

**VIBRATION ANALYSIS OF A PLATE WITH
THREE PARALLEL HORIZONTAL CRACKS**

TIEW YONG LEEK

B141710074

BMCG

alwin1234561@gmail.com

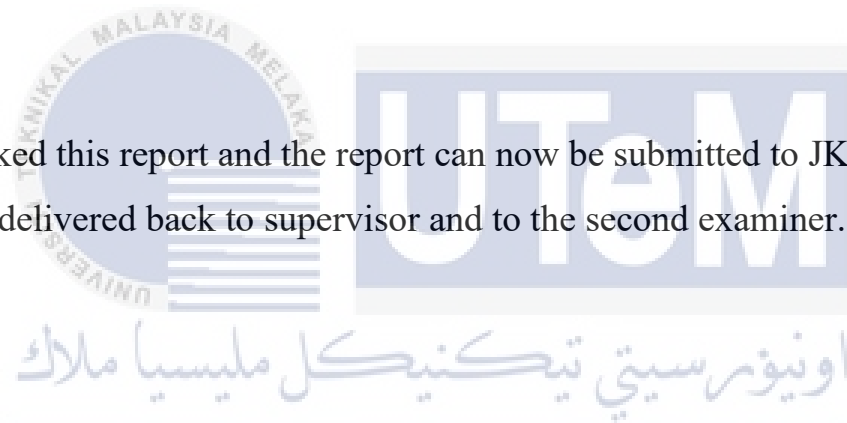


**Faculty of Mechanical Engineering
Universiti Teknikal Malaysia Melaka**

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SUPERVISOR'S DECLARATION

I have checked this report and the report can now be submitted to JK-PSM to be delivered back to supervisor and to the second examiner.



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Signature

Name of Supervisor :.....

Date :.....

APPROVAL

I hereby declare that I have read this project report and in my opinion this report is sufficient in terms of scope and quality for the award of the degree of Bachelor of Mechanical Engineering.

	Signature :
	Name of Supervisor :
	Date :

اونيورسيتي تیکنیکل ملیسيا ملاک

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Abstract

In this report, the equation of motion of vibration analysis of a thin isotropic plate with three parallel horizontal cracks for a given set of boundary conditions is developed. Kirchoff's classical plate theory is applying to obtain the principles equilibrium and equation motion of an isotropic plate. Crack formulation on the plate is then presented from the application of Line Spring Model (LSM). After, Galerkin's method is used to find the solution of governing equation by applying weighted residual technique and convert the transverse function into time dependent function. The governing equation is then finalized by applying equation of three boundary conditions. Next, the Berger's formulation is applied to transverse the deflection terms of in-plane forces into a non-linear ordinary equation. The natural frequency of the plate is then become a variable by parameters of half crack length, length of plate, boundary conditions and number of cracks (single and multiple). The mathematical model result is presented in term of natural frequency are totally affect by half crack length and number of cracks and the result is validated by comparison with finite element analysis result.

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ABSTRAK

Dalam report ini, persamaan analisis gerakan getaran plat isotropik nipis dengan tiga retakan mendatar selari untuk satu set syarat sempadan telah dikembangkan. Teori plat klasik Kirchoff diterapkan untuk mendapatkan keseimbangan prinsip dan gerakan persamaan plat isotropik. Formulasi retakan pada plat kemudian disajikan dari aplikasi Line Spring Model (LSM). Setelah itu, kaedah Galerkin digunakan untuk mencari jalan penyelesaian persamaan dengan menerapkan teknik baki bobot dan mengubah fungsi melintang menjadi fungsi bergantung pada waktu. Persamaan yang mengatur kemudiannya diselesaikan dengan menerapkan persamaan tiga syarat sempadan. Seterusnya, rumusan Berger digunakan untuk meleraiakan istilah pesongan daya dalam satah menjadi persamaan tidak linear. Kekekapan semula jadi plat kemudian menjadi pemboleh ubah mengikut parameter panjang retak separuh, panjang plat, keadaan sempadan dan bilangan retakan (tunggal dan berganda). Hasil model matematik yang dikemukakan dari segi frekuensi semula jadi benar-benar dipengaruhi oleh panjang retak separuh dan jumlah retakan dan hasilnya disahkan dengan perbandingan dengan hasil analisis elemen hingga.

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List of abrevation

CCSS Opposite edges simply supported.

CCFF Adjacent edges clamped.

SSSS Simply supported on all edges.

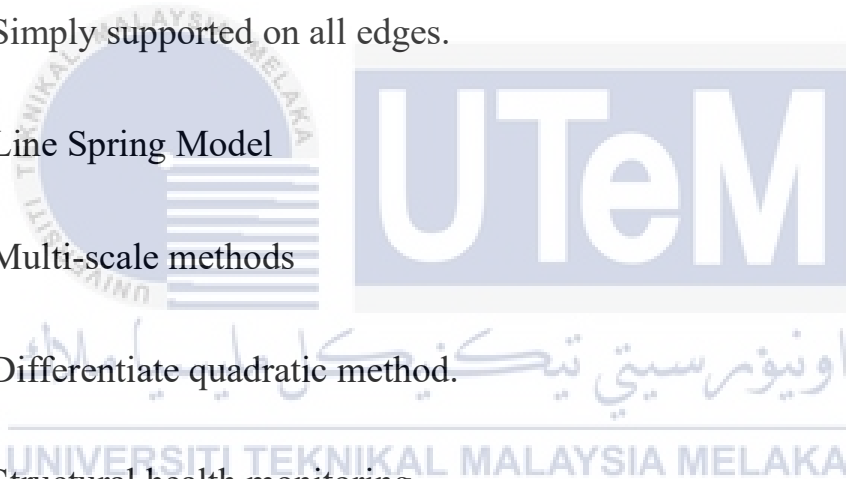
LSM Line Spring Model

MMS Multi-scale methods

DQM Differentiate quadratic method.

SHM Structural health monitoring

FEM Finite Element



LIST OF SYMBOLS

Q	Force per unit length
M	Bending moment per unit length
$2a_x$	Crack length on each crack
\bar{M}	Bending moment per unit length due to crack
q_z	Lateral force
ρ	Density of plate
D	Flexural rigidity
ν	Poisson ratio
n	In-plane force per unit length
\bar{n}	In-plane force per unit length due to crack
σ	Bending stress
h	Thickness of plate
$a_{tt}, a_{bb}, a_{tb}, a_{bt}$	Crack compliance coefficients
$\bar{\sigma}_{mn}$	Normal tensile stress at crack location
\bar{m}_{mn}	Bending moment at crack location
$w(x, y, t)$	Time dependent transverse function
X_i	Modal function in x direction of crack plate
Y_j	Modal function in y direction of crack plate
ε	Strain
E	Young modulus
A_{ij}	Arbitrary amplitude
$\Psi_{ij}(t)$	Time dependent modal coordinate
l	Length of plate
P_{ij}	Complex in-plane force function
Ω_{ij}	Excitation frequency

CHAPTER 1

INTRODUCTION

1.1 BACKGROUND



Safety is one of the most important for every auto-mobile, especially airplane. During the flight, the airplane is not allowable landing until the destination is reached, thus the appearing failure of any part of the airplane is very dangerous. “A short analysis may give a long-term safety journey for life”. Consider many engineering components used in the aerospace industries. The component’s design should be more investigate and analysis such as a vibrating structure under many random cycles of pressure.

Sandwich panels are a structure made by a light-weight core, and a flat surface layer bonded to each lateral. In this case, light weight is one of the main important matter on the sandwich panels. Its light-weight properties are function to minimize the overall weight of the aerospace’s body in order built up a highly thrust force for the transport.

The one of the main applications of sandwich panel is on aircraft and called as aircraft panels. The outermost layer of the aircraft panel of the structure is made from a aluminum plate which considers a light weight, thin and high durability which can perform well during the flight, take-off or landing. It performs a highly resistance on surface wear, pressure attack, or damage and having a better lifting during the flight.

Vibration is a mechanical phenomenon in which a structure of the component undergoes oscillations that occur about an equilibrium point. Generally, vibration will bring a lot of negative effect that occur on every component especially in the lightweight structure with a thin geometric plate. The issue of applying the lightweight structure as the material may lead to produce a high vibration on its body. Over time, the occurring of the high vibrational effects on the lightweight structure, especially on the wing panels, may have long-term as well as short-term damaging or cracks. These effects may cause the wings produce the difference output on both side of the aerospace wings such as unable to balance the body of the aerospace. Thus, the condition of the plate should be analyzed and compared the health of the plate in every maintenance schedule.

Cracks happen on the aircraft wing panels is very dangerous that may bring bad accident to us. From the latest news in the July 2019, many cracks on the aircraft wings were found on an Airbus A380 superjumbo jets. The aviation company was requested to have a full-body analysis and checking for 25 of the early production of Airbus A380 superjumbo jets to ensure the safety of their aircraft.

In this project, we concentrate and evaluate on the aircraft wing structures that had been constructed as an isotropic plate with the presence of the vibration behavior.

The plate panels are generally under transverse pressure on the outermost layer of aircraft wings and are subjected to natural and shear forces occurred in the plate plane. The plate panels may not perform as planned if they contain even a small crack will result in a complete loss of balance and cause failure.

1.2 Problem Statement

The rapid advancement of technologies has changed the way of every industry operates, where the structures of the components become lighter and thinner to be designed. One of the examples is the isotropic thin plate on the aircraft wing panels. However, the trend for the structure of the plate may lead to unwanted instances of high vibration and caused the crack to occur on the plate. In this case, it is very important to detect any defects or failures that occur inside the plate or on the surface of plate at the early stage of development. Vibration health monitoring is one of the methods used to detect the defects, it defined as damage detection and detect the degradation of mechanical integrity by using the vibration signal to generate the data. One of the examples is install the vibration sensor, this may give the benefit that engineer allowed to spend fewer time taking manual measurement and having more time focus on problems that could lead to downtime or equipment failure. From the previous analysis on the research, many of the researchers only focus on the single type of the crack problem such as the crack-oriented surface or crack depth on the fixed axis. In the phenomena every crack has their different behavior, it is impossible that there will be only a single crack that was happen on one time. The crack on the plate might occurring

in many minor cracks at the same time. In this project, we are going to enhance the crack formulation developed by Ismail (2012) and Joshi (2014) and analyze the crack of the plate with three parallel cracks on the plate surface.

1.3 Objectives:

The objective of this project are as follows:

1. To develop mathematical modelling of a plate with a three parallel cracks with horizontal to x-direction.
2. To study the influence of the parallel cracks on the vibration characteristics of the plate
3. To verify the developed model through comparison of the results with finite element method

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1.4 Scopes:

The scopes of this project are:

1. An aircraft panel structure is model as an isotropic thin flat plate with a multiple cracks. The cracked plate is subjected to horizontal and parallel to each other with the chosen CCSS, SSSS and CCFF boundary conditions and the derivation of mathematical modelling is based on classical plate theory.
2. Type of material used in this investigation is an aluminum alloy of 5083 grade with dimension of plate study is limited to three plates with plate A (0.5 x 1.0 m x 0.01 m), plate B (1.0m x 1.0m x 0.01m) and plate C (0.5m x 0.5m x 0.01m).
3. Load acting on the surface of the plate, q is assumed to be the same as previously used, which is 10 N.
4. The crack is located at the center, one-third and two-third from the y-axis of the plate and at all are in the middle on the x-axis of the plate with having 0.01 m and 0.025 m half-crack lengths.

CHAPTER 2

Literature review

2.1 Plate theory

Plate theories are numerical representation of the fundamentals of flat plates in solid mechanics that take on beam theory. Plates are defined as a flat structural with a thin structure compared to its planar sizes. Basically, the usual thickness to width ratio was about $\frac{h}{w} \ll 0.1$ of a plate structure. The thin behavior of the plate was helpful which able to reduce the complete three-dimensional structure mechanic issue to a two-dimensional problem by applying Line Spring Model. The purpose of the plate theory is to determine the deflection and stresses in a plate with a force was applied on it.

From the 19 century, Kirchoff-Love theory was the most famous used in engineering. It was a two-dimensional numerical model that was used to measure the stresses and deflection in an isotropic plate which obtained to forces and moments. Kirchoff (1888) was using an assumption to develop Kirchoff-Love theory by expanding the Euler-Bernuolli beam theory. In

Kirchoff's assumption, it was suggested that the mid-surface plane can reduce the three-dimensional plates into two-dimensional plates.

For the following kinematics assumption are as below:

- straight lines normal to the mid-surface remain straight after deformation.
- straight lines normal to the mid-surface remain normal to the mid-surface after deformation.
- the thickness of the plate stays remain during a deformation.

Since the Kirchoff-love theory was minimized the numerical equation of plate. These assumptions had been widely used for plate analysis and had been further discuss on 19-21 century.

2.2 Past research regarding to plates.

2.2.1 Vibration of plate

Leissa (1969) was published a book "Vibration of plate". The fundamental equation of plate theory had been discussed in term of elastic plate. The classical equation was form by using Laplacian operator, the relationship of bending and twisting moment with displacement. By obtain the general solution of classical equation, fourier series was assumed into a variable. Besides that, the classical equation had been applied into many forms of plate such as polar, elliptical and rectangular. After that Leissa (1969) was undergoing a series of study on the analysis the plate in polar, elliptical and rectangular form. Leissa (1969) was analysis along 21-types of boundary condition which occur on the plate such as the simply supported boundary