COMPARISON BETWEEN DOUBLE-PID+LQR AND LQR CONTROLLERS FOR ROTARY INVERTED PENDULUM

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2019

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A report submitted
in partial fulfillment of the requirements for the degree of
Bachelor of Electrical Engineering with Honours

Faculty of Electrical Engineering

UNIVERSITI TEKNIKAL MALAYSIA MELAKA

2019

DECLARATION

I declare that this thesis entitled "COMPARISON BETWEEN DOUBLE-PID+LQR AND LQR CONTROLLERS FOR ROTARY INVERTED PENDULUM is the result of my own research except as cited in the references. The thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

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APPROVAL

I hereby declare that I have checked this report entitled "Comparison Between double-PID +LQR And LQR Controllers For Rotary Inverted Pendulum " and in my opinion, this thesis it complies the partial fulfillment for awarding the award of the degree of Bachelor of Electrical Engineering with Honours

Signature : ....

Supervisor Name : ....

Date : ....

DEDICATIONS

This report is dedicated to my father and my mother who taught me the best knowledge which they learn for their own sake and always give moral supports to finish this largest tasks.

ACKNOWLEDGEMENTS

Firstly, I want to express my appreciation to my parents for their advice to face the problem and whose always give support in term of economical and mental. Without them, I will have difficult challenge to face and finish this task.

Next, I would like to express my deepest gratitude and special thanks to my supervisor, Dr Azrita Binti Alias who always give moral support, guide and keep me on the correct path to do my Final Year Project. Without his advices and guidance, this task would have never been finished.

I wish to express also to my indebtedness appreciation to the panels Dr Muhammad Nizam bin Kamarudin and Ir Mohd Safirin bin Karis for their guidance, advices and motivation. Without their support and interest this project would not have been same as presented here.

Last but not least, I would like to extend my thankfulness to my fellow friends for their support and guidance. Their views and tips are useful indeed.

ABSTRACT

The Rotary Inverted Pendulum is a classic problem of theory. This report focuses on the design, stability analysis and modeling of the rotor pendulum inverted. It also provides the design steps for the controllers for an inverted rotary motion pendulum operated by the rotary servo plant of the SRV 02 Series. A control system is designed using classic and modern control methods. The classic root locus method is the design of two compensators for the PID controller. The second method of the modern method control technique is the Linear Quadratic Regulator (LQR). The linear square regulator is thus tested for the upright and swing mode of the pendulum. The mathematical derivatives also showed that the designed controller needs to stabilize the pendulum system. Simulation studies are conducted to demonstrate the efficiency of the designed controller and the result shows that the controller can maintain a stable reverse position.

ABSTRAK

Pendulum berputar Rotary adalah masalah teori klasik. Laporan ini memberi tumpuan kepada reka bentuk, analisis kestabilan dan pemodelan pendulum pemutar terbalik. Ia juga menyediakan langkah-langkah reka bentuk untuk pengawal untuk pendulum gerakan putar terbalik yang dikendalikan oleh kilang servo putar SRV 02 Series. Sistem kawalan direka menggunakan kaedah kawalan klasik dan moden. Kaedah lokus akar klasik adalah reka bentuk dua pemampat untuk pengawal PID. Kaedah kedua kaedah kawalan kaedah moden ialah Pengatur Kuasa Lajur Linier (LQR). Oleh itu pengawal selia persegi linear diuji untuk mod tegak dan swing pendulum. Derivatif matematik juga menunjukkan bahawa pengawal yang direka untuk menstabilkan sistem pendulum. Kajian simulasi dijalankan untuk menunjukkan kecekapan pengawal yang direka dan hasilnya menunjukkan bahawa pengawal boleh mengekalkan kedudukan terbalik yang stabil.

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LIST OF SYMBOLS AND ABBREVIATIONS

SYMBOL	DESCRIPTION
L	Length to Pendulum's Center of Mass
m	Mass of Pendulum Arm
r	Rotating Arm Length
θ	Servo load gear angle (radians)
α	Pendulum Arm Deflection (radians)
h	Distance of Pendulum Center of mass from ground
Jcm	Pendulum Inertia about its center of mass
Vx	Velocity of Pendulum Center of mass in the x-direction
Vy	Velocity of Pendulum Center of mass in the y-direction
m_1	Mass of Arm
m_2	Mass of Pendulum
l_1	Length of Arm
l_2	Length of Pendulum
c_1	Distance to Centre of Arm Mass
c_2	Distance to Centre of Pendulum Mass
J_1	Inertia of Arm
J_2	Inertia of Pendulum
g	Gravitational Acceleration
α	Angular Position of Arm
$\dot{\alpha}$	Angular Velocity of Arm
β	Angular Position of Pendulum
$\dot{\beta}$	Angular Velocity of Pendulum

C_1	Viscous Friction Coefficient of Arm
C_2	Viscous Friction Coefficient of Pendulum
K_t	Motor Torque Constant
K_b	Motor Back-Emf Constant
K_u	Motor Driver Amplifier Gain
R_m	Armature Resistance
L_m	Armature Inductance

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CHAPTER 1

INTRODUCTION

This chapter will discuss on the background of the project study, motivation, problem statement, objectives and the project scope.

1.1 Background

An inverted pendulum is a pendulum with a mass center above the pivot point. It is unstable and will decrease without further help. It can be stable by using a control system to monitor the pole angle and move the pivot point horizontally back below the mass center when it begins to fall. Figure 1.1 (i) shows that the inverted pendulum system is motivated by the need to design controllers for rocket balancing during vertical takeoff. Similar to the launching rocket, the inverted pendulum requires a continuous correction mechanism to remain upright, as the system is unstable in open loop configuration [1]. This problem can be compared with the launch rocket, where rocket boosters must be fired in a controlled manner to keep the rocket upright.

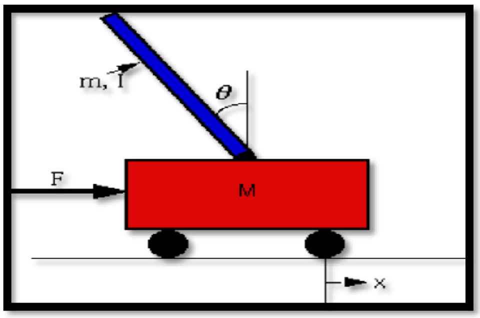

Figure 1.1 Inverted Pendulum[1]

Rotary inverted pendulum is one of the simple but hard systems to balance the upright position. Because the pendulum naturally would fall from the upright vertical position. Design of a modified PID controller and Linear Quadratic Regulator (LQR)[2][3] as a solution. Mathematical modeling is required to obtain a precise feedback. MATLAB / SIMULINK is therefore to be used in this project to control the rotary inverted pendulum by mathematical modeling of the modified PID and Linear Quadratic Regulator.

Figure 1.1 shows the inverted rotary pendulum, consisting of a pendulum that rotates in the vertical plane and is attached to a pendulum arm mounted on the servo motor shaft. At the end of the pendulum arm, the pendulum is attached to a hinge with an encoder [3]. The pendulum arm itself will rotate in horizontal plane and the pendulum will always hanging downwards. This type of pendulum, which is rotary inverted pendulum is an unstable system and required a controller to be actively balanced in order to remain the pendulum in upright vertical position.

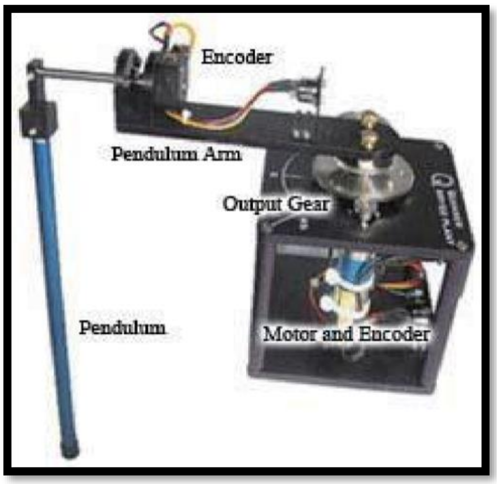

Figure 1.2 Rotary Inverted Pendulum[2]

1.2 Motivation

The rotary inverted pendulum system was motivated by the design of the controller that stabilized the rockets during the vertical take-off, because the rocket was very unstable at the start. In order to keep the rocket upright, rocket boosters must be fired in a controlled way during the start. Based on the launching process of the rocket, the inverted pendulum requires a continuous correction mechanism, as the system is unstable in open loop configuration.

1.3 Problem Statement

Inverted pendulum is one of the key issues in the theory of control. When it comes to rotating the pendulum, however, to ensure that the pendulum remains vertically upright, it is quite difficult, as it is naturally unstable and has an open loop setup. The controllers used are therefore double-PID and LQR, which must be designed and simulated successfully to switch the pendulum up.

1.4 Objectives

There are 3 objectives need to be achieved in this project:

a) To obtain the mathematical model of rotary inverted pendulum in transfer function and in state function.

b) To design and simulate double Proportional Integral Derivative with Linear Quadratic Regulator (double-PID+LQR) and Linear Quadratic Regulator (LQR) controller for balancing the rotary inverted pendulum.

c) To compare the performance of stability between double Proportional Integral Derivative with Linear Quadratic Regulator (double-PID+LQR) and Linear Quadratic Regulator (LQR) controllers.

1.5 Scopes of Project

This project required some scopes that need to achieve the objectives. The scopes of work are as follows:

1. Modeling the system to obtain the mathematical model for rotary inverted pendulum system.
2. The double Proportional Integral Derivative (double-PID) and Linear Quadratic Regulator (LQR) controllers need to be developed after the mathematical model have been derived.
3. The design requirement are Ts < 5 second and %OS < 10%.
4. The performance of the designed controllers is simulated using MATLAB SIMULINK software.

CHAPTER 2

LITERATURE REVIEW

After research has been done, a literature review will be discussed in this chapter 2. A literature review can be stated in a number of ways to complete the project as a discussion of information.

2.1 Theory and Basic Principle of Rotary Inverted Pendulum

Figure 2.1 shows Rotary Inverted Pendulum systems an under-actuated system which consists of one actuator and double Perpendicular Integral Derivative (double-PID). The only one actuator in the system the DC motor. The rotary arm is driven by the DC motor where electrical energy is converted into mechanical energy, the torque to move it. The angular motion of the rotary arm gives energy to the pendulum to swing up and maintain stable at vertical upright position. The pendulum is set to be always perpendicular to the rotary arm. When the pendulum is at vertical upright position, the system is highly unstable, where a controller is needed to achieve stabilization and swing up mechanism of Rotary Inverted Pendulum system. The amplitude of the supply voltage to the DC motor is proportional to the magnitude of the angular displacement of the rotary arm. Then, the greater the supply voltage to the actuator, the greater the angular displacement of the rotary arm.

The angular displacement of Rotary Inverted Pendulum is indirectly moved by the DC motor torque. There are two types of movement mechanism in RIP system, which are swing-up mechanism and stabilize mechanism. In this project, swing-up mechanism is not discussed, position of Rotary Inverted Pendulum is assumed to be at upright position as initial condition. Second type of movement mechanism is stabilization mechanism of rotary inverted pendulum system which is the motion maintaining the Rotary Inverted Pendulum at vertical upright position and avoiding the pendulum falling down in its free fall of nature way.

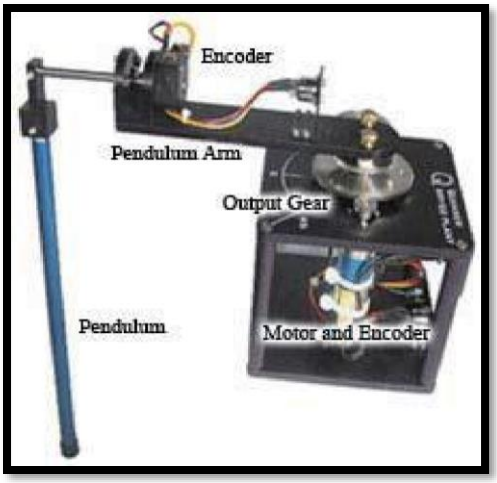

Figure 2.1 Rotary Inverted Pendulum[2]

2.2 Controller Design

The important factor of the rotary inverted pendulum system was to develop the control techniques to make the pendulum in upright position to maintain the stabilization of the system. . There were two techniques in designing the controller which are by using Linear Quadratic Regulator (LQR) controller and Proportional, Integral and Derivative (PID) cotroller. The function of this controller is to develop linear model to stabilize the position of rotary inverted pendulum in upright position. In this paper, the method to derive the LQR controller was by using Algebraic Ricatti Equation and for PID controller, the Ziegler Nicholas Tuning method was used. [3]

Meanwhile, there were 3 design of controller technique which comprising in this journal [3]. The state space equation is required by deriving the mathematical modelling of the rotary inverted pendulum where the inverted pendulum system is attached to a servo plant motor and simulation by MATLAB. The alpha represents the pendulum angle and theta represents the pendulum arm.

2.3 Types of Controllers

There is 2 types of controller in this project that are:

Table 1 Comparison Between Two Controller

Double-PID controller	LQR controller
i. α depends on β that can detected can described by using transfer ii. function. Pendulum arm position, iii. θ is activated by input voltage, V. There are two degrees of freedom used to make the speed constant and the value of β is zero. The other one will be activated by following the feedback of α.	i. The value of gain, K can be calculated by lowering a quadratic cost function with the used of Matlab function.

2.3.1 PID Controller

PID is Proportional-Integrated-Derivative. This is a type of feedback controller whose output, a control variable, is generally based on the error between a set point defined by the user and a process variable measured. Each PID controller element refers to a specific action taken against the error. The basic idea for a PID controller is to examine signals from sensors in the system called feedback signals. [2]

$$K_p + \frac{K_i}{s} + K_p s = \frac{K_P + K_i + K_d s}{s} \tag{2.1}$$

K_p= Proportional Gain

K_i= Integral Gain

K_d= Differential Gain (2.2)

The error signal is sent to the PID controller and the controller calculates the error signal derivative and integral. The signal (u) just past the controller is now equal to the proportional gain (K_p) times the error magnitude plus the integral gain (K_i) times the error integral plus the derivative gain (K_d) times the error derivative.

$$u = K_p\, e + K_I \int e(t)dt + K_D \frac{de}{dt} \tag{2.3}$$

The controller takes the new error signal and computes it derivative and integral again. This process goes continues and continues.

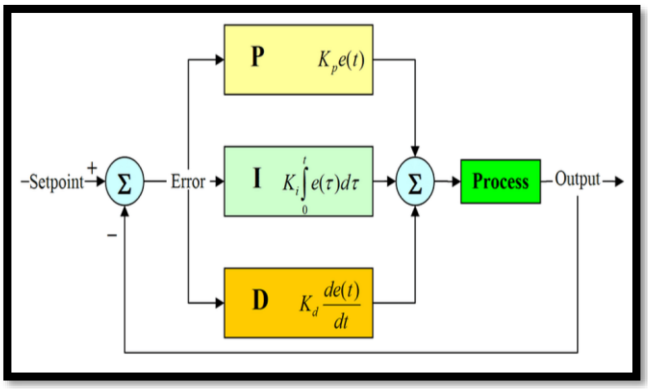

Figure 2.2 Block Diagaram of PID Controller[2]

2.3.2 LQR Controller

In order to overcome some problems faced by the PID controller, optimal control can be developed for other types of control methods, such as the Linear Quadratic Regulator (LQR). LQR is a control system that delivers the best possible performance in relation to certain performance measurements. The measurement of performance is a quadratic function consisting of state vector and control input.

Using LQR, the representation of the state space is required where this controller is based on the dynamic model that produces a high system response. This controller moves the initial pole from right to left in a complex diagram. The aim of this shift is to improve the system stability and the damping response.