CONTROL OF AN INVERTED PENDULUM:

Comparison between Bode Plot Techniques and Root Locus technique when applied to lead/lag compensator

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DEDICATION

Dedicated to my beloved family (En. Ashruddin, Pn. Maimunah, Ashril, Ashraf, Ashraz, Ashira Firadila) and friends

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ABSTRACT

This project is about control of an Inverted Pendulum system where a Bode Plot technique and Root Locus technique will be compared to see which one is better in terms of performance when applied to a controller. In this case, a lead/lag controller is chosen to perform the application. Bode plot is a plots of frequency response where the gain and phase are displayed in separate plots. The root locus is a way of presenting graphical information about a system's behaviour when the controller is working and is a widely used tool for design of closed loop systems. Lead compensator is a device that provides phase lead in it's' frequency response while lag compensator is a device that provides phase lag in it's' frequency response. This project deals with the Non-Linear system of an inverted pendulum system and it will control both the pendulum's angle and the cart's position. The task is a literature study, non-linear mathematical modelling, controller design and simulation study. The simulation is using MATLAB software.

ABSTRAK

Projek ini menerangkan tentang *Inverted Pendulum System* di mana teknik *bode plot* dan teknik *root locus* akan dibandingkan untuk menilai teknik yang lebih baik dari segi kemampuan apabila di aplikasikan pada *controller*. Untuk ini, lead/lag controller telah dipilih untuk menjalankan aplikasi ini. Bode plot merupakan plot bagi frequency response dimana gain dan phase di tunjukkan dalam plot yang berlainan. Root Locus merupakan satu kaedah untuk mewakili maklumat grafik bagi kelakuan system apabila controller berfungsi dan ianya sedang meluas digunakan untuk closed loop system. Lead compensator merupakan alat yang mewakilkan phase lead di dalam frequency response manakala lag compensator merupakan alat yang mewakilkan phase lag di dalam frequency response. Projek ini melibatkan sistem tidakk-linear bagi Inverted Pendulum dan ianya akan mengawal kedua-dua sudut pendulum dan kedudukan cart. Tugas ini melibatkan pembelajaran lisan, model matematik tidak-linear rekaan controller dan pembelajaran simulasi. Simulasi bagi projek ini menggunakan perisian MATLAB.

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CHAPTER I

INTRODUCTION

An inverted pendulum which also called a cart and pole consists of a thin rod attached at its bottom to a moving cart. Whereas a normal pendulum is stable when hanging downwards, a vertical inverted pendulum is inherently unstable, and must be actively balanced in order to remain upright, typically by moving the cart horizontally as part of a feedback system.

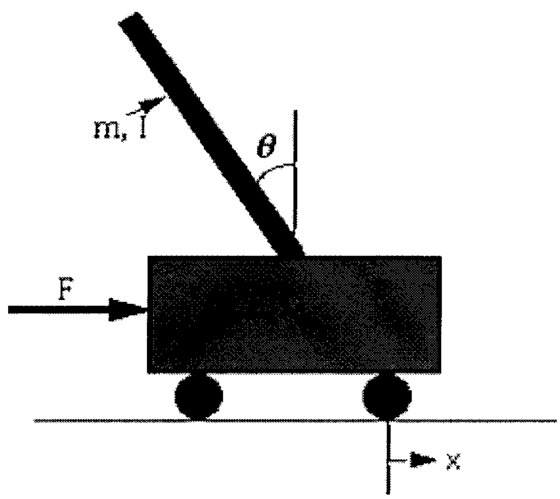

Figure 1.1: Cart with an Inverted Pendulum

The inverted pendulum is a classic problem in dynamics and control theory and widely used as benchmark for testing control algorithms such as PID controllers, neural networks, genetic algorithms and etc. Variations on this problem include multiple links, allowing the motion of the cart to be commanded while maintaining the pendulum, and balancing the cart-pendulum system on a see-saw. The inverted pendulum is related to rocket or missile guidance, where thrust is actuated at the bottom of a tall vehicle.

1.1 Project Introduction

This report will discuss the modelling and control of an inverted pendulum. The controller was implemented digitally using MATLAB, and it stabilized the rod of an inverted pendulum in the upward vertical position, while at the same time, keeping the cart in the middle of the track.

This project involved the design of a controller to stabilize an inherently unstable Inverted Pendulum. The system model used incorporated lead/lag compensator, as this type of control is determined by using Bode Plot and Root Locus technique to give the best performance. Modelling is performed in MATLAB, and then using Simulink, a control system is developed.

Initial research of the system was done using a web-based tutorial compiled by the University of Michigan and Carnegie-Mellon University. From this and other web-based research, it was determined that state space was the appropriate method for this system because the solutions shown in the PID, root locus and frequency response may not yield a workable controller for the inverted pendulum problem.

However, this project are using Transfer Function method where the x position of the cart was ignored and the pendulum can be stabilized in an inverted position if the x position is constant or if the cart moves at a constant velocity (no acceleration). Where possible in this project, the result will show what happens to the cart's position when the controller is implemented on the system. This research is important for a system such as rockets, the F-16 fighter and the newly invented Segway Scooter.

1.2 Project Objectives

To make sure this project work as planned, a few objectives were determined where these objectives will be followed as a guide through the whole completion process of this subject in order to achieve the desired output. These objectives were provided by sequence of project from beginning until the end of project. A detailed explanation for each objective will be discussed. The objectives to be achieved in this project are:

i. To develop the mathematical model of this system.
ii. To design a lead/lag compensator by using bode plot technique.
iii. To design a lead/lag compensator by using root locus technique.
iv. Comparison between bode plot and root locus techniques will be established.
v. To demonstrate design and analysis techniques by using MATLAB software through graph and animation of the system.

1.3 Problem Statement

The problem statement of this project is to control an inverted pendulum in order to control pendulum angle and the cart position. In this case, the problem was to control both of the pendulum angle and the cart position. The controller has to make sure that the pendulums should return to its upright position and the cart should stop at the desired position within the desired time. These two situations must happen in a same time.

1.4 Scope of Work

The scope of this project involve of:

i. Develop the mathematical model of an Inverted Pendulum system.
ii. Design a lead/lag compensator by using Bode Plot technique to control the system.
iii. Design a lead/lag compensator by using Root Locus technique to control the system.
iv. Compare the performance between Bode Plot and Root Locus techniques through graph and of the system by using MATLAB software.

1.5 Project Methodology

Basically, this project starts with designing the mathematical model for the system. The mathematical model was then, be linearized and the controller will be design based on the linearized mathematical model. In designing the controller, two methods will be used which is bode plot and root locus technique. A comparison will be established between these two techniques under Matlab-Simulink platform.

1.6 Report Structure

Technically, this report contains seven chapter where chapter one is an introduction of this report which explain briefly about Inverted Pendulum systems. This chapter includes the project introduction, project objective, problem statements, scope of work, brief explanation about project methodology and the last one is report structure.

Chapter two is the background study which explains about the perspective and methods being used by previous research and how far this project is related with current research and theory. This chapter shows previous theory and concept being used in solving the project problems. This chapter also stated the objective and problems of Inverted Pendulum system in a wider field of research. Every hypothesis which related with project methodology was stated clearly in this chapter.

Chapter three is project methodology which explaining the method being used to solve the problems. This chapter contains the methods used such as data collecting, process and analysis of data, model and flow chart. This chapter also include the factors for choosing those methods and finally, this chapter explain the advantage of the methods used compared to other methods.

Chapter four contains the mathematical model for the system and this chapter explain the method for obtaining transfer function for Inverted Pendulum system. The transfer function is used for bode plot and root locus technique in order to design the lead/lag compensator. The result for inverted pendulum system is shown by using figure, graph and table.

Chapter five is about consumption for the lead/lag compensator and each technique used which is bode plot and root locus. This chapter explain the theory and how it will be used to obtain the desired result for this project.

Chapter six is about result and discussion. This chapter shows how the compensator is designed by using both techniques. It shows how the data is been analyze and the steps taken to make sure the output results are achieving the objectives.

The last chapter is conclusion and suggestion where this chapter stated the summary for the project. This chapter contains the project analysis achievement and suggestion for future research.

CHAPTER II

BACKGROUND STUDY

The inverted pendulum problem is a widely-used benchmark for comparing different types of controllers, especially neural network controllers. It is a difficult nonlinear control task to balance an inverted pendulum. A linear controller can be implemented for the inverted pendulum, but it has limited range for the initial conditions and is sensitive to parameter changes.

2.1 Control System

A control system is an arrangement of physical components connected or related in such a manner as to command, direct, or regulate itself or another system. Control system has two important terms, which is define as input and output. The input is the stimulus, excitation or command applied to a control system. Typically from an external energy source, usually in order to produce a specified response from the control system. The output is the actual response obtained from a control system. It may or may not be equal to specified response implied by the input. [1]

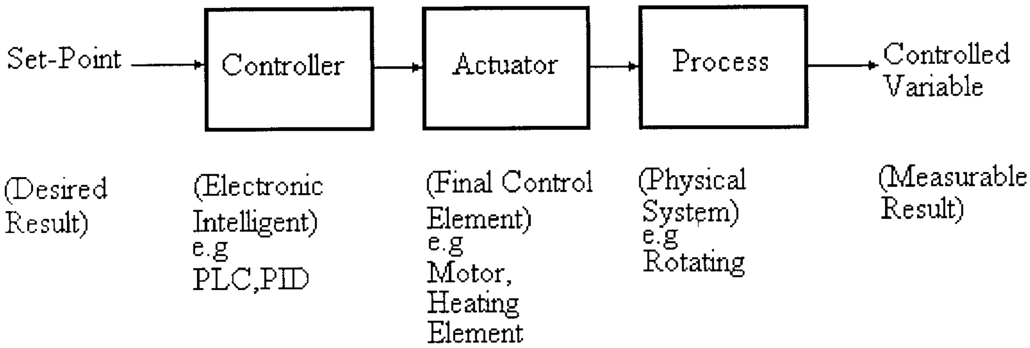

Figure 2.1: A Block Diagram of Control System

2.1.1 Open-Loop and Closed-Loop Control Systems

Control systems are classified into two general categories, open-loop and closed-loop systems. An open-loop system is one which the control action is independent of the output. A closed-loop control system is one in which the control action is somehow dependent on the output.

Open-Loop System is a control system that does not use feedback. The controller sends a measured signal to the actuator, which specifies the desired action. This type of system is not self-correcting. If some external disturbance changes the load on machine or process being performed, some degree of physical effort of human operator is required to make necessary modifications. The system manually controlled by the human. For an example, is the speed of a car controlling by a driver.

Closed-Loop System is a control system that uses feedback. A sensor continually monitors the output of the system and sends a signal to the controller, which makes adjustment to keep the output within specification. This automatic closed-loop configuration performs the self-correcting function by employing a feedback loop to keep track of how well the output actuator is doing the job it was commanded to do. A feedback signal is produced by a sensing component that measures the status of the output. This signal is then fed back to the controller. Since the controller knows what the system is actually doing, it can make any adjustments necessary to keep the output where it belongs. [1]

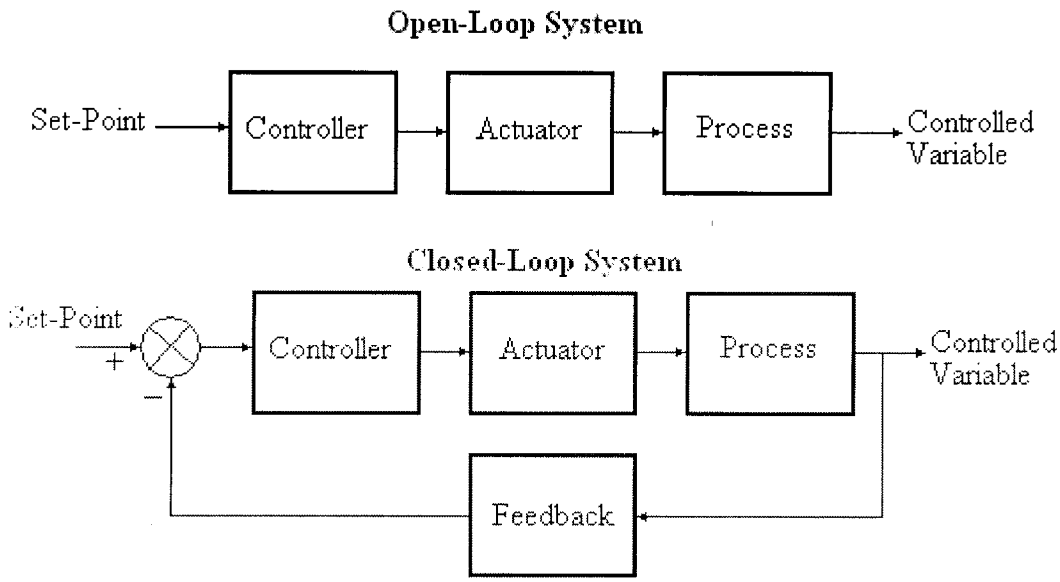

Figure 2.2: Open-Loop and Closed-Loop Control Systems

2.2 Project Literature Review

Early studies of the inverted pendulum system were motivated by the need to design controllers to balance rockets during vertical take-off. At the instance of time during launch, the rocket is extremely unstable. Similar to the rocket at launch, the inverted pendulum requires a continuous correction mechanism to stay upright, since the system is unstable in open loop configuration. This problem can be compared to the rocket during launch. Here, rocket boosters have to be fired in a control manner, to maintain the rocket upright.

A simple linear pendulum has long proved a useful model for more complicated physical systems, and its behaviour in a small amplitude limit provides a realistic yet solvable example for student in introductory classes. While the force-free, frictionless pendulum can be solved exactly for all amplitudes in terms of elliptic integrals, the solution is hardly illuminating, rarely found useful, and when damping and external driving are included, the equation of motion become intractable (except for small oscillations). With the advent of desktop computers, however, it has become possible to study in some detail in rich nonlinear dynamic of the damped, force-driven pendulum angle gain significant insight into its sensitivity to initial conditions for certain values of the system parameters.

Equal interesting, but much less studied in physical texts, is the state in which the plan pendulum can be stabilized in a small-oscillation mode about the vertical by means of sinusoidal driving force. Although the external forces can be applied to the pivot in either vertical of horizontal direction or in some liner combination, a vertically oscillating is pivot in the most common scenario as in this case.

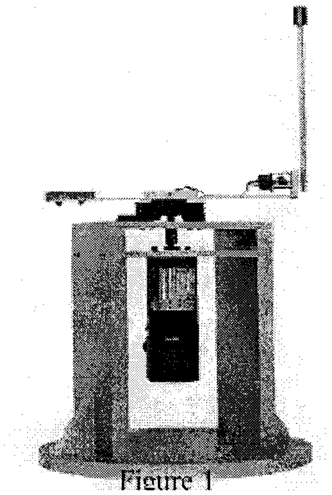

Figure 2.3: Inverted Pendulum Model

Figure 2.3 above shows the Inverted Pendulum hardware model which has been designed before. The purpose of modelling the hardware is to show clearly the motion of pendulum with cart. The performances of stabilization process can be seen clearly with hardware models.

CHAPTER III

PROJECT METHODOLOGY

3.1 Introduction

This chapter explains about the methodology that has been used in order to complete this project. This project used flow chart as a methodology. The reason of using flowchart as a methodology was because the sequence of this project can be seen clearly by steps.

3.2 Project Methodology

First task is to establish a mathematical model for Inverted Pendulum System. The reason for obtaining the mathematical model is to know the variable that will be used for the systems. The mathematical model was based on the free body diagram of the system. It is required in order to design the controller and obtaining equation for the system to get the transfer function.

The second method is linearise the mathematical model for designing controller. Linearization is required to make it easy for model based controller configuration.

Third task is to design a controller for the system by using bode plot technique. The next task is to design a controller for the system by using root locus technique. The controller that will be used is lead/lag controller. The reason for designing this controller is to control the pendulum angles within the desired angle with at the same time, placing the cart in the desired position.

Fifth task is to perform a comparison between bode plot and root locus technique when applied to a lead/lag controller. This comparison will show which technique is better in terms of performance.

The last task is to perform the comparison between bode plot technique and root locus technique to see which technique is better in terms of performance and stability. In order to complete the project, MATLAB software is used to design and simulate the output of controller for each technique.

In order to complete the task, a block diagram of project methodology is included. The block diagrams of flow chart used were shown as in figure 3.1 below:

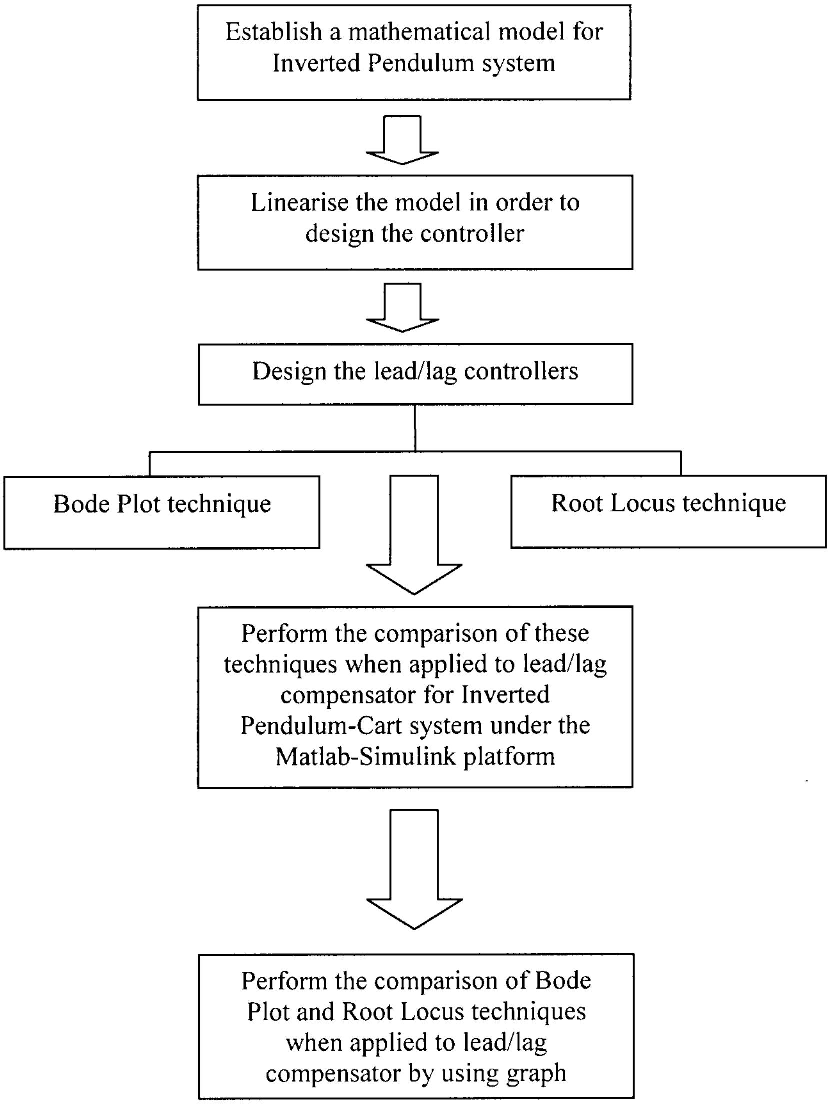

Figure 3.1: Project Methodology Flow Chart

CHAPTER IV

MATHEMATICAL MODEL

4.1 Introduction

This chapter will discuss about the mathematical model or mathematical equation for the system which have been derived from the free body diagram of the system. The mathematical model is divided into subtopic which is free body diagram, physical data, general equation and mathematical equation for the system. The free body diagram will show the representation of variables for each data required. Physical data shows the assumption of variables used. The mathematical equation for the system shows the derivation of general equation to obtain the transfer function. [2]

Before designing the controller of this Inverted Pendulum system, research and study must be done in order to understand the application and working principle of this system. From this, a mathematical model for the systems was derived. The reason for obtaining the mathematical equation was to examine whether the system is stable or not by proving it using a graph. This project was based on a research from Carnegie-Mellon University and University of Michigan as a reference. Each assumptions of value were referred to the value used in their research.

4.2 Mathematical Model

The mathematical model represents a process, device or concepts of variables which were defined to represents the inputs, outputs and a set of equations describing the interaction of these variables.

4.2.1 Free Body Diagram

Firstly, to design a controller, a mathematical model of the system must be established. Figure 4.2 below shows the two Free Body Diagrams of the system.

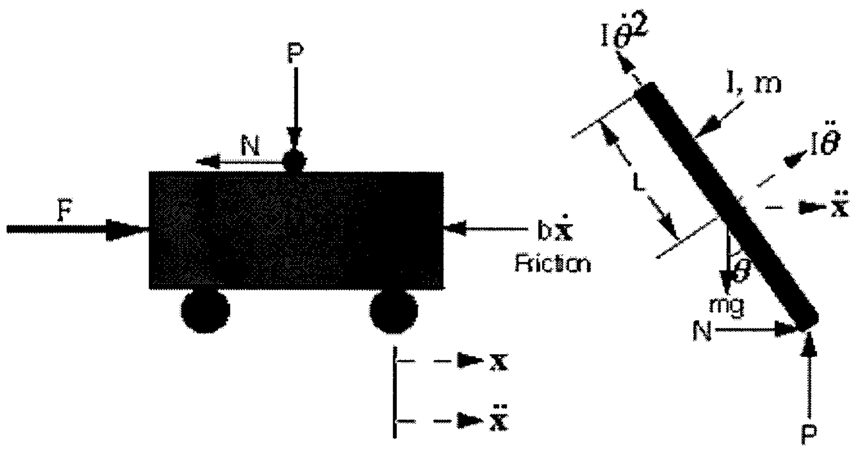

Figure 4.1: Free body diagrams of the system

4.2.2 Physical Data

It was assumed that the system starts at equilibrium, and experiences an impulse force of 1N. The pendulum should return to its upright position within 5 seconds, and never move more than 0.05 radians away from the vertical. This project will attempt to control both the pendulum's angle and the cart's position and a step input will be applied to the cart. The cart should achieve its desired position within 5 seconds and have a rise time under 0.5 seconds. Besides, the pendulums overshoot will be limited to 20 degrees or 0.35 radians and it should also settle within 5 seconds.

The design requirements for this system are:

i. Settling time of less than 5 seconds.
ii. Pendulum angle never more than 0.05 radians from the vertical.
iii. Settling time for x and theta of less than 5 seconds.
iv. Rise time for x of less than 0.5 seconds.
v. Overshoot of theta less than 20 degrees (0.35 radians).

The cart with an inverted pendulum, shown in free body diagram is bumped with an impulse force, F. Dynamic equations of motion for the system will be determined, and the equation is then, linearized about the pendulum's angle, where theta = Pi or in other words, assume that the pendulum does not move more than a few degrees away from the vertical which chosen to be at an angle of Pi. First, a set of constant/variable were assumed as follows:

Table 4.1: Representative of constant/variable for the system

Constant/Variable	**Description**	**Value**
M	Mass of the cart	0.5 kg
m	Mass of the pendulum	0.5 kg
b	Friction of the cart	0.1 N/m/sec
l	Length to pendulum center of mass	0.3 m
I	Inertia of the pendulum	0.006 kg*m^2
F	Force applied to the cart	-
X	Cart position coordinate	-
θ	Pendulum angle from vertical	-

4.2.3 General Equation

From the free body diagram of the system, a set of equations were derived. The modelling began with determining the equations of motion for the system. The equation of motion for the cart is as follows:

$$M\ddot{x} + b\dot{x} + N = F \quad [4.1]$$

The equation of motion for the pendulum is:

$$N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\theta^{2}\sin\theta \quad [4.2]$$

4.2.4 Mathematical Modelling For the System

The first equation is the motion for the cart in horizontal direction and the second equation is motion of the pendulum in vertical direction. Equations (4.1) and (4.2) are combined to form:

$$(M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^{2}\sin\theta = F \quad [4.3]$$

Equation (4.4) is derived when the forces are summed along the line of the pendulum, while equation (4.5) was derived by summing the moments around the centroid.

$$P\sin\theta + N\cos\theta - mg\sin\theta = ml\ddot{\theta} + m\dot{x}\cos\theta \quad [4.4]$$

$$-Pl\sin\theta - Nl\cos\theta = I\ddot{\theta} \quad [4.5]$$

These two equations are combined to form:

$$(I + ml^{2})\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta \quad [4.6]$$

Since MATLAB can only work with linear functions, this set of equations should be linearized. Using the following assumptions:

(a) cos(θ) = -1

(b) sin(θ) = $-\Delta$ and

(c) $(d\theta/dt)^2 = 0$

Equation (4.6) and equation (4.3) becomes:

$$(I + ml^2)\ddot{\phi} - mgl\phi = ml\ddot{x}$$ [4.7]

$$(M + m)\ddot{x} + b\dot{x} - ml\ddot{\phi} = u$$ [4.8]

Laplace transform of the system equations must be taken to obtain the transfer function of the linearized system equations analytically. The Laplace transforms of equations (4.7) and (4.8) are as follows:

$$(I + ml^2)\phi(s)s^2 - mgl\phi(s) = mlX(s)s^2$$ [4.9]

$$(M + m)X(s)s^2 - ml\phi(s)s^2 = U(s)$$ [4.10]

While solving equation (4.9) for X(s) gives:

$$X(s) = \left[\frac{I + ml^2}{ml} - \frac{g}{s^2}\right]\phi(s)$$

[4.11]

Re-arranging, the transfer function is:

$$\frac{\phi(s)}{U(s)} = \frac{\frac{ml}{q}s^2}{s^4 + \frac{b(I+ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{bmgl}{q}s} \quad [4.12]$$

Where,

$$q = \left[(M+m)(I+ml^2) - (ml)^2\right] \quad [4.13]$$

From the transfer function it can be seen that there is both a pole and a zero at the origin. These can be cancelled and the transfer function becomes:

$$\frac{\phi(s)}{U(s)} = \frac{\frac{ml}{q}s}{s^3 + \frac{b(I+ml^2)}{q}s^2 - \frac{(M+m)mgl}{q}s - \frac{bmgl}{q}} \quad [4.14]$$

After entering the value of assumption for this system, the equation would become:

$$\frac{\phi(s)}{U(s)} = \frac{4.545s}{s^3 + 0.1818s^2 - 31.18s - 4.455} \quad [4.15]$$

CHAPTER V

CONSUMPTION OF CONTROLLER

5.1 Introduction

This chapter will show how to design lead and lag compensators used for bode plot and root-locus design method. Lead and lag compensators are used quite extensively in control design. A lead compensator can increase the stability or response speed of a system, a lag compensator can reduce the steady-state error. Depending on the effect desired, one or more lead and lag compensators may be used in various combinations.

5.2 Lead-Lag Compensator

A lead-lag compensator is a component in a control system that improves an undesirable frequency response in a feedback and control system. It is a fundamental building block in classical control theory. Lead-lag compensators are everywhere influencing disciplines as varied as robotics, satellite control, automobile diagnostics, laser frequency stabilization, and many more. They are an important building block in analogue control systems, and can also be used in digital control. [2]

5.2.1 Theory of Lead-Lag Compensator

Both lead compensators and lag compensators introduce a pole-zero pair into the open loop transfer function. The transfer function can be written in the Laplace domain as:

$$\frac{Y}{X} = \frac{s - z}{s - p} \qquad [5.1]$$

Where X is the input to the compensator, Y is the output, s is the complex Laplace transform variable, z is the zero frequency and p is the pole frequency. The pole and zero are both typically negative. In a lead compensator, the pole is left of the zero in the Argand plane, $| z | < | p |$, while in a lag compensator $| z | > | p |$. A lead-lag compensator consists of a lead compensator cascaded with a lag compensator. The overall transfer function can be written as:

$$\frac{Y}{X} = \frac{(s - z_1)(s - z_2)}{(s - p_1)(s - p_2)} \qquad [5.2]$$

Typically $| p_1 | > | z_1 | > | z_2 | > | p_2 |$, where z_1 and p_1 are the zero and pole of the lead compensator while z_2 and p_2 are the zero and pole of the lag compensator. The lead compensator provides phase lead at high frequencies. It shifts the poles to the left, which enhances the responsiveness and stability of the system. The lag compensator provides phase lag at low frequencies which reduces the steady state error.

The precise locations of the poles and zeros depend on both the desired characteristics of the closed loop response and the characteristics of the system being controlled. However, the pole and zero of the lag compensator should be close together so as not to cause the poles to shift right, which could cause instability or slow convergence. Since their purpose is to affect the low frequency behaviour, they should be near the origin. [2]

5.2.2 Explanation

To begin designing a lead-lag compensator, an engineer must consider whether the system needing correction can be classified as a lead-network, a lag-network, or a combination of the two: a lead-lag network or also called as lead-lag compensator. The electrical response of this network to an input signal is expressed by the networks Laplace-domain transfer function, a complex mathematical function which itself can be expressed as one of two ways: as the Current-gain ratio transfer function or as the Voltage-gain ratio transfer function. Remember that a complex function can be in general written as $F(x) = A(x) + iB(x)$, where $A(x)$ is the "Real Part" and $B(x)$ is the "Imaginary Part" of the single-variable function $F(x)$.

The phase angle of the network is the argument of $F(x)$, in the left half plane this is $tan^{-1}(B(x) / A(x))$. If the phase angle is negative for all signal frequencies in the network then the network is classified as a lag network. If the phase angle is positive for all signal frequencies in the network then the network is classified as a lead network. If the total network phase angle has a combination of positive and negative phase as a function of frequency then it is a lead-lag network.

Depending upon the nominal operation design parameters of a system under an active feedback control, a lag or lead network can cause instability and poor speed and response times. A lead-lag compensator is then a component of the active feedback control system responsible for correcting the phase of a lead-lag network as desired to obtain a steady-state.

5.3 Bode Plot

A Bode Plot is a useful tool that shows the gain and phase response of a given LTI system for different frequencies. Bode Plots are generally used with the Fourier Transform of a given system.

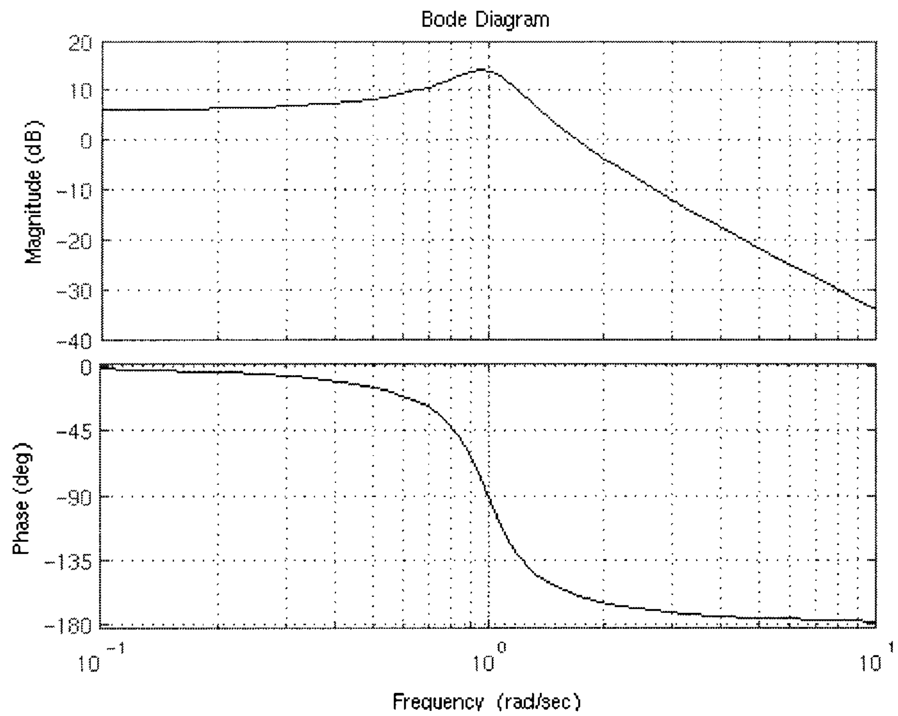

Figure 5.1: Bode Plot

Figure 5.1 above shows an example of a Bode magnitude and phase plot set. The Magnitude plot is typically on the top, and the Phase plot is typically on the bottom of the set. The frequency of bode plots are plotted against a logarithmic frequency axis. Every tick mark on the frequency axis represents a power of 10 times the previous value. For instance, on a standard Bode plot, the values of the markers go from (0.1, 1, 10, 100, 1000...) because each tick mark is a power of 10, they are referred to as a decade. Notice that the length of a decade increases as it move to the right on the graph. Bode Magnitude plot measures the system Input/Output ratio in special units called decibels. The Bode phase plot measures the phase shift in degrees typically, but radians are also used. [2]

5.3.1 Decibels

A Decibel is a ratio between two numbers on a logarithmic scale. A Decibel is not itself a number, and cannot be treated as such in normal calculations. To express a ratio between two numbers (A and B) as a decibel, the following formula was applied:

$$dB = 20\log\left(\frac{A}{B}\right) \quad [5.3]$$

Where, dB is the decibel result. Or, if we just want to take the decibels of a single number C, we could just as easily write:

$$dB = 20\log(C) \quad [5.4]$$

5.3.2 Frequency Response

If we have a system transfer function T(s), we can separate it into a numerator polynomial N(s) and a denominator polynomial D(s). We can write this as follows:

$$T(s) = \frac{N(s)}{D(s)} \quad [5.5]$$

To get the magnitude gain plot, we must first transition the transfer function into the frequency response by using the change of variables:

$$s = j\omega \quad [5.6]$$

From here, we can say that our frequency response is a composite of two parts, a real part, R and an imaginary part, X:

$$T(j\omega) = R(\omega) + jX(\omega) \quad [5.7]$$

5.3.3 Straight-Line Approximations

The Bode magnitude and phase plots can be quickly and easily approximated by using a series of straight lines. These approximate graphs can be generated by following a few short, simple rules. Once the straight-line graph is determined, the actual Bode plot is a smooth curve that follows the straight lines, and travels through the breakpoints. [4]

5.3.4 Break Points

If the frequency response is in pole-zero form:

$$T(j\omega) = \frac{\Pi_n |j\omega + z_n|}{\Pi_m |j\omega + p_m|} \quad [5.8]$$

We say that the values for all z_n and p_m are called break points of the Bode plot. These are the values where the Bode plots experience the largest change in direction. Break points are sometimes also called break frequencies, cuttoff points, or corner points.

5.3.5 Bode Phase Plots

Bode phase plots are plots of the phase shift to an input waveform dependant on the frequency characteristics of the system input. Again, the Laplace transform does not account for the phase shift characteristics of the system, but the Fourier Transform can. The phase of a complex function, in real + imaginary form is given as:

$$\angle T(j\omega) = \tan^{-1}\left(\frac{X}{R}\right) \quad [5.9]$$