COMPARISON BETWEEN DOUBLE-PID+LQR AND LQR CONTROLLERS FOR ROTARY INVERTED PENDULUM

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A report submitted in partial fulfillment of the requirements for the degree of Bachelor of Electrical Engineering with Honours

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2019

DECLARATION

I declare that this thesis entitled "COMPARISON BETWEEN DOUBLE-PID+LQR AND LQR CONTROLLERS FOR ROTARY INVERTED PENDULUM is the result of my own research except as cited in the references. The thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

Signature :

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Date : 27 May 2019

APPROVAL

I hereby declare that I have checked this report entitled "Comparison Between double-PID +LQR And LQR Controllers For Rotary Inverted Pendulum " and in my opinion, this thesis it complies the partial fulfillment for awarding the award of the degree of Bachelor of Electrical Engineering with Honours

Signature : ....

Supervisor Name : ....

Date : ....

DEDICATIONS

This report is dedicated to my father and my mother who taught me the best knowledge which they learn for their own sake and always give moral supports to finish this largest tasks.

ACKNOWLEDGEMENTS

Firstly, I want to express my appreciation to my parents for their advice to face the problem and whose always give support in term of economical and mental. Without them, I will have difficult challenge to face and finish this task.

Next, I would like to express my deepest gratitude and special thanks to my supervisor, Dr Azrita Binti Alias who always give moral support, guide and keep me on the correct path to do my Final Year Project. Without his advices and guidance, this task would have never been finished.

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Last but not least, I would like to extend my thankfulness to my fellow friends for their support and guidance. Their views and tips are useful indeed.

ABSTRACT

The Rotary Inverted Pendulum is a classic problem of theory. This report focuses on the design, stability analysis and modeling of the rotor pendulum inverted. It also provides the design steps for the controllers for an inverted rotary motion pendulum operated by the rotary servo plant of the SRV 02 Series. A control system is designed using classic and modern control methods. The classic root locus method is the design of two compensators for the PID controller. The second method of the modern method control technique is the Linear Quadratic Regulator (LQR). The linear square regulator is thus tested for the upright and swing mode of the pendulum. The mathematical derivatives also showed that the designed controller needs to stabilize the pendulum system. Simulation studies are conducted to demonstrate the efficiency of the designed controller and the result shows that the controller can maintain a stable reverse position.

ABSTRAK

Pendulum berputar Rotary adalah masalah teori klasik. Laporan ini memberi tumpuan kepada reka bentuk, analisis kestabilan dan pemodelan pendulum pemutar terbalik. Ia juga menyediakan langkah-langkah reka bentuk untuk pengawal untuk pendulum gerakan putar terbalik yang dikendalikan oleh kilang servo putar SRV 02 Series. Sistem kawalan direka menggunakan kaedah kawalan klasik dan moden. Kaedah lokus akar klasik adalah reka bentuk dua pemampat untuk pengawal PID. Kaedah kedua kaedah kawalan kaedah moden ialah Pengatur Kuasa Lajur Linier (LQR). Oleh itu pengawal selia persegi linear diuji untuk mod tegak dan swing pendulum. Derivatif matematik juga menunjukkan bahawa pengawal yang direka untuk menstabilkan sistem pendulum. Kajian simulasi dijalankan untuk menunjukkan kecekapan pengawal yang direka dan hasilnya menunjukkan bahawa pengawal boleh mengekalkan kedudukan terbalik yang stabil.

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LIST OF SYMBOLS AND ABBREVIATIONS

SYMBOL	DESCRIPTION
L	Length to Pendulum's Center of Mass
m	Mass of Pendulum Arm
r	Rotating Arm Length
θ	Servo load gear angle (radians)
α	Pendulum Arm Deflection (radians)
h	Distance of Pendulum Center of mass from ground
Jcm	Pendulum Inertia about its center of mass
Vx	Velocity of Pendulum Center of mass in the x-direction
Vy	Velocity of Pendulum Center of mass in the y-direction
m_1	Mass of Arm
m_2	Mass of Pendulum
l_1	Length of Arm
l_2	Length of Pendulum
c_1	Distance to Centre of Arm Mass
c_2	Distance to Centre of Pendulum Mass
J_1	Inertia of Arm
J_2	Inertia of Pendulum
g	Gravitational Acceleration
α	Angular Position of Arm
$\dot{\alpha}$	Angular Velocity of Arm
β	Angular Position of Pendulum
$\dot{\beta}$	Angular Velocity of Pendulum

C_1	Viscous Friction Coefficient of Arm
C_2	Viscous Friction Coefficient of Pendulum
K_t	Motor Torque Constant
K_b	Motor Back-Emf Constant
K_u	Motor Driver Amplifier Gain
R_m	Armature Resistance
L_m	Armature Inductance

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CHAPTER 1

INTRODUCTION

This chapter will discuss on the background of the project study, motivation, problem statement, objectives and the project scope.

1.1 Background

An inverted pendulum is a pendulum with a mass center above the pivot point. It is unstable and will decrease without further help. It can be stable by using a control system to monitor the pole angle and move the pivot point horizontally back below the mass center when it begins to fall. Figure 1.1 (i) shows that the inverted pendulum system is motivated by the need to design controllers for rocket balancing during vertical takeoff. Similar to the launching rocket, the inverted pendulum requires a continuous correction mechanism to remain upright, as the system is unstable in open loop configuration [1]. This problem can be compared with the launch rocket, where rocket boosters must be fired in a controlled manner to keep the rocket upright.

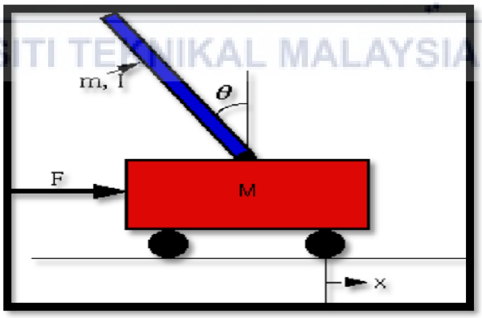


Figure 1.1 Inverted Pendulum[1]

Rotary inverted pendulum is one of the simple but hard systems to balance the upright position. Because the pendulum naturally would fall from the upright vertical position. Design of a modified PID controller and Linear Quadratic Regulator (LQR)[2][3] as a solution. Mathematical modeling is required to obtain a precise feedback. MATLAB / SIMULINK is therefore to be used in this project to control the rotary inverted pendulum by mathematical modeling of the modified PID and Linear Quadratic Regulator.

Figure 1.1 shows the inverted rotary pendulum, consisting of a pendulum that rotates in the vertical plane and is attached to a pendulum arm mounted on the servo motor shaft. At the end of the pendulum arm, the pendulum is attached to a hinge with an encoder [3]. The pendulum arm itself will rotate in horizontal plane and the pendulum will always hanging downwards. This type of pendulum, which is rotary inverted pendulum is an unstable system and required a controller to be actively balanced in order to remain the pendulum in upright vertical position.

Figure 1.2 Rotary Inverted Pendulum[2]

1.2 Motivation

The rotary inverted pendulum system was motivated by the design of the controller that stabilized the rockets during the vertical take-off, because the rocket was very unstable at the start. In order to keep the rocket upright, rocket boosters must be fired in a controlled way during the start. Based on the launching process of the rocket, the inverted pendulum requires a continuous correction mechanism, as the system is unstable in open loop configuration.

1.3 Problem Statement

Inverted pendulum is one of the key issues in the theory of control. When it comes to rotating the pendulum, however, to ensure that the pendulum remains vertically upright, it is quite difficult, as it is naturally unstable and has an open loop setup. The controllers used are therefore double-PID and LQR, which must be designed and simulated successfully to switch the pendulum up.

1.4 Objectives

There are 3 objectives need to be achieved in this project:

a) To obtain the mathematical model of rotary inverted pendulum in transfer function and in state function.

b) To design and simulate double Proportional Integral Derivative with Linear Quadratic Regulator (double-PID+LQR) and Linear Quadratic Regulator (LQR) controller for balancing the rotary inverted pendulum.

c) To compare the performance of stability between double Proportional Integral Derivative with Linear Quadratic Regulator (double-PID+LQR) and Linear Quadratic Regulator (LQR) controllers.

1.5 Scopes of Project

This project required some scopes that need to achieve the objectives. The scopes of work are as follows:

1. Modeling the system to obtain the mathematical model for rotary inverted pendulum system.
2. The double Proportional Integral Derivative (double-PID) and Linear Quadratic Regulator (LQR) controllers need to be developed after the mathematical model have been derived.
3. The design requirement are Ts < 5 second and %OS < 10%.
4. The performance of the designed controllers is simulated using MATLAB SIMULINK software.

CHAPTER 2

LITERATURE REVIEW

After research has been done, a literature review will be discussed in this chapter 2. A literature review can be stated in a number of ways to complete the project as a discussion of information.

2.1 Theory and Basic Principle of Rotary Inverted Pendulum

Figure 2.1 shows Rotary Inverted Pendulum systems an under-actuated system which consists of one actuator and double Perpendicular Integral Derivative (double-PID). The only one actuator in the system the DC motor. The rotary arm is driven by the DC motor where electrical energy is converted into mechanical energy, the torque to move it. The angular motion of the rotary arm gives energy to the pendulum to swing up and maintain stable at vertical upright position. The pendulum is set to be always perpendicular to the rotary arm. When the pendulum is at vertical upright position, the system is highly unstable, where a controller is needed to achieve stabilization and swing up mechanism of Rotary Inverted Pendulum system. The amplitude of the supply voltage to the DC motor is proportional to the magnitude of the angular displacement of the rotary arm. Then, the greater the supply voltage to the actuator, the greater the angular displacement of the rotary arm.

The angular displacement of Rotary Inverted Pendulum is indirectly moved by the DC motor torque. There are two types of movement mechanism in RIP system, which are swing-up mechanism and stabilize mechanism. In this project, swing-up mechanism is not discussed, position of Rotary Inverted Pendulum is assumed to be at upright position as initial condition. Second type of movement mechanism is stabilization mechanism of rotary inverted pendulum system which is the motion maintaining the Rotary Inverted Pendulum at vertical upright position and avoiding the pendulum falling down in its free fall of nature way.

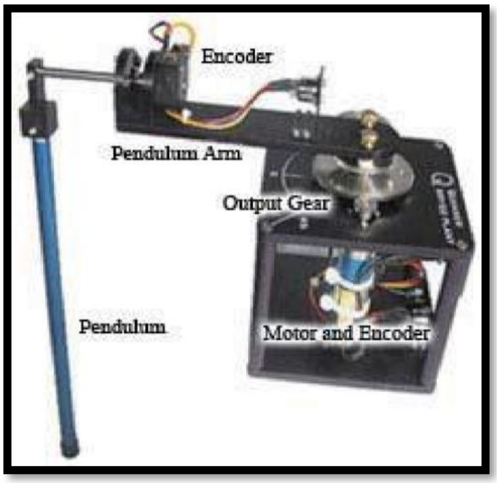


Figure 2.1 Rotary Inverted Pendulum[2]

2.2 Controller Design

The important factor of the rotary inverted pendulum system was to develop the control techniques to make the pendulum in upright position to maintain the stabilization of the system. . There were two techniques in designing the controller which are by using Linear Quadratic Regulator (LQR) controller and Proportional, Integral and Derivative (PID) cotroller. The function of this controller is to develop linear model to stabilize the position of rotary inverted pendulum in upright position. In this paper, the method to derive the LQR controller was by using Algebraic Ricatti Equation and for PID controller, the Ziegler Nicholas Tuning method was used. [3]

Meanwhile, there were 3 design of controller technique which comprising in this journal [3]. The state space equation is required by deriving the mathematical modelling of the rotary inverted pendulum where the inverted pendulum system is attached to a servo plant motor and simulation by MATLAB. The alpha represents the pendulum angle and theta represents the pendulum arm.

2.3 Types of Controllers

There is 2 types of controller in this project that are:

Table 1 Comparison Between Two Controller

Double-PID controller	LQR controller
i. α depends on β that can detected can described by using transfer ii. function. Pendulum arm position, iii. θ is activated by input voltage, V. There are two degrees of freedom used to make the speed constant and the value of β is zero. The other one will be activated by following the feedback of α.	i. The value of gain, K can be calculated by lowering a quadratic cost function with the used of Matlab function.

2.3.1 PID Controller

PID is Proportional-Integrated-Derivative. This is a type of feedback controller whose output, a control variable, is generally based on the error between a set point defined by the user and a process variable measured. Each PID controller element refers to a specific action taken against the error. The basic idea for a PID controller is to examine signals from sensors in the system called feedback signals. [2]

$$K_p + \frac{K_i}{s} + K_p s = \frac{K_P + K_i + K_d s}{s} \qquad (2.1)$$

K_p= Proportional Gain

K_i= Integral Gain

K_d= Differential Gain (2.2)

The error signal is sent to the PID controller and the controller calculates the error signal derivative and integral. The signal (u) just past the controller is now equal to the proportional gain (K_p) times the error magnitude plus the integral gain (K_i) times the error integral plus the derivative gain (K_d) times the error derivative.

$$u = K_p e + K_I \int e(t)dt + K_D \frac{de}{dt} \tag{2.3}$$

The controller takes the new error signal and computes it derivative and integral again. This process goes continues and continues.

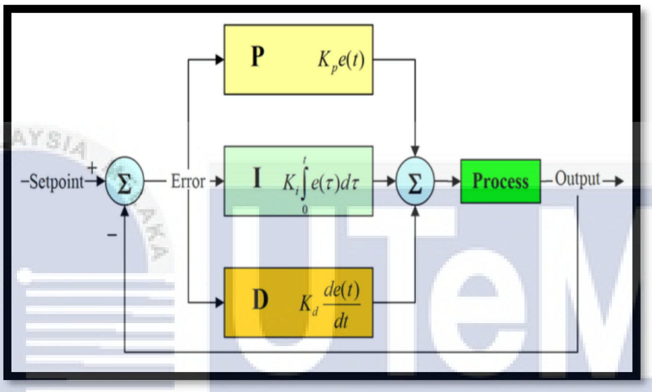


Figure 2.2 Block Diagaram of PID Controller[2]

2.3.2 LQR Controller

In order to overcome some problems faced by the PID controller, optimal control can be developed for other types of control methods, such as the Linear Quadratic Regulator (LQR). LQR is a control system that delivers the best possible performance in relation to certain performance measurements. The measurement of performance is a quadratic function consisting of state vector and control input.

Using LQR, the representation of the state space is required where this controller is based on the dynamic model that produces a high system response. This controller moves the initial pole from right to left in a complex diagram. The aim of this shift is to improve the system stability and the damping response.

The time response characteristics, such as time increase, settling time and transient oscillations, are directly affected by the closed loop pole location where additional parameters are used in this project to control all closed loop pole locations. Figure 2.6 shows the LQR status feedback system.

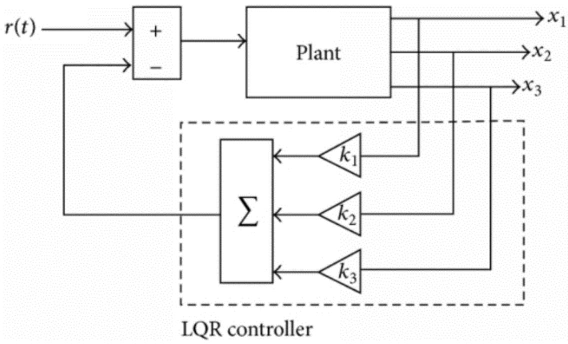


Figure 2.3 Block Diagram of LQR Controller [3]

2.4 Application of Inverted Pendulum

In real life, using pendulum principles, there are many types of applications that are pendulum clock, machine crane system and part of the amusement park. In order to improve this application, modeling and design control can be described as an important factor and can be developed very usefully. [4]

Figure 2.4 shows the pendulum clock. This pendulum clock was prepared by Christian Huygens in 1656, focusing on the pendulum supplied by Galileo Galilei around the clock. The function of this pendulum clock is that it is the instrument of decision to keep time accurate for a considerable length of time, comprising towards the end of the clock the Short free pendulum cheeks developed in 1921 and the Edward Hall pendulum clock was said to be the end of the generation of the pendulum check in the period as the most solid time keeping standard.

Figure 2.4 The clock Pendulum

A machine crane system where the crane is a mechanical gadget lift consisting of a winder, wire ropes and bundles that can be used to lift and carry the material lower and can be moved on a horizontal plane Further strengthening makes mechanical good fortune, the machine used is one or more straight forward machines and can then move steadily. The capacity of this crane machine in the industry is used for the stacking and emptying of cargo, in development division it uses the overwhelming supplies for the development of materials industry.

2.5 Review of Previous Case Study

There were many researchers studied Rotary Inverted Pendulum system from different aspects especially in modelling and designing different controller of the system. For stabilization and swing-up of Rotary Inverted Pendulum system, numerous designs of controller approach have been suggested to achieve better stabilization performance. Therefore, the study of research that had been done by other researchers is important to get a rough idea of designing controller in this Rotary Inverted Pendulum system.

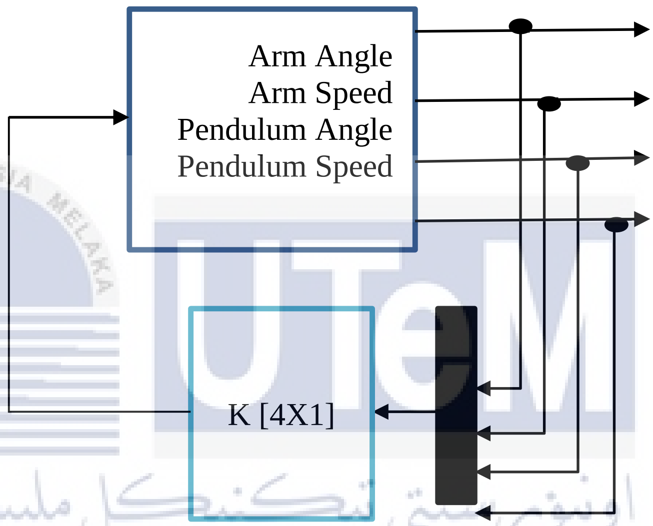
2.5.1 Double-PID of Proportional Inverted Derivative

There are basically divided to classical controller and advanced controller in proposed controllers to stabilize the Rotary Inverted Pendulum system. Double Proportional Integral Derivative (PID) controller is one of the most widely used controller in field of control engineering. As the Rotary Inverted Pendulum system is an under actuated and non-linear system, PID controller is common to be designed in the Rotary Inverted Pendulum system, as it improves overshoot percentage and steady state error of the system with an easy approach. PID controller can be although mathematical model of the system is not known. When the mathematical model is known, Ziegler Nichols rules can be applied. Ziegler Nichols tuning rules give an educated guess for the parameter values and provide a starting point for fine tuning. Thus, from year 2009-2012, 2DOF PID or Doubled-PID was designed as a controller in the Rotary Inverted Pendulum. As a stabilization controller in the Rotary Inverted Pendulum system, the controller stabilized the inverted pendulum and as well as the rotary arm. [3][5]

For advance controller, hybrid strategy was applied on the controllers. Combining two types of controller into the Rotary Inverted Pendulum system was considered in designing controller procedure. The advanced controllers that were used in hybrid strategy in previous works such as Full State Feedback and Linear Quadratic Regulator.

2.5.2 Full State Feedback

Figure 2.7 shows a block diagram of the Full State Feedback (FSF) controller in the Rotary Inverted Pendulum system. The Rotary Inverted Pendulum was designed in the state space model. Poles of the closed loop system may be placed at any desired locations by means of the state feedback through an approximate state feedback gain matrix K. There are a few approach to tune FSF controller, one of them is by pole placement method. According to Md. Akhtaruzzaman, he designed FSF controller by placing stable poles of the Rotary Inverted Pendulum, then used Ackermann's formula and Integral of Time weighted Absolute Error (ITAE) table, state feedback control gain matrix, K which is a 4x1 matrix was obtained. FSF controller was considered relatively ease of design and effective procedure to obtain the gain matrix K. There are some drawbacks of designing FSF controller. It requires successful measurement of all state variables or a state observer in the system, where it needs control system design in state space model. It also requires experienced researcher to determine the desired closed loop poles of the system, especially when the system has a higher order system than second and third order.[5][3]



2.5 Block Diagram of Full State Feedback

2.5.3 Proportional Integrated Derivative

A double-PID controller instead of single PID controller, it was because single conventional PID controller can control only one variable of the system. So, Rotary Inverted pendulum system has one input and two outputs which single PID is incapable to control the system. 2DOF PID was arranged calculated. PID arm was to maintain the rotary arm as zero. PID pendulum maintained the speed and position of the pendulum to remain stable. PID Arm was tuned first the follow by PID Pendulum. Root locus analysis was used to tune the both PID.

The paper [3][1]describes how to stabilize the pendulum and how to increase the pendulum and arm rotation speed. This paper uses the Euler-Lagrange equation to derive the rotary pendulum system's mathematical model. The paper also presents how to design a controller using both the pendulum and the arm with a PID controller and pole placement method. This paper proposed two methods to increase the pendulum and arm rotation speed, which is one loop and two closed. The result shows that the PID controller is better than a pole positioning method for both the pendulum and the arm in terms of overflow and transient response. The result also shows that a closed loop method has a greater advantage in speeding up the speed of rotation without reducing its stability

In the final paper [2], it shows how to model the physical structure of the inverted rotary pendulum according to its basic principle. It also presents in this paper how to design a balancing controller with a PID controller. The difference from other papers is that the electrical and dynamic part of the system is derived by circuit analysis and a free body diagram. The block diagram is then constructed using SIMULINK in MATLAB based on these equations. The systems are closed to the actual system because it is a non - linear model.

2.5.4 Linear Quadratic Regulator (LQR)

In this project [5][6][17]LQR was used to control the dc motor on the PIC microcontroller. This controller's main objective is to minimize the variation in DC motor speed. The driving voltage of DC motor speed is controlled. The higher the voltage, the higher the engine speed. The motor speed specifies that the motor input voltage is the motor and that the output is compared to the input. The output therefore has to be the same or about the same as the input voltage. The LQR algorithm has been implemented on the PIC microcontroller in this project. Before implementing the PIC, the space of the dc motor status must be derived. Then, from the state space, the LQR controller can be designed using the MATLAB software. The stable system is achieved by adjusting the Q and R values that the simulation. DC Motor Controller Linear Quadratic Regulator (LQR) Implementation of the PIC Algorithm.

In this research [3][7], the two modern control techniques used to stabilize the inverted rotary pendulum system were mainly discussed. The controls used are the Full State Feedback (FSF) and Linear Quadratic Regulator (LQR) controllers. The researcher tested the FSF and LQR controllers in this paper for the pendulum upright and swing-up mode. A discrete 2DOF (two degrees of freedom) PID controller was designed to digitize the plant using root locus technology. The system was then simulated using MATLAB and the result shows that the LQR controller is better suited for switching the pendulum up to its vertical position and for balancing the pendulum at the unstable balance point. However, both FSF and LQR controllers can keep the inverted rotary pendulum stable effectively.

2.5.5 Artificial Neural Network (ANN)

This paper [8] presents the physical structure, the dynamic model of the rotary inverted pendulum system and the method by which an artificial neural network (ANN) controls this system. The paper states the problem of controlling the inverted pendulum so that it can maintain its state of balance when directed upwards. The researcher proposed an ANN controller because this controller has many advantages, such as that controller can be implemented for nonlinear objects and can be adapted to any change of system parameters. Based on the SIMULINK simulation in MATLAB, the ANN controller keeps the pendulum stable vertically and upwards. The controller also manages to adjust well when the system parameters change. The paper therefore concludes that ANN can be applied to real physical systems with high nonlinearity and, in particular, physical systems with the center of gravity on the rotation axis, such as rockets, spacecraft, and skyscrapers.

2.5.6 Lyapunov

In this study [9], you will find out how to stabilize the Rotary Inverted Pendulum system by using Lyapunov. The building of Lyapunov function is the key to the design of control laws using Lyapunov control method. The Lyapunov function is built on the algorithmic function and theoretically compared to the usual quadratic function. The legislation on design control is justified by experiments to determine the efficiency of the control law. This is compared to the LQR controller, which is based on the square function. To determine the system's robustness, the system is analyzed if the system parameters contain uncertainties under the designed control legislation. The comparative results show that the built logarithmic function has greater numerical precision and faster convergence speed usual quadratic feature. The researcher therefore concluded that the Lyapunov control method applies to nonlinear systems such as rotary pendulum inverted.

2.5.7 Particle Swarm Optimization (PSO)

In this study [10], an optimal approach is presented to design a rotary inverted pendulum system using a Particle Swarm Optimization (PSO) control. The goal is to balance the pendulum to the upright position and minimize the absolute system angle error. The solution to this task is to stabilize the system using a state feedback controller. Based on the results of SIMULINK in MATLAB simulated, the researcher concludes that the PSO method is a promising way for the nonlinear control system in general, since the pendulum can be efficiently stabilized.

2.5.8 LQG / LTR (Linear Quadratic Gauss Ian / Loop Transfer Recovery)

This research [11] is a rigid connection of pendulum that rotates in a upright position. The rigid connection is connected to a pivot arm that is attached on a DC motor load shaft. The pivot arm can be rotated in the horizontal plane by the DC motor. A potentiometer inspires by the DC motor. In addition, a potentiometer is also mounted to measure the angle of the pendulum on the pivot arm. The main objective of this experiment is to balance the pendulum in upright position. Since the plant has two degrees of freedom but only one actuator, the system is under actuated and shows considerable nonlinear behavior during large pendulum excursions. The aim is to design a robust controller to control the pendulum position in real - time using a Quanser PC board and power module and the appropriate real - time Win Con software. A well-known robust method called LQG / LTR (Linear Quadratic Gauss Ian / Loop Transfer Recovery) is used for controller design.

2.6 Summary of Literature Review

Based on the previous work review, there are a lot of different method which implemented in rotary inverted pendulum system such as fuzzy-logic control, linear quadratic regulator (LQR), full state feedback (FSF) control, Proportional Integral Derivative (PID), Lyapunov algorithm etc. In addition, there are many different types of tuning methods for each control technique discussed in the review. Based on the results of the previous paper, the controllers manage to keep the pendulum upright. However, for each control technique, the output system response in terms of overflow, adjustment time and stable state error is different. Although there is a lot of advance controller, the pendulum position PID controller can adjust to have less overflow, no stable state error, faster settling and increasing rising time. It can also stabilize the motor arm to rotate in the desired position while keeping the pendulum in the desired position with the help of the LQR controller. Therefore, based on the previous research, it shows that the PID and LQR controller have been selected because this controller can fulfill the rotating inverted pendulum requirement.

CHAPTER 3

METHODOLOGY

This chapter discusses mathematical modeling and the design of the controller. Furthermore, the methodologies followed to obtain a better result using the flowchart. Research has been done to ensure the success of the project. This includes research on the design and the method used to model and control the inverted rotary pendulum. This project uses the methodology and approaches to study the theories and the concept of balancing the rotary inverted pendulum with double-PID and LQR. The mathematical modeling based on the inverted rotary pendulum must be verified. Simulink and MATLAB are used to develop the rotary inverted pendulum block diagram and simulate the output.

3.1 Project Flow Chart

Flow chart is used to make sure that the project will be done successfully without any overlapping of works occurs. For the first step to do this project is to studies about the suitable mathematical modelling and also need to research from the other sources such as journals, books and internet about double-PID and also LQR controller. Next, to model the block diagram of rotary inverted pendulum using two controllers that is double-PID and LQR. Next, after finish model the block diagram the model needs to simulate using MATLAB/Simulink. If the system was functioning, the next step need to design two controllers that is double-PID and LQR and after that the results need to analyze and compare their stability.

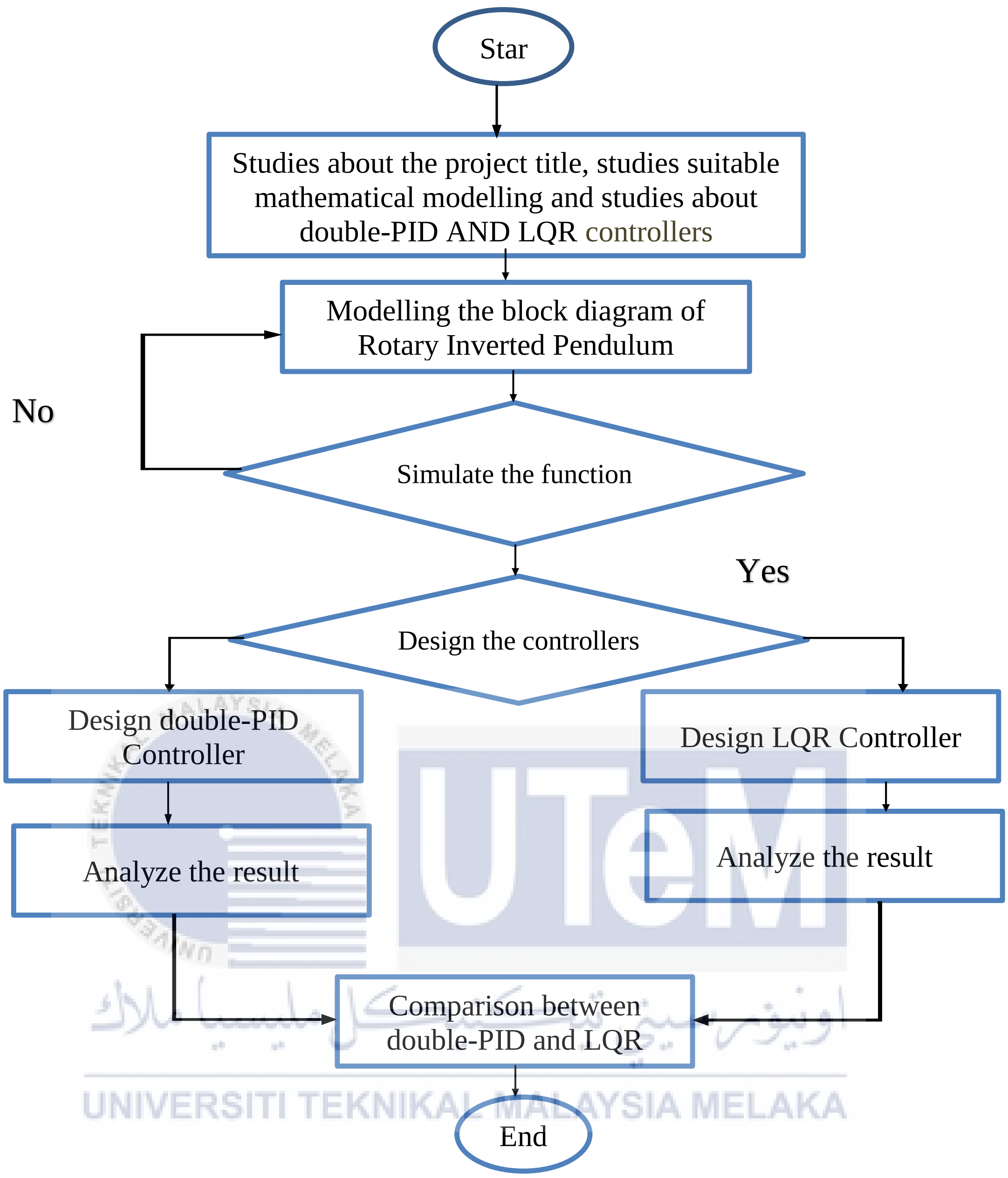
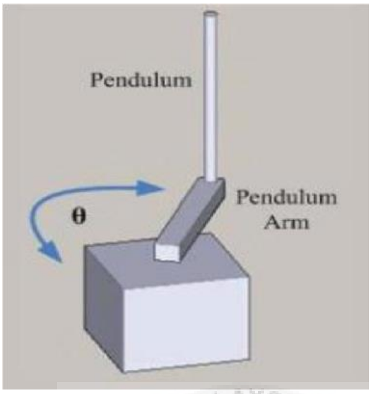


Figure 3.1 Flowchart of double-PID and LQR Controller

3.2 Mathematical Modelling of Rotary Inverted Pendulum

There is state space equation and transfer function must be used for this project. The transfer function in this project is very important parameters, such as gravity and mass, and the transfer function must also be correct to ensure that the objective was achieved.



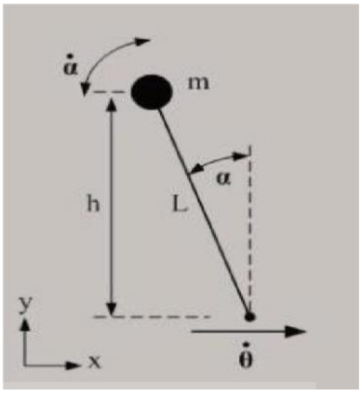


Figure 3.2 Arm Rotational Direction and Free Body Diagram of The Pendulum [9]

Figure 3.2 shows the direction of rotation of the inverted pendulum arm and the pendulum as a lump at half the length of the pendulum. The pendulum is displaced with an angle, α while in the x-direction of this illustration the direction of illustration. The mathematical model can be derived by checking the speed of the center of the pendulum mass.

The following assumptions are important in modeling of the system:

i) The system starts in a state of equilibrium meaning that the initial conditions are therefore assumed to be zero.

ii) The pendulum does not move more than a few degrees away from the vertical to satisfy a linear model.

iii) A small disturbance can be applied on the pendulum.

As the requirement of the design, the settling time (Ts) is less than 5 seconds. The system overshoot value of the system most 10%. The following table is the terminology list used in model system derivation.

Table 2 Symbol and Description of the parameters

SYMBOL	DESCRIPTION
L	Length to Pendulum's Center of Mass
m	Mass of Pendulum Arm
r	Rotating Arm Length
θ	Servo load gear angle (radians)
α	Pendulum Arm Deflection (radians)
h	Distance of Pendulum Center of mass from ground
Jcm	Pendulum Inertia about its center of mass
Vx	Velocity of Pendulum Center of mass in the x-direction
Vy	Velocity of Pendulum Center of mass in the y-direction

3.3 Physical Analysis

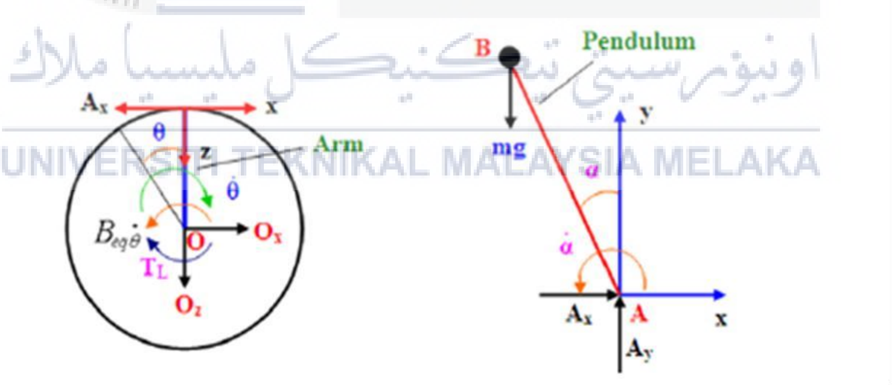


Figure 3.3 Free Body Diagram of Arm and Pendulum [5][6]

There are two components for the velocity of the pendulum lumped mass. So,

$$Vpen.center\ of\ mass = -(\dot{\alpha})\hat{x} - Lsin\alpha(\dot{\alpha})\hat{y} \tag{3.1}$$

The pendulum arm also moves with the rotating arm at a rate of,

$$V_{arm} = r\theta \quad (3.2)$$

The equations (3.1) and (3.2) can solve the x and y velocity components as,

$$Vx = r\theta - (\dot{\alpha}) \quad (3.3)$$

$$Vx = -(\dot{\alpha}) \quad (3.4)$$

3.4 Deriving the system dynamic equations

Having the velocities of the pendulum, the system dynamic equations can be obtained using the Euler-Lagrange formulation

- Potential Energy: The only potential energy in the system is gravity. So,

$$V = P.\,E_{pendulum} = mgh = mgLcos\alpha \quad (3.5)$$

Kinetic Energy: The kinetic energy in the system originates from the moving hub, the speed of the point mass in the x-direction, the speed of the mass in the y direction and the rotating pendulum around its center of mass.

$$T = K.\,E_{Hub} + K.\,E_{Vx} + K.\,E_{Vy} + K.\,E_{pendulum} \quad (3.6)$$

The moment of inertia of a rod about its center of mass is,

$$J_{cm} = \left(\frac{1}{12}\right)MR^2 \quad (3.7)$$

Since L is defined as the half of the pendulum length, R=2L. Therefore, the moment of inertia of the pendulum about its center of mass is,

$$J_{cm} = \left(\frac{1}{12}\right) MR^2 = \left(\frac{1}{12}\right) M(2L)^2 = (\frac{1}{3})ML^2 \quad (3.8)$$

$$T = (\frac{1}{2})_{Jeq}\dot{\theta}^2 + \left(\frac{1}{2}\right) m((r\theta - L\cos\alpha(\alpha))^2 + \left(\frac{1}{2}\right) m(-L\sin\alpha(\dot{\alpha}))^2 + (\frac{1}{2})J_{cm}\dot{\alpha}^2 \quad (3.9)$$

After expanding the equation and collecting terms, the Lagrangian can be formulated as,

$$L = T - V = (\frac{1}{2})_{Jeq}\dot{\theta}^2 + \left(\frac{2}{3}\right) mL2\dot{\alpha}^2 - mlrcos\alpha(\dot{\alpha})(\dot{\theta}) + \left(\frac{1}{2}\right) mr2\dot{\theta}^2 - mgLcos\alpha \quad (3.10)$$

The two generalized co-ordinates are θ and α. So, another two equations are,

$$\frac{\delta}{\delta t}\left(\frac{\delta L}{\delta\dot{\theta}}\right) - \frac{\delta L}{\delta\theta} = T_{output} - B_{eq}\dot{\theta} \quad (3.11)$$

$$\frac{\delta}{\delta t}\left(\frac{\delta L}{\delta\dot{\alpha}}\right) - \frac{\delta L}{\delta\alpha} = 0 \quad (3.12)$$

Solving the equations and linearizing about $\alpha = 0$ equations become,

$$(J_{eq} + mr^2)\theta - mLr\ddot{\alpha} = T_{output}B_{eq}\theta \quad (3.13)$$

$$\frac{4}{3}mL^2\ddot{\alpha} - mLr\ddot{\theta} - mgL\alpha = 0 \quad (3.14)$$

$$T_{output} = \frac{n_m n_{gK_t} K_g (V_m - K_g K_m \dot{\theta})}{R_m} \quad (3.15)$$

Non linear Dynamic Model

The non linear dynamic model describes the system by giving the exact relationships among all the variables involved. The parameters of corresponding symbols represent, the dynamic model with the pendulum, with motor torque characteristic, in the upright position is:

$$[A]\begin{bmatrix}\ddot{\alpha}\\ \ddot{\beta}\end{bmatrix} = [D]u - [B]\begin{bmatrix}\dot{\alpha}\\ \dot{\beta}\end{bmatrix} - [C] \tag{3.16}$$

Matrix A is inertia matrix of the system, matrix B represents Corolis and gyroscopic of the system. While Matrix C represents gravity terms in Cartesian space of the system, and Matrix D is the torque on the end-effector of the pendulum. Thus, the Rotary Inverted Pendulum system nonlinear state space model is:

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}\ddot{\alpha}\\ \ddot{\beta}\end{bmatrix} = \begin{bmatrix}j\\ 0\end{bmatrix}u - \begin{bmatrix}e & f\\ g & h\end{bmatrix}\begin{bmatrix}\dot{\alpha}\\ \dot{\beta}\end{bmatrix} - \begin{bmatrix}0\\ i\end{bmatrix} \tag{3.17}$$

Where,

$a = j_0 + m_1 l_0^2 + m_1 l_1^2 sin^2\beta$ $\quad$ $f = -m_1 L_0 l_1 \dot{\beta} sin\beta + \frac{1}{2} m_1 l_1^2 \dot{\alpha} sin2\beta$

$b = -m_1 L_0 l_1 cos\beta$ $\quad$ $g = -\frac{1}{2} m_1 l_1^2 \dot{\alpha} sin2\beta$

$c = -m_1 L_0 l_1 cos\beta$ $\quad$ $h = C_1$

$d = J_1 + m_1 l_1^2$ $\quad$ $i = m_1 g l_1 sin\beta$

$e = C_0 + \frac{K_t K_b}{R_a} + \frac{1}{2} m_1 l_1^2 \dot{\beta} sin2\beta$ $\quad$ $j = \frac{K_t K_u}{R_a}$

Linearized Rotary Inverted Pendulum Model

The model is then linearized by expanding the non linear model into a Taylor Series about the operating point and retention of only the linear terms. The model is linearized about the upright position of the pendulum, whereby the pendulum is at static, angle and velocity of the pendulum, β and $\dot{\beta}$ are zero and the rotary arm is not moving as well, velocity of arm $\dot{\alpha}$ is zero.

$$\begin{bmatrix} \dot{\alpha} \\ \ddot{\alpha} \\ \dot{\beta} \\ \ddot{\beta} \end{bmatrix} = \frac{1}{af - c^2} \begin{bmatrix} 0 & af - c^2 & 0 & 0 \\ 0 & -df & ch & -cC_1 \\ 0 & 0 & 0 & -c^2 \\ 0 & -cd & ah & -aC_1 \end{bmatrix} \begin{bmatrix} \alpha \\ \dot{\alpha} \\ \beta \\ \dot{\beta} \end{bmatrix} + \frac{1}{af - c^2} \begin{bmatrix} 0 \\ ef \\ 0 \\ ce \end{bmatrix} u$$

Where,

$a = J_0 + m_1 L_0^2$ $\quad$ $d = C_0 + \frac{K_t K_b}{R_a}$ $\quad$ $h = m_1 g l_1$

$b = m_1 l_1^2$ $\quad$ $e = \frac{K_t K_u}{R_a}$

$c = m_1 l_0 l_1$ $\quad$ $f = J_1 + m_1 l_1^2$

$b41 = b \frac{n_m n_g K_t K_g}{R_m EE}$

By having values of all the parameters the quation is rearranged:

$$\begin{bmatrix} \dot{\alpha} \\ \ddot{\beta} \\ \dot{\alpha} \\ \ddot{\beta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 5.98 & -0.05267 & 0 \\ 0 & 57.68 & -0.04514 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 28.84 \\ 24.72 \end{bmatrix} u \qquad (3.18)$$

$$y = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + [0]u$$

Equation 3.16 shows the mathematical state space model of Rotary Inverted Pendulum system

3.5 LQR Design

Five steps to design LQR

- Develop Linear model of system dynamic
- Specified by adjusting Q and R
- Running LQR in Matlab
- Simulate the system
- Adjust Q and R if necessary

The values of gain are represented by K1, K2, K3 and K4. The system must be checked to determine the controllability by checking the controllability matrix, Q indicated Appendix A. By calling the function

$$K = \text{lqr}(A, B, Q, R) \qquad (3.16)$$

Where A and B are state representation matrix and desired pole contains the desired closed loop poles. The controllability matrix Q must not be equal to zero for the Pole Placement method where the value of Q can be checked by the formula controllability matrix, Q = $[B\ AB\ A2B\ A3B]$ by using at command MATLAB .

CHAPTER 4

RESULTS AND DISCUSSIONS

In this chapter, the pendulum arm and the angle of the pendulum must be transferred to the state-space. The step response performance is performed by using MATLAB. This chapter is divided into two controllers that is double-PID Controller and also LQR Controller.

Table 3 Parameters of Rotary Inverted Pendulum

SYMBOL	DESCRIPTION	VALUE
m_1	Mass of Arm	0.01826
m_2	Mass of Pendulum	0.01826
l_1	Length of Arm	0.16
l_2	Length of Pendulum	0.16
c_1	Distance to Centre of Arm Mass	0.08
c_2	Distance to Centre of Pendulum Mass	0.08
J_1	Inertia of Arm	0.00215058
J_2	Inertia of Pendulum	0.00018773
g	Gravitational Acceleration	9.81
α	Angular Position of Arm	-
$\dot{\alpha}$	Angular Velocity of Arm	-
β	Angular Position of Pendulum	-
$\dot{\beta}$	Angular Velocity of Pendulum	-
C_1	Viscous Friction Coefficient of Arm	0
C_2	Viscous Friction Coefficient of Pendulum	0
K_t	Motor Torque Constant	0.01826
K_b	Motor Back-Emf Constant	0.01826
K_u	Motor Driver Amplfier Gain	850
R_m	Armature Resistance	2.56204
L_m	Armature Inductance	0.0046909

4.1 Simulation without Controller

Rotary inverted pendulum in state space equations is shown at below:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & a32 & a33 & 0 \\ 0 & a42 & a43 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b31 \\ b41 \end{bmatrix} Vm \tag{4.1}$$

Parameters of the Rotary Inverted Pendulum were defined in m-file script. The step response was found in the appendix A using MATLAB code. From the step response, it found that the systems are not stable and therefore the value of state-space A, B, C and D in the MATLAB is obtained.

$$\begin{bmatrix} \dot{\boldsymbol{\alpha}} \\ \ddot{\boldsymbol{\beta}} \\ \dot{\boldsymbol{\alpha}} \\ \ddot{\boldsymbol{\beta}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{5.98} & \mathbf{-0.05267} & \mathbf{0} \\ \mathbf{0} & \mathbf{57.68} & \mathbf{-0.04514} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \\ \dot{\boldsymbol{\alpha}} \\ \dot{\boldsymbol{\beta}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{28.84} \\ \mathbf{24.72} \end{bmatrix} \tag{4.2}$$

$$\boldsymbol{y} = [\mathbf{0}\ \ \mathbf{1}\ \ \mathbf{0}\ \ \mathbf{0}] \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{bmatrix} + [\mathbf{0}]\boldsymbol{u}$$

Based on the above state space equation it is possible to derive the following two transfer functions.

$$\frac{\alpha}{u} = \frac{28.84s^2 - 2.562e^{-14}s - 1516}{s^4 + 0.05267s^3 - 57.68s^2 - 2.768s} \tag{4.3}$$

$$\frac{\beta}{u} = \frac{24.72s^2 + 0.0001648s + 5.734e^{-14}}{s^4 + 0.05267s^3 - 57.68s^2 - 2.768s} \tag{4.4}$$

By applying root locus equation from 4.3 and 4.4, it can get the value of Ki, Kd and Kp.

4.2 Designing Proportional Integral Derivative (PID) Controller by using Root Locus technique

4.2.1 Pendulum's arm and Pendulum's

Pendulum's arm

In order to design the controller, the pendulum's arm of transfer function is obtain

$$\frac{\alpha}{u} = \frac{28.84s^2 - 2.562e^{-14}s - 1516}{s^4 + 0.05267s^3 - 57.68s^2 - 2.768s} \tag{4.5}$$

The design requirement of plant system must have 5s of settling time and maximum 5% overshoot. It means that pendulum's arm must be stable in vertical position which 0 degrees and in 5s it can be oscillate but in less oscillation.

Table 4 Values of zero and pole gain

Zero Gain	Pole Gain
7.2494	0
-7.2494	7.5924
0	-7.5971
0	-0.0480

To design PID controller, PD and PI controller need to obtain. The transfer function of PID controller is shown in equation 4.5 and 4.6.

$$G_{PI}(s) = G_{PD} \times G_{PI} \tag{4.6}$$

$$G_{PID}(s) = K_1 + \frac{K_2}{s} + K_3(s) \tag{4.7}$$

First damping ratio, ζ need to be calculated from the overshoot. The formula of damping ratio is

$$\zeta = \frac{\left|In\left(\frac{os}{100}\right)\right|}{\sqrt{\pi^2 + In^2\left(\frac{os}{100}\right)}} \tag{4.8}$$

Where: OS = Overshoot

ζ = *damping ratio*

Value of requirement overshoot is 5% which is 0.05. Hence the value of damping ratio, ζ

$$\zeta = \frac{|In(0.05)|}{\sqrt{\pi^2 + In^2(0.05)}} = 0.69 \quad (4.9)$$

Next, to find the natural frequency, ωn is calculated using equation 4.9 below. The value of settling time is 5second.

$$\omega_n = \frac{4}{T_s \zeta}$$

$$\omega_n = \frac{4}{(5)(0.69)} = 1.1594 \quad (4.10)$$

Where: Ts = Rise Time

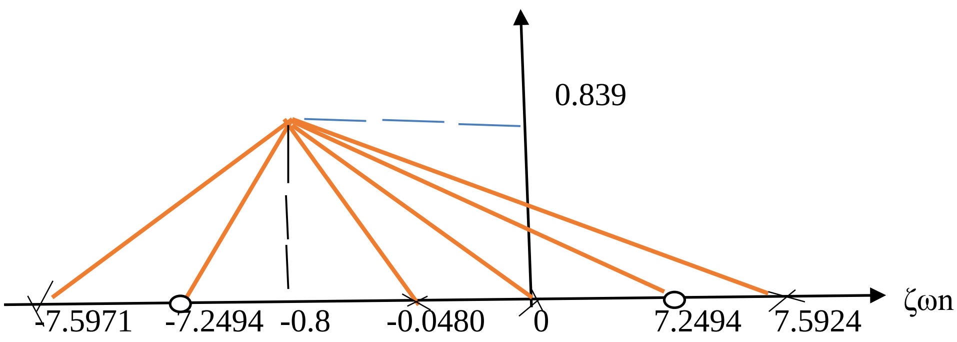
ω_n = Natural Frequency

Dominant pole can be obtained from the equation 4.11 below

$$D.P = -\zeta\omega_n \pm {}^{\omega_n}\sqrt{1 - \zeta^2}$$

$$D.P = -(0.69)(1.1594) \pm {}^{1.1594}\sqrt{1 - (0.69)^2}$$

$$\boldsymbol{D.P = -0.8 \pm j0.839} \quad (4.11)$$



For zero angle efficiency, θ_z it can be calculated as below

$$\theta_{Z1} = 180^{\circ} - tan^{-1}\left(\frac{0.839}{8.0494}\right) = 174.0495 \quad (4.12)$$

$$\theta_{Z2} = tan^{-1}\left(\frac{0.839}{6.4494}\right) = 7.4120 \quad (4.13)$$

For pole efficiency, θ_p it can be calculated as below

$$\theta_{p1} = 180^{\circ} - tan^{-1}\left(\frac{0.839}{8.3924}\right) = 174.2910 \quad (4.14)$$

$$\theta_{p2} = 180^{\circ} - tan^{-1}\left(\frac{0.839}{0.8}\right) = 133.6369 \quad (4.15)$$

$$\theta_{p3} = 180^{\circ} - tan^{-1}\left(\frac{0.839}{0.752}\right) = 131.870 \quad (4.16)$$

$$\theta_{p4} = tan^{-1}\left(\frac{0.839}{6.7971}\right) = 7.0367 \quad (4.17)$$

In order to locate the zero for PD Controller, θ_{zero} the angle deficiency of for pole zero is calculated as below

$$\sum \theta_{zeroes} - \sum \theta_{poles} = -180^{\circ} \quad (4.18)$$

$$(174.0495 + 7.4120 + \theta_z) - (174.2910 + 133.6369 + 131.8700 + 7.0367) = -180 \quad (4.19)$$

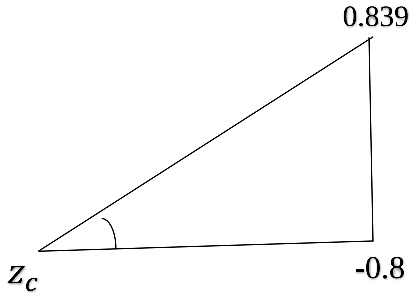
$$\theta_z = 85.3731 \quad (4.20)$$

Where: $\sum \theta_{zeroes}$ = total of zero angle deficiency

$\sum \theta_{poles}$ = total of pole angle deficiency

θ_z = Zero Compensator

Zero compensator can be calculated as below:



$$85.3731 = tan^{-1}\left(\frac{0.839}{z_c - 0.8}\right) \tag{4.21}$$

$$z_c = 0.8679 \tag{4.22}$$

Hence the transfer function of PD controller is:

$$(s) = (s + 0.8679) \tag{4.23}$$

In order to find zero compensator of PI Controller an open loop pole placed at the origin and the pole must be nearest to origin. To find the constant gain, K the value must be calculated using equation 4.25 below

$$K = \left(\frac{\sum poles}{\sum zeroes}\right) \tag{4.24}$$

In order to get the value for each hypotenuse for pole and zero, it can be obtained by calculation below

$$L_{P1} = \sqrt{(7.5924 + 0.8)^2 + (0.839)^2} = 8.4342 \tag{4.25}$$

$$L_{P2} = \sqrt{(0.8 - 0.0480)^2 + (0.839)^2} = 1.1267 \tag{4.26}$$

$$L_{P3} = \sqrt{(7.5971 - 0.8)^2 + (0.839)^2} = 6.848 \tag{4.27}$$

$$L_{z1} = \sqrt{(7.2494 + 0.8)^2 + (0.839)^2} = 8.0930 \tag{4.28}$$

$$L_{z2} = \sqrt{(7.2494 - 0.8)^2 + (0.839)^2} = 6.5037 \tag{4.29}$$

$$L_{zc} = \sqrt{(0.8679 - 0.8)^2 + (0.839)^2} = 0.8417 \quad (4.30)$$

So, to find gain from equation

$$K = \frac{\sum poles}{\sum zeros} = \frac{8.4342 \times 1.1267 \times 6.8487}{8.0930 \times 6.5037 \times 0.8417} = 1.4690 \quad (4.31)$$

Hence the transfer function of PID Controller is

$$G_{PID}(s) = \frac{1.4690(s + 0.001)(s + 0.8417)}{s} \quad (4.32)$$

$$G_{PID}(s) = 1.4690s + 1.2379 + \frac{0.0012}{s} \quad (4.33)$$

Finally, based on the equation of 4.41 the gain of PID Controller for K_P, K_i and K_D is 1.4690, 1.2379 and 0.0012

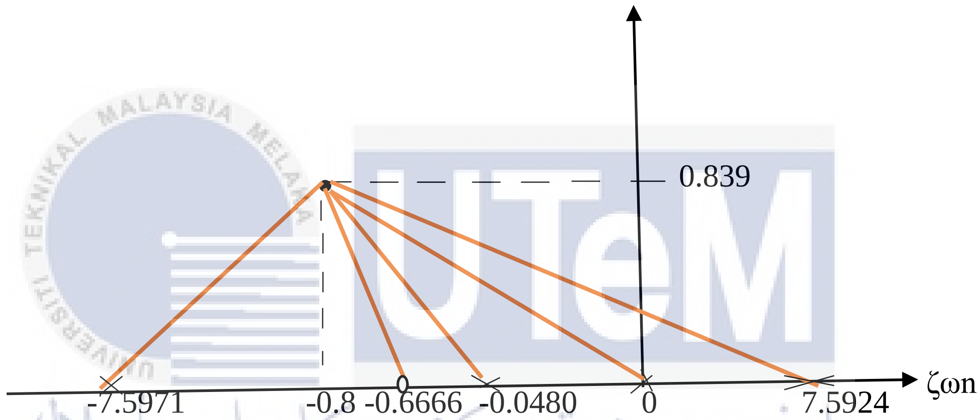
Pendulum's

In order to design the controller, the pendulum of transfer function is obtain

$$\frac{\beta}{u} = \frac{24.72s^2+0.0001648s+5.734e^{-14}}{s^4+0.05267s^3-57.68s^2-2.768s} \tag{4.34}$$

Table 5 Values of zero and pole gain

Zero Gain	Pole Gain
-0.6666	0
0	-7.5971
	-0.0480



For zero angle efficiency, θ_z it can be calculated as below

$$\theta_{Z1} = 180^{\circ} - tan^{-1}\left(\frac{0.839}{0.1334}\right) = 99.0343 \tag{4.35}$$

For pole efficiency, θ_p it can be calculated as below

$$\theta_{p1} = 180^{\circ} - tan^{-1}\left(\frac{0.839}{8.3924}\right) = 174.2910 \tag{4.36}$$

$$\theta_{p2} = 180^{\circ} - tan^{-1}\left(\frac{0.839}{0.8}\right) = 133.636 \tag{4.37}$$

$$\theta_{p3} = 180^{\circ} - tan^{-1}\left(\frac{0.839}{0.752}\right) = 131.8700 \tag{4.38}$$

$$\theta_{p4} = tan^{-1}\left(\frac{0.839}{6.7971}\right) = 7.0367 \quad (4.39)$$

In order to locate the zero for PD Controller, θ_{zero} the angle deficiency of for pole zero is calculated as below

$$\sum \theta_{zeroes} - \sum \theta_{poles} = -180^{\circ} \quad (4.40)$$

$$(99.0343+\theta_z) - (174.2910+133.6369+131.8700+7.0367) = -180 \quad (4.41$$

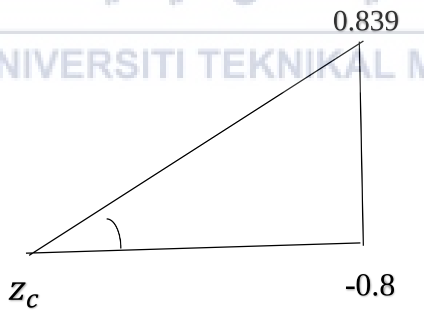
$$\theta_z = 167.8003$$

Where: $\sum \theta_{zeroes}$ = total of zero angle deficiency

$\sum \theta_{poles}$ = total of pole angle deficiency

θ_z = Zero Compensator

Zero compensator can be calculated as below:



$$85.3731 = tan^{-1}\left(\frac{0.839}{z_c - 0.8}\right) z_c = 0.8679 \quad (4.42)$$

Hence the transfer function of PD controller is:

$$(s) = (s + 0.8679) \quad (4.43)$$

In order to find zero compensator of PI Controller an open loop pole placed at the origin and the pole must be nearest to origin. To find the constant gain, K the value must be calculated using equation 4.25 below

$$K = \left(\frac{\sum poles}{\sum zeroes}\right) \tag{4.44}$$

In order to get the value for each hypotenuse for pole and zero, it can be obtained by calculation below

$$L_{P1} = \sqrt{(7.5924 + 0.8)^2 + (0.839)^2} = 8.4342 \tag{4.45}$$

$$L_{P2} = \sqrt{(0.8 - 0.0480)^2 + (0.839)^2} = 1.1267 \tag{4.46}$$

$$L_{P3} = \sqrt{(7.5971 - 0.8)^2 + (0.839)^2} = 6.8487 \tag{4.47}$$

$$L_{z1} = \sqrt{(0.8 - 0.6666)^2 + (0.839)^2} = 0.8495 \tag{4.48}$$

$$L_{zc} = \sqrt{(0.8679 - 0.8)^2 + (0.839)^2} = 0.8417 \tag{4.49}$$

So, to find gain from equation

$$\mathrm{K} = \frac{\sum \mathrm{poles}}{\sum \mathrm{zeros}} = \frac{8.4342 \times 1.1267 \times 6.8487}{0.1334 \times 0.8495} = 574.3030 \tag{4.50}$$

Hence the transfer function of PID Controller is

$$\mathrm{G_{PID}(s)} = \frac{574.3030(\mathrm{s} + 0.001)(\mathrm{s} + 0.8417)}{\mathrm{s}} \tag{4.51}$$

$$\mathrm{G_{PID}(s)} = 574.3030\mathrm{s} + 0.8427 + \frac{0.00084}{\mathrm{s}} \tag{4.52}$$

Finally, based on the equation of 4.41 the gain of PID Controller for K_P, K_i and K_D is 574.3030, 0.8427 and 0.00084.

Figure 4.1 shows the block diagram of double-PID controller and the calculation have been calculated to get the value of gain Kp, Ki and Kd using root locus method. There are two systems that need to control that is pendulum (α) and pendulums arm (β).

The main target is to maintain pendulum angle alpha, α as zero so that the rotary inverted pendulum will be stable. One will maintain the speed and position of beta while other controller will function based on the feedback of alpha. Alpha and beta indicates the plant of model of alpha and theta output while there is have two PID compensators that represent to PID Alpha and PID Beta.

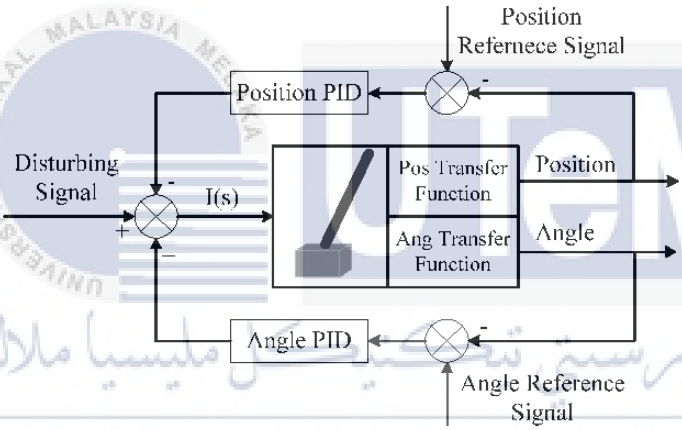


Figure 4.1 Block Digram of double-PID Controller [16]

It is basically a test and error method where the damping ratio and frequency values must sometimes be higher than the calculated one in order to meet the desired requirements. A small program is written using the Root Locus algorithm where the program takes the damping ratio and frequency value to calculate both compensators. Kp= 1.4690, Ki= 1.2379 and Kd= 0.0012 and Kp= 574.3030, Ki= 0.8427 and Kd= 0.00084 are the calculated gains for alpha beta.

To analyze the behavior of the output response to the angle position of the pendulum when implementing the PID controller. The gain is 574.3030, 0.8427 and 0.00084 for Kp, Ki and Kd. The results of the pendulum angular position response for each initial position are shown in Figure 4.3, 4.4, 4.5 and 4.6. First, the pendulum is tested at the initial position at 180 degrees and the pendulum result is stable as it stops at the angle of 0 degrees with 4,300 for ascending time and 7,762 for settling time. This is because the pendulum needs to swing up the pendulum from its hanging position to the upright position of 0 degree when the initial angle position is too large for the pendulum. However, this experiment is involve PID controller as balancing controller.

The PID controller for the pendulum is varied in order to analyze the output response for the pendulum as the design of the controllers gives the best system performance result. The pendulum gain value is Kp= 574.3030, Ki= 0.8427 and Kd= 0.00084 and the arm angle value is Kp= 1.4690, Ki= 1.2379 and Kd= 0.0012 based on the design. Table 5 and Table 6 show the gain value being tuned using the manual tuning method to assess how different gain value affects the behavior of the rotary inverted pendulum system in terms of overshoo, rise time and settling time.

Table 6 Tuning Arm Angle (Alpha)

	P	I	D
Calculated	1.4690	1.2379	0.0012
Tuning	0.000042	0.0000015	0.00001

Table 7 Tuning Pendulum Angle (Beta)

	P	I	D
Calculated	574.3030	0.8427	0.00084
Tuning	0.000042	0.0000015	0.00001

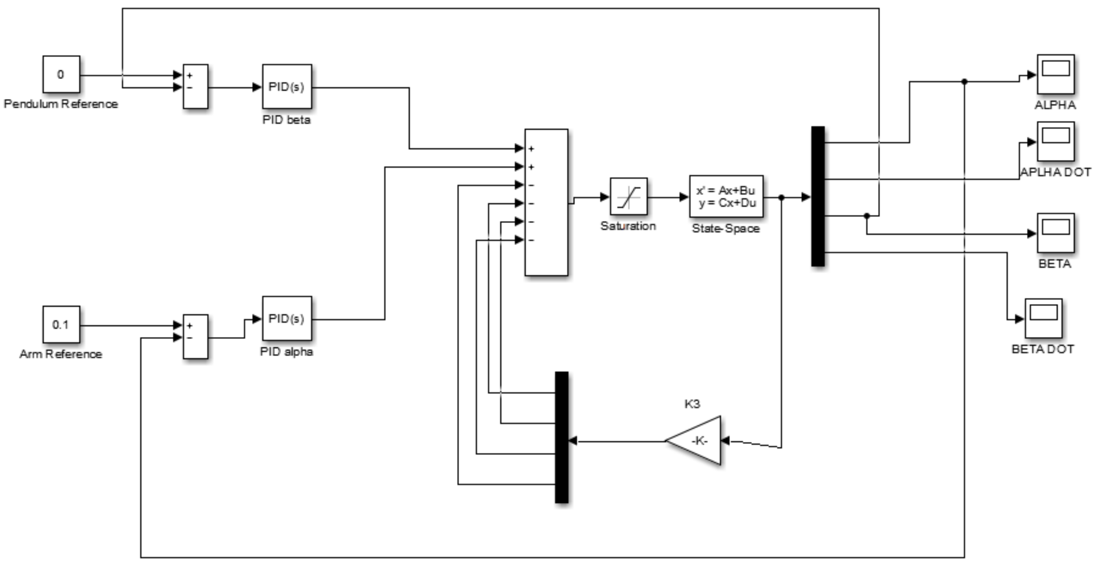


Figure 4.2 Simulink of Block Diagram of Double-PID+LQR controller for rotary inverted pendulum

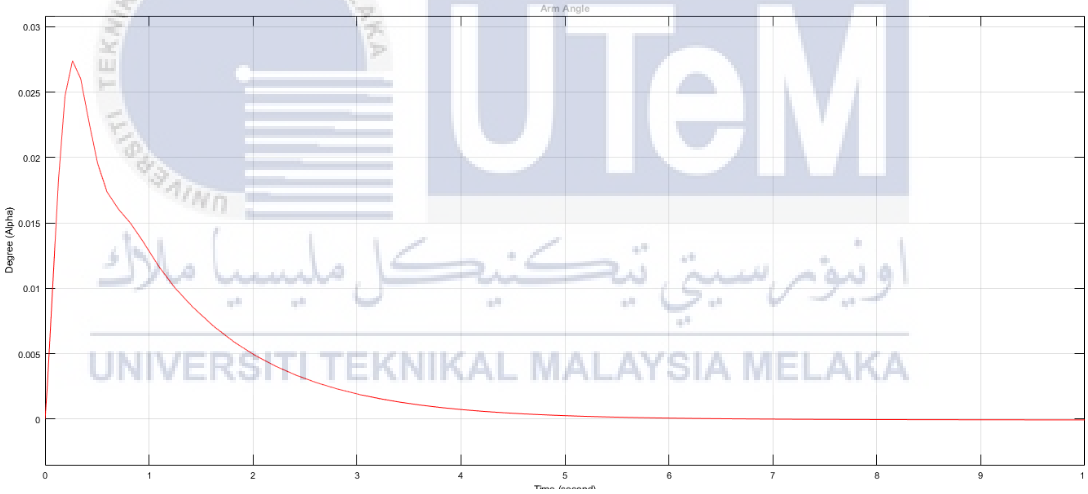


Figure 4.3 Step Response of Arm Angle (Alpha) in double-PID+LQR

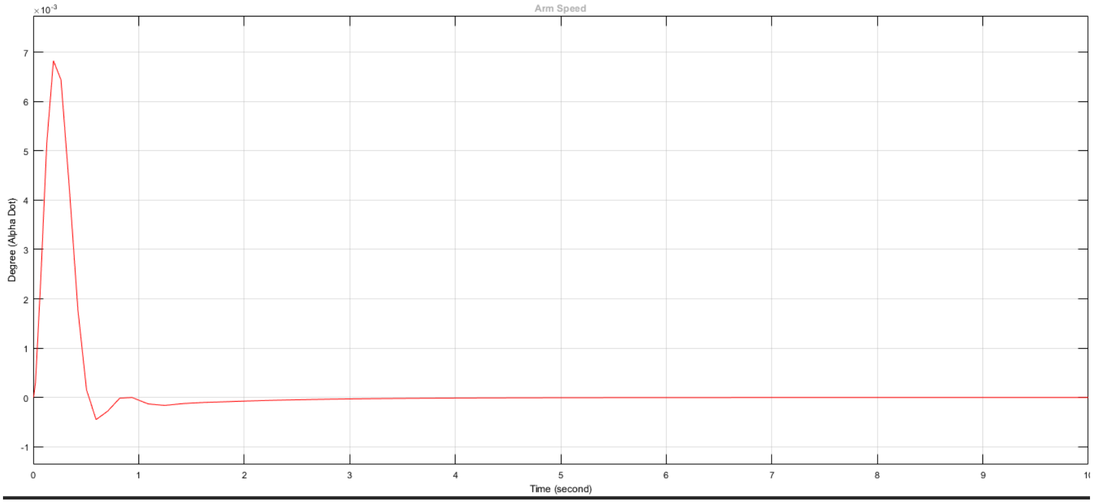


Figure 4.4 Step Response of Arm Speed (Alpha Dot) in double-PID+LQR

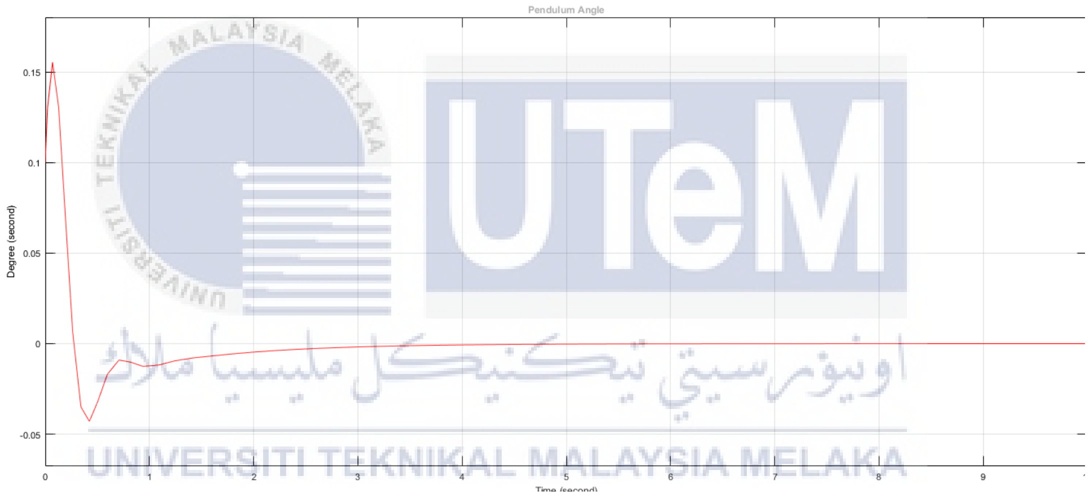


Figure 4.5 Step Response of Pendulum Angle (Beta) in double-PID+LQR

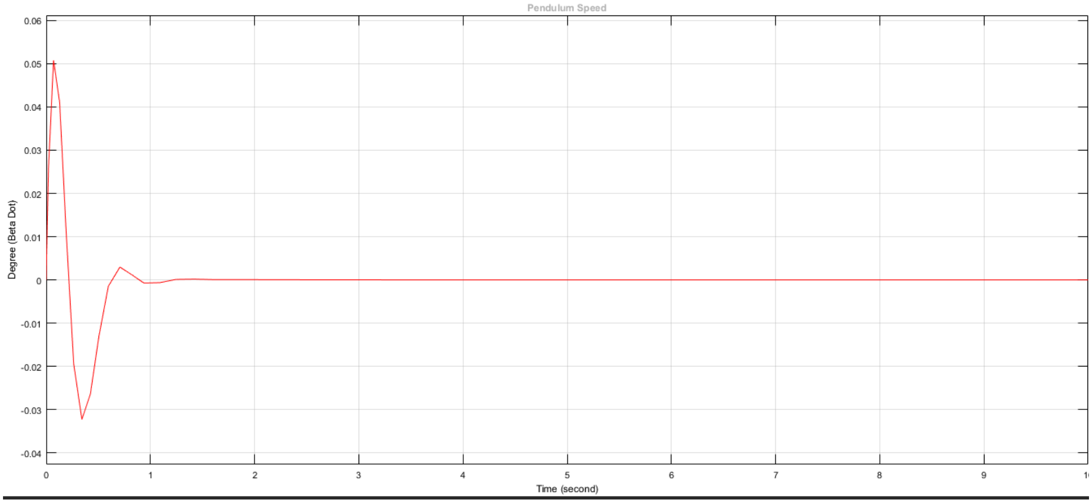


Figure 4.6 Step Response of Pendulum Speed (Beta Dot) in double-PID + LQR

4.3 System Without Controller

By inserting the value of state space that have been calculated in order to obtain the result of simulation. The block diagram of the rotary inverted pendulum system is shown in figure 4.7 below

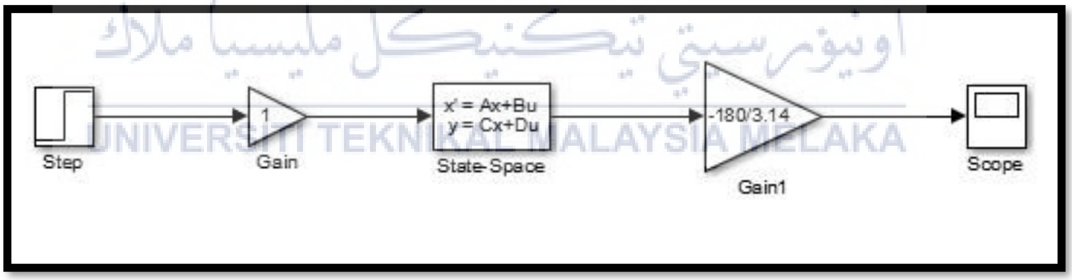


Figure 4.7 Block diagram simulation of Rotary Inverted Pendulum without controller

Figure 4.8 shows the graph of step response of rotary inverted pendulum without controller. The graph is not stable because the pendulum does not swing up. The x-axis represent time in second while the y-axis represents the angle of the pendulum.

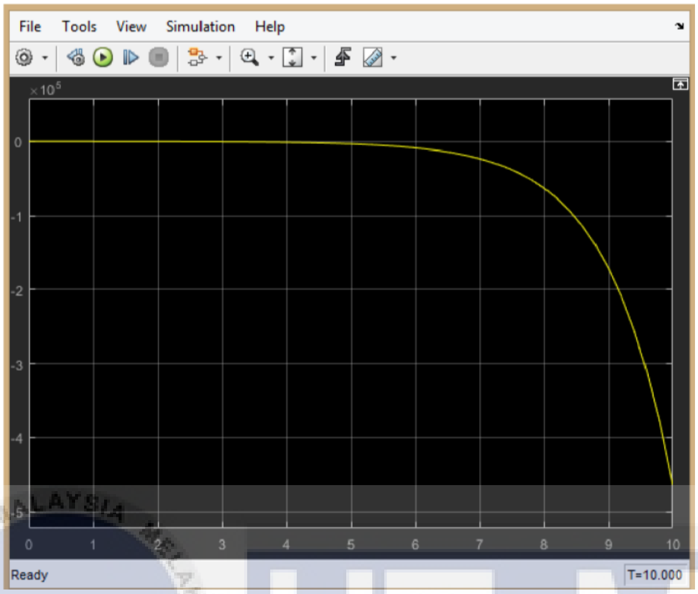


Figure 4.8 Graph step response of rotary inverted pendulum without controller

4.4 LQR Controller

Matrix Q is obtained using the method of testing and error. Linear Quadratic Regulator is an optimal controller used with minimal control effort and time to achieve the desired target value. Q and R weigh matrices that allow individual state variables and individual control inputs to be weighed relatively. In MATLAB, the [K]= lqr (A, B, Q, R) command calculates the optimal feedback matrix K to minimize the cost function subject to the state equation constraint. The system's response to different set of state gain matrix feedback is determined by varying Q values, keeping R= 1 and choosing the one that gives the best performance. LQR controller can be considered as robust controller. The simulation result for pendulum position and arm position for controller are shown in figure.

$$Q = \begin{bmatrix} 50 & 0 & 0 & 0 \\ 0 & 26.9 & 0 & 0 \\ 0 & 0 & 55 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.53}$$

$$R = [1\ ]$$

K =[−0.435 10.752 −0.577 1.350] (4.54)

Rotary inverted pendulum system need controller to make sure that the pendulum move in upward position. After the calculation have been calculate using MATLAB (coding in Appendix) to get the value of gain. The Simulink must be build and the parameters need to be insert in the system. Figure 4.9 shows that the simulation of LQR.

Figure 4.10, 4.11, 4.12 and 4.12 shows that the result of rotary inverted pendulum using MATLAB. Rotary inverted pendulum system need controller to make sure that the pendulum move in upward position. After the calculation have been calculate using MATLAB (coding in Appendix) to get the value of gain.

Since the controllability matrix is 4x4, the matrix rank must be 4, using the MATLAB command ctrb to generate the controllability matrix and the MATLAB command rank to test the matrix rank. The following output will be generated by adding the following additional commands to m-file and running in the MATLAB command window.

Controllability = 4

After verifying that the system can be controlled, the state-feedback gain control matrix K was then determined. The LQR MATLAB function allows two parameters R and Q to be selected to balance the relative importance of the control effort and error in the cost function that needs to be optimized. The simplest case is that the diagonal matrix is assumed by R=1 and Q. The cost function corresponding to this R and Q places equal importance on the variables of control and status that output the angle of the arm and angle of the pendulum. The attempt and error diagonal Q matrix is used to observe Q's structure.

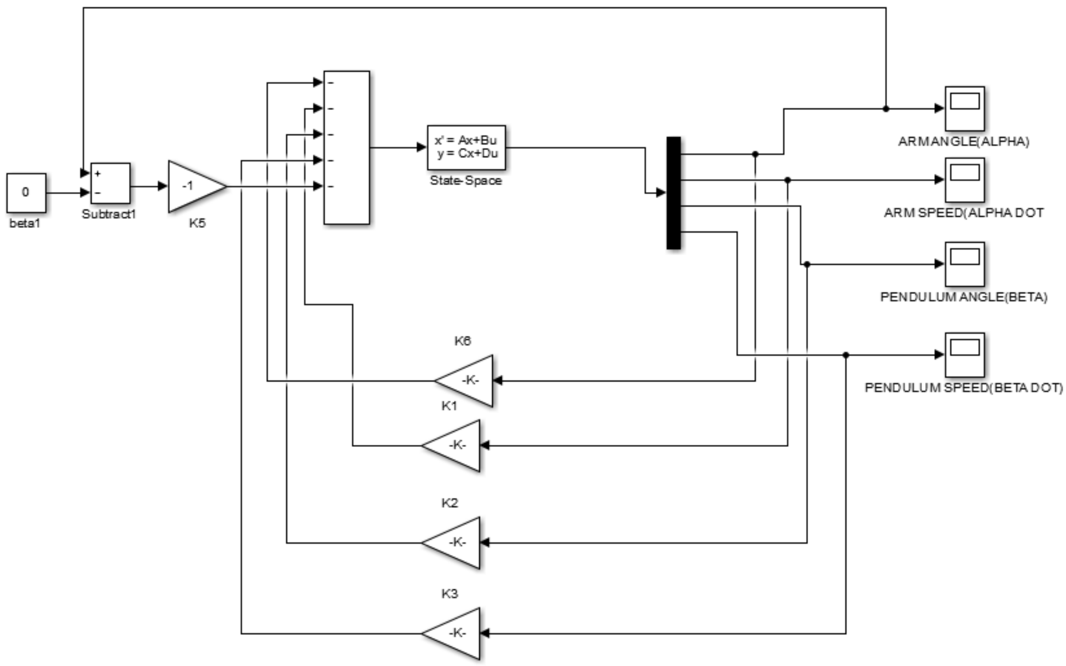


Figure 4.9 Simulink Diagram of LQR Controller

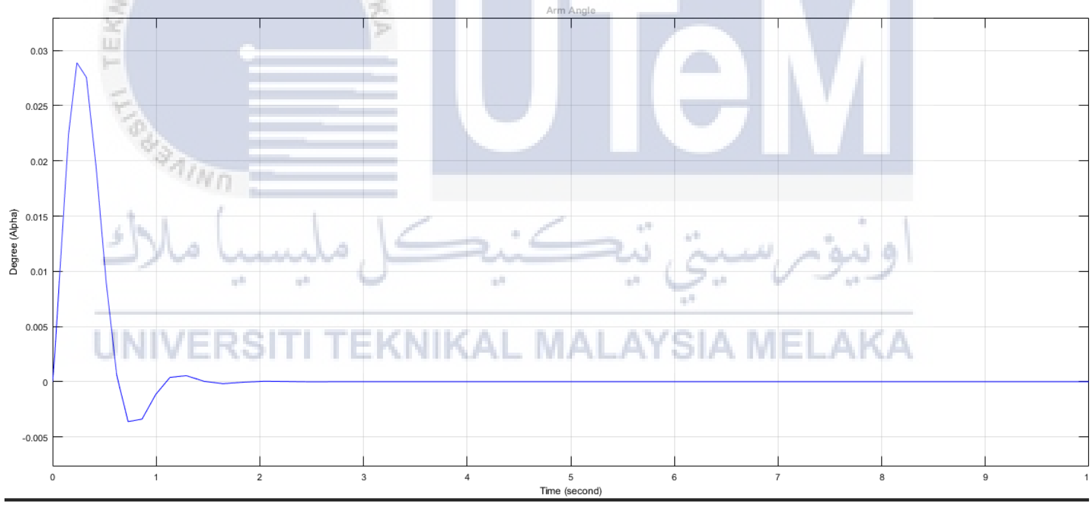


Figure 4.10 Step Response of Arm Angle (Alpha) in LQR Controller

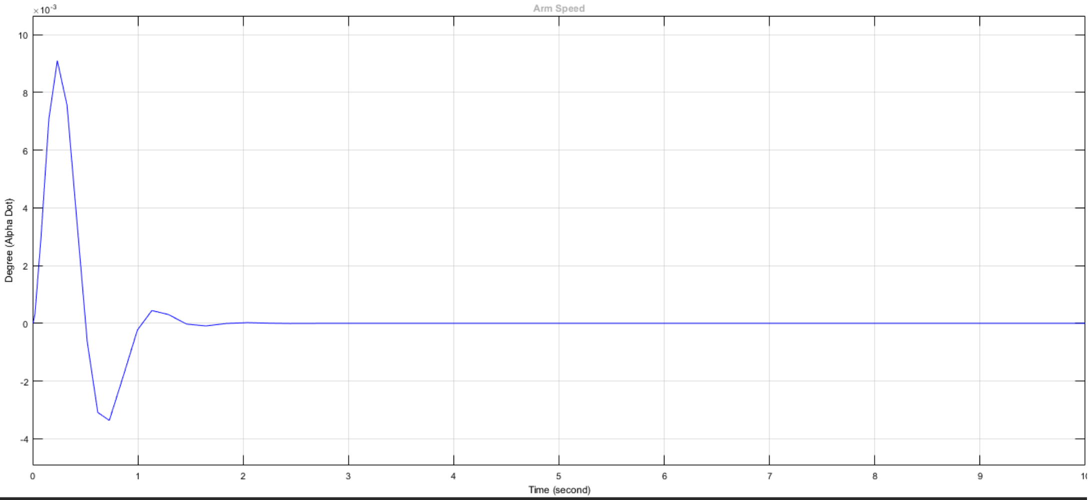


Figure 4.11 Step Response of Arm Speed (Alpha Dot) in LQR Controller

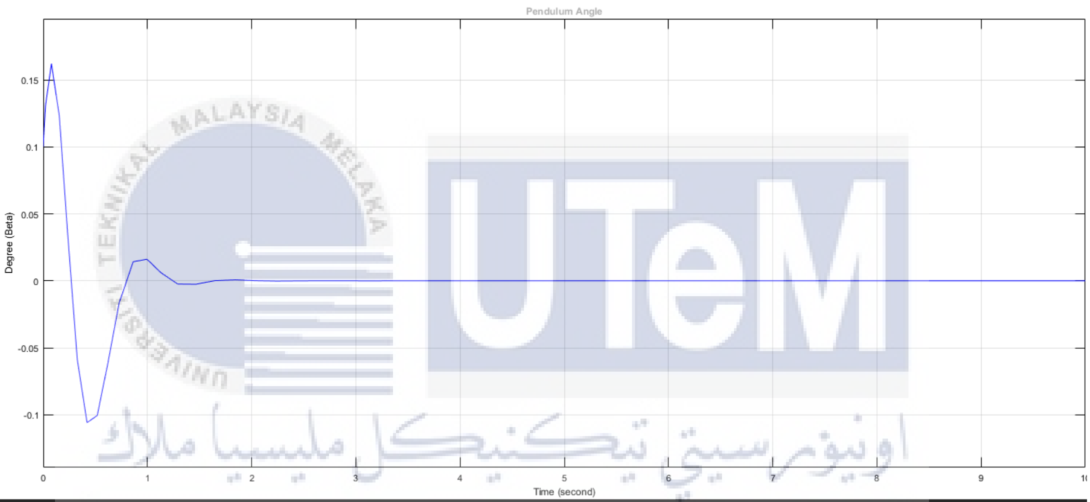


Figure 4.12 Step Response of Pendulum Angle (Beta) in LQR Controller

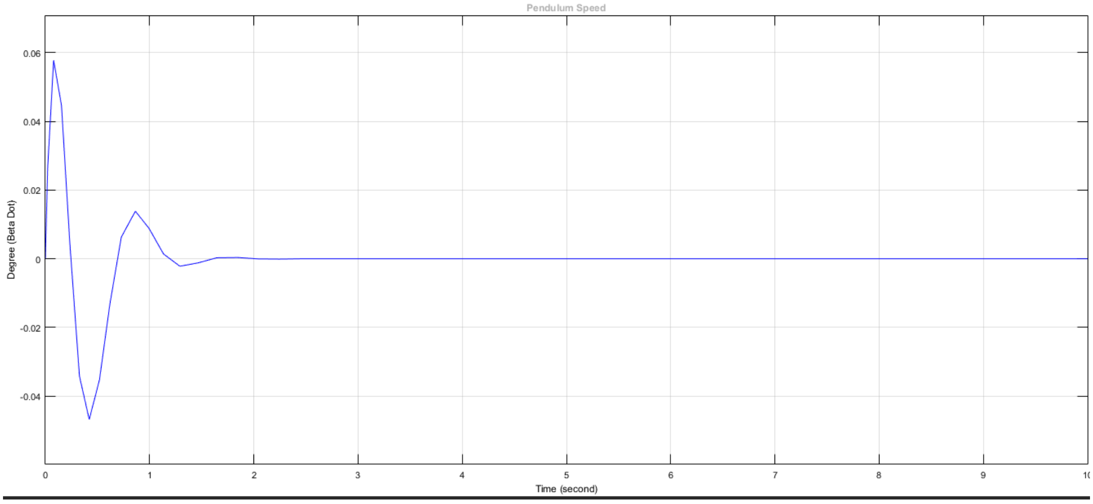


Figure 4.13 Step Response of Pendulum Speed (Beta Dot) in LQR Controller

LQR tries to maintain the system stability. In the practical case the controller is capable to maintain the pendulum vertically up but not robust. The other controller that is double-PID+LQR can be considered as robust. For both controllers double-PID+LQR and LQR, comparative simulated results of alpha, alpha dot, theta and theta dot sre shown in figure 4.15, 4.16, 4.17 and 4.18. As shown in figure below, the maximum overshoot of LQR is larger than double-PID+LQR controller. Furthermore, double-PID+LQR have a faster response, which can be seen from rising and settling time. Table 5 shows the comparison between two controllers. The rising time and settling time of LQR is more than twice of double-PID+LQR controller, which suggest that the double-PID+LQR controller has a faster transient response. Besides, the LQR has an advantage over double-PID+LQR in maximum overshoot.

4.5 Comparison Between double-PID+LQR and LQR Controller

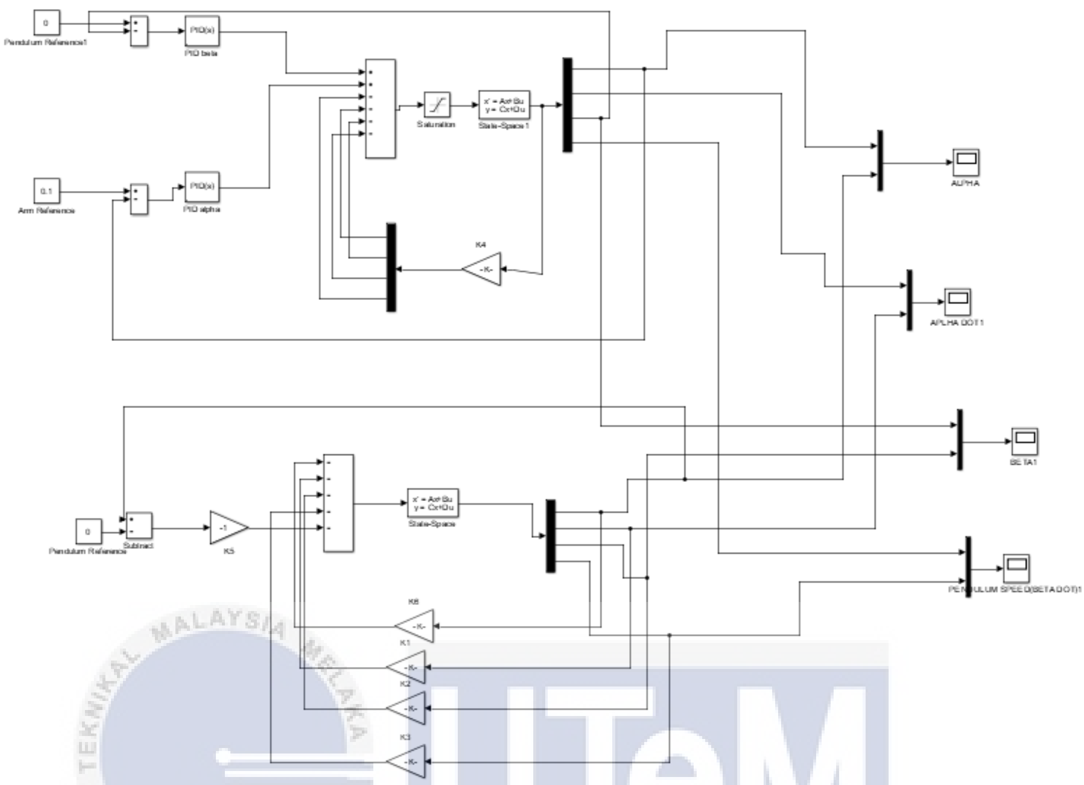


Figure 4.14 Simulation Diagram of double-PID+LQR and LQR

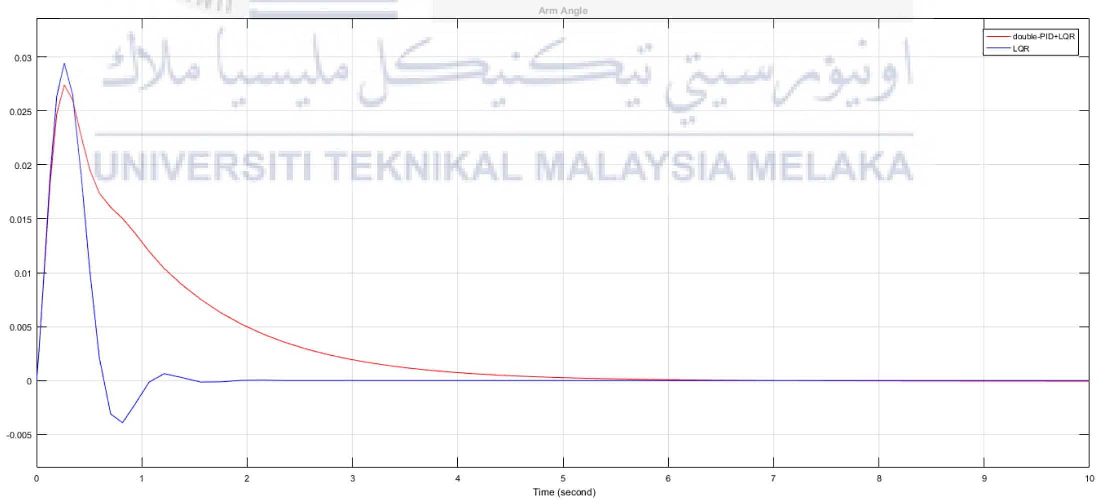


Figure 4.15 Comparative results of Arm Angle (Alpha)

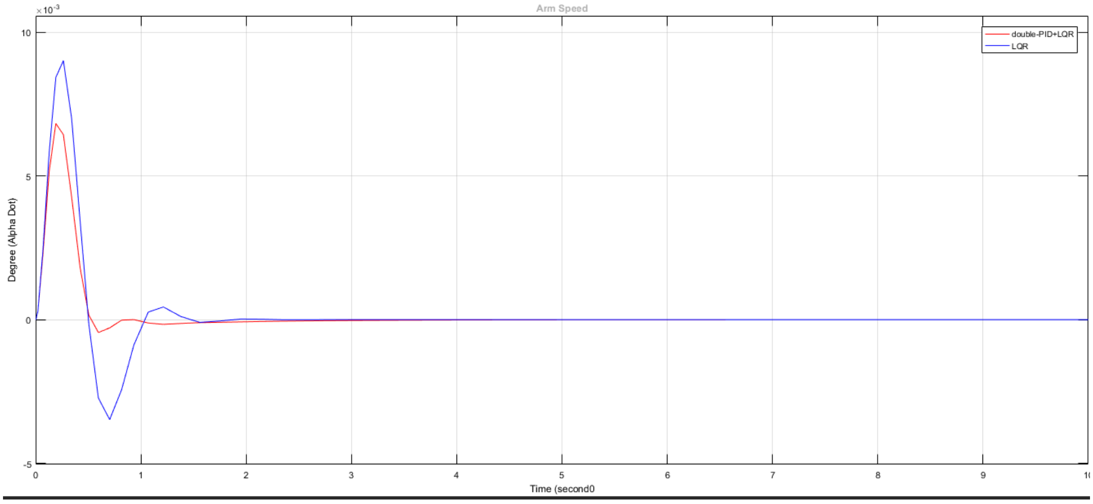


Figure 4.16 Comparative results of Arm Speed (Alpha Dot)

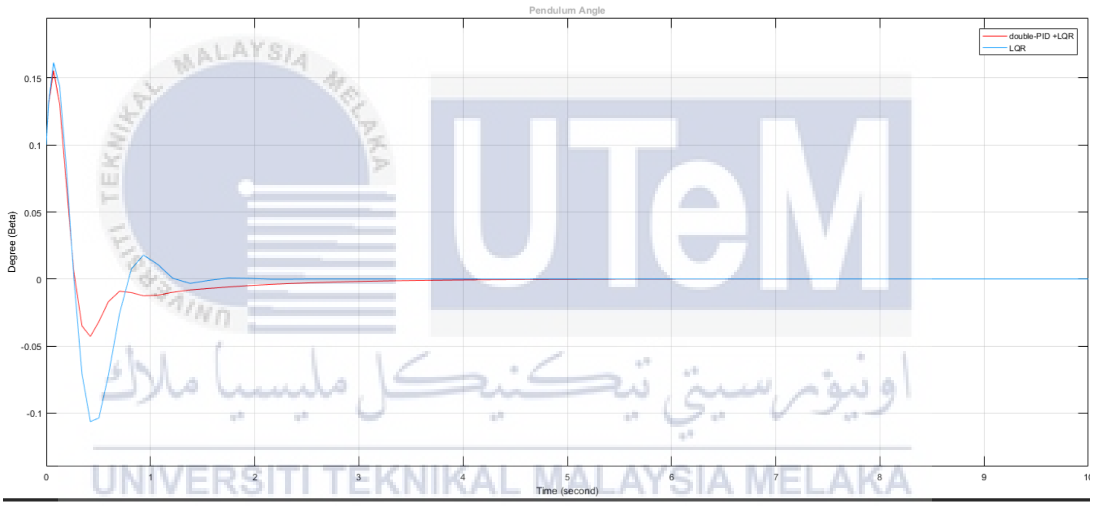


Figure 4.17 Comparative results of Pendulum Angle (Beta)

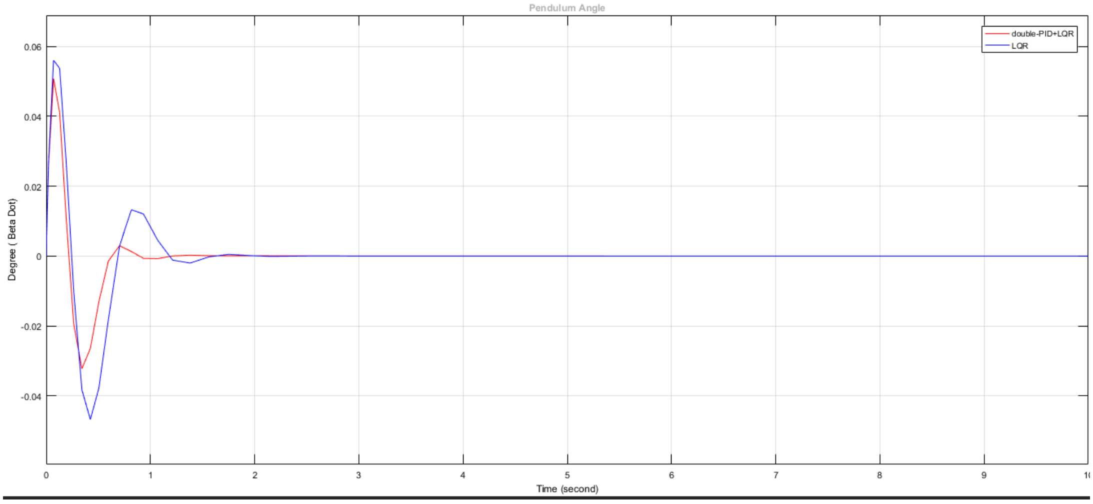


Figure 4.18 Comparative results of Pendulum Speed (Beta Dot)

From table 8 shows that the comparison between two methods that be used in order to maintain the pendulum in upright position.

Table 8 Comparison Between 2 Methods

Controllers	Rise Time	Setling Time	Overshoot	Peak Value
Double-PID+LQR	0.160	5.342	99%	0.0274
LQR	0.131	2.342	55%	0.0294

From figure above it is clearly seen that for both outputs such as alpha, alpha dot, beta and beta dot, double-PID+LQR controller shows the better result where rising time, settling time, overshoot and peak value are more acceptable than LQR controller. On observing above figures, double-PID+LQR controller is better to settle the response with in less time than LQR and double-PID+LQR controller is better in reducing the overshoot of the system. However by proper choice of weighing matrices Q and R, LQR response can further improved and is more dominant than LQR controller. The gain matrix K chosen LQR is almost perfect for stabilizing the pendulum. It is always not easy to obtain the gain matrix LQR. But in the case of LQR controller the gain matrix K can be tuned easily to obtain desired response.

LQR control methods are discussed for the stabilization of Rotary Inverted Pendulum system. From the simulation results, it is found that both double-PID+LQR and LQR method are efficient in satisfying the design requirements and robust to the parameter variations. The double-PID+LQR control shows better results in minimizing the steady state value compared to LQR while double-PID+LQR method is better to improve transient response of the system. Both double-PID+LQR and LQR controllers are capable of maintaining the pendulum in its upright position.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

In this chapter 5, the project will be concluded, and the suggestion and recommendation will be discussed. The suggestion is needed to make sure that there is some improvement of this projects

5.1 Conclusion

The application of concept of Rotary Inverted Pendulum system has become wider in the industry, robotics and field of research due to its simplicity of system setup with highly unstable and underactuated characteristics. To achieve the first objective previous researches are studied, the stabilization controller of LQR and Double-PID+LQR is proposed to be the stabilization controller to maintain upright position of Rotary Inverted Pendulum. Lagrange's equation is one of the suggested approaches to model the Rotary Inverted Pendulum system. With the consideration of kinetic energy and potential energy of the Rotary Inverted Pendulum system, a mathematical model of Rotary Inverted Pendulum is obtained. Without a controller, a Rotary Inverted Pendulum cannot be stable at upright position. The second objective, To design and simulate double Proportional Integral Derivative Double-PID and Linear Quadratic Regulator (LQR) controller for balancing the rotary inverted pendulum. Designing of double-PID+LQR controller and LQR controller is carried out respectively. Double-PID controller is designed by using root locus method while LQR controller is designed by using optimal gain characteristics. After designing, the comparison the performance of stability between double-PID+LQR controller and LQR controller need to figure out. The stabilization performance is evaluated and analyzed. As conclusion, the double-PID+LQR controller has improved the stabilisation performance of the rotary inverted pendulum compared to LQR.

5.2 Future Works

From the repeatability test on the stabilization performance of Rotary Inverted Pendulum system, it showed the long control time affected the result of stabilization performance. Shorter control time and longer time interval between each attempt in repeatability test can result a better stabilization performance. There are many idea to suggest for improvement the control of rotary inverted pendulum. For future work and development, another controller can be used instead of 2DOF-PID and LQR that have been used in this research. PSO algorithm method can solved and tuning the controller parameters more efficiently. For future research to utilize other controller and must improving the performance of real time.

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APPENDICES

APPENDIX A CODE IN MATLAB TO OBTAIN THE VALUE OF GAIN

```
A=[ 0     0          1       0;
     0     0          0        1;
     0     5.98   -0.05267   0;
     0     57.68 -0.04514    0]


 B=[   0;
       0;
       28.84;
       24.72]

  C= [0 1 0 0]

    D=[0]
    sys = ss(A,B,C,D)
poles = eig (A)
co = ctrb (A,B)
controllability = rank (co)
R = [1]
Q = [  50      0          0        0;
     0      26.9         0         0;
     0       0          55         0;
     0       0           0         1]

[K]=lqr(sys,Q,R)
sys_cl=ss(A-B*K,B,C,D)
```

APPENDIX B THE CONTROLLABILITY MATRIX Q AND THE VALUE OF GAIN

Command Window

```
A =

         0         0    1.0000         0
         0         0         0    1.0000
         0    5.9800   -0.0527         0
         0   57.6800   -0.0451         0


B =

         0
         0
   28.8400
   24.7200


C =

     0     1     0     0


D =

     0


sys =
```

Command Window

```
sys =

  A = 
             x1        x2        x3        x4
   x1         0         0         1         0
   x2         0         0         0         1
   x3         0      5.98  -0.05267         0
   x4         0     57.68  -0.04514         0

  B = 
          u1
   x1      0
   x2      0
   x3  28.84
   x4  24.72

  C = 
       x1  x2  x3  x4
   y1   0   1   0   0

  D = 
       u1
   y1   0

Continuous-time state-space model.
```

```
Command Window

poles =

         0
    7.5924
   -7.5971
   -0.0480


co =

   1.0e+03 *

         0    0.0288   -0.0015    0.1479
         0    0.0247   -0.0013    1.4259
    0.0288   -0.0015    0.1479   -0.0156
    0.0247   -0.0013    1.4259   -0.0818


controllability =

     4


R =
```

```
Command Window
   1.0e+03 *

         0    0.0288   -0.0015    0.1479
         0    0.0247   -0.0013    1.4259
    0.0288   -0.0015    0.1479   -0.0156
    0.0247   -0.0013    1.4259   -0.0818


controllability =

     4


R =

     1


Q =

   50.0000         0         0         0
         0   26.9000         0         0
         0         0   55.0000         0
         0         0         0    1.0000


K =
```

APPENDIX C GHANT CHART