

**WIDEBAND NONLINEAR DYNAMIC VIBRATION ABSORBER USING  
COMBINED SOFTENING AND HARDENING STIFFNESS MECHANISM**

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**A report submitted  
in fulfillment of the requirements for the degree of  
Bachelor of Mechanical Engineering**

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## DECLARATION

I declare that this project report entitled “Wideband Nonlinear Dynamic Vibration Absorber Using Combined Softening and Hardening Stiffness Mechanism” is the result of my own research except as cited in the references.

Signature: .....

Name: ALEX MADI CHUNG

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## **SUPERVISOR DECLARATION**

I hereby declare that I have read this project report and in my opinion this report is sufficient  
in terms of scope and quality for the award of the degree in Bachelor of Mechanical  
Engineering

Signature: .....

Supervisor: ASSOCIATE PROFESSOR DR. ROSZAIDI BIN RAMLAN

Date: .....

## **DEDICATION**

I dedicate this work to:

My parents,

Chung Ah Onn and Sylvia ak Abas

My siblings,

Supervisor who always give support and encouragement,

Associate Professor Dr. Roszaidi bin Ramlan

## ABSTRACT

The goal of the present work is to assess the performances of a nonlinear dynamic vibration absorber (NDVA) with combined softening and hardening stiffness mechanism in reducing the vibrations of a beam structure which is fixed at its end. In particular, three mechanisms of DVA are considered which are linear stiffness, softening stiffness and hardening stiffness. The purpose is to clarify if the combined of both nonlinear stiffness shows improvements with respect to the linear dynamic vibration absorber (LDVA) and single NDVA device. The nonlinear stiffness mechanism in NDVA is produced by the interaction force between the magnets. The strength of the nonlinearity can be varied by adjusting the separation gap between the magnets. Quasi-static measurement is used to estimate the linear and nonlinear stiffness of the system through force-deflection graph. Then, experimental results from dynamic measurement are presented to characterize the NDVA when the parameters are varied such as the separation gap and input excitation. The DVA is then attached on a beam structure where it is base-excited using a shaker and the reduction bandwidth on the beam response is measured. From this research, it is found that the bandwidth of a NDVA using combined softening and hardening stiffness mechanism is wider than the bandwidth of the LDVA and individual NDVA.

## ABSTRAK

*Matlamat penyelidikan ini dilakukan adalah untuk menilai prestasi penyerap getaran dinamik bukan linear dengan gabungan mekanisme memperlambatkan dan memperkeras untuk mengurangkan getaran struktur rasuk yang diapit pada hujungnya. Terdapat tiga mekanisme penyerap getaran dinamik iaitu proses linear, memperlambatkan dan memperkeras. Tujuannya adalah untuk menjelaskan sama ada gabungan kedua-dua mekanisme bukan linear menunjukkan penambahbaikan daripada mekanisme linear atau satu mekanisme bukan linear. Proses bukan linear dalam penyerap getaran dinamik dihasilkan oleh daya interaksi antara magnet. Kekuatan pengaruh nonlinear boleh diubah dengan mengubah jarak pemisahan antara magnet. Pengukuran kuasi statik digunakan untuk menganggarkan kekejuran linear dan bukan linear sistem melalui graf daya-defleksi. Kemudian, data eksperimen dari pengukuran dinamik telah dibentangkan untuk menunjukkan sifat prestasi penyerap getaran bukan linear apabila parameter berubah seperti jurang pemisahan antara magnet dan amplitud getaran. Peranti itu kemudian diletakkan pada struktur rasuk di mana ia telah menerima getaran daripada alat penggoncang dan kadar pengurangan pada tindakbalas rasuk telah diukur. Kajian ini telah mendapati bahawa prestasi penyerap getaran dinamik bukan linear dengan gabungan mekanisme memperlambatkan dan memperkeras adalah lebih baik daripada linear atau hanya satu mekanisme bukan linear.*

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## LIST OF ABBREVIATIONS

DVA	Dynamic Vibration Absorber
FRF	Frequency Response Function
LDVA	Linear Dynamic Vibration Absorber
NDVA	Nonlinear Dynamic Vibration Absorber
TMD	Tuned Mass Damper

## LIST OF SYMBOLS

$F$	-	Excitation force
$k$	-	Stiffness constant
$c$	-	Damping coefficient
$\zeta$	-	Damping ratio
$m$	-	Mass
$\omega$	-	Operating frequency
$\omega_n$	-	Natural frequency
$\Omega$	-	Frequency ratio
$x$	-	Displacement
$\dot{x}$	-	First derivatives of $x$
$\ddot{x}$	-	Second derivatives of $x$
$A$	-	Excitation amplitude
$b$	-	Harmonic term
$\beta$	-	Phase angle
$\mu$	-	Mass ratio
$\alpha$	-	Nonlinearity value
$L$	-	Length



$b$	-	Width
$d$	-	Thickness
$\rho$	-	Density
$E$	-	Modulus of rigidity
$A$	-	Area
$I$	-	Second moment of area

# CHAPTER 1

## INTRODUCTION

### 1.0 Background

Generally, a vibration is a periodic motion, i.e., a motion which repeats itself in all its particulars after a certain interval of time, called the period of the vibration (Den Hartog, 1956). Vibration is undesirable in most cases since it creates noise and damage a machine component. These effects motivate the engineers from different fields to find a method to control vibration levels since the demands of reliability, comfort, accuracy, and noise reduction are increasing. Vibration problems however are still encountered in this modern era mainly in applications like rotating machinery, construction, and transportation. One of the possible reasons is deviation from the usual operating range of the machine for example a sudden increase operating speed. This will increase the vibration amplitude of the machine and transmit the vibration force to the structure where it is placed. When the vibrating frequency of the machine reaches the natural frequency of the structure, resonance occurs. Resonance is the common cause of structural failure. Schilling (2013) reported in a news article that a building collapsed in Bangladesh was because of the vibration generated from sudden restart of huge generators and thousands of sewing machines after brief power outage. Therefore, it is crucial to master the knowledge of vibration in to encounter and control them.

The dynamic vibration absorber was first invented about a century ago by Frahm (1911). The purpose of the invention was to damp a vibration at resonance which arise in bodies subjected to certain period impacts. It is one of the very simple, low cost and efficient solution invented to dissipate excessive vibration energy. Until today, a lot of research have been conducted to improve the performance of dynamic vibration absorber and a new type of vibration absorber is keep increasing.

## **1.2 Problem Statement**

Vibration control measures can be classified into passive vibration control, active vibration control and semi-active vibration control. Passive vibration control is achieved by altering the mass, stiffness or damping to match the natural frequency of the vibrating structure. It does not require extra energy to be added to the system. Active vibration control is the opposite of vibration control as it requires an energy source to drive active devices continuously such as actuators, sensor and some other electronic controller. The sensor will measure the vibration amplitudes produced and an actuator will act to minimize the vibration amplitudes using a well-defined control strategy. Semi-active vibration control can be defined as a passive vibration control measure in which the stiffness and damping of the absorber is altered by the application of a control signal. It requires smaller amount of energy compare with active vibration control to tune the system. The types, applications, advantages and disadvantages of these vibration absorbers will be discussed in the next chapter.

Many researchers are still trying to explore all the possibilities of the passive vibration absorbers. Previous research papers have described different tuning procedures and optimization techniques depending on the type of primary system. However, the major drawback of a linear

vibration absorber is it has a narrow bandwidth operating frequency. A small change in frequency will result in another resonance. Therefore, a nonlinear vibration absorber with combined hardening and softening mechanism is proposed to increase the bandwidth. The mechanism on how hardening and softening mechanism are generated will be extensively in the thesis.

### **1.3 Objective**

The objectives of this project are as follows:

- i. To characterize the dynamic vibration absorber.
- ii. To design and develop nonlinear vibration absorber.
- iii. To investigate the performance of the nonlinear dynamic vibration absorber.

## CHAPTER 2

### LITERATURE REVIEW

#### 2.1 Dynamic Vibration Absorber

Basically, a dynamic vibration absorber comprises of an additional weight portion, a rigidity element which is usually a spring and a damping element which is a damper. A rigidity element acts as to generate a reaction force to the inertia force applied to the additional weight portion and a damping element acts as to absorb vibration energy. A single dynamic vibration absorber will only provide a reduction or suppression of vibration in one direction only. In other words, two dynamic vibration absorbers placed on a x – and y- axis of a structure will suppress a vibration in both horizontal and vertical direction (Aida et al., 1994). The desire of a dynamics vibration absorber is to provide a reduction of a vibration in a large rotating elements, particularly such elements that exerts periodic unbalance forces on the foundation (Brewer et al., 1975). The absorber is tuned so that its operating frequency is near to a resonant frequency of a system so that when a resonance happens, the vibration is suppressed or reduced to an acceptable level.

#### 2.2 Linear Vibration Absorber

Roberson (1952) states that “In principle, a linear dynamic vibration absorber is an inertia member coupled to a vibrating system by suitable linear coupling members (usually a

spring and viscous damper).” It provides counteract against a vibration of a system by providing a reaction force equal and opposite to the exciting force which causes the vibration. Den Hartog (1956) described a vibration absorber is a device consisting of a linear spring with a stiffness  $k$ , a mass and a damper with a damping constant system. The system was attached on a support machine as shown in Figure 2.1. The vibration absorber provides an equal counteract force at resonance when it is properly tuned to match the excitation frequency of the machine. This equation is shown below. The negative sign indicates that the absorber mass is vibrating at a displacement of  $x$  and providing an equal but in opposite direction of counteract force which reduces the machine’s vibration.

$$F = -kx \quad (2.1)$$

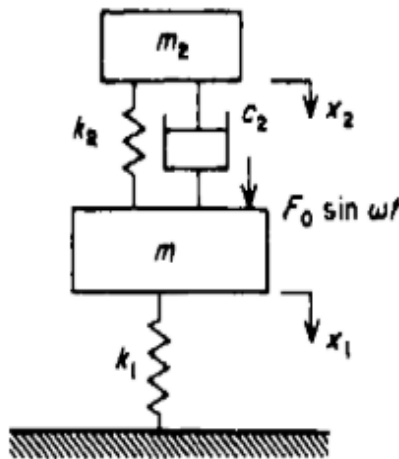


Figure 2.1: Model of a linear dynamic vibration absorber attached to a structure [Source: (Den Hartog, 1956)].

### 2.2.1 Passive Vibration Absorber

Passive Vibration Absorber is a simple and effective device which is widely used to suppress undesirable machine vibrations excited by harmonic forces. A classic form of passive

vibration absorber has mass-spring-damper configuration. Passive vibration absorber does not require external force or energy. It is activated by the vibrating motion of the structure and acts immediately to dissipate the input energy. Gafsi et al. (2017) presented a design of passive linear vibration absorber to improve the stability of milling tools. The absorber comprises a spring-mass-damper is attached to the milling spindle in x and y axis as shown in Figure 2.2.

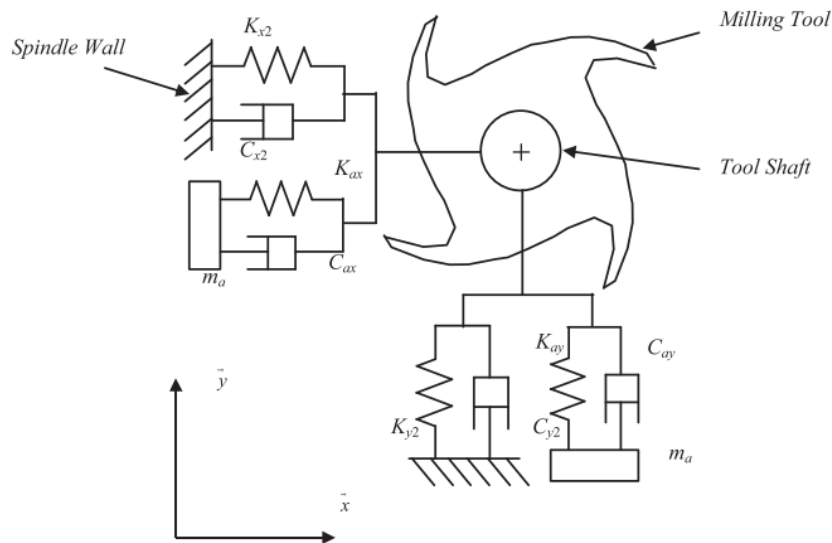


Figure 2.2: Model of the absorber attached on milling tools [Source: (Gafsi et al., 2017)].

The results confirm that the vibration of the milling tool and dynamic loads applied inside the gear transmission is effectively reduced with the vibration absorber. The implementation of the absorber also improves the quality of the work piece surface and reduce the cutting time as the feed rate could be doubled with a slight decrease of tool vibration and the dynamic effort. Fischer (2007) in his journal compare the advantages and disadvantages of different types passive vibration absorber to suppress wind-excited vibration structure. Figure 2.3 shows absorber with translatory mass movement. Any vibration in vertical direction are going to be absorbed by the absorber mass that is suspended vertically on a spring. The mentioned absorber is intended for

bridges. The required damping force is provided by a cylinder immersed in a specific fluid which is well-defined viscosity and little dependent on the temperature.

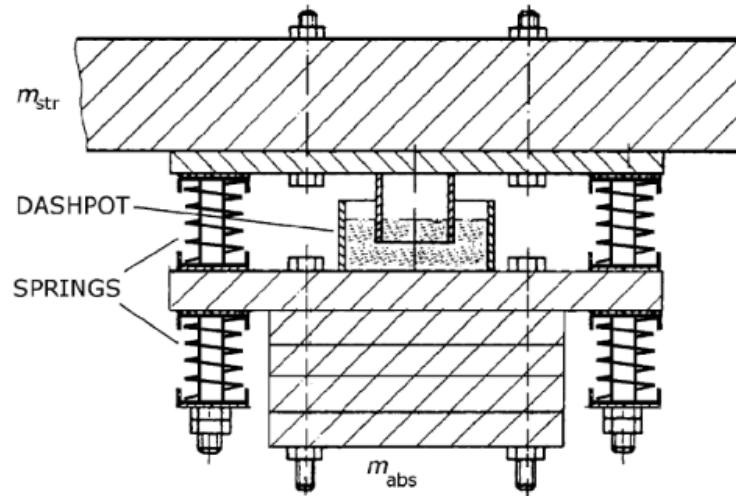


Figure 2.3: Vertical translation vibration absorber [Source: (Fischer, 2007)].

Second type of absorber discussed Fischer is pendulum absorber. Some possible arrangements of the pendulum schematic are shown in Figure 2.4. The damping force required to absorb the vibrations is provided by the motion of a pendulum. However, pendulum absorbers are very difficult to build in tall structures of low natural frequency which they require extremely long or short suspensions.