash project.

Mrs. Aida Fadzliana, Mrs. Jurifa and Mr. Zairusfaizery Jopeny has made Computer Aided Education on Three Phase Transformer. They discuss about three phase transformer including theory and the lab simulation.

Another e-learning produced by Mrs. Jurifa and Mrs. Aida Fadzliana are on topic My Intelligent Electric Cost Systems. They show the best power consumption to use the electrical equipment to decrease the cost of electricity.

Others example of e-learning that has been done by UTeM FKEs lecturer is Sistem Tiga Fasa by Prof. Dr. Marizan Sulaiman, Mr. Zainuddin Mat Isa and Mrs. Azrita Alia.

2.3 Conclusion of review

There are e-learning on basic math and algebra course but no e-learning yet on Heat Equation course. So, this project will put another math course on e-learning. From the studies, researches and software development above, it were obvious that e-learning systems have provided a better strategy and understanding for teaching, learning, administration and system development. E-learning will become one of the most important means for the education and as a medium delivering knowledge and information, especially for universities and the outside world in the future.

CHAPTER 3

HEAT EQUATION THEORY

3.1 Introduction

The heat equation is an important partial differential equation which describes the distribution of heat (or variation in temperature) in a given region over time. The heat equation is of fundamental importance in diverse scientific fields. In mathematics, it is the prototypical parabolic partial differential equation. In statistics, the heat equation is connected with the study of Brownian motion via the Fokker–Planck equation. The diffusion equation, a more general version of the heat equation, arises in connection with the study of chemical diffusion and other related processes.

3.2 Heat Conduction Model

The **one *dimensional heat equation*** is an equation of the form

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, 0 < x < L, 0 < t < \infty,$$

which relates the quantity $\frac{\partial u}{\partial t} =$ rate of temperature change with respect to time (measured in terms of degree/second unit) and $\frac{\partial^2 u}{\partial x^2} =$ concavity of the temperature profile $u(x,t)$ (which compares the temperature of two neighbouring points). Here, the constant c^2 is called as the ***diffusion constant.***

As an example, assume a rod of length L insulated on its sides with an insulator. The rod is placed in an environment of fixed temperature (say T_0 degrees) for a certain length of time. As the temperature of the rod becomes steady at T_0 degrees, the rod is removed from the environment. The time at which it is removed is denoted as $t_0 = 0$. We then have an initial condition as

$$u(x,0) = T_0, 0 \le x \le L, \tag{1}$$

This mean that at time $t_0 = 0$, the temperature at any point x on the rod is T_0.

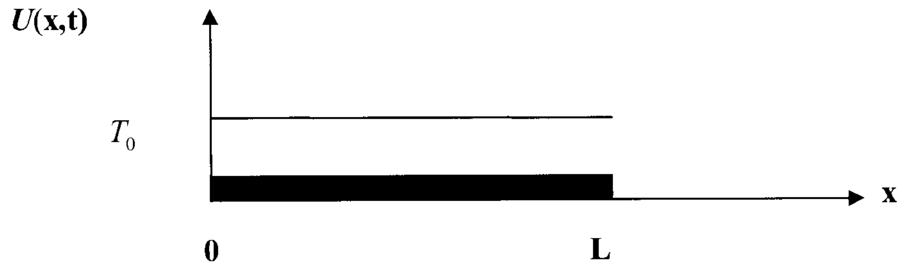

Figure 3.1 Graph U(x,t) against x

Now, place two heating elements, say T_1 and T_2, at the two ends of the rod so that the temperatures

Figure 3.2 A rod placed with two heating element T_1 and T_2

at the two ends of the rod are fixed. We call this the boundary conditions, namely,

$$u(0,t) = T_1$$

$$u(L,t) = T_2 \qquad 0 \le t < \infty \tag{2}$$

This means, at any time t, the temperatures at the two ends of the rod, that is, at points 0 and L are T_1 and T_2 degrees respectively.

As an illustration, consider the heat equation model

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$u(x,0) = f(x) \qquad \textbf{(3)}$$

$$u(0,t) = u(L,t) = 0$$

where $f(x)$ is the initial temperature of the rod.

3.3 Separable Method

We will now try to determine the function $u(x,t)$ which satisfies all the three condition **(3)** above. A simple solution is $u(x,t) = 0$. This solution is of no interest to us since it gives us no interpretation which is of worthy discussion.

We will try to use the separable method to analyze the solution of the problem. Assume that

$$u(x,t) = F(x)G(t) \qquad \textbf{(4)}$$

where $F(x)$ and $G(t)$ are functions of the single variables x and t respectively. Note that we separate the solution as the product of two functions $F(x)$ and $G(t)$. Thus,

$$\frac{\partial u}{\partial x} = G\frac{dF}{dx}$$

$$\frac{\partial^2 u}{\partial x^2} = G\frac{d^2F}{dx^2} \qquad \textbf{(5)}$$

$$\frac{\partial u}{\partial t} = F\frac{dG}{dt}$$

To further simplify our process, we use the following notations in equation **(5)**:

$\frac{dF}{dx}$ is denoted by F', $\frac{dG}{dt}$ is denoted by $\dot{G}$

and

$\frac{d^2F}{dx^2}$ is denoted by F''

Thus, equation (**5**) becomes

$$\frac{\partial u}{\partial t} = GF'$$

$$\frac{\partial^2 u}{\partial x^2} = GF'' \qquad \textbf{(6)}$$

$$\frac{\partial u}{\partial t} = F\dot{G}$$

Substitute (**6**) in (**3**), we obtain

$$F\dot{G} = c^2 GF'' \qquad \textbf{(7)}$$

Separate G and F to obtain

$$\frac{\dot{G}}{c^2 G} = \frac{F''}{F'} \qquad \textbf{(8)}$$

Note that the expression on the left side of (**8**) is in term of t while the expression on the right side of (**8**) is in term of x. This can only happen if both expressions in (**8**) are constant, say k. Therefore,

$$\frac{\dot{G}}{c^2 G} = \frac{F''}{F} = k \qquad \textbf{(9)}$$

Now, we have two ordinary differential equations

$$\dot{G} - kc^2 G = 0 \qquad \textbf{(10)}$$

and

$$F'' - kF = 0 \qquad \textbf{(11)}$$

There are three possibilities for the value of k, namely a negative number, zero or a positive number.

(a) **Positive Value of *k***

Suppose that $k = a^2$ is a positive number. Therefore, from **(11)** we obtain

$$F'' - a^2 F = 0 \quad \textbf{(12)}$$

The solution of **(12)** is

$$F(x) = (c_1 e^{ax} + c_2 e^{-ax}) \quad \textbf{(13)}$$

Substituting **(13)** into **(4)**, we obtain

$$u(x,t) = G(t)(c_1 e^{ax} + c_2 e^{-ax}) \quad \textbf{(14)}$$

By using the boundary conditions $u(0,t) = u(L,t) = 0$ in **(3)**, we obtain

$$u(0,t) = 0 = G(t)(c_1 + c_2) \quad \textbf{(15)}$$

If $G(t) = 0$ therefore we have $u(x,t) = F(x) \cdot 0 = 0$ but we are not interested with the solution $u(x,t) = 0$. Thus, $c_1 + c_2 = 0$ or $c_2 = -c_1$. Substitute this again in the second equation of **(15)**. You can try and show that $c_1 + c_2 = 0$ and consequently we obtain the solution which we are not interested in, that is $u(x,t) = 0$. As a conclusion we are not interested in the positive value of *k*.

(b) **Zero Value of *k***

Assume that the value of *k* is zero. Then, from **(11)** we have

$$F''(x) = 0 \quad \textbf{(16)}$$

The solution of **(16)** is

$$F(x) = c_1 x + c_2 \quad \textbf{(17)}$$

where and are constants. Substituting **(17)** into **(4)**, we obtain

$$u(x,t) = G(t)(c_1 x + c_2) \quad \textbf{(18)}$$

Using the boundary conditions in **(3)** $u(0,t) = u(L,t) = 0$, we obtain

$$u(0,t) = 0 = G(t)c_2$$

$$u(L,t) = 0 = G(t)(c_1 L + c_2) \quad \textbf{(19)}$$

If $G(t) = 0$, we obtain $u(x,t) = F(x) \cdot 0 = 0$ but we are not interested in the solution $u(x,t) = 0$. Therefore, we conclude that in the first equation of **(19)**, that $c_2 = 0$. Next, substitute in the second equation of **(19)**, you can try and show that $c_1 = 0$ and consequently we obtain a solution which we are not interested in, that is $u(x,t)=0$. As a conclusion, we are also not interested in the zero value of k.

(c) **Negative Value of *k***

Suppose that $k=-a^2$ is a negative number. Then, from **(11)** we obtain

$$F'' + a^2 F = 0 \quad \textbf{(20)}$$

and from **(10)** we obtain

$$\dot{G} + c^2 a^2 G = 0 \quad \textbf{(21)}$$

The solution of **(20)** is

$$F(x) = c_1 \cos(ax) + c_2 \sin(ax) \quad \textbf{(22)}$$

From the boundary conditions $u(0,t)=u(L,t)=0$, we obtain

$$0 = c_1 G(t)$$

$$0 = (c_1 \cos(aL) + c_2 \sin(aL))G(t) \quad \textbf{(23)}$$

Therefore, we can conclude that $c_1 = 0$ and $c_2 \sin(aL) = 0$. If we assume that $c_2 = 0$, then $F(x)=0$ and consequently, we obtain the solution which we are not interested in, that is $u(x,t)=0$. Therefore,

$$\sin(aL) = 0 \quad \textbf{(24)}$$

that is

$$a = \frac{n\pi}{L} \qquad (n = 1,2,3,\ldots) \quad \textbf{(25)}$$

This means that there exist infinitely many solutions of F(x). We denote the nth solution as

$$F_n(x) = c_2 \sin\left(\frac{n\pi x}{L}\right) \qquad (n = 1,2,3\ldots) \quad \textbf{(26)}$$

For simplicity, we set $c_2 = 1$.

Note that we do not consider negative values of n since $\sin(\theta) = -\sin(\theta)$. Therefore, the solutions are the same except that the sign is changed. The coefficient c_2 can be added later.

The equation (**21**) is rewritten as

$$\dot{G} + c^2\left(\frac{n\pi}{L}\right)^2 G = 0 \qquad (n = 1,2,3\ldots) \qquad \textbf{(27)}$$

The solution of (**27**) is

$$G_n(t) = B_n e^{-q_n^2 t} \qquad (n = 1,2,3\ldots) \qquad \textbf{(28)}$$

where B_n is a constant and $q_n = \frac{cn\pi}{L}$.

We conclude that the general solution of (**3**) is

$$u_n(x,t) = F_n(x)G_n(t) = B_n \sin\left(\frac{n\pi x}{L}\right)e^{-\left(\frac{cn\pi}{L}\right)^2 t} \qquad (n = 1,2,3\ldots) \qquad \textbf{(29)}$$

3.4 Complete Solution

The complete solution of

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$u(x,0) = f(x)$$

$$u(0,t) = u(L,t) = 0$$

is the sum of all $u_n(x,t)$ in (**29**) that is

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{L}\right)e^{-\left(\frac{cn\pi}{L}\right)^2 t} \qquad \textbf{(30)}$$

By using the initial condition $u(x,0) = f(x)$, we obtain

$$u(x,0) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{L}\right) f(x) \qquad \textbf{(31)}$$

Note that the equation (**31**) is a sine Fourier series. Therefore,

$$B_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$