PLANT & DYNAMICAL MODELING DEVELOPMENT OF INVERTED PENDULUM ON ROTATING DISC

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A report submitted in partial fulfillment of requirements for the degree of
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"I declare that this report entitle "Plant & Dynamical Modeling Development of Inverted Pendulum on Rotating Disc" is the result of my own research except as cited in the references. The report has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

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Date : 8 MAY 2009

For my beloved father and mother
Fauzi bin Abdullah and Che Satinah binti Che Isa
In appreciation of supported and understanding.

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ABSTRACT

Inverted Pendulum system is a multivariable, unstable, high order and nonlinear system. The inverted pendulum system has a single-input-multiple-output (SIMO) structure where one single input force has to control both angles of the pendulum and the position of the cart at the same time. So far, many kinds of inverted pendulum were implemented such as inverted pendulum on cart and rotational inverted pendulum. The rotational inverted pendulums are mounted on motor shaft and the pole is maintained in vertical position with restricted angle oscillation by controlling the motor. Modeling and system identification are vital part of control engineering theory. The quest for a model can include a very detailed analysis of the physics of the system. In this project, the dynamic and mathematical modeling of inverted pendulum system should be determined and obtained so that the controller and entire system will work efficiently. Complete dynamical modeling will be developed, analyzed and demonstrate using MATLAB/SIMULINK as an aided software tool.

ABSTRAK

Sistem pembalikan bandul adalah satu multipembolehubah, tidak stabil, tertib tinggi dan sistem tidak lurus. Sistem pembalikan bandul mempunyai satu input dan banyak output (SIMO) dimana satu daya input mengawal kedua-dua sudut pada bandul dan kedudukan pembawa (cart) pada masa yang sama. Setakat ini, banyak jenis bandul terbalik telah dilaksanakan seperti bandul terbalik di pembawa (cart) dan bandul terbalik yang berputar. Bandul terbalik yang berputar berada pada aci motor dan sudut dipastikan dalam keadaan tegak dengan ayunan sudut terhad oleh pengawalan motor. Pemodelan (modeling) dan sistem kenalpasti (Identification System) adalah bahagian yang penting teori kejuruteraan kawalan. Pencarian bagi sebuah model boleh dimasukkan dalam satu analisis yang terperinci bagi system fizik yang sebenar. Dalam projek ini, pemodelan dinamik dan matematik bagi sistem bandul terbalik perlu ditentukan dan diperolehi supaya pengawal dan keseluruhan sistem akan bekerja dengan cekap. Pemodelan dinamik yang lengkap akan dibina, dianalisis dan dipersembahkan menggunakan perisian MATLAB / SIMULINK.

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CHAPTER 1

INTRODUCTION

1.1 Project Background

The control of an inverted pendulum is a well-known and a challenging problem that serves as a popular benchmark in modern control system studies. It was quite popular because the many variations of the pendulum often represent different kinds of robotic arms. When the pendulum is in the upside-down position, the operators move the cart and enlarge the angle of pendulum. The relation between the angle and the power moving the cart is studied by moving the cart left and right. Based on predicting of its movement, the swing up to the upright position is done. The cart is moved in the direction where the pendulum is laying to maintain the upright pendulum.

The standard problem related to inverted pendulum is stabilization of either downward or upright equilibriums at a prescribed position of the cart. The relation between the movement of the pendulum and the power moving the cart is studied by moving the cart left and right. As the angular velocity and the power moving the cart become 0° in the upright point, the adding power is determined by predicting of its movement. Moreover, if the position of the cart is different from the target position of the cart, the pendulum is put in the direction of the target position to approach the target position.

The mathematical model is a set of dynamic equations that provide an accurate description for the motion of a particular system. Without a proper mathematical modelling, the controller cannot be designed and control's objective of inverted pendulum cannot be achieved. The dynamic non-linear equation is needed to be linearized before state space equation state variable and transfer function are used for simulation and design controller. Deriving a mathematical model for the inverted pendulum is the first step in the design process. The model must be able to describe the response of the actual system as closely as it can. Lagrange's equation of motion was used in the derivation of the dynamic non-linear model. The model gives the exact relationships among all the variables involved. Using the dynamic non-linear model, we can derive the linear model.

An inverted pendulum is an inherently unstable system by nature. Force must be properly applied to keep the pendulum in an upright position. The system including pendulum, cart, and motor can be modeled as a linear system if all the parameters are known such as masses, lengths in order to find a controller to stabilize it. If not all the parameters are known, one can however try to 'reconstruct'' the system parameters using measured data on the dynamics of the pendulum. The plant of inverted pendulum on rotating can be constructing based on mathematical modeling.

1.2 Problem Statements

Inverted pendulums have been widely used by scientists to illustrate the ideas of control technology. In the beginning these were ideas in linear control. Because of their nonlinear behavior, nowadays they are also used to illustrate new ideas in nonlinear control. The problems that are faced:

i. Inverted pendulum is an inherently unstable system. Force must be properly applied to keep the system intact.
ii. Difficult to stabilizing inverted pendulum on vertical position, basically Rotational Inverted Pendulum in horizontal arm driven.
iii. It is not easy to model a physical system perfectly. The main point is to find important factors, which influence the system, and apply the factors properly to the system equation. The controller design can be done easily once the system model is well established.

iv. Similar to the rocket at launch, the inverted pendulum requires a continuous correction mechanism to stay upright, since the system is unstable in closed loop configuration.

1.3 Project Objectives

The specific objectives for this project are:

i. To derive the mathematical equation involves for Inverted Pendulum (IP) on rotating disc.
ii. To obtain a transfer function of Inverted Pendulum (IP) on rotating disc.
iii. To develop and analyze dynamical modeling using Matlab / Simulink.
iv. To set up the hardware plant of inverted pendulum on rotating disc.

1.4 Project Scope

While doing the project, the scope of work plays a very important role. In order to do in guideline method, student should fulfill the project requirement. The scope of this project is listed as below:

i. The SIMULINK/MATLAB software version 7.6.0.324 (R2008a) developed was able to analyze the inverted pendulum stability and get the value of parameter for this project.
ii. The dynamical modeling was used to design plant of Inverted Pendulum on rotating disc based on hardware proposed.
iii. The plant of inverted Pendulum proposed consist DC motor, encoder, normal disc and pendulum strip.
iv. This project not covers the controller of Inverted Pendulum on Rotating Disc.
v. It also not includes the interface module between SIMULINK software with plant of this project.

1.5 LITERATURE REVIEW

In this section, related previous project are reviews to get an idea about the project design, conception and any information related in order to improve this project.

1.5.1 Rotational Inverted Pendulum

Inverted pendulum system, which has the characteristics of high order, instability, multivariable, nonlinear and strong coupling [10].The standard problem related to inverted pendulum is stabilization of either downward or upright equilibriums at a prescribed position of the cart. In [8], the vertical positioning of the inverted pendulum requires a continuous correction mechanism to stay upright because this set-point is unstable in an open-loop configuration. Both problems are widely investigated and solved by different methods as discussed in [1]-[11]. A different problem related to this under-actuated system is a stabilization of a given rotational behavior of the pendulum with a prescribed position of the cart.

An inverted pendulum also is an unstable system and the developments of its controller have been a great interest for many researchers as mentioned in [1]-[7]. So far many kinds of inverted pendulum were implemented such as inverted pendulum on cart and rotational inverted pendulum. In [2] and [3], some authors proposed nonlinear controller especially for those having strong nonlinearity. Many researchers have applied the neural network theory to control the inverted pendulum systems based on [7]–[12]. Most of them were focused on the control problem of how to maintain an inverted pendulum in the neighborhood of an unstable equilibrium position.

In addition to being the simplest unstable nonlinear system imaginable, it also provides are simple model for rocket control, control of a space rocket during takeoff and occurs in practice in the areas of missile stabilization as in [2]. The control performance of inverted pendulum can be measured directly by the angle of pendulum, the displacement of cart and the transition time. It can be use inverted pendulum to verify and compare the effectiveness of controller when an innovative theory or method of control comes out based on [6]-[7]. Therefore, the research for control techniques for inverted pendulum has important theoretic and practical meaning as discussed in [1]-[15].

Nowadays inverted pendulum becomes an important experimental device for studying, researching and validation of different control theory as in [1]. Traditionally, typical inverted pendulum is driven by a rotating servo motor which drives the cart via transfer mechanism to keep the balance of inverted pendulum based on [7].

1.5.2 Dynamical Modeling

The important part in plant of inverted pendulum studies is mathematical modeling of the system and without a proper mathematical modeling, the controller cannot be designed and control' objective of inverted pendulum cannot be achieved as in [1]. The successful application of classical controller design techniques required considerable knowledge of the accurate system dynamic model and desired system behaviors with the expression of an objective function based on [6]-[8]. However, the mathematical model derived from physical relationships or identified from experimental results is only an approximated model.

Modeling is the process of identifying the principal physical dynamic effects to be considered in analyzing a system, writing the differential and algebraic equations from the conservative laws and property laws of the relevant discipline, and reducing the equations to a convenient differential equation model as in. In order to develop the control system, mathematical model is established to predict the behavior before applied into real system. Actually, the dynamics refer to a situation, which is varying with time as in [11]. This non-linear equation is needed to be linearized before state space equation and state variable are used for simulation and design of controller based on [1].

According to [12], for classical translational inverted pendulum, the mathematical modeling was divided into 3 models were inverted pendulum and cart model, DC motor model and transmission model. Wilfredo in [1] divide the system modeling into two parts which is the ideal system modeling and laboratory plant modeling. The ideal system modeling was assumed motor dynamic friction, physics, geometry, rigid objects and non-linear detail. The Laboratory plant modeling is includes with dynamics and non-linear term.

As discussed and mentioned in [2]-[12], the most popular approach to obtained the non-linear mathematical model and dynamic equation of the plant is using Euler-lagrance equation based on [1].To establish the mathematic model of the system, then make simulating experiments, at last real system experiments will be made. There are some features of Lagrange Equation as in [15]:

i. Use generalized coordinates to express equation of motion in any complete system; the amount of equation and the freedom of system are accordant.
ii. The ideal constraint counterforce would not appear in the equations set. In the process of establishing the movement equations, it only need to analysis the known drive force, there is no need to analysis the unknown counterforce.
iii. Lagrange equation is the motion equations which established from the view of energy. In order to list the motion equations of system, it is need to analyze from two aspects. One is the kinetic energy of the system which represents the kinetic variance of the system's motion; the other one is generalized force which represents the kinetic variance of the drive force's action.

M.Roman, E.Bobasu and D.sendrescu in [12] indicate that the system of dynamic equations describing a Rotary Inverted Pendulum can be derived by Newtonian methods using free body diagrams or the Euler-Lagrange method. The Euler-Lagrange method derives the system equations by applying the Euler-Lagrange equation to the Lagrangian. The Lagrangian is defined as the difference between potential and kinetic energy (*L*=*K*-*P*). The Lagrange method simplifies the mathematical derivation substantially.

Regardless to translational inverted pendulum, most of the horizontal rotational inverted pendulum as in [1]-[5] which similar to inverted pendulum on rotating disc have the following parameter or dynamics variable:

i. Pendulum angle
ii. Pendulum Mass
iii. Length of Pendulum
iv. Moment of inertia of pendulum
v. Arm position angle
vi. Length of rotating arm
vii. Moment of inertia of rotating arm
viii. DC Motor armature voltage
ix. DC motor friction constant
x. Torque of DC motor.

According to [13], in general, controlling nonlinear mechanical systems is not possible without a model of the system. Derivation of a mathematical model can be time-consuming and is not always possible. If finally a structure of a model is determined, unknown physical parameters remain which have to be determined from measurements of the system. In practice, this is a difficult process.

1.5.3 Plant Set-up

To set the plant of inverted pendulum on rotating disc, a proper plant set up is needed. Review studies of [1]-[12] noted that the plant of rotational inverted pendulum system is consist of components such as;

i. Rotary Inverted Pendulum – consist of pendulum angle, mass of pendulum, pendulum length and moment of inertia of pendulum.

ii. Arm-Driven Inverted Pendulum – consists about length of rotating arm & length of rotating arm

iii. DC Motor – Specification about DC Motor (DC Motor armature voltage, torque of DC motor & DC motor friction constant)

W.Moeljono and Y.Stephen in [8] set up the plant of "Intelligent Control for Inverted Pendulum System' Swing Up and dancing" which consist of DC Motor, rotating base and pendulum. The values of the relevant parameters for this plant are DC motor parameter (K_p = 74.89rad-$s^{-2\text{-}v\text{-}1}$), DC motor parameter (a_p = 33.04s^{-2}), torque constant *(K_l)* 1.9 x 10^{-3} kg-m/rad, frictional constant (C_l)2.9790x10^{-3} N-m-s/rad, pendulum mass *(m_l)* 0.086184 kg, pendulum length *(I_l),* 0.113 m, inertia (J_l) 1.3010 x 10^{-3} N-m-s^2.

The set up the plant of inverted pendulum was constructed the parameter similar to reference [8]. Its structure consists of a cart and pendulum where the pendulum is hinged to the cart via a pivot and only the cart is actuated, where θ is the pendulum angle (rad), x is the cart position (m), M is the mass of the cart (kg), m is the mass of the pendulum (kg), I is the distances from turning center to center of mass of, the pendulum (m), f is the cart's friction coefficient (kg/s) and F is the force applied to the cart (N).

The physical parameters of the rotational inverted pendulum system in the laboratory can be indicated in [4] and [5].Most of the values can easily be determined by measuring the dimensions and masses such as the pendulum length, mass of the cart and other. The input-to-force gain km can be computed from the motor specifications and the gains of the interface amplifiers. The value of the damping coefficient b is including viscous friction and the back-emf of the motor is not known a priori and can only be determined experimentally. It is your task to devise and carry out an identification experiment to obtain a more accurate value for this damping coefficient *b*.

The inverted pendulum setup consists of a cart driven by a DC motor. The motor can steer the cart left and right on a track approximately one meter long. On the cart, a pendulum is mounted such that it can freely rotate around an axis that is perpendicular to the direction of