

**CONTROL OF A REAL INVERTED PENDULUM USING PID CONTROLLER**

**AWANG MUHAMMAD MURSYIDIN BIN AHAT**

**This report is submitted in partial fulfillment of the requirements for the  
award of Bachelor of Electronic Engineering  
(Industrial Electronics) With Honours**

**Faculty of Electrical Engineering  
Universiti Teknikal Malaysia Melaka**

**APRIL 2009**



**UNIVERSITI TEKNIKAL MALAYSIA MELAKA**  
**FAKULTI KEJURUTERAAN ELEKTRONIK DAN KEJURUTERAAN KOMPUTER**

**BORANG PENGESAHAN STATUS LAPORAN**  
**PROJEK SARJANA MUDA II**

**Tajuk Projek** : CONTROL OF A REAL INVERTED PENDULUM USING PID  
**CONTROLLER**  
**Sesi** : 2008/2009  
**Pengajian** :

Saya ..... **AWANG MUHAMMAD MURSYIDIN BIN AHAT** .....  
 (HURUF BESAR)

mengaku membenarkan Laporan Projek Sarjana Muda ini disimpan di Perpustakaan dengan syarat-syarat kegunaan seperti berikut:

1. Laporan adalah hakmilik Universiti Teknikal Malaysia Melaka.
2. Perpustakaan dibenarkan membuat salinan untuk tujuan pengajian sahaja.
3. Perpustakaan dibenarkan membuat salinan laporan ini sebagai bahan pertukaran antara institusi pengajian tinggi.
4. Sila tandakan (  ) :

**SULIT\***

(Mengandungi maklumat yang berdarjah keselamatan atau kepentingan Malaysia seperti yang termaktub di dalam AKTA RAHSIA RASMI 1972)

**TERHAD\***

(Mengandungi maklumat terhad yang telah ditentukan oleh organisasi/badan di mana penyelidikan dijalankan)

**TIDAK TERHAD**

Disahkan oleh:

(TANDATANGAN PENULIS)

(COP DAN TANDATANGAN PENYELIA)

Alamat Tetap: Lot 1010, Lorong Piala 4B

Jalan Sultan Tengah, Taman Sukma,

93050, Kuching, Sarawak.

Tarikh 30/04/09

**MOHD SHAKIR BIN MD SAAT**

Pensyarah

Fakulti Kejuruteraan Elektronik Dan Kejuruteraan Komputer

Universiti Teknikal Malaysia Melaka (UTeM)

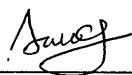
Karung Berkunai No 1752

Pejabat Pos Durian Tunggal

76100 Durian Tunggal, Melaka

Tarikh: 30/04/09

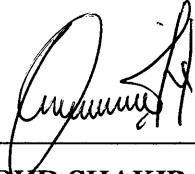
**“I hereby declare that this report is the result of my own work except for quotes as cited in the references.”**

**Signatures** :  \_\_\_\_\_

**Author** : AWANG MUHAMMAD MURSYIDIN BIN AHAT

**Date** : 30 APRIL 2009

**“I hereby declare that I have read this report and in my opinion this report is sufficient in terms of the scope and quality for the award of Bachelor of Electronic Engineering (Industrial Electronic Engineering) With Honours.”**

**Signature** :   
**Author** : EN MOHD SHAKIR B. MD SAAT  
**Date** : 30 APRIL 2009

**For you, my mom and dad**  
**For your truly support and undivided love**  
**For making me the person**  
**Who I am today**

## ACKNOWLEDGEMENT

First of all, I am greatly indebted to Allah SWT on His blessing to make this project successful. This text could not have been written and produced without the help of many lecturer and colleagues. Appreciation is expressed to those who have reviewed and/or made contributions to this final year project report.

This work is a synergistic product of many good minds. It began since the first day I got this final project. I am grateful for the inspiration and friendly culture of peoples here. I would like to thanks to all of peoples here especially to my supervisor and my team work. Guidance and co-operation and encouragement from all below are appreciated by me in sincere. Although I cannot repay the kindness from them, I would like to wish them to be well and happy always.

For the development and production of this final report I feel deep sense of gratitude:

- To my parents for their loves, prays and encouragements.
- To my lecturers for their guides and advices.
- To my supervisor, Mr. Mohd Shakir for his carefulness, brilliant ideas and flexibility.
- To my team work, Zaini, Safuan, Zulfadhli, Mariani for their full support, wisdom thinking and enjoy moments working with.

## ABSTRACT

This project is about an Inverted Pendulum that been control by the by PID controller. The inverted pendulum is a classical control system problem because of its nonlinear characteristics and unstable behavior. Control of inverted pendulum is a Control Engineering project based on the flight simulation of rocket or missile during the initial stages of flights. The inverted pendulum (which is simulating the rocket, here) is mounted on a moving cart. The PID controller used the trial and error in order to get the value of gain in the PID controller. For the inverted pendulum system, it is typical nonlinear system that is inherently unstable under the open-loop state. For the inverted pendulum system, it is a typical nonlinear system that is inherently unstable under the open-loop state. This system can be a linear system using linearization near the equilibrium (inverted) point. The complexity of the system can be increased by adding extra links and it is simple to implement an actual system. This controller will control both angle and position of the inverted pendulum. The performance test results is then presented and compared with closely matching simulation results which will verify the successful design and implementation of the control loop in the real system. The Real Time Windows Target (RTWT) application is used as a host target that enables to connect Simulink models and execute in real time meanwhile the Data Acquisition Card (DAQ card) is used to generate data from computer to the real hardware.

## ABSTRAK

Projek ini menerangkan tentang *Inverted Pendulum System* di mana system ini dikawal oleh *PID controller*. *Inverted Pendulum* adalah satu masalah sistem kawalan yang klasik disebabkan oleh ciri-cirinya yang tak linear dan mempunyai tingkah laku tidak stabil. Kawalan bagi *Inverted Pendulum* adalah satu projek Kejuruteraan Kawalan berdasarkan simulasi penerbangan roket atau peluru berpandu semasa peringkat awal penerbangan. Bandul terbalik (yang dianggap sebagai roket, di sini) dipasang di atas sebuah kereta atau troli. Bagi kawalan ini, kaedah *trail and error* bagi mendapatkan nilai gandaan bagi *PID controller*. Bagi system *inverted pendulum*, system ini adalah system tidak linear di mana system ini tidak stabil pada keadaan *open loop*. Sistem ini boleh menjadi linear dengan menggunakan kaedah *linearization* pada keadaan keseimbangan. Kawalan ini akan mengawal kedua-dua sudut pendulum dan kedudukan cart. Keputusan projek ini akan dibandingkan di mana kejayaan keputusan rekabentuk simulasi akan diaplikasikan ke dalam system sebenar. Aplikasi Real Time Windows Target (RTWT) digunakan seperti satu tuan rumah sasaran yang membolehkan untuk model Simulink berkait dan dijalankan dalam masa nyata sementara itu Data Acquisition Card (Kad DAQ) bertujuan menjana data daripada komputer untuk perkakasan yang sebenar.



## TABLE OF CONTENTS

<b>CHAPTER</b>	<b>TITLE</b>	<b>PAGE</b>
	<b>PROJECT TITLE</b>	i
	<b>ACKNOWLEDGEMENT</b>	vi
	<b>ABSTRACT</b>	vii
	<b>ABSTRAK</b>	viii
	<b>TABLE OF CONTENTS</b>	ix
	<b>LIST OF FIGURE</b>	xii
	<b>LIST OF TABLE</b>	xiii
<b>I</b>	<b>INTRODUCTION</b>	<b>1</b>
	1.1 Project Introduction	2
	1.2 Project Objective	3
	1.3 Problem Statement	4
	1.4 Scope of Work	4
	1.5 Project Methodology	5
	1.6 Report Structure	5
<b>II</b>	<b>LITERATURE REVIEW</b>	<b>7</b>
	2.1 Introduction	7
	2.2 Definition of control	7
	2.2.1 Control System	8
	2.2.2 Open-Loop Control Systems	8
	2.2.3 Closed-Loop Control Systems	10

	2.3 Project Literature Review	11
<b>III</b>	<b>PROJECT METHODOLOGY</b>	<b>13</b>
	3.1 Introduction	13
	3.2 Project Methodology	13
	3.3 Flow Chart model	15
<b>IV</b>	<b>MATHEMATICAL MODEL</b>	<b>16</b>
	4.1 Introduction	16
	4.2 Mathematical Model	16
	4.2.1 Free Body Diagram	17
	4.2.2 Physical Data	17
	4.2.3 General Equation	18
	4.2.4 Mathematical Modelling For the System	19
<b>V</b>	<b>CONSUMPTION OF CONTROLLERS</b>	<b>21</b>
	5.1 Introduction	21
	5.2 PID Controller	21
	5.2.1 The Characteristics of P, I and D Controllers	23
	5.2.2 Method of Tuning the PID Controller	24
	5.2.3 Step using PID controller in Project design	25
<b>VI</b>	<b>INTERFACING</b>	<b>27</b>
	6.1 Introduction	27
	6.1.1 Real Time Window Target (RTWT)	27
	6.1.1.1 Real-Time Kernel	29
	6.1.1.2 Real-Time application	29

6.1.1.3	Signal Acquisition and Analysis	31
6.1.1.4	Parameter Tuning	31
6.1.1.5	Hardware Environment	32
6.1.1.6	Software Environment	32
6.1.1.7	System Concepts	34
6.1.2	Advantech PCI 1711	35
6.1.3	Input/Output port	37
6.1.4	Inverted Pendulum Hardware	39
6.1.5	Interfacing Procedure	41
<b>VII</b>	<b>RESULT AND DISCUSSION</b>	<b>44</b>
7.1	Introduction	44
7.2	Result	45
7.2.1	Simulation	45
7.2.2	Interfacing	47
7.4	Discussion	50
<b>VIII</b>	<b>CONCLUSION AND SUGGESTION</b>	<b>53</b>
8.1	Conclusion	53
8.2	Suggestion	54
	<b>REFERENCES</b>	<b>55</b>
	<b>APPENDIX</b>	<b>56</b>

## LIST OF FIGURE

NO	TITLE	PAGE
1.1	Cart with an Inverted Pendulum	1
2.1	Types of Control	7
2.2	A Block Diagram of Control System	8
2.3	A Block Diagram of Open-Loop System	9
2.4	A Block Diagram of Closed-Loop System	10
2.5	Inverted Pendulum Model	12
3.1	Project Methodology Flow Chart	15
4.1	Free body diagrams of the system	17
5.1	PID system configurations	22
5.2	Open Loop systems	25
5.3	Closed-Loop systems	26
6.1	Advantech PCI 1711	36
6.2	Port for I/O connector	37
6.3	Wiring I/O port to Hardware	38
6.4	Inverted Pendulum	39
6.5	Features of Inverted Pendulum	39
6.6	Flowchart Interfacing procedure	41
6.7	Check Sensor block	42
6.8	Interfacing Block	43
7.1	Unstable system graph	45
7.2	Stable system graph	45
7.3	Check Sensor position	48
7.4	Stable system graph for cart position	49

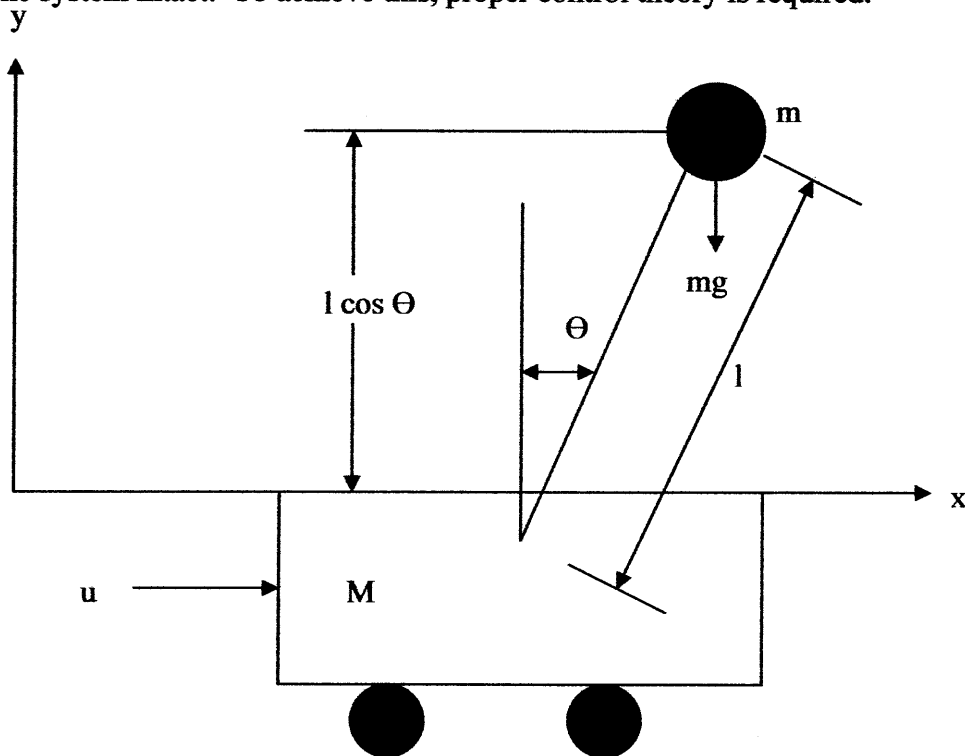
**LIST OF TABLE**

<b>NO</b>	<b>TITLE</b>	<b>PAGE</b>
4.1	Representative of constant/variable for the system	18
5.1	Effect of increasing $K_p$ , $K_i$ and $K_d$	23
6.1	I/O Connector Signal Description	38
7.1	Value of $K_p$ , $K_i$ and $K_d$ for PID controller	46
7.2	Reference Voltage for Pendulum Angle	47
7.3	Reference Voltage for the Cart position	47
7.4	Value PID for interfacing block	48

## CHAPTER I

### INTRODUCTION

For the inverted pendulum, it just likes a broom-stick on your index finger that need to be balance so that the object will be in upright position at the actual position. However, inverted pendulum is limited in that it only moves in one dimension, while your hand could move up, down, sideways and etc. An inverted pendulum is an inherently unstable system. Force must be properly applied to keep the system intact. To achieve this, proper control theory is required.



**Figure 1.1** Cart with an Inverted Pendulum

The inverted pendulum is a classic problem in dynamics and control theory and widely used as a benchmark. It is among the most difficult systems to be control in the field of control engineering. The main task of this project is to design an inverted pendulum system which keeps the pendulum angle and cart position in stable condition using the PID controller. System identification is the procedure that develops the models of a dynamic system based on the input and the output signals from the system. The input and the output data must show some of the dynamics of the process. The parameters of the model are adjusted until the output from the model is approximately to the output of the process.

Control theory of determining the stability will be used as to design a better controller. This project is including literature study, non linear and linear mathematical modelling, controller design and simulation study that using the MATLAB-Simulink software.

## **1.1 Project Introduction**

This report will discuss the modelling and control of an inverted pendulum. The controller was implemented digitally using MATLAB, and it stabilized the rod of an inverted pendulum in the upward vertical position, while at the same time, keeping the cart in the middle of the track. The minimum specifications for control was that the control system must be theoretically closed-loop stable, and the pendulum rod must be balanced vertically for at least 20 seconds and has a overshoot at 20% while keeping the cart at desired position.

This project involved the design of a controller to stabilize an inherently unstable inverted pendulum. For the inverted pendulum system, it is a typical nonlinear system that is inherently unstable under the open-loop state. This system can be a linear system using linearization near the equilibrium (inverted) point. The complexity of the system can be increased by adding extra links and it is simple to implement an actual system.

Due to above features, an inverted pendulum is used to prove or verify new control theories in many theses. The objective of this research is to use PID control on the real inverted pendulum which focusing on using the Real Time Window Target(RTWT) as the interfacing to connect the Simulink and Stateflow models to physical systems and execute them in real time on PC-compatible hardware. This application is use in order to stabilize the real inverted pendulum at vertical position and at the same time place a cart at desired position.

Then simulate the PID control scheme using MATLAB/Simulink for proper tuning and variations after that implementing the controller on the physical inverted pendulum model by using RTW target. The data acquisition (DAQ) card is use as I/O devices to enable to communicate with the real inverted pendulum model. The performance test results is then presented and compared with closely matching simulation results which will verify the successful design and implementation of the control loop. This research is important for a system such as simulation of dynamic of a robotic arm, model of a human standing still, launching a rockets and the F-16 fighter.

## **1.2 Project Objectives**

To make sure this project work as planned, a few objectives were determined where these objectives will be followed as a guide through the whole completion process of this subject in order to achieve the desired output. These objectives were provided by sequence of project from beginning until the end of project. A detailed explanation for each objective will be discussed.

The objectives to be achieved in this project are:

- i. To design controller based on PID control scheme to control the inverted pendulum.
- ii. To maintain the pendulum at the upright position and place the cart at the desired position.



- iii. To interface the controller from MATLAB to the physical inverted pendulum by using the RTW Target application.
- iv. To compare the simulation result and practical result.

### **1.3 Problem Statement**

The problem statement for this project is to control an inverted pendulum in order to control pendulum angle and the cart position. In this case, the problem was to control both of the pendulum angle and the cart position. The controller has to make sure that the pendulum should return to its upright position and the cart should stop at the desired position within the desired time. These two situations must happen in a same time. Another problem regarding to the inverted pendulum is unstable system, nonlinear system and non minimum behaviour phase system.

### **1.4 Scope of Work**

The scope of this project involve of:

- i. Design mathematical model of a inverted pendulum system
- ii. Design PID controller to control this system and implement the designing controller in MATLAB
- iii. Interfacing between hardware and software using RTW Target application and the data acquisition (DAQ) card will be using as the I/O devices to enable the interfacing.
- iv. Compare the result between the simulation and the practical result.

## **1.5 Project Methodology**

In general, this project starts with designing the mathematical model for the system of the inverted pendulum. The mathematical model was then, be linearized and the controller will be design based on the linearized mathematical model. In designing the controller, trial and error method will be use in getting the gain for the controller PID. This result from the Simulink will be compare with the real system result of the inverted pendulum.

## **1.6 Report Structure**

In general, this report contains six chapters where chapter one is an introduction of this report which explain briefly about inverted pendulum systems. This chapter includes the project introduction, project objective, problem statements, scope of work, brief explanation about project methodology and the last one is report structure.

Chapter two is the literature review which explains the details of the research on the inverted pendulum system background. In this chapter also explained about the relationship between the project with the theory and concept by using a figure or suitable model.

Chapter three is project methodology which explaining the method being used to solve the problems. This chapter contains the methods used such as data collecting, process and analysis of data, model and flow chart.

Chapter four is about the mathematical model of the system. This chapter will cover all the equation that related to the project. The mathematical model will be separate into several subtopic starts with free body diagram, physical data, general equation and lastly the mathematical model of the system.

Chapter five is the consumption of the controller. This chapter briefly explain the PID Controller. In this chapter also it explains the Characteristics of P, I and D Controllers and how is the method used for tuning the PID Controller.

Chapter six is regarding the interfacing between the interfacing block with the real inverted pendulum. This chapter will explain how the procedure of interfacing is done. Type of application that used to connect to the inverted pendulum also being explain that is the Real Time Window Target (RTWT) application.

Chapter Seven will discuss about the result of the Simulink from the MATLAB and the real system of the inverted pendulum. Comparison and discussion will be made in order to analysis result of the data obtained. The result for inverted pendulum system is shown by using figure, graph and table.

The last chapter is conclusion and suggestion where this chapter stated the summary for the project. This chapter contains the project analysis achievement and suggestion for future research.

## CHAPTER II

### LITERATURE REVIEW

#### 2.1 Introduction

In this chapter will discuss about the literature review that will discusses published information in a particular area and sometimes information in a particular subject area within a certain time period. This is also include the simple summary of the sources, an organizational pattern and combines both summary and synthesis. The inverted pendulum problem is a widely-used benchmark for comparing different types of controllers, especially neural network controllers. It is a difficult nonlinear control task to balance an inverted pendulum. A linear controller can be implemented for the inverted pendulum. This literature review will also evaluate the sources.

#### 2.2 Definition of Control

Control is define the power to make decisions about something and decide what should happen.

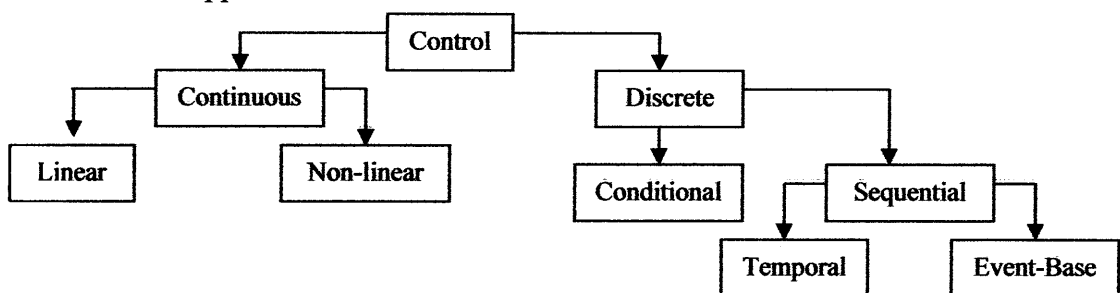
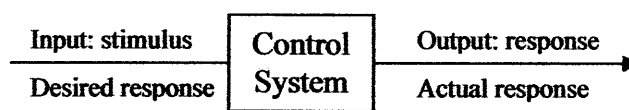


Figure 2.1 Types of Control

### 2.2.1 Control System

A control system consists of subsystems and processes (or plant) assembled for the purpose of controlling the outputs of the processes. For example, a furnace produces heat as a result of the flow of fuel. In this process, subsystems called fuel valves and fuel-valve actuators are used to regulate the temperature of a room by controlling the heat output from the furnace. Other subsystems, such as thermostats, which act as sensors, measure the room temperature. In this simplest form, a control system provides an output or response for a given input or stimulus [1].

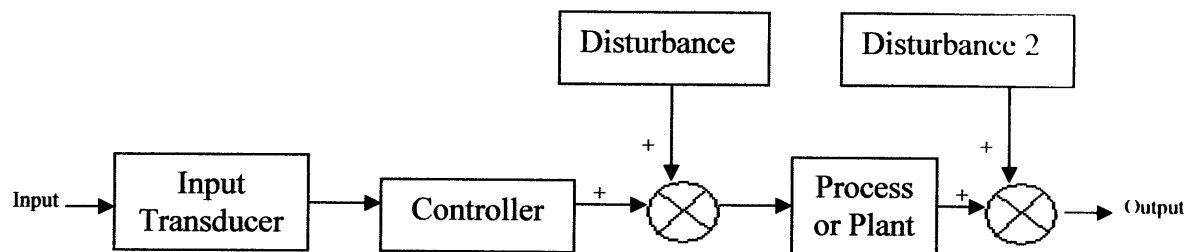


**Figure 2.2** A Block Diagram of Control System

### 2.2.2 Open-Loop Control Systems

Open-Loop System start with a subsystem called an input transducer which convert the form of the input to the used by the controller. The controller drives a process of plant. The input is sometimes called the reference, while the output can be called the controlled variable. Other signals, such as disturbances, are shown added to the controller and process outputs with summing junctions, which yield the algebraic sum of their inputs signal using associated signs [1].

Open-Loop System is a control system that does not use feedback. The controller sends a measured signal to the actuator, which specifies the desired action. This type of system is not self-correcting. If some external disturbance changes the load on machine or process being performed, some degree of physical effort of human operator is required to make necessary modifications. The system manually controlled by the human. For an example, is the speed of a car controlling by a driver [2].

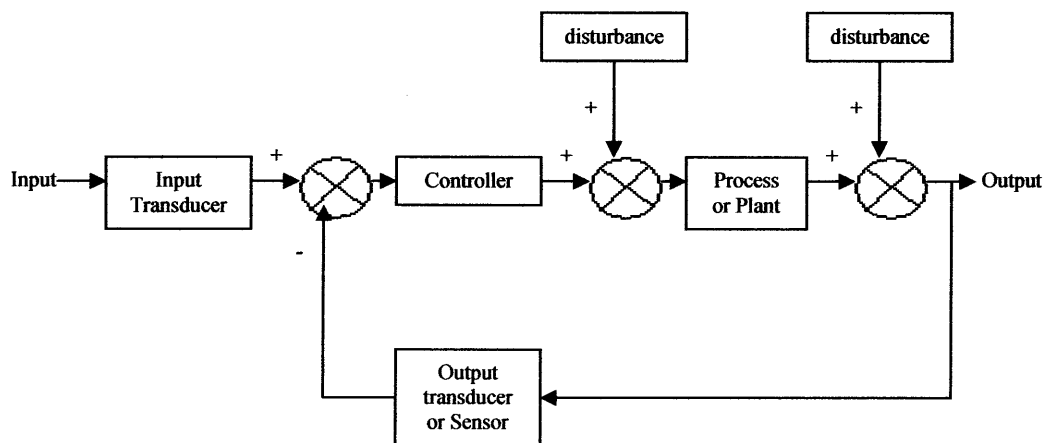


**Figure 2.3** A Block Diagram of Open-Loop System

### 2.2.3 Closed-Loop Control Systems

The closed-loop system compensates for disturbances by measuring the output response, feeding that measurement back through a feedback path, and comparing that response to the input at the summing junction. If there is any difference between the two responses, the system drives the plant, via the actuating signal, to make a correction. If there is no difference, the system does not drive the plant, since the plant's response is already the desired response [1].

Closed-Loop System is a control system that uses feedback. A sensor continually monitors the output of the system and sends a signal to the controller, which makes adjustment to keep the output within specification. This automatic closed-loop configuration performs the self-correcting function by employing a feedback loop to keep track of how well the output actuator is doing the job it was commanded to do. A feedback signal is produced by a sensing component that measures the status of the output. This signal is then fed back to the controller. Since the controller knows what the system is actually doing, it can make any adjustments necessary to keep the output where it belongs [2].



**Figure 2.4** A Block Diagram of Closed-Loop System

### 2.3 Inverted Pendulum System

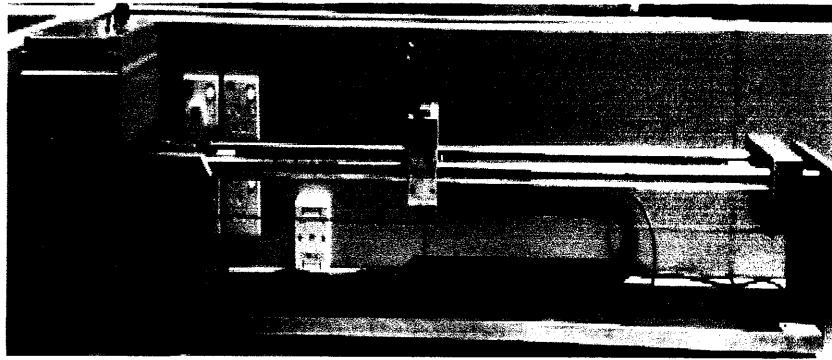
The inverted pendulum system is a challenging problem in term of controlling a system. This is due to the nonlinear system, unstable system and non-minimum phase characteristic presented by an inverted pendulum system. The control objectives of a inverted pendulum system are to balance the pendulum to upright position and at the same time place the cart at a desired position.

Furthermore, the inverted pendulum system were motivated by the need to design controllers to balance rockets during vertical take-off. At the instance of time during launch, the rocket is extremely unstable. Similar to the rocket at launch, the inverted pendulum requires a continuous correction mechanism to stay upright, since the system is unstable in open loop configuration. This problem can be compared to the rocket during launch [2].

A simple linear pendulum has long proved a useful model for more complicated physical systems, and its behaviour in a small amplitude limit provides a realistic yet solvable example for student in introductory classes. While the force-free, frictionless pendulum can be solved exactly for all amplitudes in terms of elliptic integrals, the solution is hardly illuminating, rarely found useful, and when damping and external driving are included, the equation of motion become intractable (except for small oscillations) [2].

Many researches were carried out researches to control an inverted pendulum system. Various control strategies have been proposed by numerous researchers for controlling the inverted pendulum such that the system is stable as well as the cart is move to the desired position. The approaches varied from the classical control to the advanced control. PID controller was design to control the inverted pendulum problem [3]. The drawback of the PID controller is it only can control for a Single-Input-Single-Output (SISO) system. It means that the PID controller only can control either for the position of the cart or angle of the ball at a one time [3].





**Figure 2.5** Inverted Pendulum Model

## **CHAPTER III**

### **PROJECT METHODOLOGY**

#### **3.1 Introduction**

To complete this project, flow chart was used as a method and approach to solve the problems. Before choosing the method and approaches, the factor considered is to list the process sequence for completing this project in a clear definition. The reason for using this method was because the process flow can be observed clearly and additional information can be added where possible. This chapter explains about the methodology that has been used in order to complete this project. This project used flow chart as a methodology. The reason of using flowchart as a methodology was because the sequence of this project can be seen clearly by steps.

#### **3.2 Project Methodology**

First task is to establish a mathematical model for Inverted Pendulum System. The reason for obtaining the mathematical model is to know the variable that will be used for the systems. The mathematical model was based on the free body diagram of the system. It is required in order to design the controller and obtaining equation for the system to get the transfer function.

The second method is linearise the mathematical model for designing PID controller. Linearization is required to make it easy for model based controller configuration.

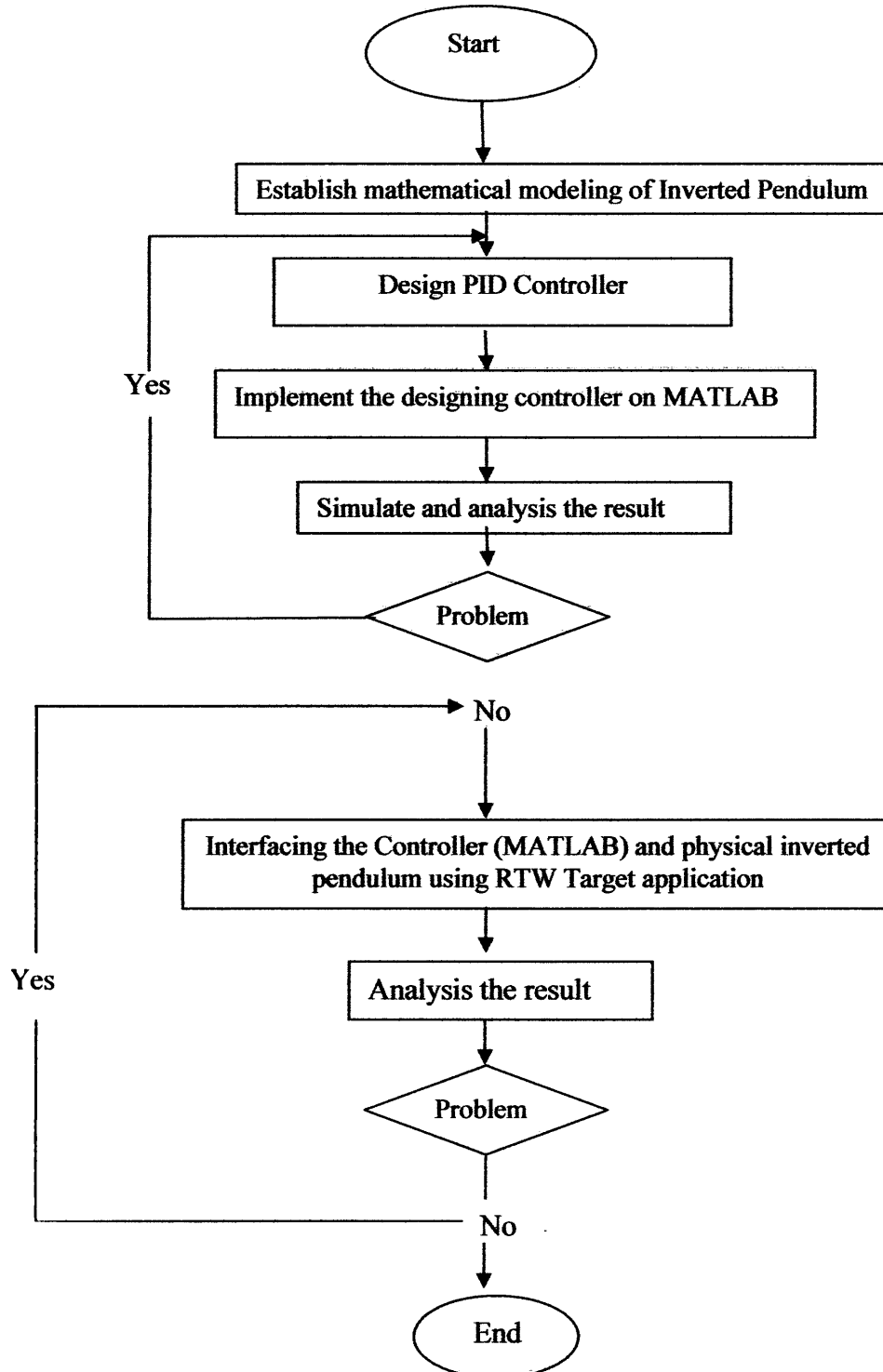
Third task is to design the PID controller for the system using the MATLAB software. For this task, the open loop and closed loop system will be design using this software. This is to get the Simulink result for the inverted pendulum system. If any problem occurs during the simulation, we will reconstruct the circuit again until we get the result that approximately with our requirement.

Seventh task is to interfacing the controller in the MATLAB and physical inverted pendulum using RTW Target application. The result that we get will be analyse and if the result is not satisfy with the requirement, we will try to interfacing the system again until we get the satisfy result.

The last task is to perform the comparison between the Simulink result and the real inverted pendulum result for controlling system. This comparison is important in order to prove that the inverted pendulum can be control.

### 3.3 Flow Chart Model

In order to complete the task, a block diagram of project methodology is included. The block diagrams of flow chart used were shown as in **Figure 3.1**:



**Figure 3.1** Project Methodology Flow Chart

## **CHAPTER IV**

### **MATHEMATICAL MODEL**

#### **4.1 Introduction**

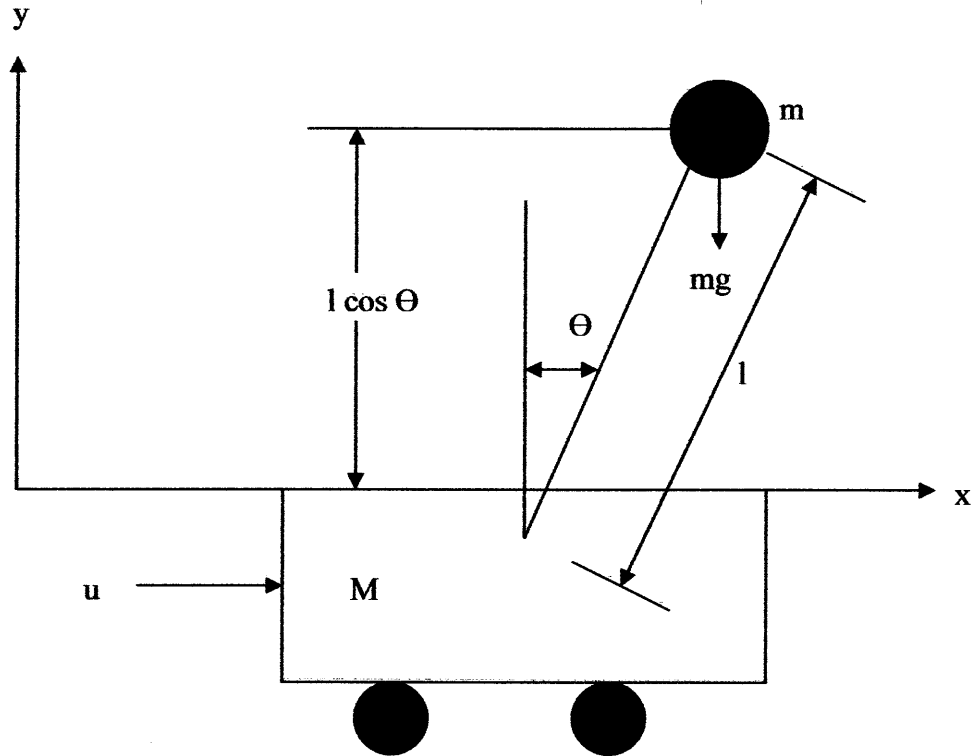
In this chapter, it will discuss about the mathematical model or mathematical equation for the system which have been derived from the free body diagram of the system. The mathematical model was divided into subtopic which is free body diagram, physical data, general equation and mathematical equation for the system. The free body diagram will show the representation of variables for each data required. Physical data shows the assumption of variables used. The mathematical equation for the system shows the derivation of general equation to obtain the transfer function.

#### **4.2 Mathematical Model**

The mathematical model represents a process, device or concepts of variables which were defined to represents the inputs, outputs and a set of equations describing the interaction of these variables.

### 4.2.1 Free Body Diagram

Firstly, to design a controller, a mathematical model of the system must be established. Figure 4.1 shows the Free Body Diagrams of the system.



**Figure 4.1** Free body diagrams of the system

### 4.2.2 Physical Data

It is assumed that the system starts at equilibrium, and experiences an impulse force of 1N. The pendulum should return to its upright position within 20 seconds, and cannot move than 0.05 radians away from the vertical. This project will attempt to control both the pendulum's angle and the cart's position and a step input will be applied to the cart. The overshoot of the pendulums will be limited to 20% and the settling time is 20 seconds.

The design requirements for this system are:

- i. Pendulum angle never more than 0.05 radians from the vertical.

- ii. Settling time for position of cart,  $x$  and angle of pendulum,  $\theta$  is less than 20 seconds.
- iii. Overshoot of angle less than 20%

The cart with an inverted pendulum, shown in free body diagram is bumped with an impulse force,  $F$ . Dynamic equations of motion for the system will be determined, and the equation is then, linearized about the pendulum's angle, where  $\theta = \pi$  or in other words, assume that the pendulum does not move more than a few degrees away from the vertical which chosen to be at an angle of  $\pi$ . First, a set of constant/variable were assumed as follows:

**Table 4.1** Representative of constant/variable for the system

Constant/Variable	Description	Value
M	Mass of the cart	0.85 kg
m	Mass of the pendulum	0.19 kg
l	Length to pendulum centre of mass	0.07 m
g	Gravity	9.81
F	Force applied to the cart	-
X	Cart position coordinate	-
$\theta$	Pendulum angle from vertical	-

### 4.2.3 General Equation

From the free body diagram of the system, a force balance in the  $x$ -direction gives that the mass times acceleration of the cart plus the mass times the  $x$ -directed acceleration of the point mass must equal the external force on the system. Referring for the Figure 4.1. The equation of motion for the cart is as follows:

$$M \frac{d^2}{dt^2} x + m \frac{d^2}{dt^2} x_G = u \quad [4.1]$$

Expanding the equation:

$$(M + m)\ddot{x} - ml \sin \theta \dot{\theta}^2 + ml \cos \theta \ddot{\theta} = u \quad [4.2]$$

The resultant equation for the balance force components:

$$(F_x \cos \theta) \mathcal{V} - (F_y \sin \theta) \mathcal{V} = (mg \sin \theta) \mathcal{V} \quad [4.3]$$

#### 4.2.4 Mathematical Modelling For the System

Nonlinear State Equations for inverted pendulum system by expanding the equation 4.3, we get:

$$m \ddot{x} \cos \theta + ml \ddot{\theta} = mg \sin \theta \quad [4.4]$$

Substituting the equation 4.4 into equation 4.2, we get:

$$(M + m - m \cos^2 \theta) \ddot{x} = u + ml \sin \theta \dot{\theta}^2 - mg \cos \theta \sin \theta \quad [4.5]$$

$$(ml \cos^2 \theta - (M + m) \mathcal{V}) \ddot{\theta} = u \cos \theta - (M + m)g \sin \theta + ml \cos \theta \sin \theta \dot{\theta}^2 \quad [4.6]$$

Substituting the state variable for the state vector  $z$  into equation 4.5 and 4.6

$$z_1 = \theta$$

$$z_2 = \dot{\theta}$$

$$z_3 = x$$

$$z_4 = \dot{x}$$

we get:

$$\frac{d}{dt} \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} z_2 \\ \frac{u \cos z_1 - (M + m)g \sin z_1 + ml(\cos z_1 \sin z_1)z_2^2}{ml \cos^2 z_1 - (M + m) \mathcal{V}} \\ z_4 \\ \frac{u + ml(\sin z_1)z_2^2 - mg \cos z_1 \sin z_1}{M + m - m \cos^2 z_1} \end{bmatrix} \quad [4.7]$$



Non-linear state equation:

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} \frac{u \cos z_1 - (M+m)g \sin z_1 + ml(\cos z_1 \sin z_1)z_2^2}{ml \cos^2 z_1 - (M+m)} \\ \frac{u + ml(\sin z_1)z_2^2 - mg \cos z_1 \sin z_1}{M+m - m \cos^2 z_1} \end{bmatrix} \quad [4.8]$$

Linear state equation by using the physical Table 4.1

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 186.3076 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3.2315 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ -16.8067 \\ 0 \\ 1.1765 \end{bmatrix} u \quad [4.9]$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u \quad [4.10]$$

## CHAPTER V

### CONSUMPTION OF CONTROLLERS

#### 5.1 Introduction

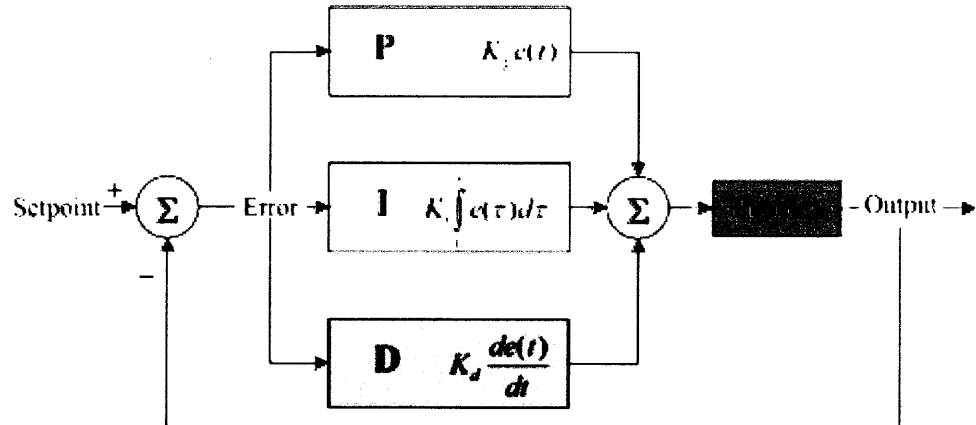
In this chapter will discuss about the controller that being use in this inverted pendulum system. The controller that been use is the PID controller. For PSM 1, we have done the Simulink control to the system using the MATLAB software. For next step of the project is to interface the controller to the real system of the inverted pendulum. This step will be done at PSM 2 and will be explain at chapter VI.

#### 5.2 PID Controller

The PID algorithm is the most popular feedback controller used within the process industries. It has been successfully used for over 50 years. It is a robust easily understood algorithm that can provide excellent control performance despite the varied dynamic characteristics of process plant. Even PID controller is commonly used; they are not always used in the best way because the controller is often poorly tuned. It is quite common that derivative action is not used. The reason is that it is difficult to tune three parameters by trial and error. Then PID can be improved with automatic tuning, automatic generation of gain schedules and continuous adaptation.

As the name suggests the, PID algorithm consists of three basic modes, the Proportional mode, the Integral and the Derivative modes. When utilizing this

algorithm it is necessary to decide which modes are to be used (P, I or D) and then specify the parameters (or settings) for each mode used. Generally three basic algorithms are used P, PI or PID.



**Figure 5.1** PID system configurations

For proportional term, is to handle the immediate error, the error is multiplied by a constant  $P$  (for "proportional"). Note that when the error is zero, a proportional controller's output is zero. However, the  $P$  controller cannot always guarantee that the set point will be reached if the set point is not fixed in time. The phenomenon called as a Steady State Error. To fix this, Integral and Derivative controllers are required. Next for integral term is to learn from the past the error is integrated and multiplied by a constant  $I$ . Without integral term, a PID controller cannot eliminate error if the process requires a non-null input to produce the desired set-point. Lastly, for derivative term is to anticipate the future, the first derivative (the slope of the error) over time is calculated and multiplied by another constant  $D$ .

### 5.2.1 The Characteristics of P, I and D Controllers

A proportional controller ( $K_p$ ) will have the effect of reducing the rise time and will reduce, but never eliminate, the steady-state error. An integral control ( $K_i$ ) will have the effect of eliminating the steady-state error, but it may make the transient response worse. A derivative control ( $K_d$ ) will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response. Effects of each of controllers  $K_p$ ,  $K_d$ , and  $K_i$  on a closed-loop system are summarized in the table shown below.

**Table 5.1** Effect of increasing  $K_p$ ,  $K_i$  and  $K_d$

<b>CL RESPONSE</b>	<b>RISE TIME</b>	<b>OVERSHOOT</b>	<b>SETTLING TIME</b>	<b>S-S ERROR</b>
<b><math>K_p</math></b>	Decrease	Increase	Small Change	Decrease
<b><math>K_i</math></b>	Decrease	Increase	Increase	Eliminate
<b><math>K_d</math></b>	Small Change	Decrease	Decrease	Small Change

Note that these correlations may not be exactly accurate, because  $K_p$ ,  $K_i$ , and  $K_d$  are dependent of each other. In fact, changing one of these variables can change the effect of the other two. For this reason, the table should only be used as a reference when you are determining the values for  $K_i$ ,  $K_p$  and  $K_d$ .

### 5.2.2 Method of Tuning the PID Controller

Tuning a control loop is the adjustment of its control parameters ( $K_p$ ,  $K_i$  and  $K_d$ ) to the optimum values for the desired control response. The optimum behavior on a process change or setpoint change varies depending on the application. Some processes must not allow an overshoot of the process variable from the setpoint. Other processes must minimize the energy expended in reaching a new setpoint. Generally stability of response is required and the process must not oscillate for any combination of process conditions and setpoints.

There are several methods for tuning a PID loop. The choice of method will depend largely on whether or not the loop can be taken "offline" for tuning, and the response speed of the system. If the system can be taken offline, the best tuning method often involves subjecting the system to a step change in input, measuring the output as a function of time, and using this response to determine the control parameters.

If the system must remain online, one tuning method is to first set the  $I$  and  $D$  values to zero. Increase the  $P$  until the output of the loop oscillates, then the  $P$  should be left set to be approximately half of that value for a "quarter amplitude decay" type response. Then increase  $I$  until any offset is correct in sufficient time for the process. However too much  $I$  will cause instability. Finally, increase  $D$ , if required, until the loop is acceptably quick to reach its reference after a load disturbance. However too much  $D$  will cause excessive response and overshoot. A fast PID loop tuning usually overshoots slightly to reach the setpoint more quickly; however, some systems cannot accept overshoot, in which case a "critically damped," tune is required, which will require a  $P$  setting significantly less than half that of the  $P$  setting causing oscillation.

Basically, the tuning method of PID controller can be categorized into two. First are closed-loop tuning methods and second are open-loop tuning methods. For closed-loop tuning methods, tuning will be accomplished while the controller is operating in the closed loop that is while it is in the automatic state. There are few