

**NUMERICAL ANALYSIS OF TRANSIENT ANALYSIS OF RLC  
CIRCUIT UNDER AC SUPPLY**

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UNDER AC SUPPLY**

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**A report submitted in partial fulfillment of the requirements for the degree of  
Mechatronics Engineering**

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## DECLARATION

“I declare that this report entitle “Numerical Analysis of Transient Analysis of RLC Circuit under AC Supply” is the result of my own research except cited in the reference. This report has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

Signature : .....

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Date : .....

## **DEDICATION**

To my beloved mother and father

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## ABSTRACT

The objective of this project is to analyze the transient analysis of transient response in RLC electric circuit under Alternate Current (AC) supply by using numerical method. Transient analysis is the analyzation of transient response in an electric circuit. The purpose of transient analysis is to measure the performance of the electric circuit. This motivates us to provide alternative solution in obtaining transient analysis by using numerical analysis. Transient response is occurred when there were voltage and current change suddenly that may cause damage to electrical component and instrument in the circuit. In order to obtain the transient analysis, analytical method and numerical method are used to solve the second order ordinary differential equation that is derived from series RLC circuit. Meanwhile, transient response is a response of a system from equilibrium state to steady state. Analytical method is commonly used in obtaining transient analysis however it use more computational time and require expertise in mathematic knowledge. The numerical methods discussed in this project are Trapezoidal method, Euler method and Runge-Kutta method. The advantage and disadvantage of each method are then being discussed. The Fourth-Order Runge-Kutta was selected because it produces high accuracy result. The results of transient analysis obtained are plotted in MATLAB software. The numerical analysis method results are then being compared with analytical solution. In chapter 4, the Fourth-Order Runge-Kutta is a highly accurate method in predicting the transient response. At the end of this report, the conclusion can be made was Fourth-Order Runge-Kutta method produce 99% accuracy and the highest error obtained is 0.0352%. This make Fourth-Order Runge-Kutta is convenient method in obtaining the transient response.

## ABSTRAK

Projek ini dilaksanakan adalah bertujuan untuk menganalisis tindak balas sementara dalam litar elektrik RLC yang menggunakan sumber arus ulang-alik dengan menggunakan kaedah berangka. Analisis sementara adalah analisis tindak balas sementara di dalam litar elektrik. Tujuan analisis sementara dilakukan adalah untuk mengukur tahap prestasi litar elektrik. Keadaan ini memberi kami motivasi untuk mencari penyelesaian alternatif dengan menggunakan kaedah berangka. Kejadian tindak balas sementara apabila terjadinya perubahan voltan dan arus elektrik secara mendadak. Perubahan secara mendadak ini boleh mengakibatkan kerosakan kepada komponen elektrik. Bagi mendapatkan analisis sementara, kaedah analitikal dan kaedah berangka dilakukan untuk menyelesaikan persamaan perbezaan yang dihasilkan dari persamaan litar manakala tindak balas sementara adalah tindak balas sistem dari keadaan tidak stabil hingga stabil. Kaedah analitikal sering digunakan untuk mendapat transient analysis namun kaedah ini memerlukan masa penyelesaian yang lama dan memerlukan kemahiran ilmu matematik. Kaedah berangka yang dibincangkan di dalam projek ini adalah kaedah Trapezoidal, kaedah Euler, dan kaedah Runge-Kutta peringkat keempat. Kekuatan dan kelemahan untuk setiap kaedah dibincangkan. Kaedah Runge-Kutta peringkat keempat dipilih kerana kaedah ini menghasilkan keputusan ketepatan yang bagus. Hasil keputusan analisis sementara dipaparkan di dalam graf menggunakan perisian MATLAB dan keputusan dari kaedah berangka dan kaedah analitikal dibandingkan. Dalam bahagian 4 menunjukkan kaedah Runge-Kutta peringkat keempat lebih tepat dalam meramal tindak balas sementara. Di penghujung laporan ini, kesimpulan yang boleh dibuat ialah Runge-Kutta peringkat keempat menghasilkan 99% ketepatan dan kesalahan yang paling tinggi dicatat ialah 0.0352%. Ini membuatkan Runge-Kutta peringkat keempat lebih mudah untuk mendapatkan tindak balas sementara.



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## CHAPTER 1

### INTRODUCTION

#### 1.1 General Overview

Electricity is generated when it has the existence of proton and electron. The movement of electron from voltage source or current source is called an electric circuit. The electrons are moving from one point to another point and the point it enters the circuit is the “source” and the point it leaves the circuit is “earth ground”[1]. Electric circuit is also defined as the interconnection of electric elements or electric device. Basically, electric circuit are divided into two categories which are the active component and passive component. An example of active components are transistor, diode, motor, inductor and capacitor. Active components are easier to define as a component that give a net of energy in the circuit but passive component is the opposite of active component which does not introduce net of energy in the circuit. An example of passive component is resistor.

Transient analysis consist of two types of circuit which are first order differential equation and second order differential equation. The different between these two types of circuit is the number of energy storage. The capacitor and inductor are the storage elements



in the circuit. Capacitor works by storing energy in electric field meanwhile inductor works by storing energy in magnetic field. The capabilities of the storage elements can contain energy are depend on the value of storage elements [2].

The first order differential equation has only one energy storage in the circuit due to capacitor (C) or inductor (L) are placed in the circuit. There are two types of simple circuit in first order differential equation and these circuit are well known as RC circuit and RL circuit. The RC circuit are comprising with resistor and capacitor meanwhile RL circuit are comprising a resistor and inductor. The second order differential equation have two energy storage in the circuit due to the capacitor and inductor in the circuit. The two storage elements in the circuit are the capacitor and the inductor together with the resistor and this circuit is called RLC circuit. All these circuits are connected in series or in parallel [3].

The purpose of numerical analysis is to provide a convenient method in obtaining a solution for mathematical problem and for extracting a useful information. In several mathematical problems there are no solution that can only be solved by using analytical method. In this case, a numerical method was used to find the approximate solution. Generally, the numerical analysis does not strive for exactness but just approximation. In numerical analysis there are several methods can be used to solve the mathematical problem specifically on differential equations such as Euler's method, Trapezoidal method and Runge-Kutta method [4].

## 1.2 Motivation

Transient analysis is the result of the analyzation of electric circuit. Transient analysis is very important to RLC circuit to analyze the performance of the circuit. The purpose of transient analysis is to avoid any damage to the component and equipment used cause the presence of sudden change of voltage and current. In RLC circuit second order differentiation is utilized to obtain the transient analysis.

The numerical method is one of the problem solver in complex mathematical problem. The benefits of using numerical method are it is less computational time and easy to use. The numerical analysis gives the approximation value and it depends on the accuracy of types of numerical method used. There are several methods in numerical method such as Runge-Kutta method, Euler's method and Trapezoidal method Runge-Kutta is most commonly used for numerical analysis due to its high accuracy of approximation. Runge-Kutta method has three types which are first order, second order and fourth order and the most accurate is the fourth order.

## 1.3 Problem Statement

- 1) In RLC circuit, second order differential equation is derived due to the presence of two storage element which inductor and capacitor. There are several methods to solve the second order differential equation in analytical method such as Laplace Transform and Undetermined Coefficient but there are limitation in it. Analytical method requires expertise in mathematical knowledge to solve the complex second order differential equation.

- 2) The reason to determine the transient analysis is to find the response of voltage and current at inductor and capacitor. However the response keep changing when different value of resistor is vary thus it need to do another calculation. Transient analysis calculation using analytical method use more computational time.
- 3) Usually in obtaining transient response of an RLC circuit, direct current is used as the power supply but in this case alternate supply is used as the power supply. This situation makes the second order ODE is more complex to be solved.

#### **1.4 Objective**

The objectives to achieve at the end of this report are:

- 1) To solve the transient response of RLC circuit under AC supply by using analytical method and Fourth-Order Runge-Kutta method.
- 2) To compare the results obtained from analytical method and Fourth-Order Runge-Kutta method.
- 3) To analyze the result obtained from the transient analysis by using Fourth-Order Runge-Kutta method.

## 1.5 Scope

The scopes of the project are listed as below: -

- 1) Calculation of second-order RLC circuit under AC supply is used for transient analysis with the fix value of inductor,  $L$  and capacitor,  $C$  and varying the value resistor,  $R$  in every case.
- 2) Undetermined coefficient and Fourth-Order Runge-Kutta method are used in obtaining the transient analysis.
- 3) The result of analytical solution and Fourth-Order Runge-Kutta solution are plotted in MATLAB software and analyzed.

## CHAPTER 2

### LITERATURE REVIEW

#### 2.1 Introduction

This chapter discussed about the transient analysis and types of numerical method that is commonly used. This chapter also explained in detail about alternate current (AC) supply and comparison between analytical method and numerical method.

#### 2.2 Alternate Current (AC) Supply

Alternate Current (AC) supply is a sinusoidal input with a frequency. The forced response,  $V_f(t)$  frequency will be the same with input frequency. However, as time goes on the transient part of the response will sooner dies out and this part of response is called steady state response. Steady state is a state or condition when it does not change with time [7]. Equation (2.1) and equation (2.2) are the AC supply.

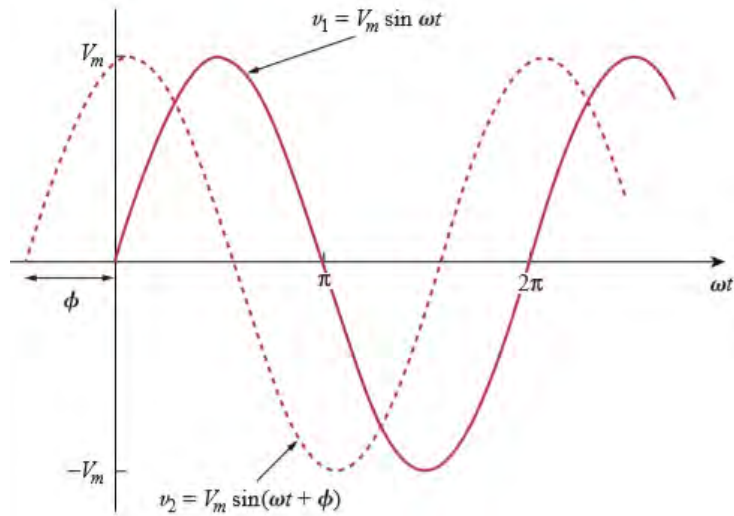


Figure 2.1: AC supply waveform[7]

Equation (2.1) is the instantaneous voltage of AC supply and the units of it is in volt.  $V_m$  is the maximum level of voltage waveform or the peak value of voltage waveform.  $V_m$  has two polarity which are in positive value or negative value.  $\Phi$  is the phase of the waveform and it is in radians or degree. The value of  $\Phi$  determine the phase angle of the waveform whether it is leading the current or lagging the current.

$$V(t) = V_m \sin (\omega t + \Phi) \quad (2.1)$$

Equation (2.2) is the instantaneous current of AC supply and the units is in ampere.  $I_m$  is the maximum level of current waveform or the peak value of current waveform.  $I_m$  has two polarity which are in positive value or negative value.

$$I(t) = I_m \sin (\omega t + \Phi) \quad (2.2)$$

Equation (2.3) is the angular frequency,  $\omega$  and it is expressed in radians.

$$\omega = 2 \pi f \quad (2.3)$$

Frequency,  $f$  is the number of period in one second and it is expressed in equation (2.4). The unit for frequency is in hertz, Hz. Period,  $T$  is the time taken for the waveform oscillate in one complete cycle and it is second.

$$f = \frac{1}{T} \quad (2.4)$$

When involving an alternating signal such as voltage and current in Alternating Current (AC) it is difficult to determine the voltage or signal magnitude. The solution of this problem is to use the Root-Mean-Square (RMS) value. There are two parameters in AC signal which is  $V_{RMS}$  and  $I_{RMS}$ . In AC signal the sinusoidal voltage and current are varying with time but in Direct Current (DC) analysis the waveform are not changed with time because the magnitude is constant [8].

$$V_{RMS} = V_m \times \frac{1}{\sqrt{2}} \quad (2.5)$$

$V_{RMS}$  or also can be referred as effective value of a sinusoidal waveform in alternating current (AC) supply is determined by equation (2.5). The  $V_{RMS}$  value can be obtained by multiplying the peak current with  $\frac{1}{\sqrt{2}}$ .

$$I_{RMS} = I_m \times \frac{1}{\sqrt{2}} \quad (2.6)$$

Equation (2.6) is the formula to find the value of current roots-mean-square,  $I_{RMS}$ .  $I_{RMS}$  is also referred as the effective value of current in AC supply. The  $I_{RMS}$  value can be obtained by multiplying the peak current with  $\frac{1}{\sqrt{2}}$  [7].

### 2.3 RLC circuit

RLC circuit is an electric circuit that consist of three component of electrical elements which are resistor, inductor and capacitor that are connected in series or parallel. The RLC name is the symbol of resistor, inductor and capacitor respectively [5]. The input of series RLC circuit is either be supply by direct current supply (DC) or alternate current supply (AC).

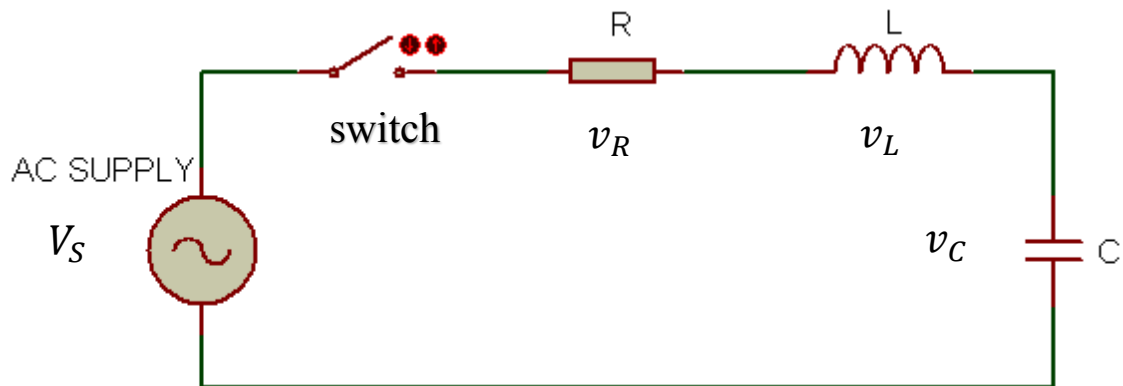


Figure 2.2 shows the series RLC circuit with AC supply.

The circuit contain three circuit element which are resistor, inductor and capacitor and is labeled R, L and C respectively. The circuit is supplied with voltage supply,  $V_S$  that in sinusoidal waveform.

$$V_S = V_m \sin(\omega t + \Phi) \quad (2.7)$$

The total current flow across the resistor,  $i_R$  inductor,  $i_L$  and capacitor,  $i_C$  is the same because it is in one loop. In series RLC circuit Kirchhoff's Voltage Law (KVL) is used. This law states that the algebraic sum of voltage across various circuit elements in a closed path or loop is equal to zero. In other words, the sum of voltage drops is equal to the sum of voltage rises. In series RLC circuit second order differential equation is