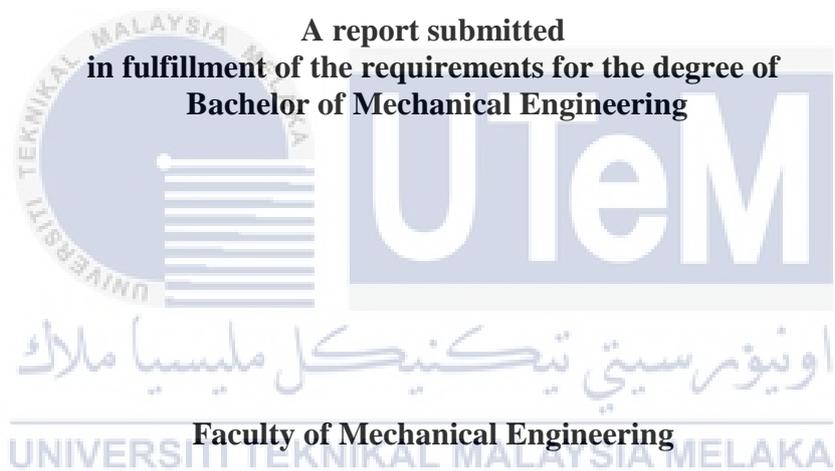


**COMPARISON OF MODELLING BETWEEN ARX AND ARMAX MODEL IN  
SYSTEM IDENTIFICATION**

**SITI NUR NADHIRAH BINTI NONCHIK**



**UNIVERSITI TEKNIKAL MALAYSIA MELAKA**

**2018**

## SUPERVISOR'S DECLARATION

I hereby declare that I have read this project report and in my opinion this report is sufficient in terms of scope and quality for the award of the degree of Bachelor of Mechanical Engineering.

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## DECLARATION

I declare that this project report entitled “Comparison of Modelling between ARX and ARMAX Model in System Identification” is the result of my own work except as cite in the references.

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## DEDICATION

To my beloved father, Nonchik Bin Adam  
and mother, Fedzilah Bt Yusof



## ABSTRACT

This research presents the comparison of modelling between ARX and ARMAX in System Identification. System Identification is a methodology to explain the dynamic behavior by building mathematical models using measurement of the system's input and output signals. The field of system identification is now widely used in most of the industrial projects in which the identification software have a wide circulation in industrial world. There are several type of general models in system identification that consist of AR model, ARX model, ARMAX model, Box-Jenkins model and Output-Error model but the main focus of this project are using ARX and ARMAX model. The aim of this research is able to simulate modelling using ARX model and ARMAX model and to compare the modelling performance of ARX and ARMAX model based on selected performance indicators. Specifically, the performance indicators that were used includes best fit value, final prediction error value and mean square error value. In a completion of the analysis, all simulation is conducted using the 'ident' graphical user interface in MATLAB R2015b and the least square method is utilized to estimate the parameters of the ARX and ARMAX models structure in this research. The results generally showed that ARX model structure is slightly better than ARMAX model structure in terms of model best fit, final prediction error and mean square error due to an additional input variable in the model. Thus, ARMAX could not provide better fit value caused by the random disturbance provided. However, by implementation the real data, the results showed that ARMAX model structure is better compared to ARX model. In conclusion, the better performance of both ARX and ARMAX model is still depending on the data distribution.

## ABSTRAK

*Kajian ini membentangkan perbandingan antara model ARX dan ARMAX menggunakan pengenalpastian sistem. Pengenalpastian sistem adalah kaedah untuk menerangkan tingkah laku dinamik dengan membina model matematik menggunakan system input dan output signal. Bidang pengenalpastian sistem kini digunakan secara meluas dalam kebanyakan projek industri di mana perisian pengenalan mempunyai peredaran yang meluas di dunia perindustrian. Terdapat beberapa jenis model dalam sistem pengenalan yang terdiri daripada model AR, model ARX, model ARMAX, Model Box-Jenkins dan model Output-Error tetapi tumpuan utama projek ini adalah menggunakan model ARX dan ARMAX. Tujuan penyelidikan ini adalah untuk mensimulasikan pemodelan menggunakan model ARX dan model ARMAX dan membandingkan prestasi pemodelan model ARX dan ARMAX berdasarkan penanda prestasi terpilih. Khususnya, penanda prestasi yang digunakan merangkumi nilai fit, FPE dan MSE. Untuk menyelesaikan analisis, semua simulasi dijalankan dengan menggunakan 'ident' GUI di MATLAB R2015b dan kaedah kuasa dua terkecil digunakan untuk menganggarkan penanda prestasi struktur model ARX dan ARMAX dalam kajian ini. Hasil keputusan secara amnya menunjukkan bahawa struktur model ARX adalah lebih baik daripada struktur model ARMAX dari segi nilai fit, FPE dan MSE disebabkan oleh input tambahan dalam model. Oleh itu, ARMAX tidak dapat memberikan nilai yang lebih baik disebabkan oleh gangguan rawak yang disediakan. Bagaimanapun, dengan pelaksanaan data sebenar, hasilnya menunjukkan bahawa struktur model ARMAX lebih baik berbanding dengan model ARX. Kesimpulannya, prestasi ARX dan ARMAX yang lebih baik masih bergantung kepada pengagihan data.*

اويور سيتي بيكنيكل مليسيا ملاك

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## LIST OF ABBREVIATIONS

ARMAX	-	Autoregressive-moving average with exogenous terms
ARX	-	Autoregressive with exogenous terms
DC	-	Direct Current
FPE	-	Final Prediction Error
GUI	-	Graphical User Interface
MATLAB	-	Matrix Laboratory
MSE	-	Mean Square Error
SISO	-	Single Input Single Output
PSM	-	Projek Sarjana Muda



## LIST OF SYMBOLS

$y$	-	Output
$u$	-	Input
$w$	-	Measured Disturbance
$y(t)$	-	Current Output
$y(t - k)$	-	Finite number of past outputs
$u(t - k)$	-	Input at lag k
$e(t)$	-	Noise
$n_a$	-	Number of poles
$n_b - 1$	-	Number of Zeros
$n_k$	-	Dead-time
$q^{-1}$	-	Delay operator
$b$	-	The slope of the regression line.
$a$	-	The intercept point of the regression line and y axis
$\bar{X}$	-	$x$ average
$\bar{Y}$	-	$y$ average
$u_2$	-	Input (heating power)
$y_2$	-	Output (temperature of the outflow air)
$R^2$	-	$R^2$ Coefficient
$\theta, Z^N$	-	Loss Function
$d$	-	Total number of estimated data
$N$	-	The length of data record
$n_a$	-	Number of past output terms used to predict the current output.
$n_b$	-	Number of past input terms used to predict the current output.
$n_k$	-	Delay from input to the output in terms of the number of samples.
$n_c$	-	Number of past values of the disturbance signal.
$\hat{Y}$	-	Vector of $n$ predictions
$Y$	-	Vector of the observed values

## CHAPTER 1

### INTRODUCTION

#### 1.1 Background

Mathematical model can take very different forms depending on the system under study, which may range from social, economic, environmental, mechanical to electrical system. Generally, the inner mechanism of economic, social or environmental systems are not widely known or recognize and often only small data sets are available, while previous understanding of mechanical and electrical systems is at high level, and experiments can easily carried out. Hence, system identification is one of the method that commonly used to develop a suitable mathematical model of a particular dynamic system (Mediliyegedara et al., 2004).

System identification is a methodology to explain the dynamic behavior by building mathematical models using measurement of the system's input and output signals (Saifizi, Ab Muin Sazali & Mohamad, 2013). In order to carry out prediction and simulation that require wide applications including biology, meteorology, mechanical engineering, economics, physiology and model-based control design, system identification tools can be used in resulting dynamic mathematical model (Singh & Ajith B, (2014). System identification is applied to many objects from huge systems involving gas turbine, reactors, airplanes and even geology of the earth to a small system which are servo DC motor and electromagnetic valves. It is mostly used in three areas that consist of modelling and simulation, control design and prediction.

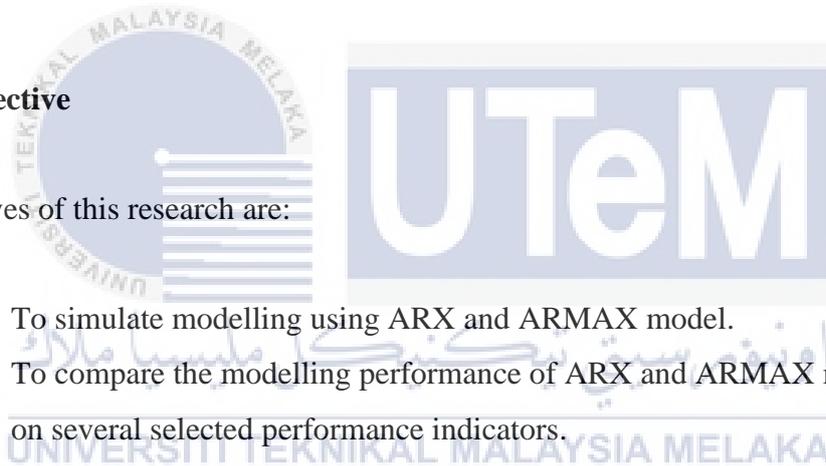
## 1.2 Problem Statement

System identification is an area of control system where the range between theory and practical is not very well pronounced. The field of system identification is now widely used in most of the industrial projects in which the identification software have a wide circulation in industrial world. Furthermore, to build a model in industry, a carefully designed identification experiment is carried out. There are several type of general models in system identification that consist of AR model, ARX model, ARMAX model, Box-Jenkins model and Output-Error model. Hence, the focus of this project is to investigate the comparison and to clarify the difference between ARX and ARMAX model in order to find a suitable model for identification.

## 1.3 Objective

The objectives of this research are:

1. To simulate modelling using ARX and ARMAX model.
2. To compare the modelling performance of ARX and ARMAX model based on several selected performance indicators.



## 1.4 Scope

In order to achieve the objective, the scopes are prepared as shown below:

1. All simulation is conducted using the 'ident' graphical user interface in MATLAB.
2. MATLAB is also used to make data acquisition based on simulated data in the form of Single-Input-Single-Output (SISO) system.
3. The performance of modelling will be decided based on several indicators provided in Graphical User Interface (GUI).
4. The least square method is utilized to estimate the parameters of the ARX and ARMAX models in this research.



## CHAPTER 2

### LITERATURE REVIEW

#### 2.1 System Identification

##### 2.1.1 Introductions

System identification is a technique to develop a mathematical model of a specific dynamic system using the measurement of systems input and output signals. Both of the systems input and output can be seen as an interface between the actual application and the world of mathematical control theory and model abstraction by using a combination of observed data; 1) Basic mechanics and dynamics, 2) Prior knowledge of relationship between signals (Rivera, 2004). The models can be divided into three types that are a white box, a black box and a gray box but the main focus of this topic will be black box. The black box is a completely empirical description of the dynamics of a system for which essentially no information is known a priori.

##### 2.1.2 Dynamic System

There are a few terms of a system that is accessible which ranged from loose description to severe mathematical formulations. One type of a system called *open systems* produces observable signals. It is commonly called *outputs* and are contemplated to be an object in which different variables connect at different types of time and space scales and it is influenced by external stimuli. *Input* can be operated by the observer and as for the *disturbances*, it can be categorized into those that is directly measured and that are only can

be detected through its influence on the output (Ljung, 2012). The graphical model of a general open system that is acceptable for system identification and the dissimilarity among measured disturbances and inputs is frequently insignificant for the modeling process as can be seen in Figure 2.1.

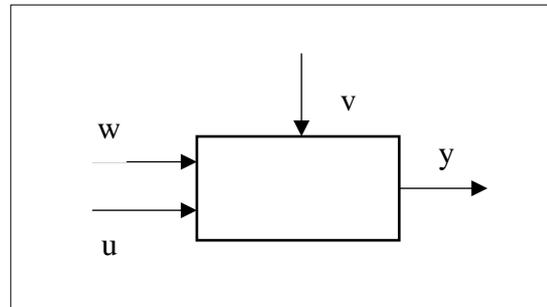


Figure 2.1 A system with output  $y$ , input  $u$ , measured disturbance  $w$ , and unmeasured disturbance  $v$  (Ljung, 2012)

As shown in Figure 2.2, the solar-heated house is considered as an example of a system. The system runs in a way that sun heats the air on the solar panel. The air then flows into heat storage which is a box filled with pebbles. The stored energy can later be transferred to the house. This system is represented in Figure 2.3 and the record of data obtained over fifty hour period and the variables sampled every ten minutes are shown in Figure 2.4 (Ljung, 2012).

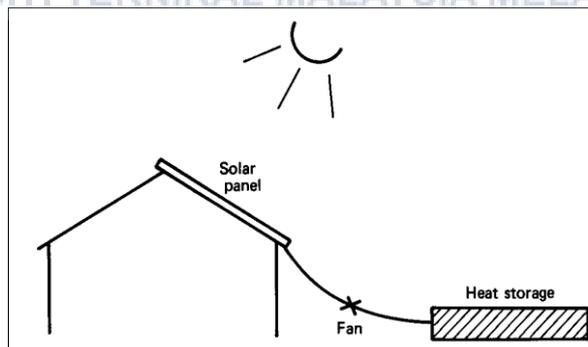


Figure 2.2 A solar-heated house.

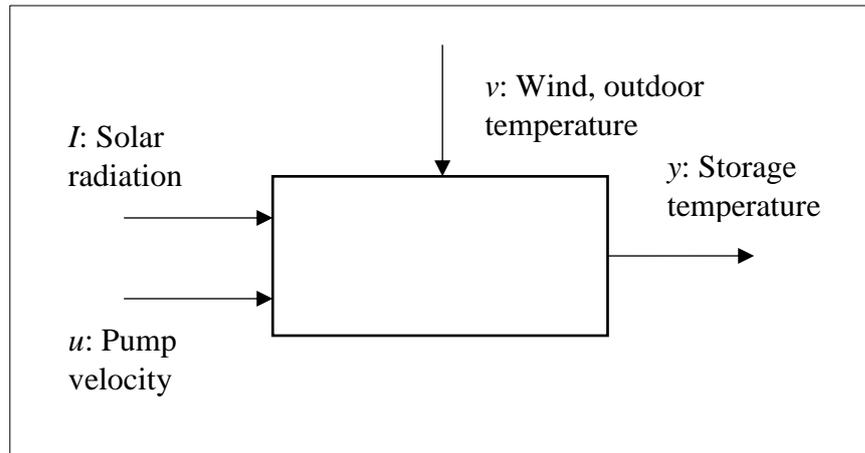


Figure 2.3 The solar-heated house system:  $u$ : input;  $w$ : measured disturbance;  $y$ : output;  $v$ : unmeasured disturbances (Ljung,2012).

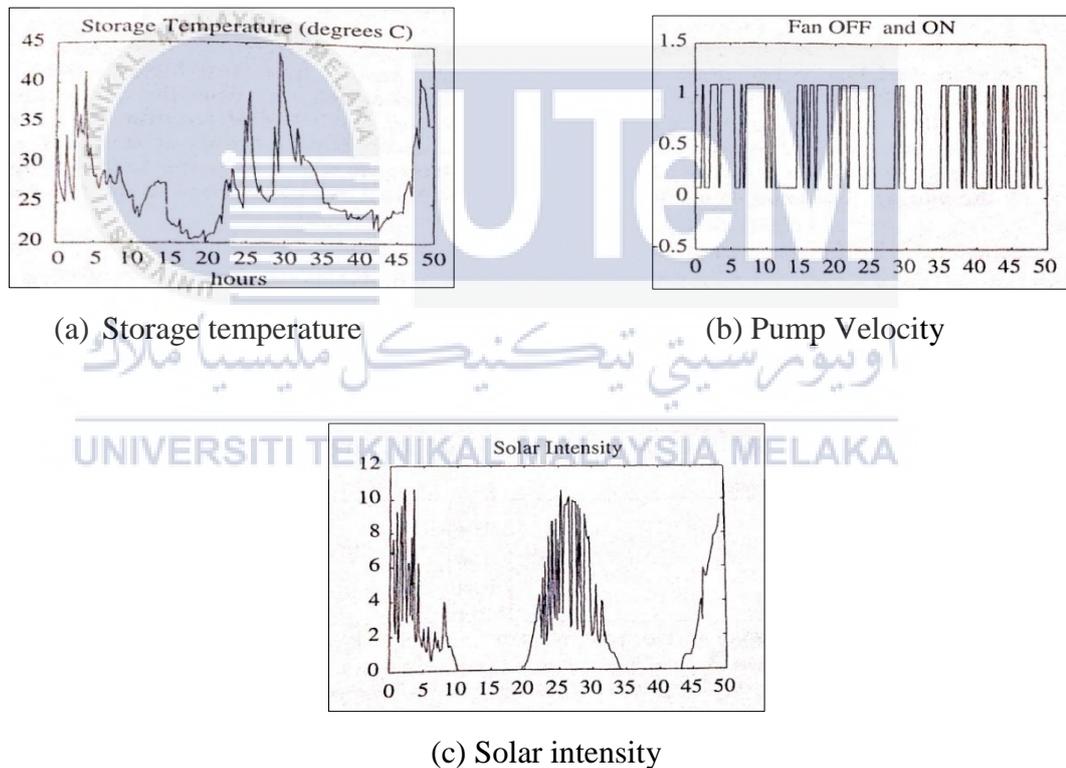


Figure 2.4 Storage temperature  $y$ , pump velocity  $u$ , and solar intensity  $I$  over a 50 hour period. Sampling Interval: 10 minutes.

### 2.1.3 Mathematical Models

Additive sensor noise term  $v(\cdot)$  in Figure 2.5 denote errors produce from the measurement process.  $W(\cdot)$  represents input disturbances while a white noise signal, it is usually presumed to represent  $v(\cdot)$ .

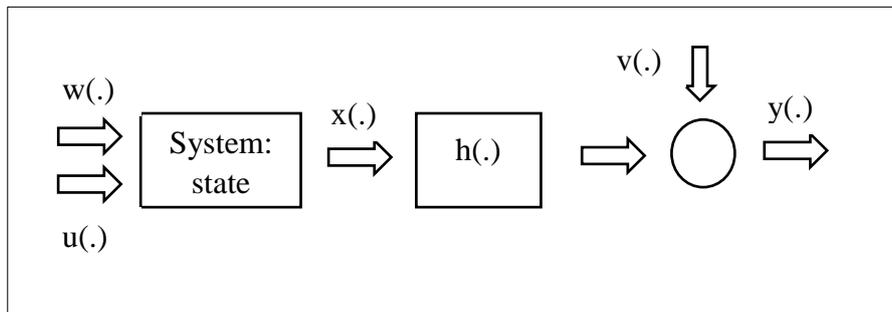


Figure 2.5 Basic structure of mathematical model (Keesman, 2011)

By considering the basic structure of system shown in Figure 2.5, set of standard differential equations with additive sensor noise shown in equation (2.1) and (2.2) as it represent standard description of a finite-dimensional system (Keesman, 2011).

*Discrete-time:*

$$\begin{aligned} x(t+1) &= f(t, x(t), u(t), w(t); \vartheta), & x(0) &= x_0 \\ y(t) &= h(t, x(t), u(t); \vartheta) + v(t), & t &\in \mathbb{Z}^+ \end{aligned} \quad (2.1)$$

*Continuous-time:*

$$\begin{aligned} \frac{dx(t)}{dt} &= f(t, x(t), u(t), w(t); \vartheta), & x(0) &= x_0 \\ y(t) &= h(t, x(t), u(t); \vartheta) + v(t), & t &\in \mathbb{R} \end{aligned} \quad (2.2)$$

Due to the availability of trial data and the ideal modification of a mathematical model into simulation code, the discrete-time form may be apply in system identification while the continuous-time will only be used for demonstration. According to Keesman (2011), these classification also tell the difference between linear and nonlinear, time-invariant and time-varying, static and dynamic systems.

Linearity: Under zero initial conditions,  $u_1(t)$  and  $u_2(t)$  as an inputs to a system with corresponding outputs of  $y_1(t)$  and  $y_2(t)$ . The system will called linear if its response to  $au_1(t) + \beta u_2(t)$ , with a constants of  $a$  and  $\beta$  is  $ay_1(t) + \beta y_2(t)$ . Specifically for linear systems, the properties of superposition or additivity and scaling hold. In equation (2.1) and (2.2), since  $f(\cdot)$  and  $h(\cdot)$  show its standard functions, it will not hold the linearity. Therefore, the nonlinear system will represents the basic model structure. (Keesman, 2011).

Time-invariance:  $u_1(t)$  is an input system to a corresponding output  $y_1(t)$ . If the response to  $u_1(t + \tau)$ , with  $\tau$  a time shift, is  $y_1(t + \tau)$ , the system is called time-invariant. It explains that the system equations do not vary in time. For time-varying systems represents the notation  $f(t, \cdot)$  and  $h(t, \cdot)$  that show both functions are explicit functions of the time variable  $t$  (Keesman, 2011).

Dynamics: dynamic system specifically can be explained in the matter of differential equation and it has memory while a static system that can be described as algebraic equations has no memory (Keesman, 2011).

#### 2.1.4 System Identification Procedure

System identification methodology has an understandable logical flow: 1) data is collected, 2) model set is chosen, 3) the 'best' model is picked. As for the first model obtained, it might not pass the model validation test. Hence, step 1 until step 3 is repeated by using the same procedure. The model may be lacking for a few reasons:

1. The numerical procedure failed to find the best model according to our criterion.
2. The criterion was not well chosen.
3. The model set was not convenient, in that it did not contain any description of the system.
4. The data set was not informative enough to provide guidance in selecting good models.

Based on the reasons above, the third reason is an inadequate iteration which is the most important part of an identification application of addressing to these problems, guided by prior information and the outcomes of the previous attempt. As can be seen in Figure 2.6 the interactive software is an important tool for handling the iterative character of this problem (Ljung, 2012).

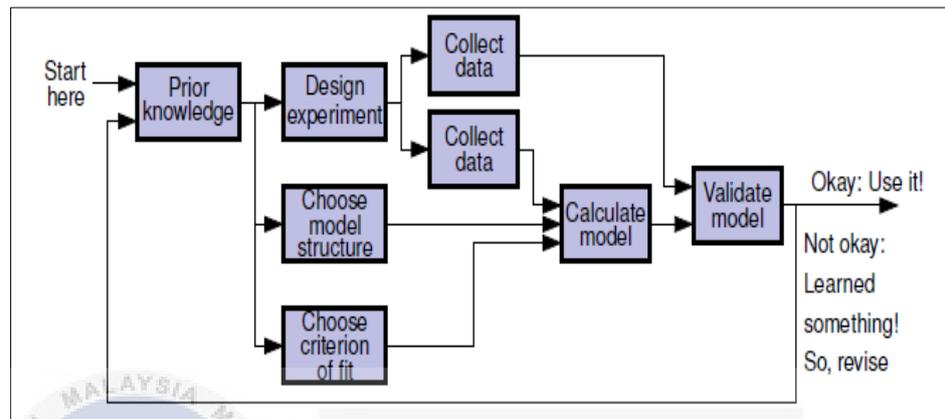


Figure 2.6 The system identification loop (Ljung, 2012).

## 2.2 ARX Model

### 2.2.1 The Model Structure

The system's input and output at time  $t$  indicates by  $u(t)$  and  $y(t)$ , respectively. The most basic relationship between input and output is the simple linear difference equation below which involves the current output  $y(t)$  to a finite number of past outputs  $y(t - k)$  and inputs  $u(t - k)$ .

$$y(t) + a_1y(t - 1) + \dots + a_{n_a}y(t - n_a) = b_1u(t - n_k) + \dots + b_{n_b}u(t - n_k - n_b) + e(t) \quad (2.3)$$

The structure is designate where  $e(t)$  refer to the noise,  $a_{n_a}$  and  $b_{n_b}$  are the model parameter, the three integers which consist of  $n_a$ ,  $n_b$  and  $n_k$ .  $n_a$  is defined as the number of poles,  $n_b - 1$  is the number of zeros that indicate the order of polynomials of the output  $A(q)$  and the input  $B(q)$ , respectively, and  $n_k$  is the dead-time as it is commonly known as

pure time-delay in the system (Chetouani, 2008). The polynomial representation of the equation (2.4) is shown below.

$$A(q)y(t) = B(q)u(t - n_k) + e(t) \quad (2.4)$$

Where  $A(q)$  and  $B(q)$  are given by:

$$A(q) = 1 + a_1q^{-1} + \dots + a_{na}q^{-na} \quad (2.5)$$

$$B(q) = b_1q^{-1-n_k} + \dots + b_nq^{-nb-n_k} \quad (2.6)$$

$q^{-1}$  is the delay operator:

$$u(t - 1) = q^{-1}u(t) \quad (2.7)$$

$A(q)$  and  $B(q)$  are estimated by the least squares identification.

### 2.2.2 Advantages and Disadvantages of ARX Model

This model is one of the uncomplicated model that integrate the stimulus signal in which some of the stochastic dynamics as a part of the system dynamics were capture by the ARX model. The transfer function have the same set of poles for both the deterministic part of the system and the stochastic part and this coupling can be impractical. Besides, ARX model has the most important advantages that can describe the unpredictability and inaccuracy or fuzzy system, since it has integrated the experts understanding into the nominal measurement data. It explains that in a society and economic system, identification is good application (Zheng, 2010).

The disadvantage of this system can be diminish if the signal-to-noise is high due to the system dynamics and stochastic dynamics of a system do not share the same set of poles continuously.

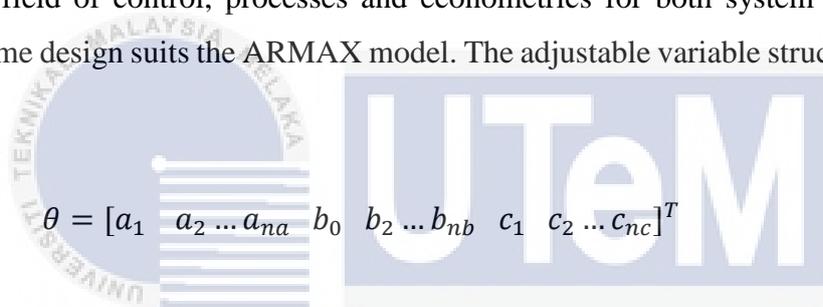
## 2.3 ARMAX Model

### 2.3.1 The Model Structure

The dissimilarity of the ARX model is the unavailability of the error term. Therefore, the ARMAX model is created and stated in a consideration of the error term (Saifizi, Ab Muin Sazali & Mohamad, 2013)

$$y(t) + a_1y(t-1) + \dots + a_{na}y(t-na) = b_0u(t-d) + \dots + b_{nb}u(t-d-nb) + e(t) + c_1e(t-1) + \dots + c_{nc}e(t-n_c) \quad (2.8)$$

In a field of control, processes and econometrics for both system modelling and control scheme design suits the ARMAX model. The adjustable variable structure are shown below:



$$\theta = [a_1 \ a_2 \ \dots \ a_{na} \ b_0 \ b_2 \ \dots \ b_{nb} \ c_1 \ c_2 \ \dots \ c_{nc}]^T \quad (2.9)$$

Given that

$$A(q^{-1})y(t) = B(q^{-1})u(t) + C(q^{-1})e(t) \quad (2.10)$$

Or

$$y(t) = \frac{B(q^{-1})}{A(q^{-1})}u(t) + \frac{C(q^{-1})}{A(q^{-1})}e(t) \quad (2.11)$$

### 2.3.2 Advantages and Disadvantages of ARMAX Model

The ARMAX model structure is incorporated with disturbance dynamics. ARMAX model is functional if it control disturbances that enter early in the process as can be seen mostly at the input. One of the advantages in this model are it has additional flexibility in the handling disturbance compared to the ARX model. Besides, forecasting values will produce a better outcome by taking into consideration of process behavior.

As for the disadvantages, noise presume to be an identically distributed random sequence. ARMAX model also exerts an excessive control effort and cannot be applied to Non-Minimum phased system for “Minimum variance control” (Ganesh, 2011).

### 2.4 Least Square Method

Bretscher & Otto (1995) stated that the least square method are generally was established by Carl Friedrich Gauss in 1821. It is one of the essential method of determining the parameters when a linear-in-the-parameter model is used. Least square method is basically used to estimate parameters by minimizing the irregular squared among observed data and expected value. The criterion of least square method is a computationally convenient measure of fit that relates to the maximum likelihood estimation when the noise is normally distributed with equal variance. However, other measure of fit are occasionally used such as least absolute deviations which is more robust. Lawson & Hanson (1974) explains the formula for least square method in equation (2.12).

$$Y = a + bX \quad (2.12)$$

Where:

- $b$  - The slope of the regression line.
- $a$  - The intercept point of the regression line and y axis

$$b = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2} \quad (2.13)$$

$$a = \bar{Y} - b\bar{X} \quad (2.14)$$

$\bar{X}$  -  $x$  average

$\bar{Y}$  -  $y$  average



## CHAPTER 3

### METHODOLOGY

#### 3.1 Introduction

This chapter will describe in detail the methodology used in this research to simulate modelling using ARX and ARMAX model using MATLAB software (R2015b). MATLAB is a multi-paradigm numerical computing environment which is programming language developed by MathWorks. The flowchart of the project is shown in Figure 3.1 in order to identify the different elements of the project and understand the relationships among the various steps.

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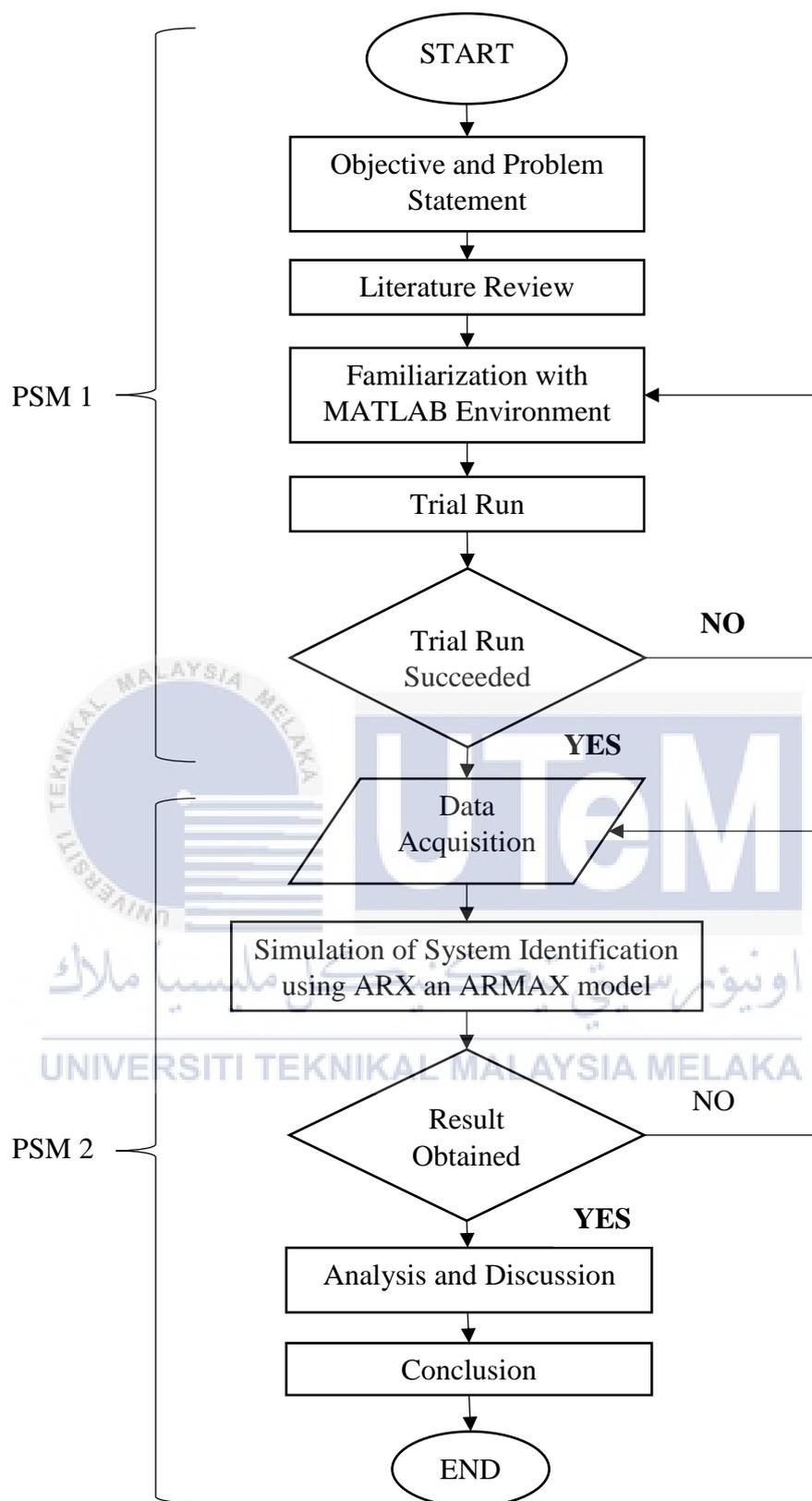


Figure 3.1 Project Flow Chart

### 3.2 Familiarization with MATLAB environment

Matrix Laboratory or generally known as MATLAB is a high performance language and it is user friendly for numerical computation, programming and visualization. It is usually used to develop algorithm, modelling, simulation, prototyping, data analysis and exploration. MATLAB has hundreds of built-in functions and toolboxes in order to allow various approaches and find a better solution compared to traditional programming language. A wide range of applications used in MATLAB consist of control system, test and measurement, signal processing, communications and computational biology.

### 3.3 System Identification Toolbox

System identification toolbox is used in this project in order to estimate and analyze linear and nonlinear models from measured input and output data by using system identification app. System identification application basically includes rectangular icons for import data and import model which can be seen in Figure 3.2 below. The imported data will be in the left side called Data Board, and the model icons will be on the other side in Model Board.

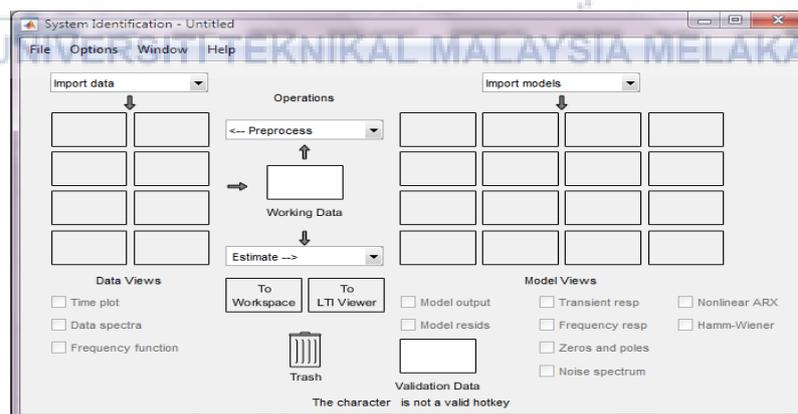


Figure 3.2 System Identification Toolbox

For the purposes of system identification, MATLAB provides a user-friendly graphical user interface. A graphical user interface is a visual display in windows involving controls that usually called components. It allows user to carry out interactive task. The graphical user interface is built using MATLAB tools that can conduct read and write data

files, type of computation, and display data such as tables and plots as shown in Figure 3.3 below.

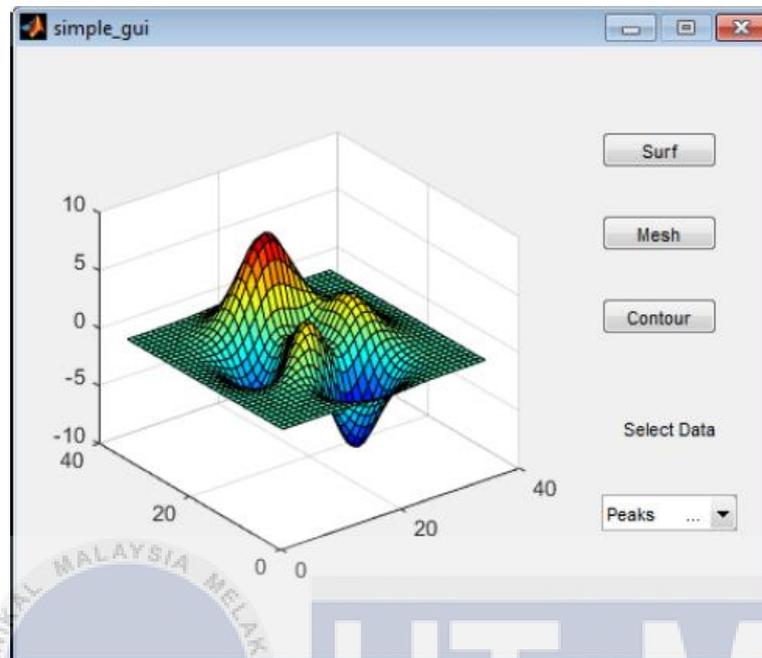
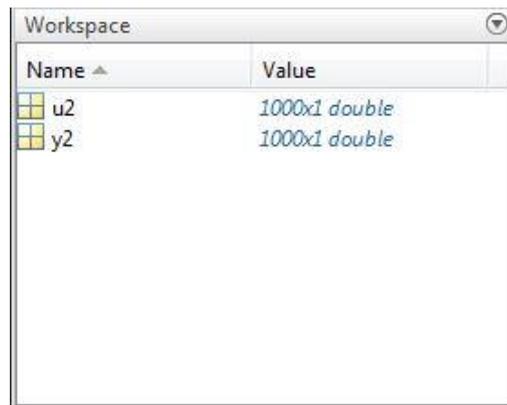


Figure 3.3 Example of User Interface

### 3.3.1 Starting GUI

System Identification Toolbox provides a graphical user interface (GUI). During the session, GUI cover all the toolbox's function and can easily access to all variables. For the trial test, the data provided from MATLAB Software is used as initial indicator since there is no valid data. This data act as pilot study and as a starting of preliminary run for system identification functionally in GUI. As for the first step, '*ident*' in the MATLAB command window and the data variables of  $u_2$  and  $y_2$  will load into the workspace as shown in Figure 3.4. The collected data is taken from an actual hair dryer to be as an example for trial run. The input ( $u_2$ ) act as a heating power while the output ( $y_2$ ) is the temperature of the outflow air. The following Figure 3.5 describes the different areas in the System Identification application.



Name	Value
u2	1000x1 double
y2	1000x1 double

Figure 3.4 Data Variables in Workspace

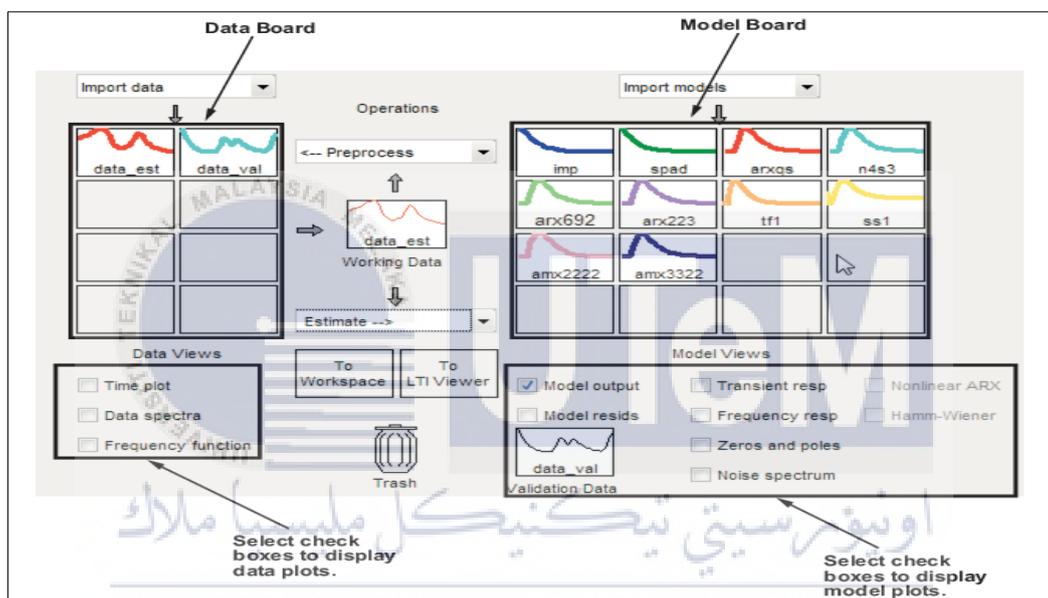


Figure 3.5 The different areas in the System Identification Application

### 3.3.2 Import Data

‘Time domain data’ is chosen under popup menu ‘Import data’ as shown in Figure 3.6. In workspace variable section on the Import Data box that can be seen in Figure 3.7, input and output is inserted as  $u2$  and  $y2$  respectively. An example of 0.08 s is inserted and the data value is assigned a sample time, which is calculated from start time and the sampling interval.

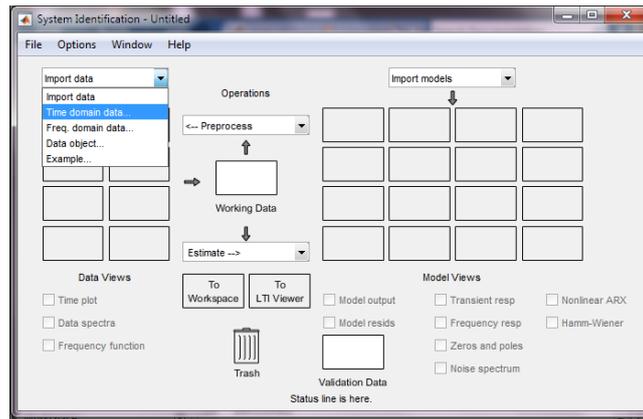


Figure 3.6 Selected 'Time domain data'.

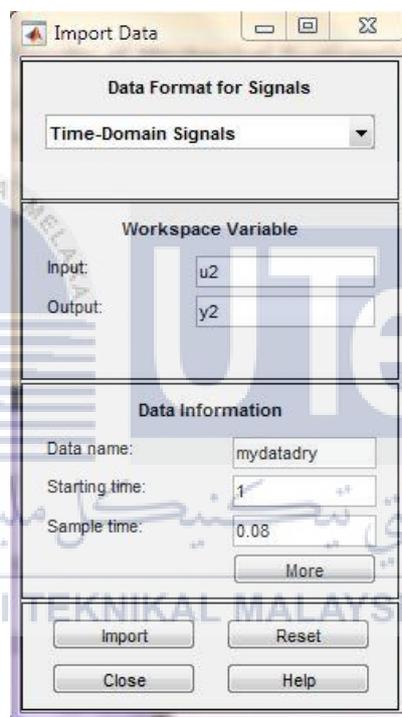


Figure 3.7 Import Data Box

After the 'import' button is pressed, the data will be presented as an icon in System Identification application data board as shown in Figure 3.8. The data are also filled in the Working Data and Validation Data boxes.

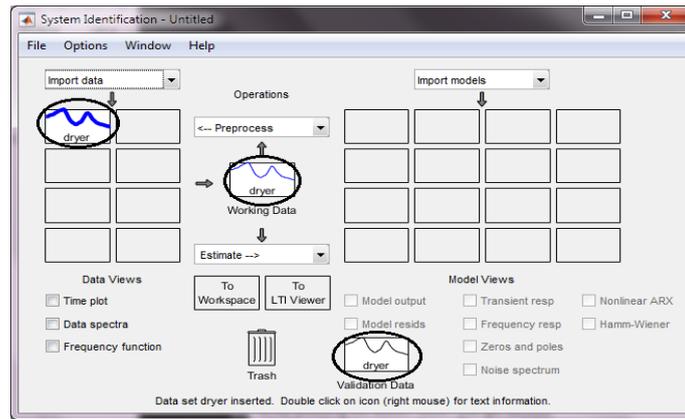


Figure 3.8 System Identification GUI

'Time plot' checkbox is clicked as shown in Figure 3.9 to open a figure as presented in Figure 3.10 to examine the plot. The time scales shows the information about sampling time and start time entered when importing the data.

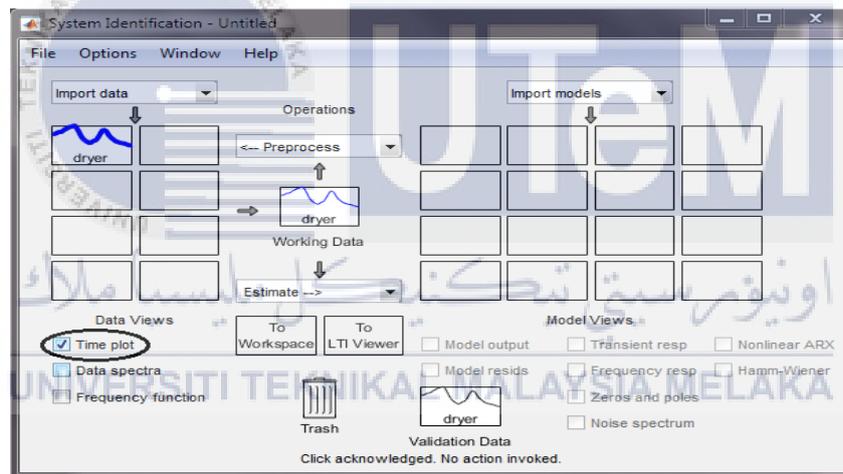


Figure 3.9 Time Plot icon

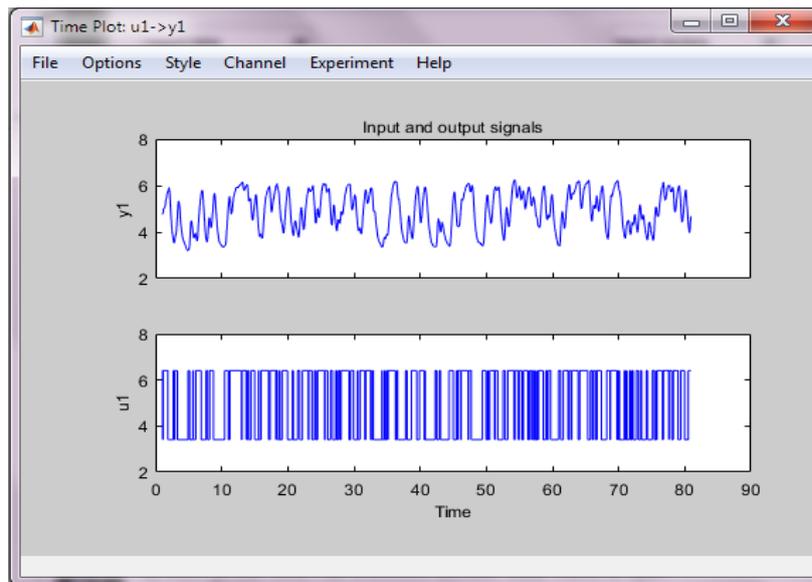


Figure 3.10 Time Plot

‘Remove means’ is selected as shown in Figure 3.11 from the ‘Preprocess’ popup menu, the constant levels in the data sequences will be removed.

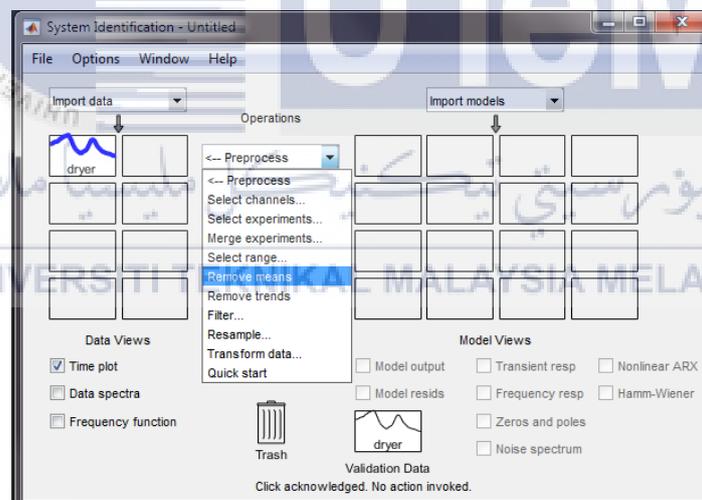


Figure 3.11 Selecting ‘Remove means’

As can be seen in Figure 3.12, the new data set has automatically been presented in Time Plot figure. Autorange may be chosen to see the new plots.

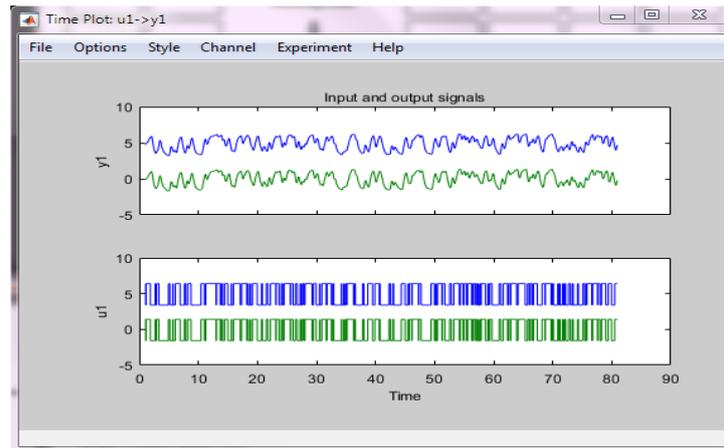


Figure 3.12 The new data set

The new data set with a 'd' attached to its name were dragged and dropped onto the 'Working Data' icon at the center of the Graphical User Interface (GUI) shown in Figure 3.13. The detail information about the data is described in Figure 3.14.

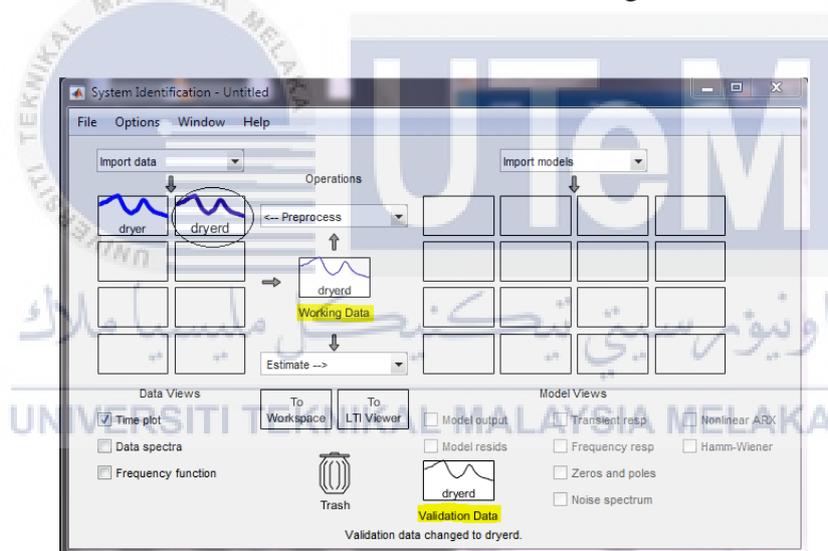


Figure 3.13 'dryerd' data

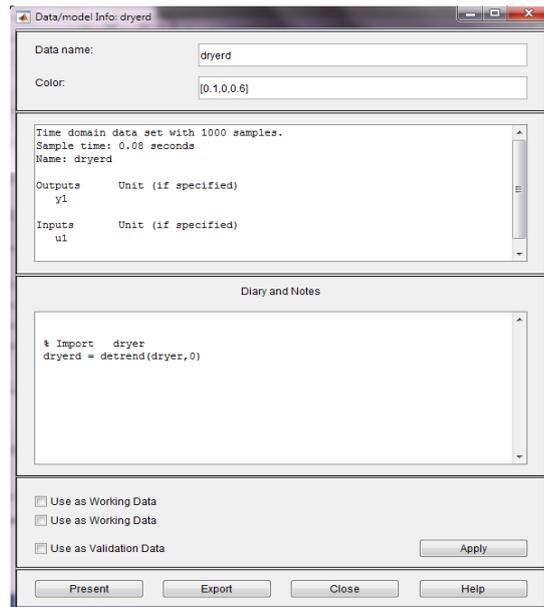


Figure 3.14 The Information of 'dryerd' data

After the previous step, the 'Select Range' option is chosen from the 'Preprocess' popup menu and a new figure will open as shown in Figure 3.15 below.

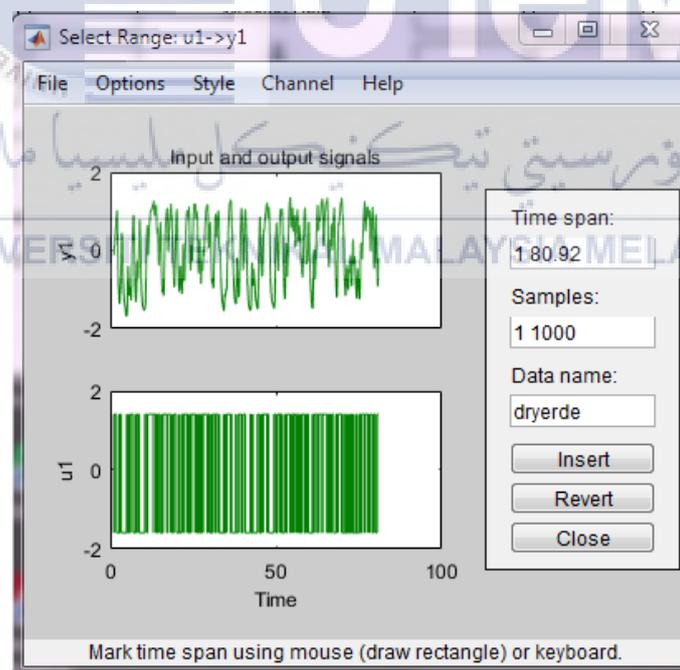


Figure 3.15 Select Range

By referring to Figure 3.16, the interval is chosen from 1 to 50 seconds and when the 'insert' button is pressed in the dialog box, the new selected data range is added to the Data board. The new data range will open as shown in Figure 3.17.

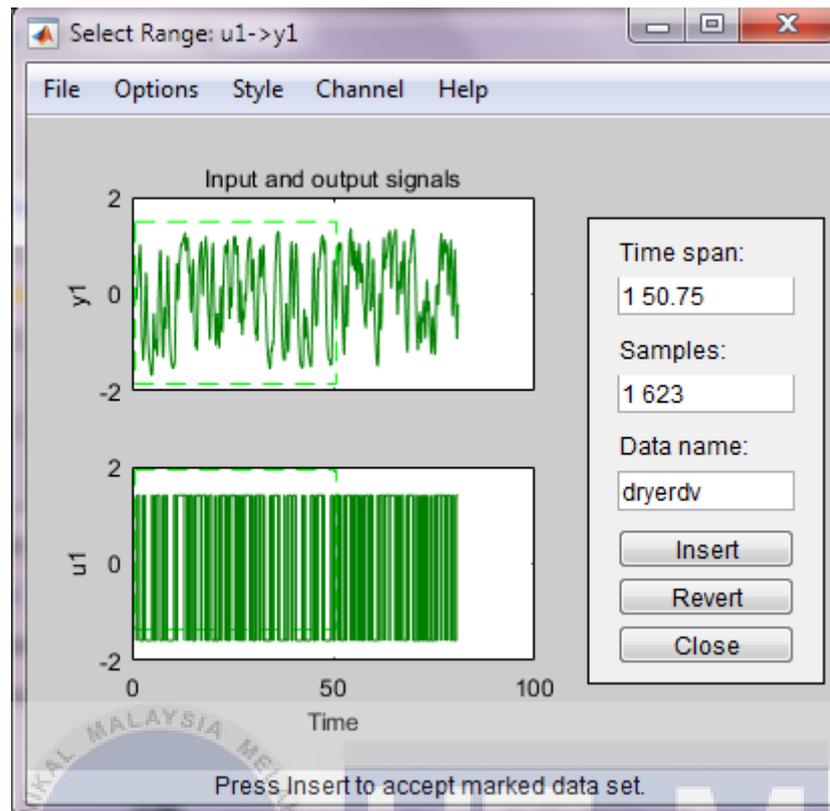


Figure 3.16 1 to 50 s interval

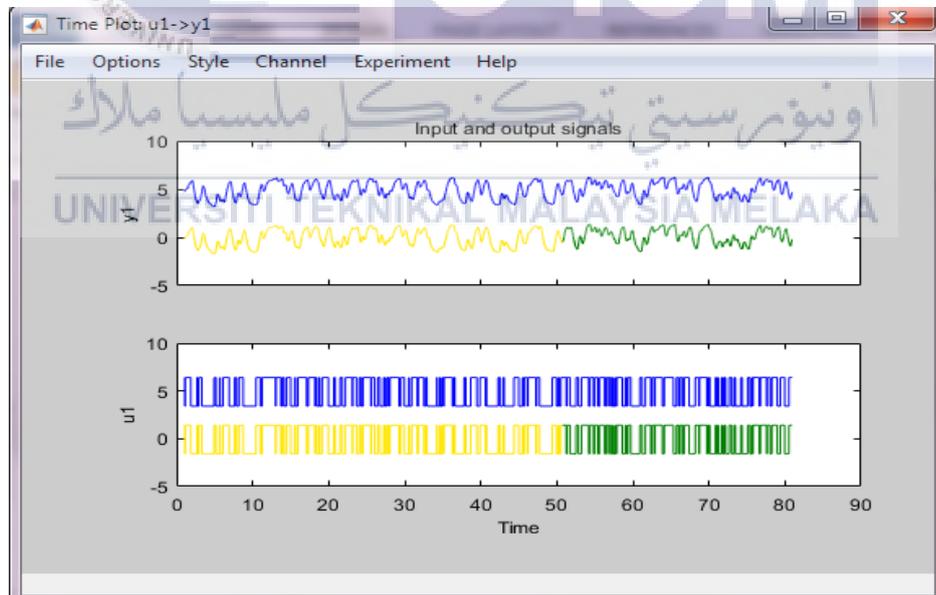


Figure 3.17 New Added Data Range

In the Data Board, the third set (*dryerde*) is selected for estimation by dragging it to the Working data whereas the fourth data set (*dryerdv*) for validation purposes by dragging it to Validation Data. These steps are shown in Figure 3.18.

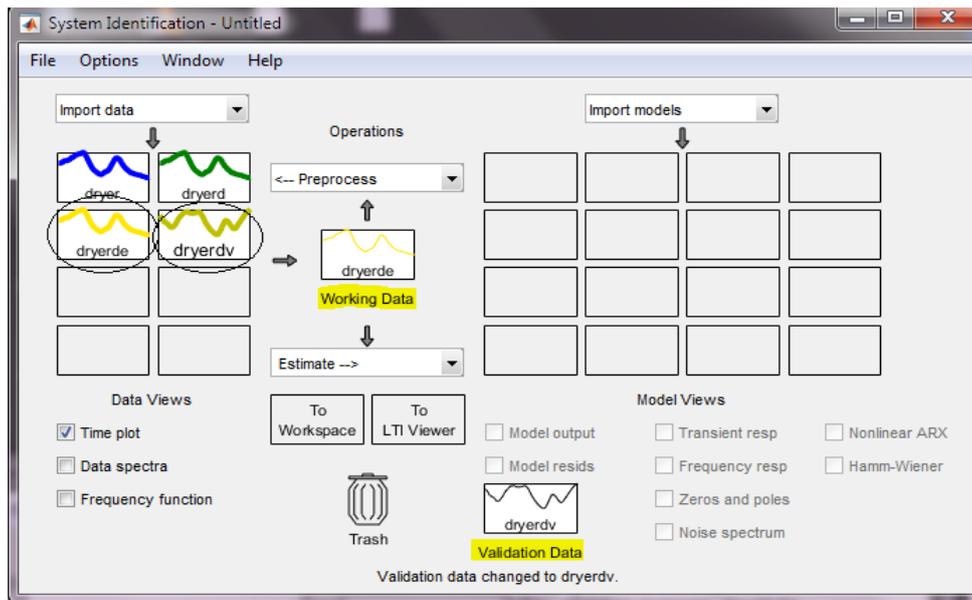


Figure 3.18 Working and Validation Data

### 3.3.3 Estimate Model

'Polynomial Models' is chosen from the 'Estimate' popup menu to identify parametric models. A dialog box of polynomial models will show up as can be seen in Figure 3.19. The required model structure is chosen to generate certain models.

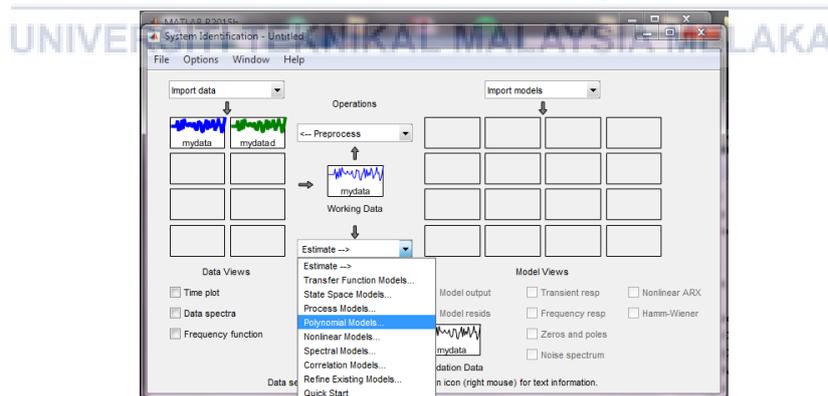


Figure 3.19 Polynomial Model

After that, the Polynomial Models window will appear as in Figure 3.20 below. In Figure 3.21, the type of model can be selected by pressing the *structure* window. As for this project, the two models that need to be analyze are ARX and ARMAX model.

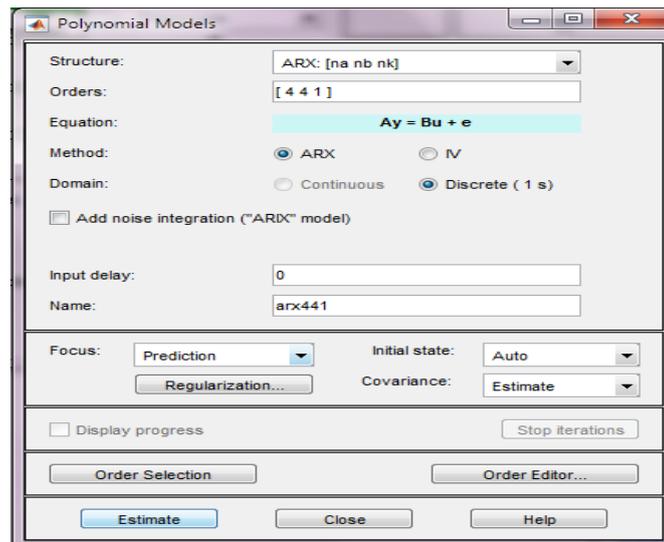


Figure 3.20 Polynomial Models window

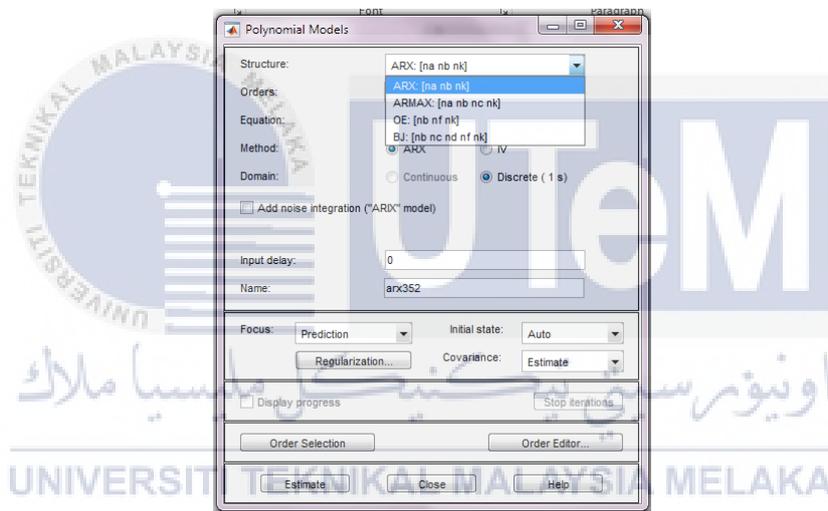


Figure 3.21 Structure window

After 'Estimate' button at the bottom of the dialog box is pressed, the model will be computed and inserted into the Model board. As for the reference, a fourth order of ARX type difference equation model has been chose. The 'Open Editor' is opened by pressing it to change the orders. The 'Order Editor will opened as can be seen in Figure 3.22.

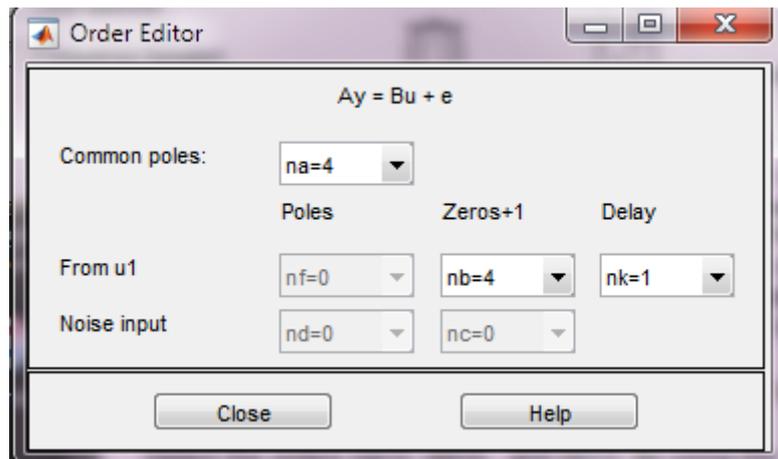


Figure 3.22 Open Editor Dialog Box

‘Estimate’ button is clicked to compute the model after the order is changed.

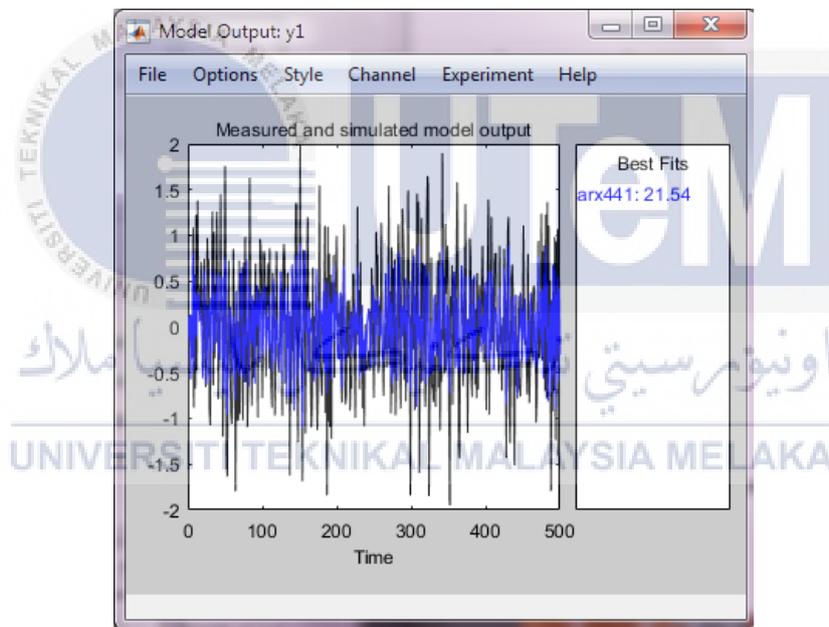


Figure 3.23 Model Output

The model output result shows the value of best fit as can be seen in Figure 3.23. The higher the value of best fit indicate the best model. To generate the quantitative diagnostic move the cursor to the model in model board and right-click in mouse is pressed. The value of Final Prediction Error value and Mean Square Error value will appear as in Figure 3.24.

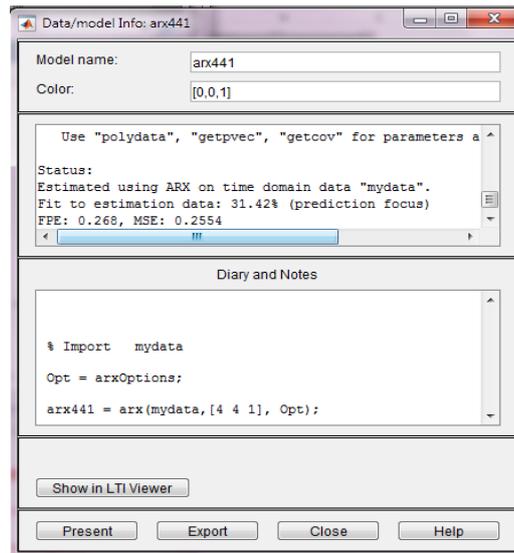


Figure 3.24 Model Info

### 3.3.4 Performance Indicator

Performance indicator is used to compare the modelling performance of ARX and ARMAX model. Based on this project, the studies will focus on three criterion which are best fit criterion, Final Prediction Error criterion and Mean Square Error criterion.

#### 3.3.4.1 Best Fit

The best model structure is the one that minimizes the prediction error. Best fit criterion is often used as a performance indicator for model validation, by considering the highest fit. The best fit is measured by the coefficient of determination denoted  $R^2$ , expressed by:

$$R^2 = 100 \times \left( 1 - \frac{\sum_{i=1}^N \varepsilon^2}{\sum_{i=1}^N (y - \hat{y})^2} \right) \% \quad (3.1)$$

### 3.3.4.2 Final Prediction Error

Final Prediction Error (FPE) evaluates model quality, where the model is tested on a new set of data. The most accurate model has the smallest FPE. The FPE equation is defined by the following equation:

$$FPE = V_N(\theta, Z^N) \left( \frac{1 + \frac{d}{n}}{1 - \frac{d}{n}} \right) \quad (3.2)$$

Where  $(\theta, Z^N)$  represents the loss function for the studied structure,  $d$  is the total number of estimated data and  $N$  is the length of data record.

### 3.3.4.3 Mean Square Error

The MSE equation is defined by the following equation:

$$MSE = \frac{1}{n} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 \quad (3.3)$$

Where:

- $\hat{Y}$  - Vector of  $n$  predictions
- $Y$  - Vector of the observed values

## CHAPTER 4

### RESULTS AND ANALYSIS

#### 4.1 Introduction

This chapter will discuss about the results obtain from the simulation that have been done using System Identification Toolbox. The analysis will explain further in detail as the simulation of different model structure were carried out using MATLAB software. Different model structure of ARX and ARMAX model are used in order to analyze the performance indicators. The performance indicators that need to analyze are fit value, final prediction value and mean square error value.

#### 4.2 ARX and ARMAX Model Orders and Delay

The orders and delays of ARX ( $n_a, n_b, n_k$ ) and ARMAX ( $n_a, n_b, n_c, n_k$ ) are used to define the standard regression of the model. The orders and delay are defined as follows:

- $n_a$  - Number of past output terms used to predict the current output.
- $n_b$  - Number of past input terms used to predict the current output.
- $n_k$  - Delay from input to the output in terms of the number of samples.
- $n_c$  - Number of past values of the disturbance signal.

### 4.3 Model 1

From Figure 4.1 below, the Input and Output Signals shows measured input and output data for Model 1 by using the equation (4.1). The preprocess method has been carried out by removing means as shown in Figure 4.2. In order to obtain the mathematical model of this system, the following parametric model of ARX is used. The true model specification used for first model is ARX 352. ARX 352 is computed by following order number;  $n_a = 3$ ,  $n_b = 5$  and  $n_k = 2$ .

$$y(t) = 0.2y(t - 1) - 0.6y(t - 3) + 0.5u(t - 2) + u(t - 6) + e(t) \quad (4.1)$$

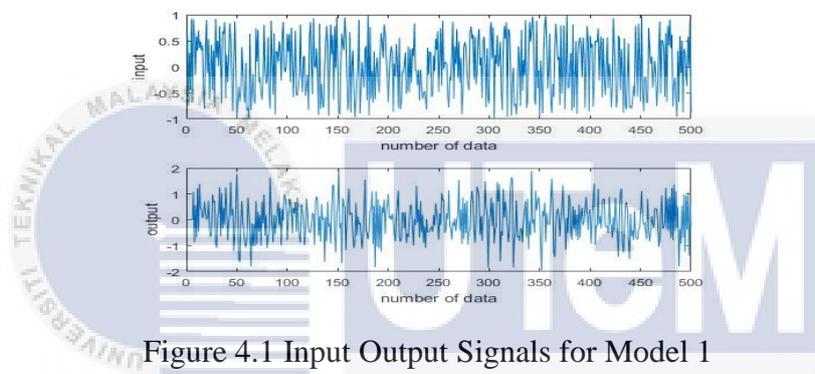


Figure 4.1 Input Output Signals for Model 1

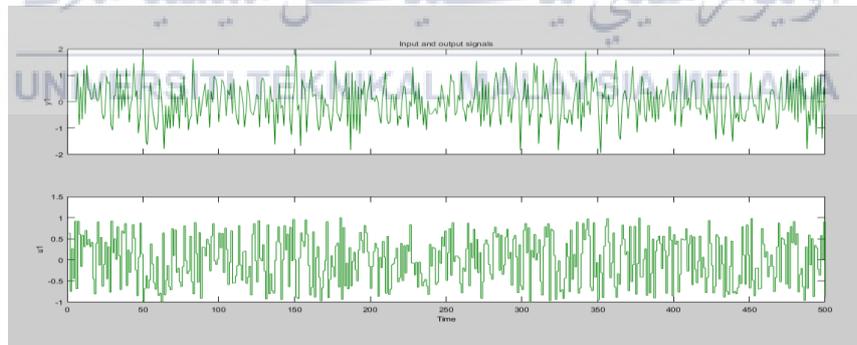


Figure 4.2 Time Plot for Model 1

The simulation result of model output for time domain data in Model 1 shown in Figure 4.3 consist of ARX 352 (true model specification), ARX 452, ARX 362, AMX 3512, AMX 3522 and AMX 3532. The model output graph shows the comparison of all Models 1 simulated output. From the graph, it shows that the highest fit of 98.92% was produced by model ARX 362. As for model ARX 352, ARX 452, AMX 3512, AMX 3522 and AMX

3532 has the same value of fit which are 98.27%. Overall, ARX 362 model is chose as the best model based on its fit, FPE and MSE value.

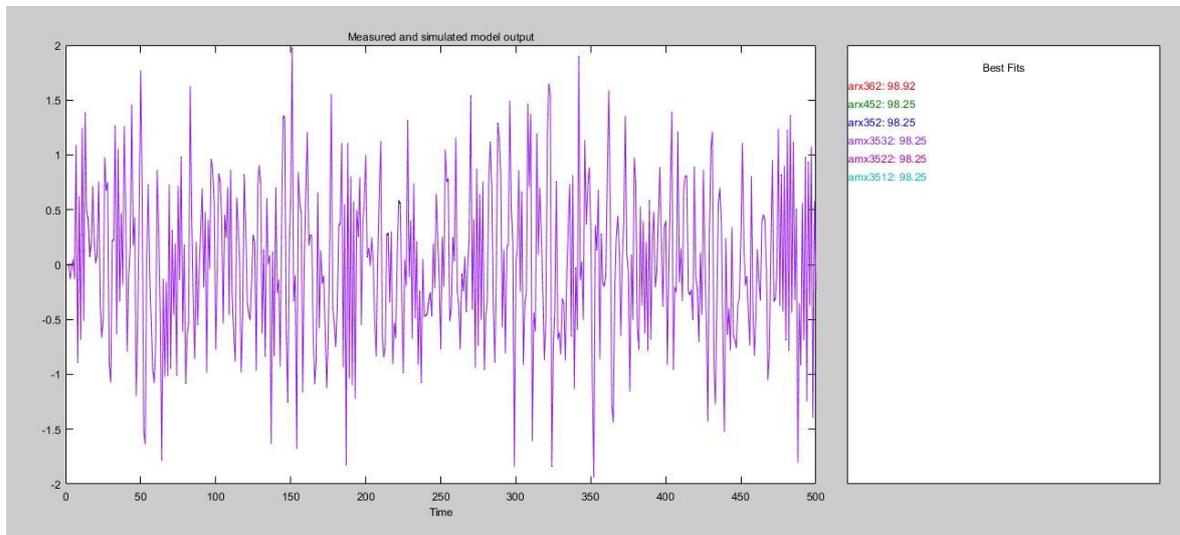
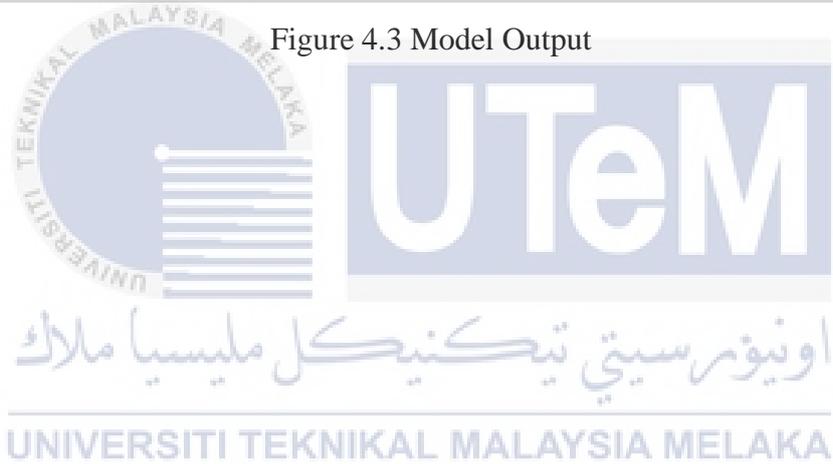


Figure 4.3 Model Output



### 4.3.1 Simulated Results

#### 4.3.1.1 ARX 352

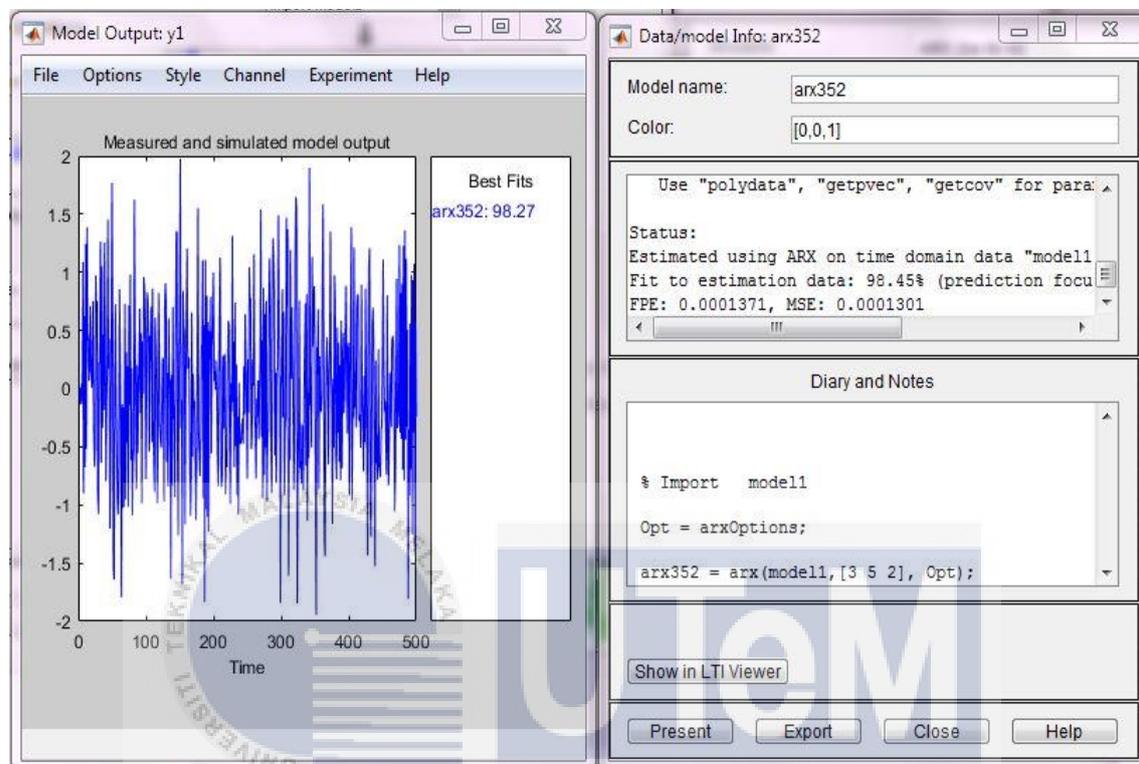


Figure 4.4 Model Output for ARX 352

In the ARX 352 model, the Final Prediction Error is 0.0001412 and Mean Square Error is 0.0001341

Discrete-time IDPOLY of ARX 352 model is:

$$A(z) = 1 - 0.2002z^{-1} - 0.0001713z^{-2} + 0.5994z^{-3}$$

$$B(z) = 0.5z^{-2} - 0.0006203z^{-3} - 0.0008136z^{-4} - 0.001365z^{-5} + z^{-6}$$

### 4.3.1.2 ARX 452

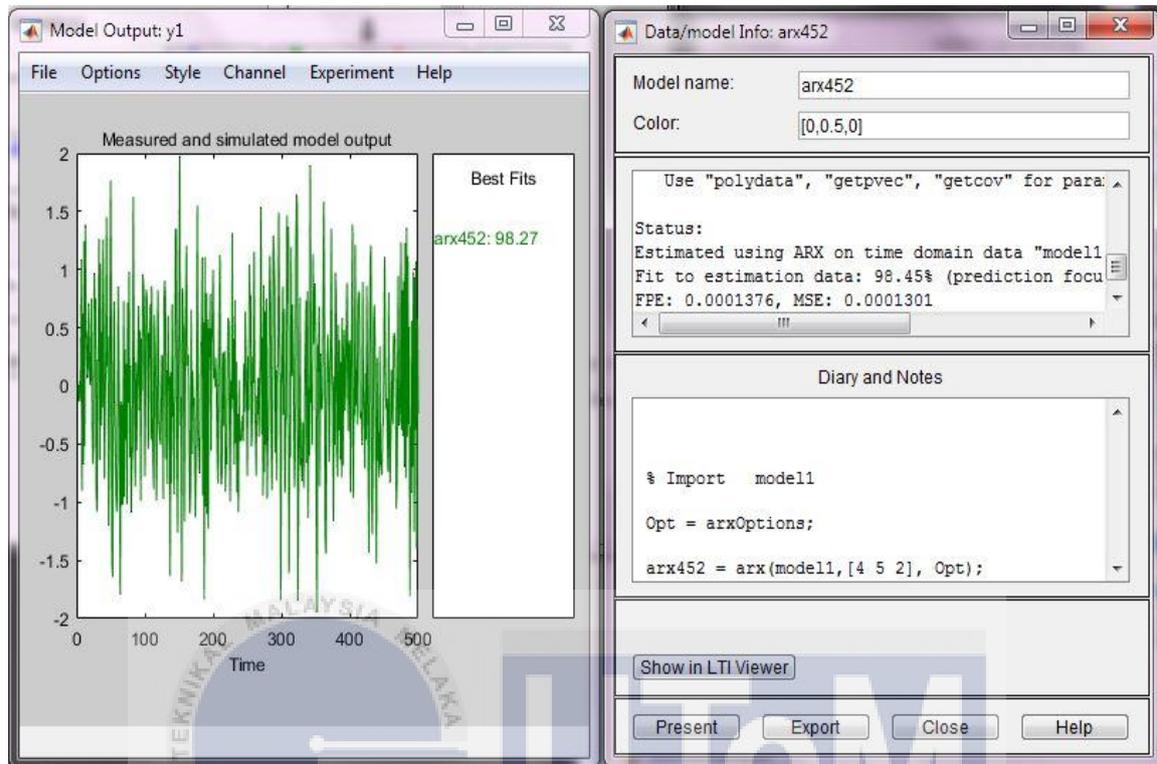


Figure 4.5 Model Output for ARX 452

In the ARX 452 model, the Final Prediction Error is 0.0001418 and Mean Square Error is 0.0001341.

Discrete-time IDPOLY of ARX 452 model is:

$$A(z) = 1 - 0.2z^{-1} - 0.0001703z^{-2} + 0.5994z^{-3} + 0.0003601z^{-4}$$

$$B(z) = 0.5z^{-2} - 0.0005056z^{-3} - 0.0007674z^{-4} - 0.001374z^{-5} + z^{-6}$$

### 4.3.1.3 ARX 362

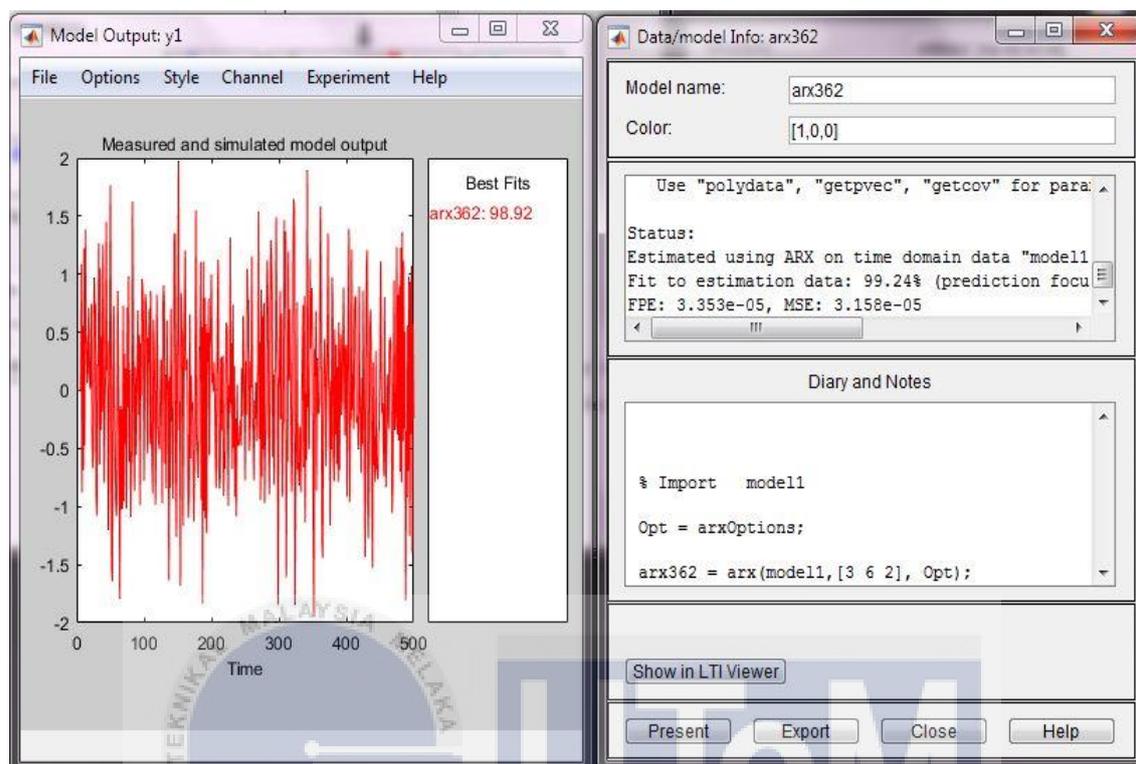


Figure 4.6 Model Output for ARX 362

In the ARX 362 model, the Final Prediction Error is 0.00003421 and Mean Square Error is 0.00003222.

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Discrete-time IDPOLY of ARX 362 model is:

$$A(z) = 1 - 0.2006z^{-1} - 0.00004459z^{-2} + 0.5994z^{-3}$$

$$B(z) = 0.5z^{-2} - 0.0008132z^{-3} - 0.0007655z^{-4} - 0.001372z^{-5} + z^{-6} + 0.0006283z^{-7}$$

#### 4.3.1.4 AMX 3512

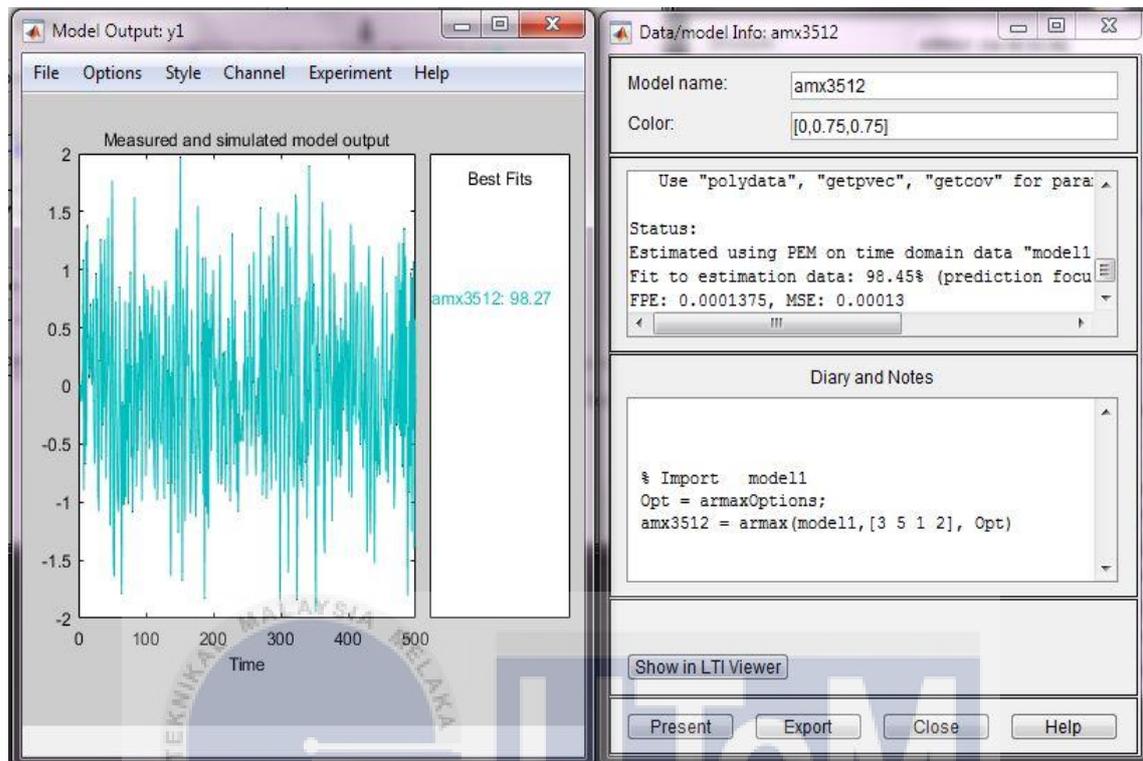


Figure 4.7 Model Output for AMX 3512

In the AMX 3512 model, the Final Prediction Error is 0.0001415 and Mean Square Error is 0.0001338.

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Discrete-time IDPOLY of AMX 3512 model is:

$$A(z) = 1 - 0.2002z^{-1} - 0.0001672z^{-2} + 0.5994z^{-3}$$

$$B(z) = 0.5001z^{-2} - 0.0006209z^{-3} - 0.0008068z^{-4} - 0.001377z^{-5} + z^{-6}$$

$$C(z) = 1 + 0.09345z^{-1}$$

### 4.3.1.5 AMX 3522

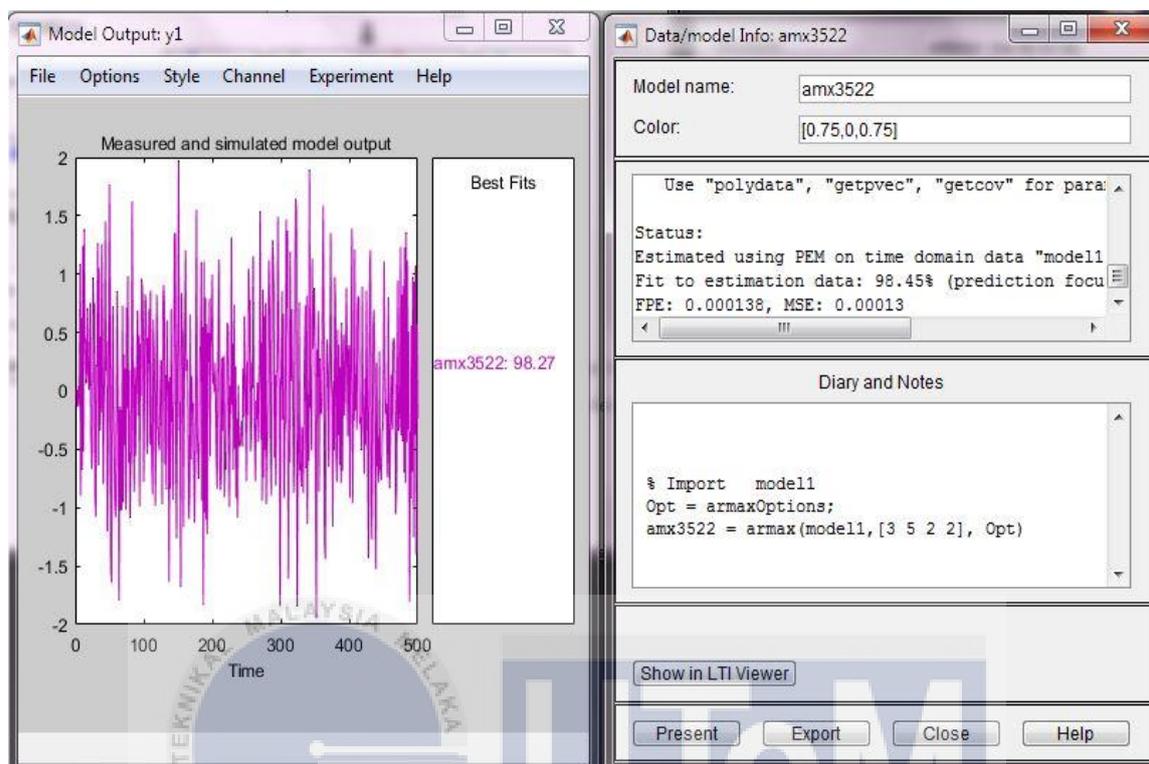


Figure 4.7 Model Output for AMX 3522

In the AMX 3522 model, the Final Prediction Error is 0.0001421 and Mean Square Error is 0.0001338.

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Discrete-time IDPOLY of AMX 3522 model is:

$$A(z) = 1 - 0.2002z^{-1} - 0.0001708z^{-2} + 0.5994z^{-3}$$

$$B(z) = 0.5001z^{-2} - 0.0006089z^{-3} - 0.0008082z^{-4} - 0.001372z^{-5} + z^{-6}$$

$$C(z) = 1 + 0.09184z^{-1} - 0.01369z^{-2}$$

### 4.3.1.6 AMX3532

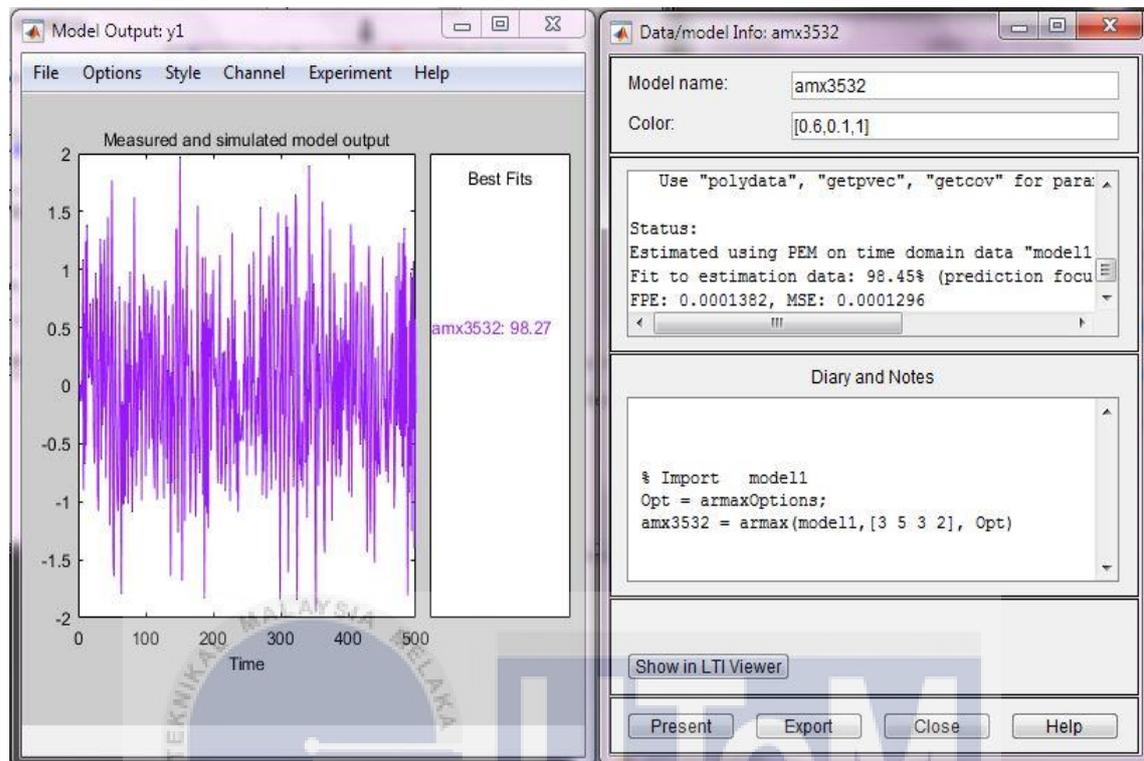


Figure 4.8 Model Output for AMX 3532

In the AMX 3532 model, the Final Prediction Error is 0.0001421 and Mean Square Error is 0.0001333.

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Discrete-time IDPOLY of AMX 3532 model is:

$$A(z) = 1 - 0.2003z^{-1} - 0.0001669z^{-2} + 0.5994z^{-3}$$

$$B(z) = 0.5001z^{-2} - 0.000671z^{-3} - 0.0007328z^{-4} - 0.00137z^{-5} + z^{-6}$$

$$C(z) = 1 + 0.0961z^{-1} + 0.002103z^{-2} + 0.07525z^{-3}$$

### 4.3.2 Discussion of Fits in Model 1

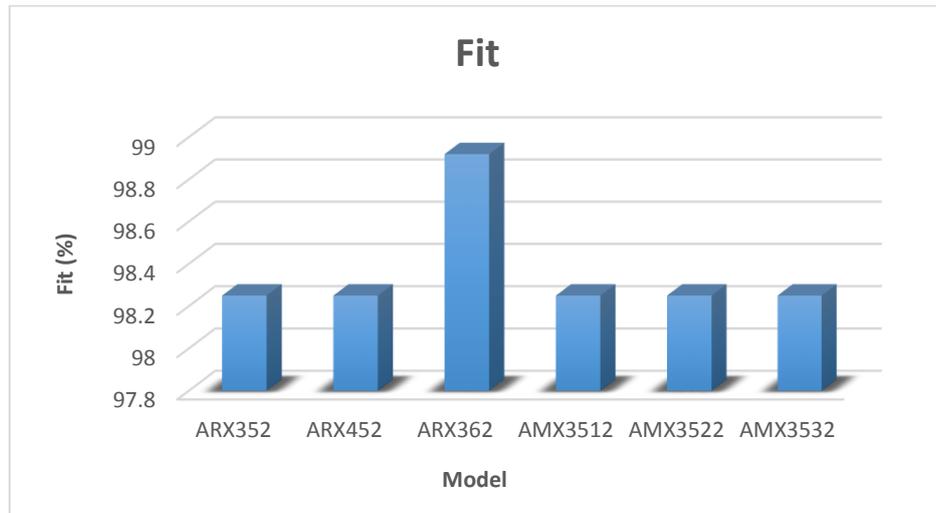


Figure 4.9 Fit Chart for Model 1

The bar chart in Figure 4.9 above illustrates the comparison value for six type of model that are simulated using MATLAB application. The model that need to be compared in Model 1 consist of ARX 352, ARX 452, ARX 362, AMX 3512, AMX 3522 and AMX 3532 with a true model specification is ARX 352. Overall, the fit value of most models are the same which is 98.25%. Apart from that, ARX 362 still has the highest fit value of 98.92%.

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### 4.3.3 Discussion of Final Prediction Error in Model 1

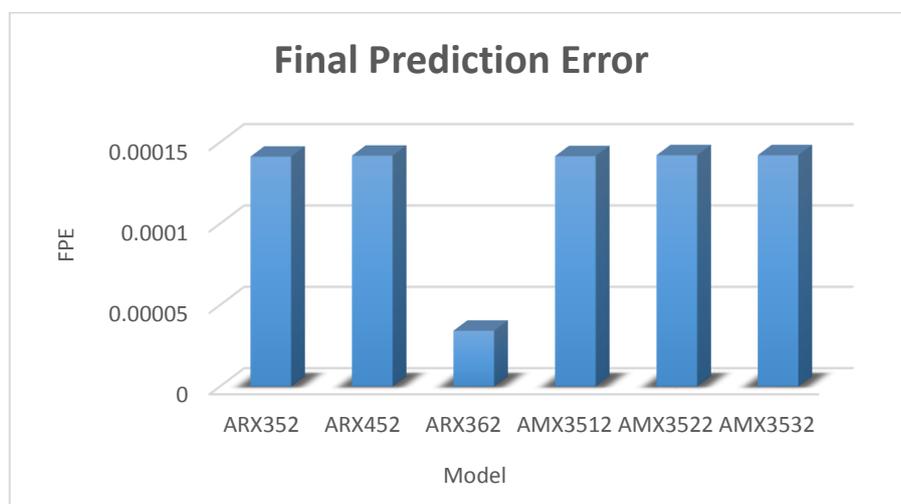


Figure 4.10 FPE Chart for Model 1

Figure 4.10 shows the value of Final Prediction Error (FPE) for six different models. From the graph, it explains that ARX 362 model has the smallest FPE value compared to other models. As for FPE value for other model, there is slight different but in comparison with ARX 362 model, there is a big difference. In conclusion, the graph shows that ARX 362 has the best value of FPE which is 0.00003421.

#### 4.3.4 Discussion of Mean Square Error in Model 1

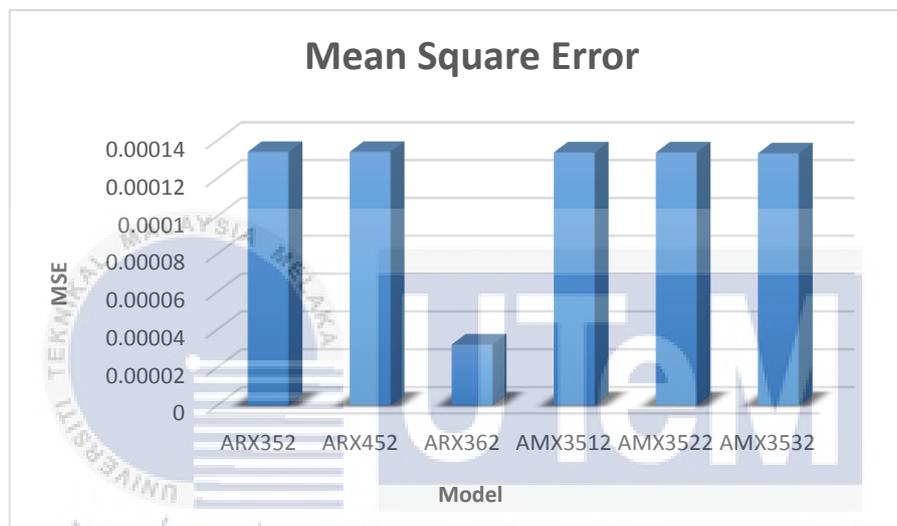


Figure 4.11 MSE Chart for Model 1

The bar chart as in Figure 4.11 represents the value of Mean Square Error (MSE) for six different types of model used in Model 1. From the graph above, ARX 362 has the smallest value of MSE. As for the MSE value for the other models, it has less difference between the models. In conclusion, the best MSE value is ARX 362 model which is 0.00003222.

### 4.3.5 Summary of Model Properties for Model 1

Table 4.1 shows the summary of model properties for Model 1.

Table 4.1 Summary of ARX and ARMAX Model Properties for Model 1

Model	Fit (%)	FPE	MSE	Discrete-time Coefficient
ARX 352	98.25	0.0001412	0.0001341	$A(z) = 1 - 0.2002z^{-1} - 0.0001713z^{-2} + 0.5994z^{-3}$ $B(z) = 0.5z^{-2} - 0.0006203z^{-3} - 0.0008136z^{-4} - 0.001365z^{-5} + z^{-6}$
ARX 452	98.25	0.0001418	0.0001341	$A(z) = 1 - 0.2z^{-1} - 0.0001703z^{-2} + 0.5994z^{-3} + 0.0003601z^{-4}$ $B(z) = 0.5z^{-2} - 0.0005056z^{-3} - 0.0007674z^{-4} - 0.001374z^{-5} + z^{-6}$
ARX 362	98.92	0.00003421	0.00003222	$A(z) = 1 - 0.2006z^{-1} - 0.00004459z^{-2} + 0.5994z^{-3}$ $B(z) = 0.5z^{-2} - 0.0008132z^{-3} - 0.0007655z^{-4} - 0.001372z^{-5} + z^{-6} + 0.0006283z^{-7}$
AMX 3512	98.25	0.0001415	0.0001338	$A(z) = 1 - 0.2002z^{-1} - 0.0001672z^{-2} + 0.5994z^{-3}$ $B(z) = 0.5001z^{-2} - 0.0006209z^{-3} - 0.0008068z^{-4} - 0.001377z^{-5} + z^{-6}$ $C(z) = 1 + 0.09345z^{-1}$
AMX 3522	98.25	0.0001421	0.0001338	$A(z) = 1 - 0.2002z^{-1} - 0.0001708z^{-2} + 0.5994z^{-3}$ $B(z) = 0.5001z^{-2} - 0.0006089z^{-3} - 0.0008082z^{-4} - 0.001372z^{-5} + z^{-6}$ $C(z) = 1 + 0.09184z^{-1} - 0.01369z^{-2}$
AMX 3532	98.25	0.0001421	0.0001333	$A(z) = 1 - 0.2003z^{-1} - 0.0001669z^{-2} + 0.5994z^{-3}$ $B(z) = 0.5001z^{-2} - 0.000671z^{-3} - 0.0007328z^{-4} - 0.00137z^{-5} + z^{-6}$ $C(z) = 1 + 0.0961z^{-1} + 0.002103z^{-2} + 0.07525z^{-3}$

#### 4.3.6 Analysis of Model 1

Table 4.1 shows the summary of ARX and ARMAX model properties for Model 1. From the table, it clearly explains that among three model of ARX 352, ARX 452 and ARX 362, model ARX 362 has the highest value of fit which is 98.92% compared to ARX 352 and ARX 452 that have a percentage of 98.25% fit. The higher the value of fit indicate the best model. Apart from observing its fit value, the Final Prediction Error (FPE) value also have to be taken into consideration as it is one of the parameters to choose the best model. As stated in Akaike's theory, one of the FPE criterion is it has the smallest value. ARX 362 proves that with a higher order number of input will results in low FPE and MSE value. Hence, the FPE value of ARX 362 gives the smallest result compared to other models which is 0.00003421 and 0.00003222 in MSE value.

As in Table 4.2, it consist the result of simulated data of AMX 3512, AMX 3522 and AMX 3532 models by using the same data. In this study, all the ARMAX model gives the same values of fit which is 98.25%. Since the models have the same value of fit, the FPE and MSE were analyzed. AMX 3512 proves that it has the lowest FPE value of 0.0001415 but for MSE value, AMX 3532 gives the smaller value which is 0.0001333.

By comparing for both ARX model and ARMAX model above, it can be observed that ARX 362 model provide best fit of 98.92% with low FPE and MSE value due to the model has an additional input variable of  $u(t - 7)$ . Generally ARMAX model would provide better fit but since the disturbance provided in the program is random, ARMAX could not provide better fit.

#### 4.4 Model 2

As can be seen in Figure 4.12 below, the Input and Output Signals shows measured input and output data for Model 2 by using the equation (4.2). The preprocess method has been carried out by removing means as shown in Figure 4.13. In order to obtain the mathematical model of this system, the following true model specification of ARX 243 is used. ARX 243 is computed by the following order number;  $n_a = 2$ ,  $n_b = 4$  and  $n_k = 3$ .

$$y(t) = 0.3y(t - 1) - 0.5y(t - 2) - 0.5u(t - 3) + 0.2u(t - 6) + e(t) \quad (4.2)$$

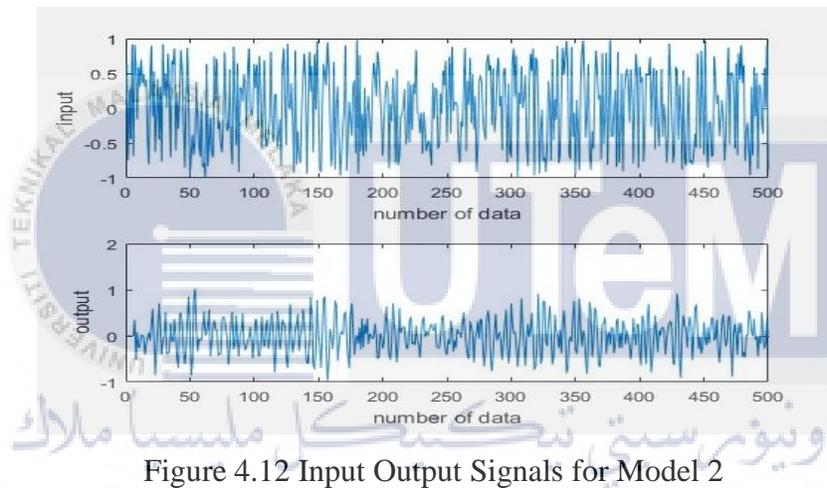


Figure 4.12 Input Output Signals for Model 2

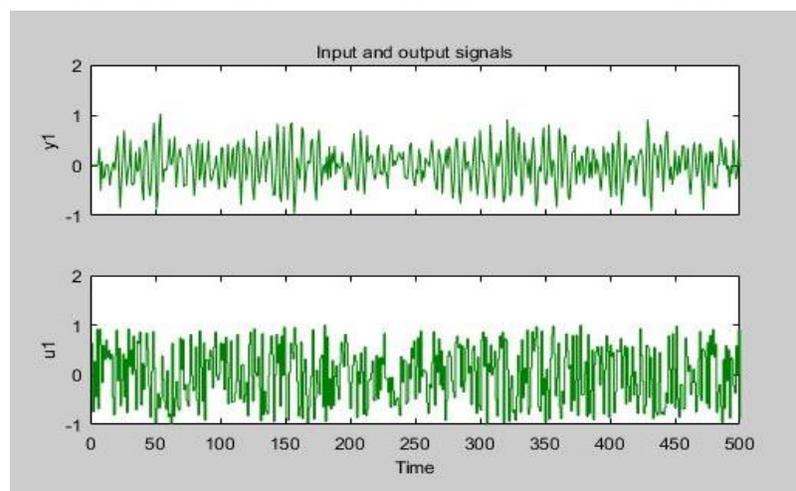


Figure 4.13 Time Plot for Model 2

The simulation result of model output for time domain data in Model 2 shown in Figure 4.14 consist of ARX 243 (true model specification), ARX 343, ARX 253, AMX 2413, AMX 2423 and AMX 2433. The model output graph shows the comparison of all Models 2 simulated output. From the graph, it shows that the highest best fit of 97.32% was produced by model ARX 253. As for model ARX 243, ARX 343, AMX 2413, AMX 2423 and AMX 2433 has the same value of fit which are 97.32%. Overall, ARX 253 model is chosen as the best model based on its fit, FPE and MSE value.

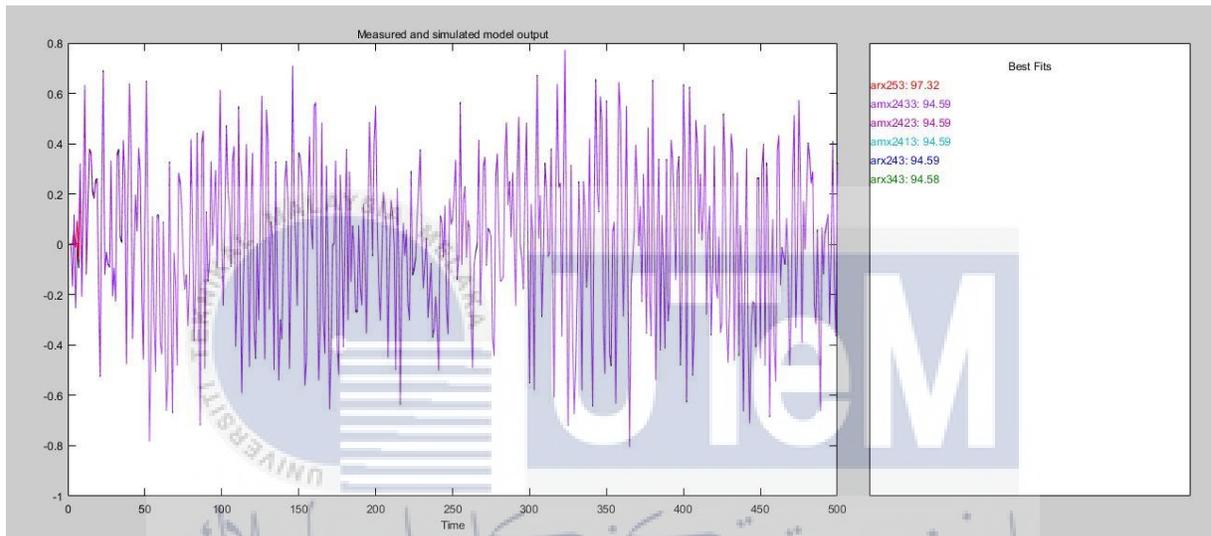


Figure 4.14 Model Output for Model 2

## 4.4.1 Simulated Results

### 4.4.1.1 ARX 243

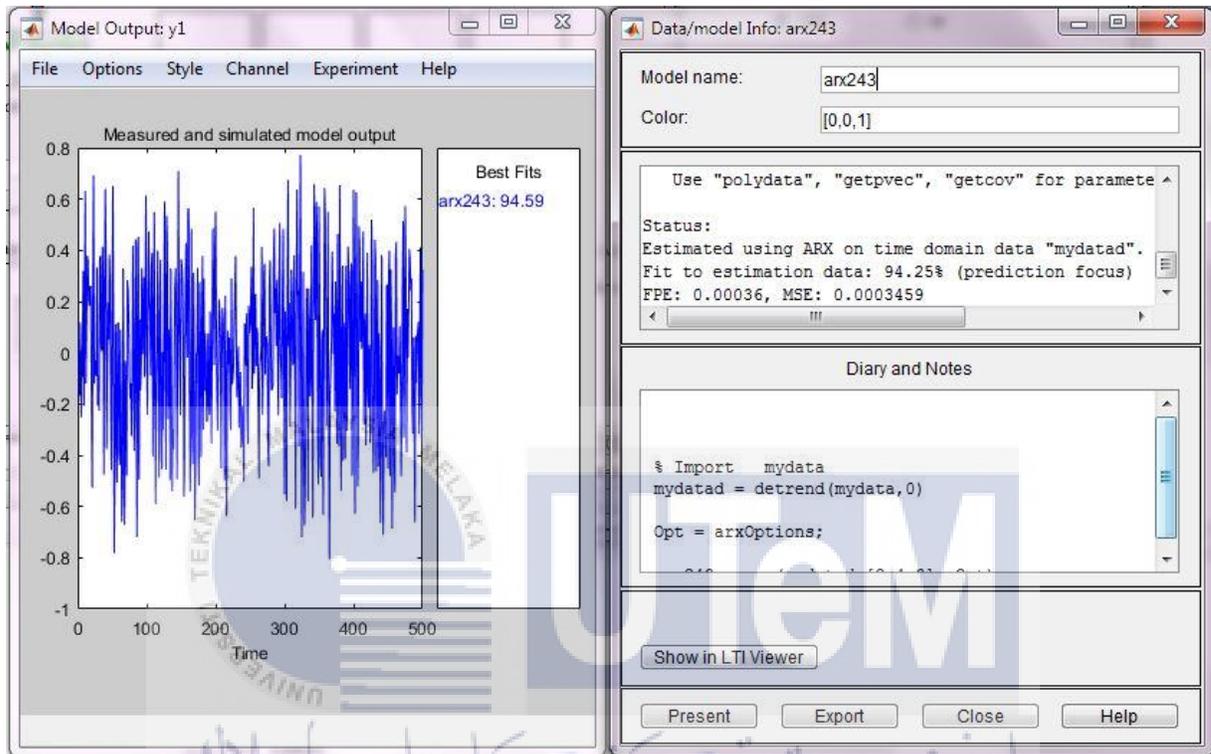


Figure 4.15 Model Output for ARX 243

In the ARX 243 model, the Final Prediction Error is 0.00036 and Mean Square Error is 0.0003459.

Discrete-time IDPOLY of ARX 243 model is:

$$A(z) = 1 - 0.2994z^{-1} + 0.4987z^{-2}$$

$$B(z) = -0.4996z^{-3} - 0.00044652z^{-4} - 0.0015911z^{-5} - 0.1992z^{-6}$$

#### 4.4.1.2 ARX 343

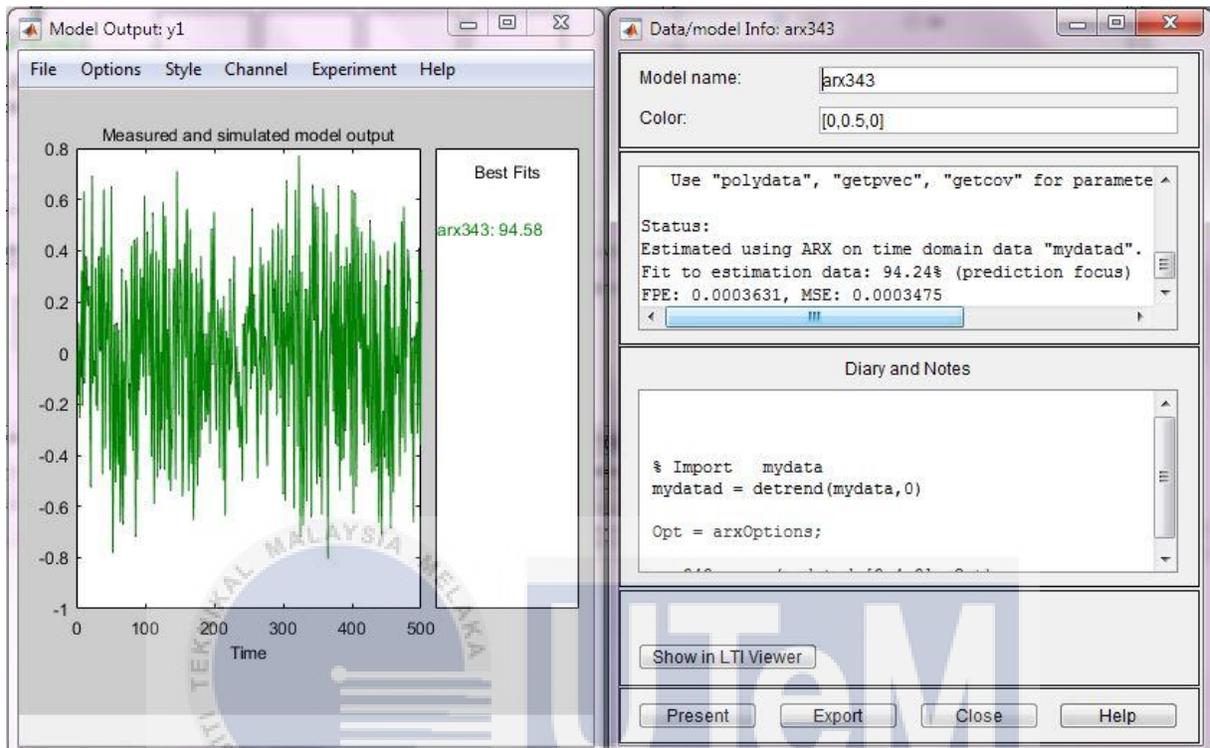


Figure 4.16 Model Output for ARX 343

In the ARX 343 model, the Final Prediction Error is 0.0003631 and Mean Square Error is 0.0003475.

Discrete-time IDPOLY of ARX 343 model is:

$$A(z) = 1 - 1.3028z^{-1} - 0.4969z^{-2} + 0.002958z^{-3}$$

$$B(z) = 0.4995z^{-3} - 0.0002136z^{-4} - 0.002975z^{-5} + 0.1981z^{-6}$$

### 4.4.1.3 ARX 253

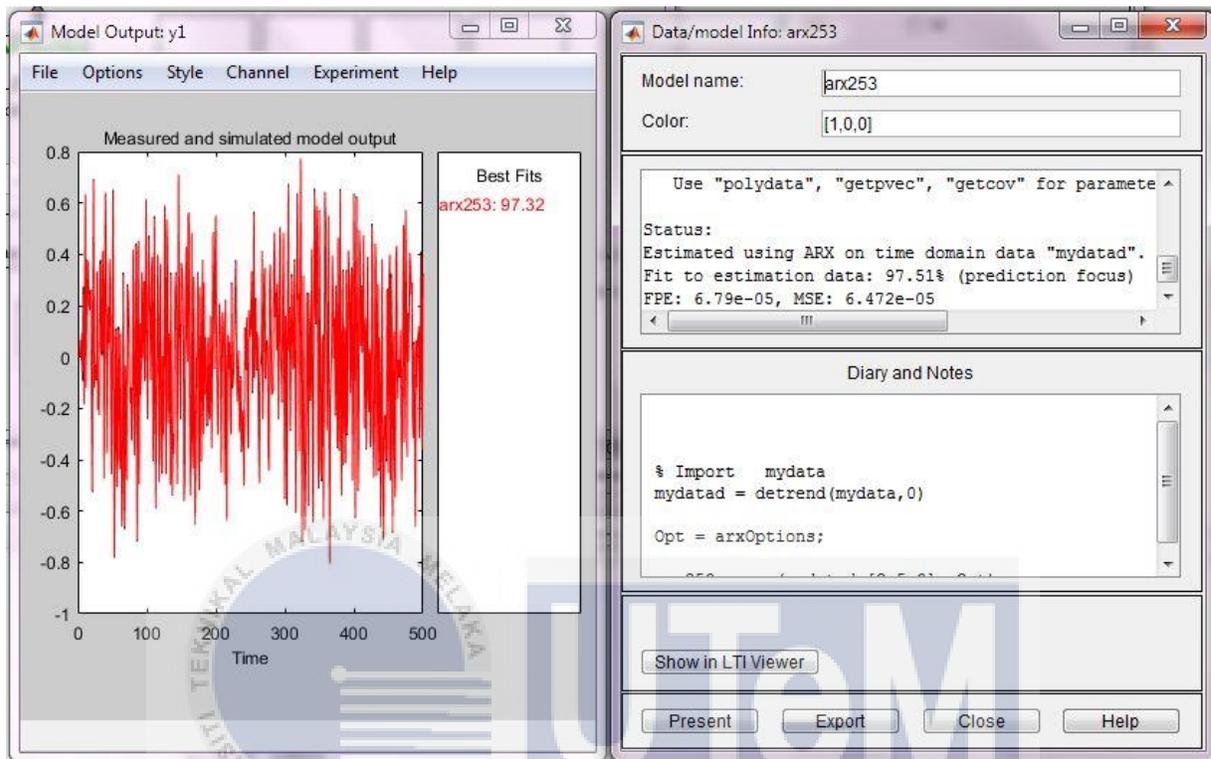


Figure 4.17 Model Output for ARX 253

In the ARX 253 model, the Final Prediction Error is 0.0000679 and Mean Square Error is 0.00006472.

Discrete-time IDPOLY of ARX 253 model is:

$$A(z) = 1 - 0.2973z^{-1} - 0.4956z^{-2}$$

$$B(z) = 0.4995z^{-3} + 0.0006528z^{-4} - 0.002808z^{-5} + 0.1983z^{-6} + 0.001032z^{-7}$$

#### 4.4.1.4 AMX 2413

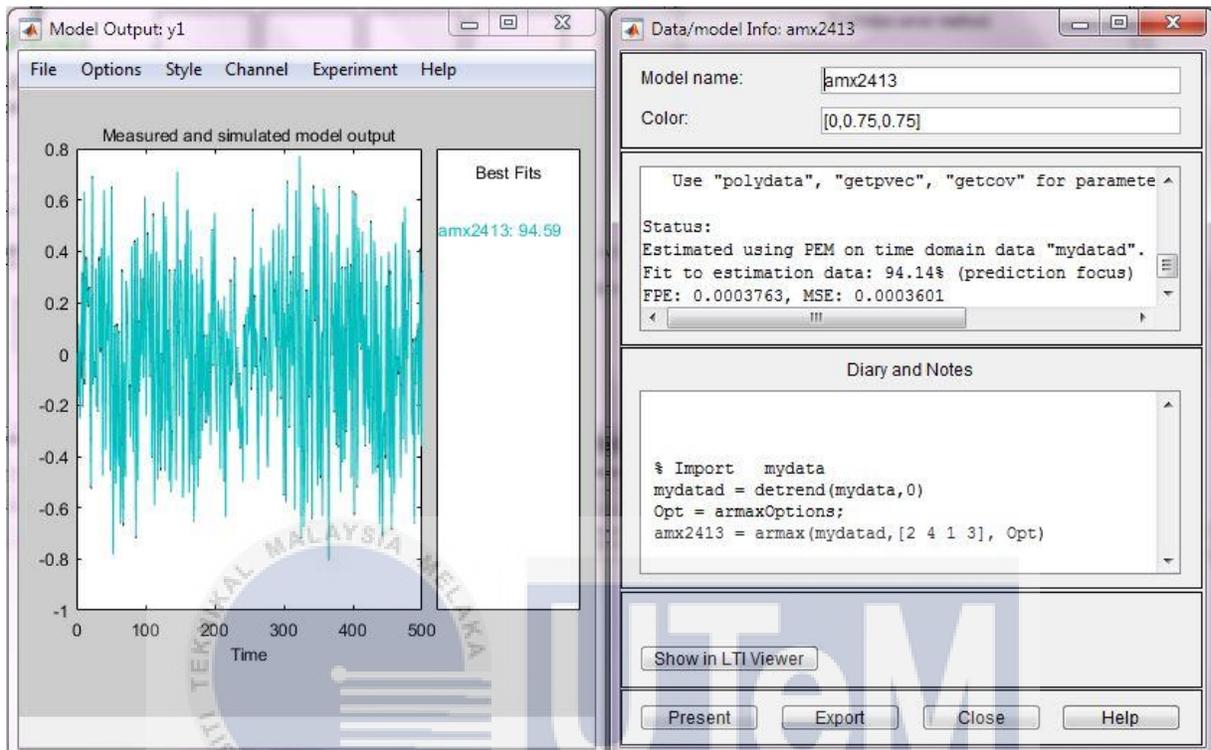


Figure 4.18 Model Output for AMX 2413

In the AMX 2413 model, the Final Prediction Error is 0.0003763 and Mean Square Error is 0.0003601.

Discrete-time IDPOLY of AMX 2413 model is:

$$A(z) = 1 - 0.2989z^{-1} - 0.4988z^{-2}$$

$$B(z) = -0.4996z^{-3} - 0.0001938z^{-4} - 0.001462z^{-5} - 0.1991z^{-6}$$

$$C(z) = 1 + 0.07433z^{-1}$$

#### 4.4.1.5 AMX 2423

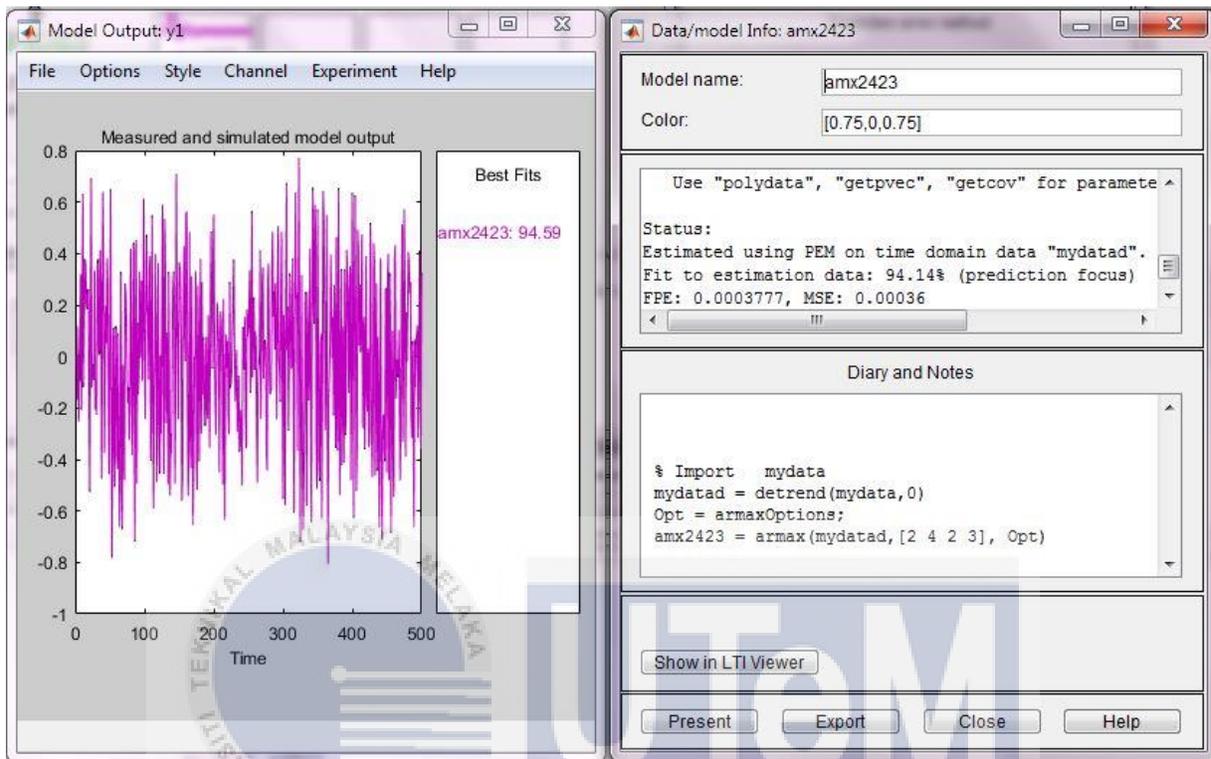


Figure 4.19 Model Output for AMX 2423

In the AMX 2423 model, the Final Prediction Error is 0.0003777 and Mean Square Error is 0.0003600.

Discrete-time IDPOLY of AMX 2423 model is:

$$A(z) = 1 - 0.2989z^{-1} - 0.4988z^{-2}$$

$$B(z) = -0.4996z^{-3} - 0.0001913z^{-4} - 0.0014642z^{-5} - 0.1991z^{-6}$$

$$C(z) = 1 + 0.07408z^{-1} - 0.001077z^{-2}$$

#### 4.4.1.6 AMX 2433

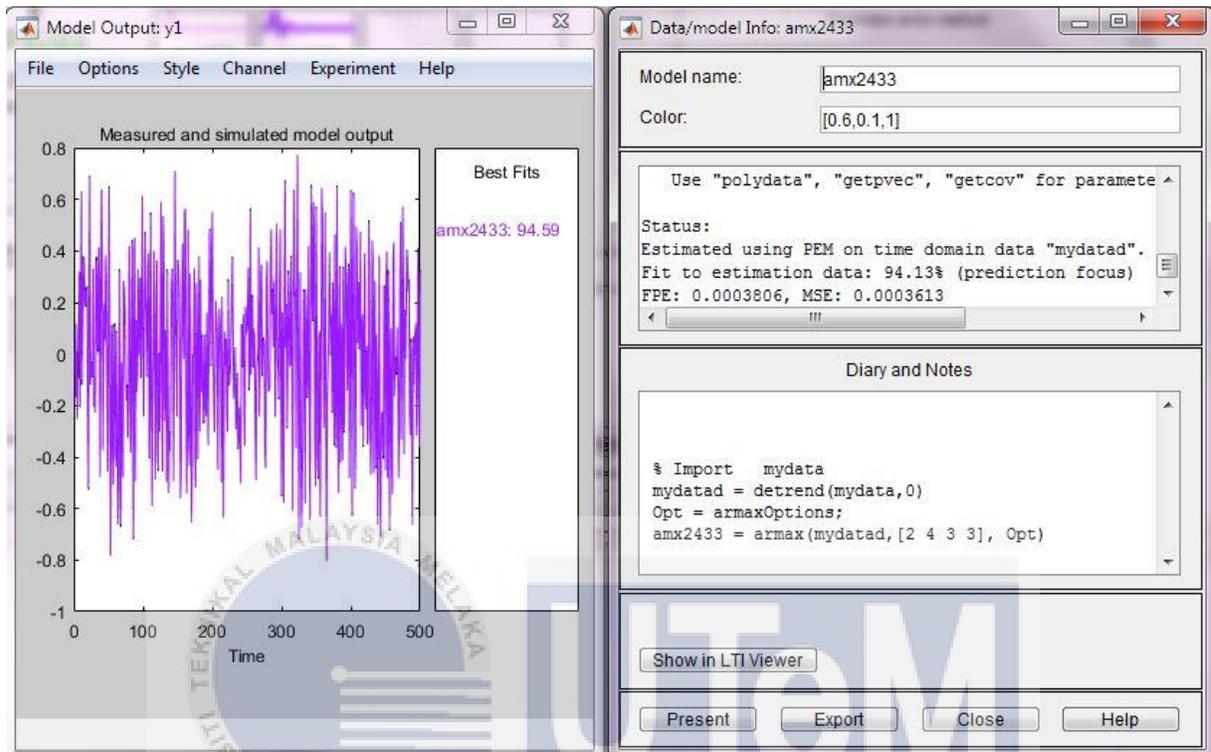


Figure 4.20 Model Output for AMX 2433

In the AMX 2433 model, the Final Prediction Error is 0.0003806 and Mean Square Error is 0.0003613.

Discrete-time IDPOLY of AMX 2433 model is:

$$A(z) = 1 - 0.2989z^{-1} - 0.498z^{-2}$$

$$B(z) = -0.4995z^{-3} - 0.0001536z^{-4} - 0.001473z^{-5} - 0.199z^{-6}$$

$$C(z) = 1 + 0.07746z^{-1} + 0.006297z^{-2} + 0.05652z^{-3}$$

#### 4.4.2 Discussion of Fits in Model 2

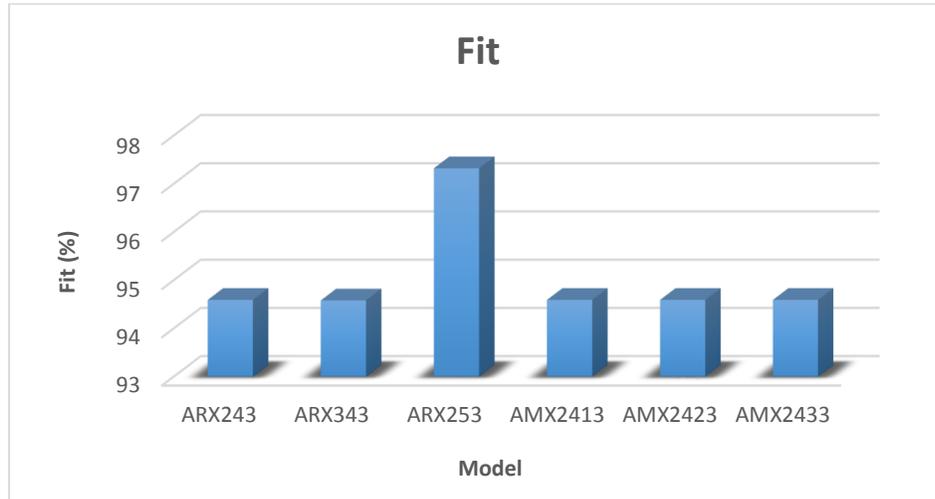


Figure 4.21 Fit Chart for Model 2

Figure 4.21 exhibit all the value of fit for six models for Model 2 which consist of ARX 243, ARX 343, ARX 253, AMX 2413, AMX 2423 and AMX 2433 model with a true model specification which is ARX 243. Based from the data obtain shows that the fit value for all models have same value of 94.59% except for model ARX 253 which have the highest value of 97.32%.

#### 4.4.3 Discussion of Final Prediction Error in Model 2

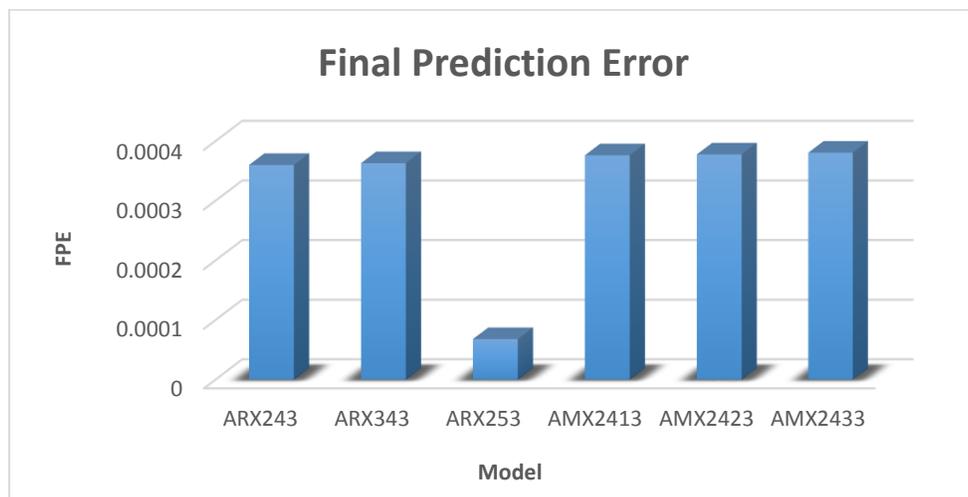


Figure 4.22 FPE Chart for Model 2

As stated in the Figure 4.22, the value of Final Prediction Error value for Model 2 also proves that ARX 253 has the lowest value compared to other models. The FPE value of ARX 253 model is 0.00006966 which indicate that it is the best model. Apart from that, there is a small difference of FPE value between the other models but in terms of FPE criterion, ARX 253 clearly shows that this model has the best FPE value.

#### 4.4.4 Discussion of Mean Square Error in Model 2

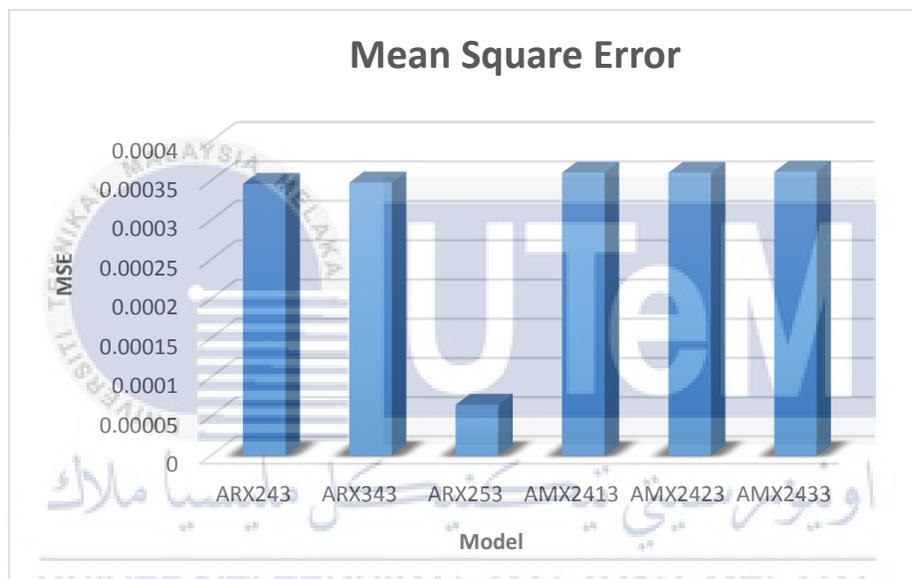


Figure 4.23 MSE Chart for Model 2

Figure 4.23 above also clearly show that ARX 253 model has the lowest MSE value of 0.0000664. Based on the result obtained from the simulation, there is a difference of MSE value between ARX 243 and other models. Holistically, ARX 253 is suggested as the best model in terms of its MSE value is the lowest.

#### 4.4.5 Summary of Model Properties for Model 2

Table 4.2 shows the summary of model properties for Model 2.

Table 4.2 Summary of ARX and ARMAX Model Properties for Model 2

Model	Fit (%)	FPE	MSE	Discrete-time Coefficient
ARX 243	94.59	0.0003600	0.0003459	$A(z) = 1 - 0.2994z^{-1} + 0.4987z^{-2}$ $B(z) = 0.4996z^{-3} - 0.00044652z^{-4} - 0.0015911z^{-5} - 0.1992z^{-6}$
ARX 343	94.58	0.0003631	0.0003475	$A(z) = 1 - 1.3028z^{-1} - 0.4969z^{-2} + 0.002958z^{-3}$ $B(z) = 0.4995z^{-3} - 0.0002136z^{-4} - 0.002975z^{-5} + 0.1981z^{-6}$
ARX 253	97.32	0.0000679	0.00006472	$A(z) = 1 - 0.2973z^{-1} - 0.4956z^{-2}$ $B(z) = 0.4995z^{-3} + 0.0006528z^{-4} - 0.002808z^{-5} + 0.1983z^{-6} + 0.001032z^{-7}$
AMX 2413	94.59	0.0003763	0.0003601	$A(z) = 1 - 0.2989z^{-1} - 0.4988z^{-2}$ $B(z) = -0.4996z^{-3} - 0.0001938z^{-4} - 0.001462z^{-5} - 0.1991z^{-6}$ $C(z) = 1 + 0.07433z^{-1}$
AMX 2423	94.59	0.0003777	0.0003600	$A(z) = 1 - 0.2989z^{-1} - 0.4988z^{-2}$ $B(z) = -0.4996z^{-3} - 0.0001913z^{-4} - 0.0014642z^{-5} - 0.1991z^{-6}$ $C(z) = 1 + 0.07408z^{-1} - 0.001077z^{-2}$
AMX 2433	94.59	0.0003806	0.0003613	$A(z) = 1 - 0.2989z^{-1} - 0.498z^{-2}$ $B(z) = -0.4995z^{-3} - 0.0001536z^{-4} - 0.001473z^{-5} - 0.199z^{-6}$ $C(z) = 1 + 0.07746z^{-1} + 0.006297z^{-2} + 0.05652z^{-3}$

#### 4.4.6 Analysis of Model 2

The summary results of best fit, FPE and MSE value is shown in Table 4.2. The results precisely explain that among three model of ARX 243, ARX 343 and ARX 253, the highest best fit value is ARX 253 which is 97.32%. By comparing with a true specification of ARX 243, model ARX 253 also show that it has the largest number of percentage. As the fit value achieve an optimum amount which closer to 100%, it can be evaluated that it is the best model.

From Table 4.2 shows that ARX 253 has the lowest value of 0.0000679 compared to ARX 243 and ARX 343. Based on Akaike's Theory, the smallest value indicates that it has better performance. The less error could be obtain as there is more variable in the system. However, it can be one of the disadvantage as with more variable, the system will be more complicated. For Mean Square Error (MSE), the lowest value among all the models was also given by ARX 253 model which is 0.00006472.

Besides, the result of simulated data of AMX 2413, AMX 2423 and AMX 2433 models also shown in Table 4.2. In this study, all the ARMAX model gives the same values of best fit which is 94.59%. Since the models have the same value of best fit, the FPE and MSE value were analyzed. AMX 2413 proves that it has the lowest FPE value of 0.0003763 and as for MSE value, AMX 2423 gives the smallest value which is 0.00036 but there is only a slight different value between the models.

By comparing for both ARX model and ARMAX model above, it can be observed that ARX 253 model provide best fit of 97.32% with low FPE and MSE value. Because of the disturbance provided in the program is random, ARMAX could not provide better fit. Other than that, the model is considered good due to additional variable of ARX 253 of  $u(t - 7)$ . There is slight difference in other models but ARX 253 is considered has better performance in terms of fit, FPE and MSE value.

#### 4.5 Model 3

The Input and Output Signals shows measured input and output data for Model 3 by using the equation (4.3) are shown in Figure 4.24 below. The preprocess method has been carried out by removing means as shown in Figure 4.25. In order to obtain the mathematical model of this system, the true model specification of AMX 3532 is used. AMX 3532 is computed by the following order number;  $n_a = 3$ ,  $n_b = 5$ ,  $n_c = 3$  and  $n_k = 2$ .

$$y(t) = 0.2y(t - 1) - 0.6y(t - 3) + 0.5u(t - 2) + u(t - 6) + e(t) - 0.4e(t - 3) \quad (4.3)$$

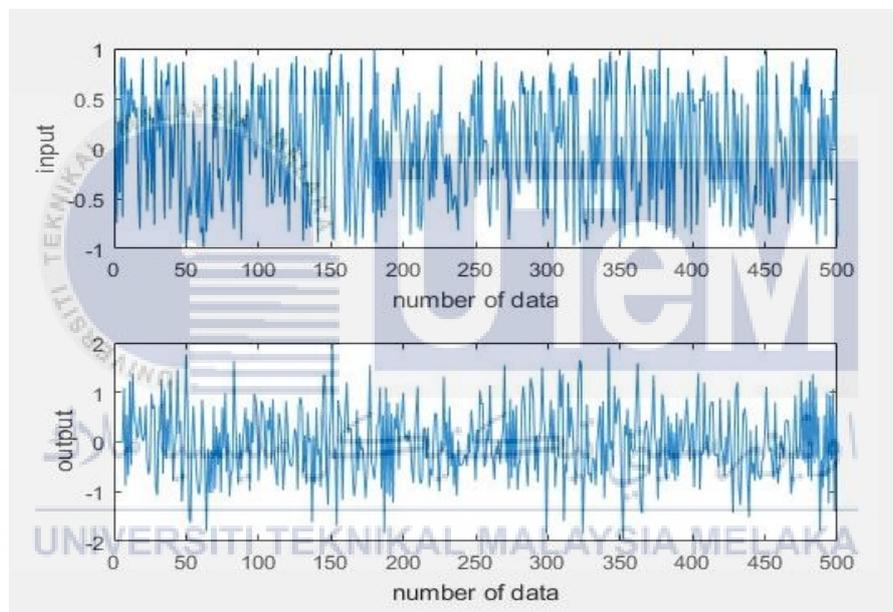


Figure 4.24 Input Output Signals for Model 3

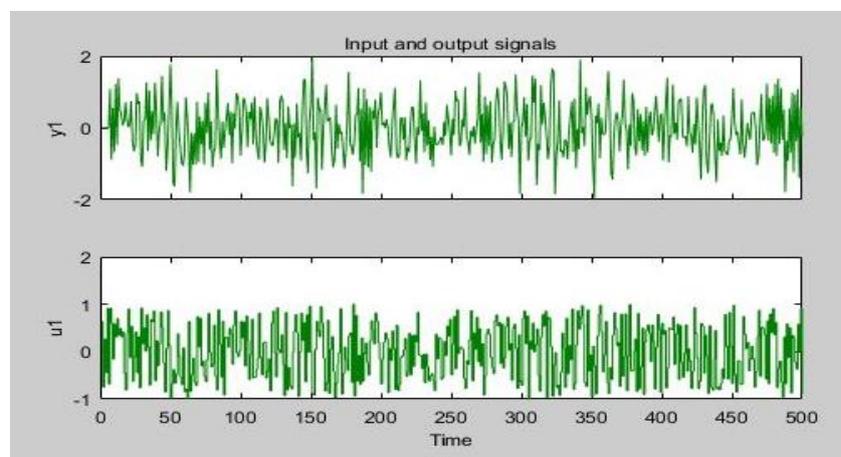


Figure 4.25 Time Plot for Model 3

The simulation result of model output for time domain data in Model 1 is shown in Figure 4.26, consist of ARX 352, ARX 452, ARX 362, AMX 3512, AMX 3522 and AMX 3532 (true model specification). The model output graph shows the comparison of all Models 3 simulated output. From the graph, it shows that the highest fit of 98.60% was produced by model ARX 362. As for model ARX 352, ARX 452, AMX 3512, AMX 3522 and AMX 3532 has the same value of fit which are 98.04%. Overall, ARX 362 model is chosen as the best model based on its fit, FPE and MSE value.

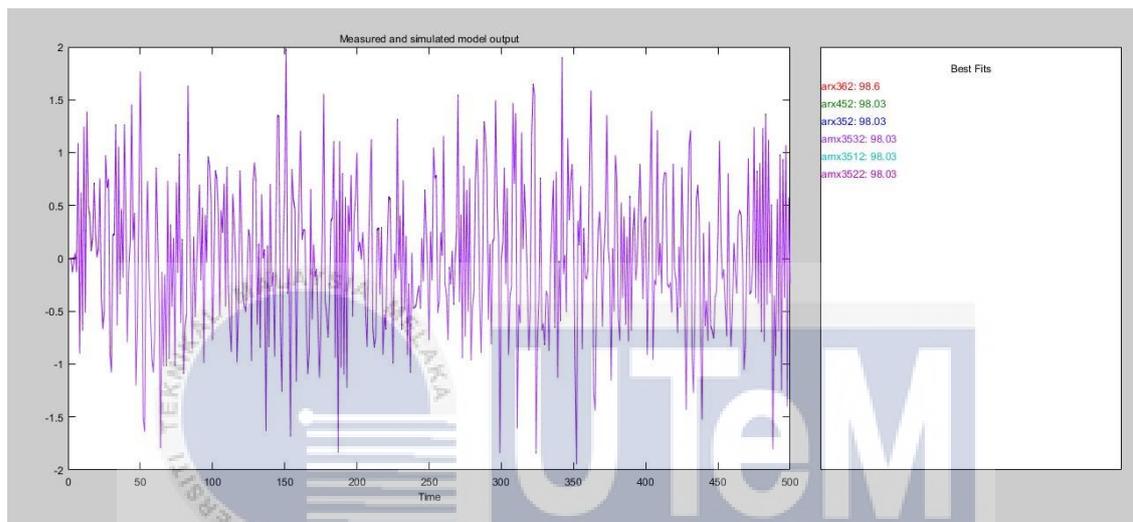


Figure 4.26 Model Output for Model 3

## 4.5.1 Simulated Results

### 4.5.1.1 ARX 352

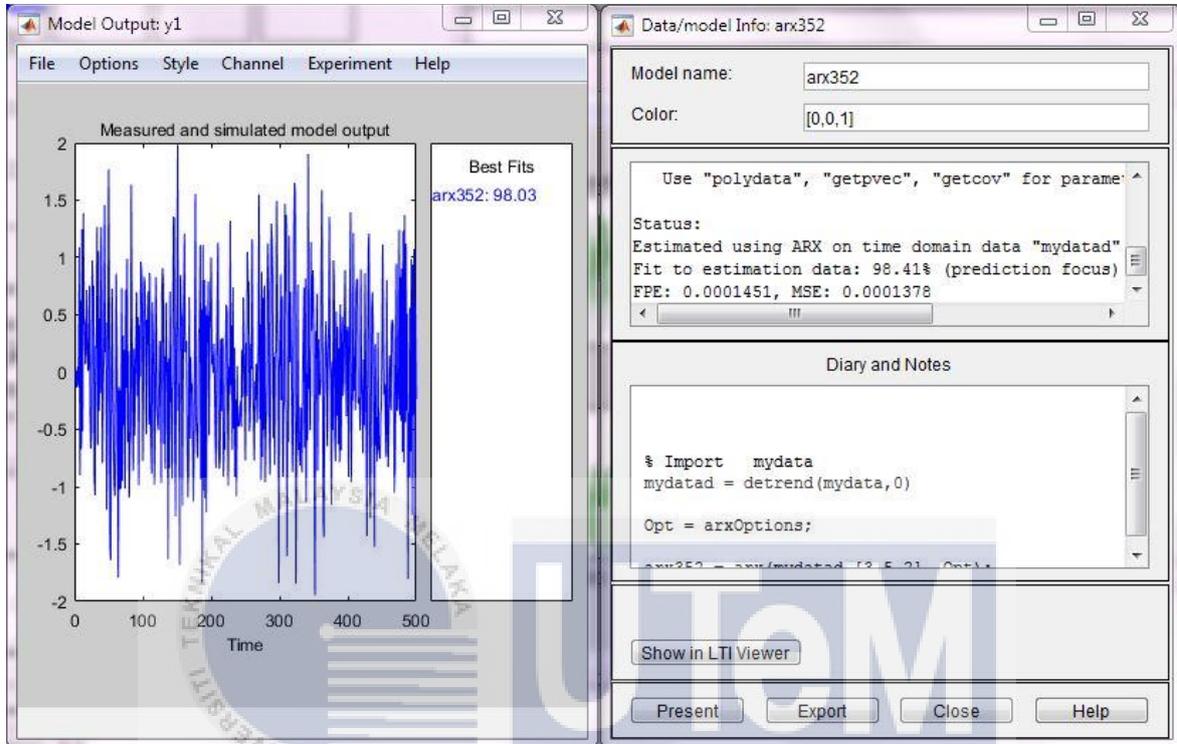


Figure 4.27 Model Output for ARX 352

In the ARX 352 model, the Final Prediction Error is 0.0001451 and Mean Square Error is 0.0001378.

Discrete-time IDPOLY of ARX 352:

$$A(z) = 1 - 0.2006z^{-1} - 0.0004166z^{-2} + 0.5993z^{-3}$$

$$B(z) = 0.5001z^{-2} - 0.001115z^{-3} - 0.0006585z^{-4} - 0.001463z^{-5} + z^{-6}$$

### 4.5.1.2 ARX 452

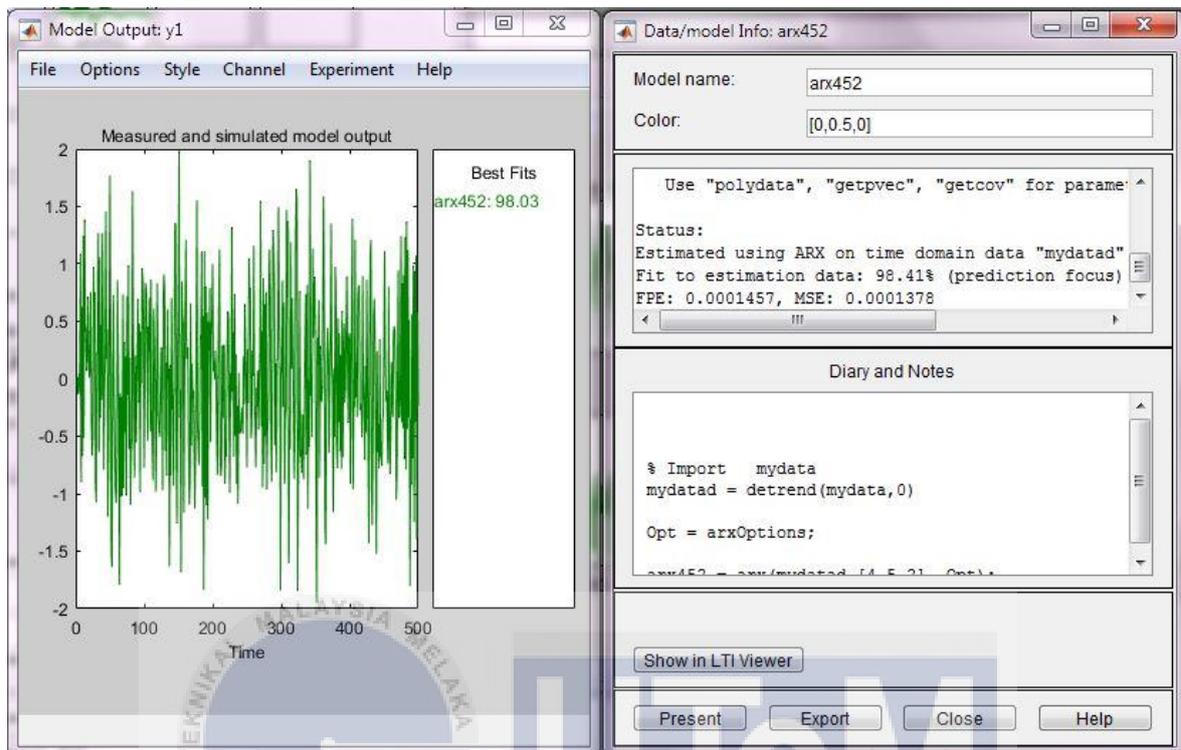


Figure 4.28 Model Output for ARX 452

In the ARX 452, the Final Prediction Error value is 0.0001457 and Mean Square Error value is 0.0001378.

Discrete-time IDPOLY of ARX 452:

$$A(z) = 1 - 0.2004z^{-1} - 0.0003195z^{-2} + 0.5993z^{-3} + 0.0002647z^{-4}$$

$$B(z) = 0.5001z^{-2} - 0.00103z^{-3} - 0.0006245z^{-4} - 0.001459z^{-5} + z^{-6}$$

### 4.5.1.3 ARX 362

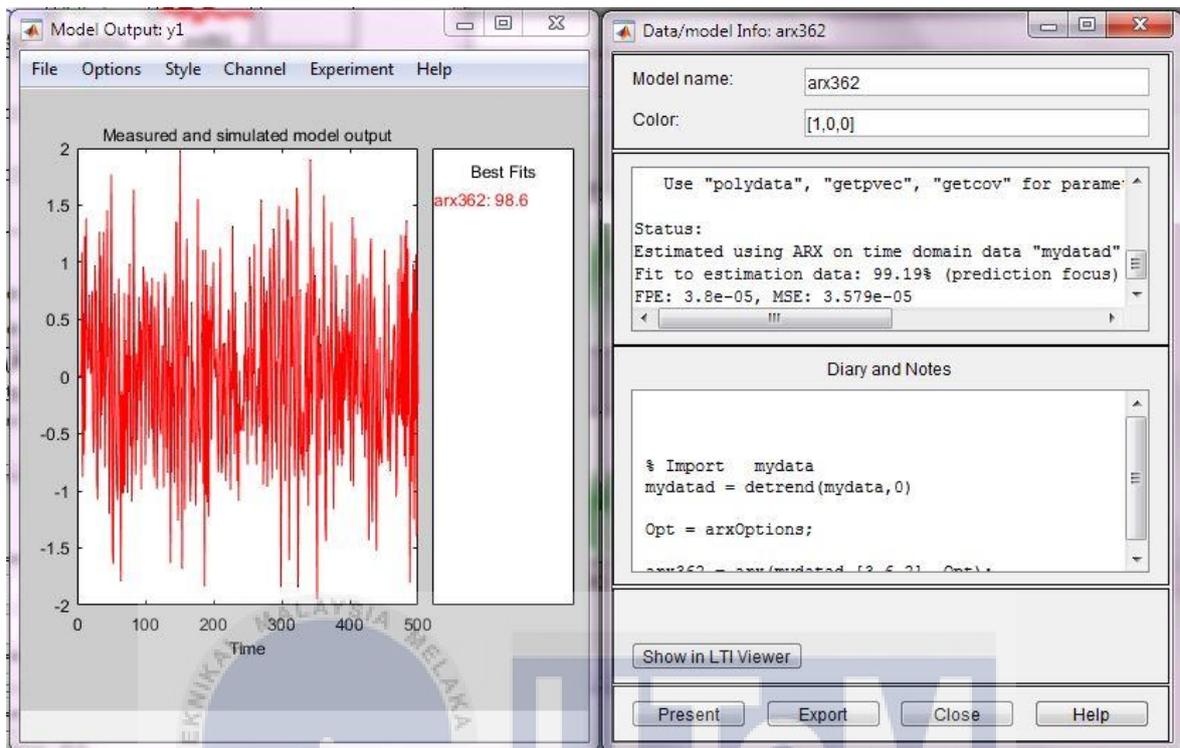


Figure 4.29 Model Output for ARX 362

In the ARX 362 model, the Final Prediction Error value is 0.000038 and Mean Square Error value is 0.00003579.

Discrete-time IDPOLY of ARX 362:

$$A(z) = 1 - 0.2009z^{-1} - 0.0003195z^{-2} + 0.5993z^{-3}$$

$$B(z) = 0.5001z^{-2} - 0.001266z^{-3} - 0.0006194z^{-4} - 0.001474z^{-5} + z^{-6} - 0.000477z^{-7}$$

#### 4.5.1.4 AMX 3512

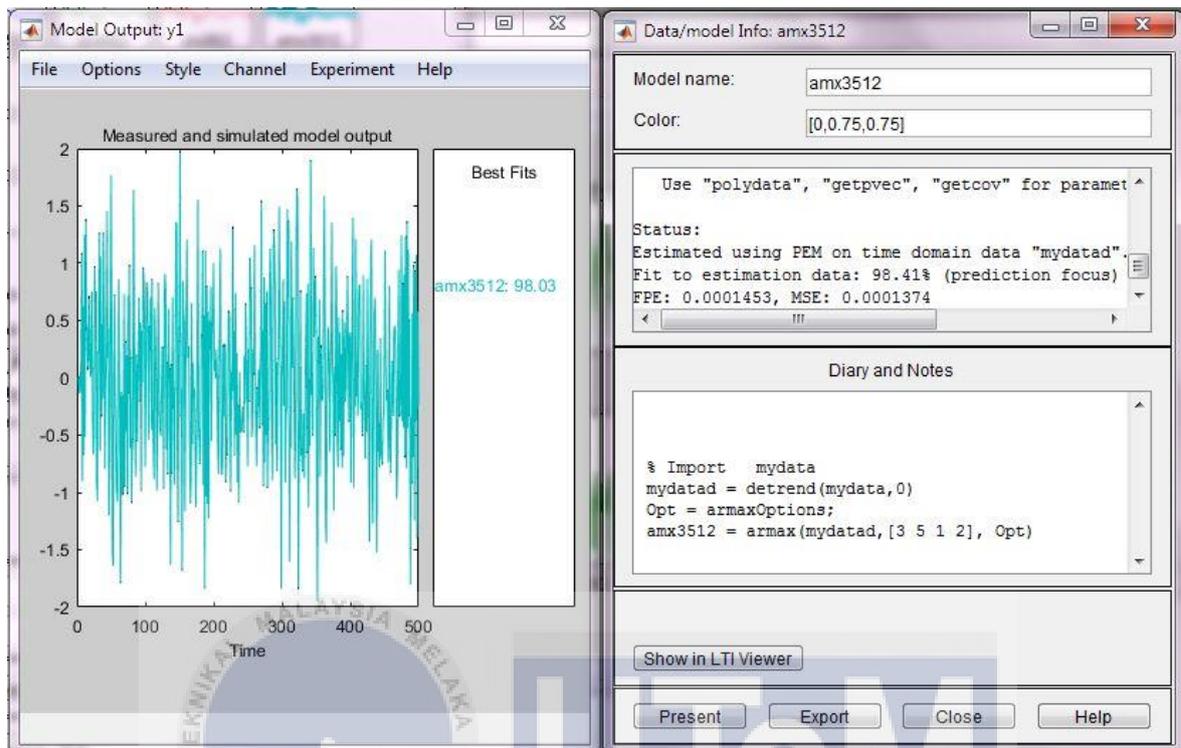


Figure 4.30 Model Output for AMX 3512

In the AMX 3512 model, the Final Prediction Error value is 0.0001453 and Mean Square Error value is 0.0001374.

Discrete-time IDPOLY of AMX 3512:

$$A(z) = 1 - 0.2006z^{-1} - 0.0004126z^{-2} + 0.5992z^{-3}$$

$$B(z) = 0.5001z^{-2} - 0.001114z^{-3} - 0.0006523z^{-4} - 0.001473z^{-5} + z^{-6}$$

$$C(z) = 1 + 0.09753z^{-1}$$

### 4.5.1.5 AMX 3522

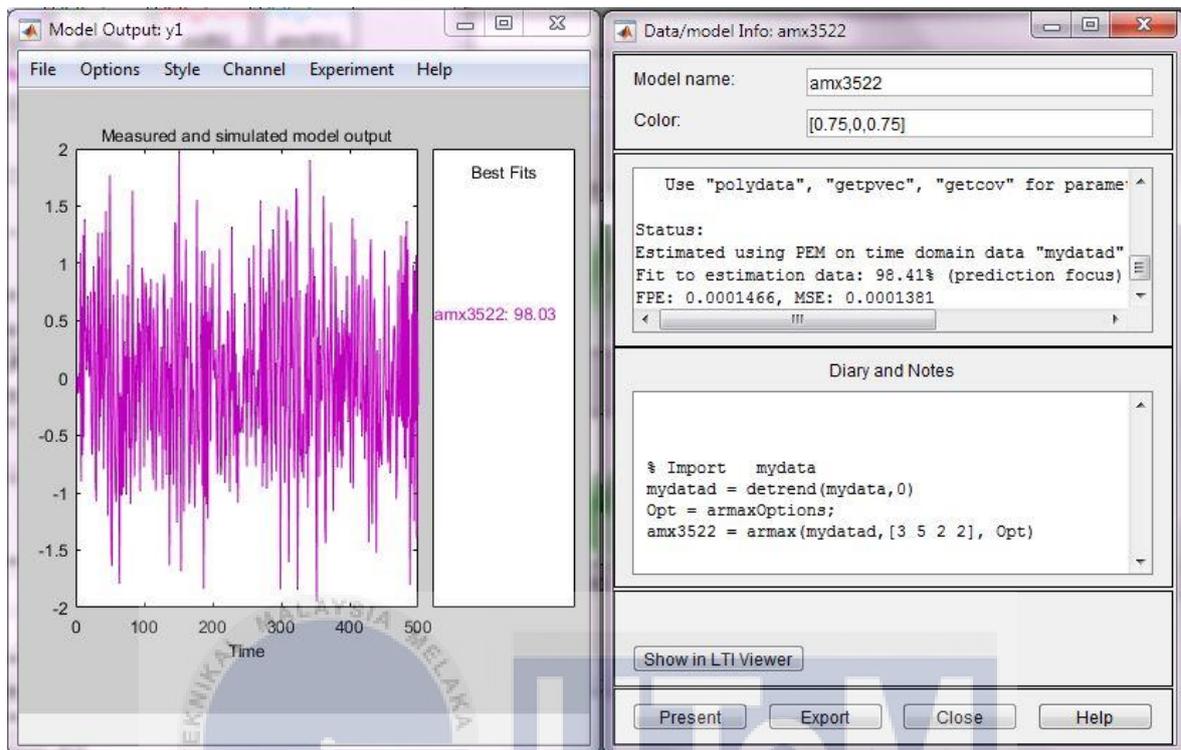


Figure 4.31 Model Output for AMX 3522

In the AMX 3522 model, the Final Prediction Error value is 0.0001466 and Mean Square Error value is 0.0001381.

Discrete-time IDPOLY of AMX3522:

$$A(z) = 1 - 0.2006z^{-1} - 0.0004238z^{-2} + 0.5992z^{-3}$$

$$B(z) = 0.5001z^{-2} - 0.001038z^{-3} - 0.0006538z^{-4} - 0.001455z^{-5} + z^{-6}$$

$$C(z) = 1 + 0.1483z^{-1} + 0.09289z^{-2}$$

### 4.5.1.6 AMX 3532

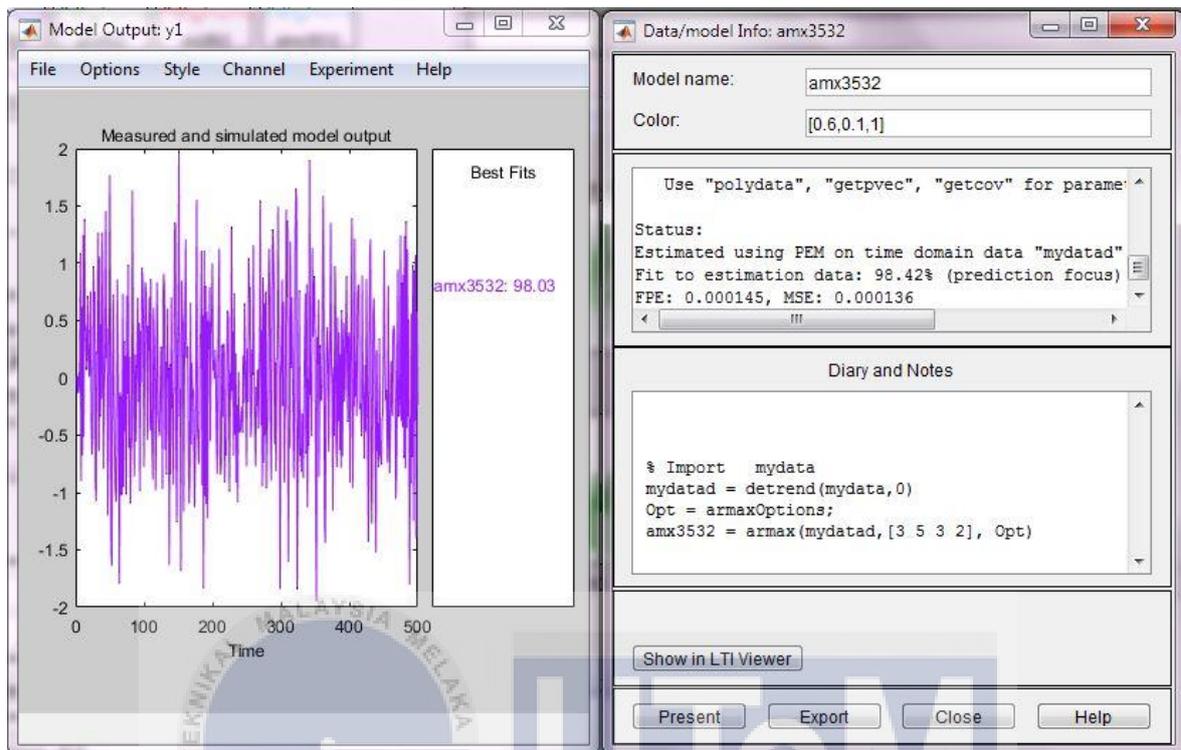


Figure 4.32 Model Output for AMX 3532

In the AMX 3532 model, the Final Prediction Error value is 0.0001450 and Mean Square Error is 0.0001360.

Discrete-time IDPOLY of AMX 3532:

$$A(z) = 1 - 0.2006z^{-1} - 0.0004903z^{-2} + 0.5992z^{-3}$$

$$B(z) = 0.5001z^{-2} - 0.0007981z^{-3} - 0.0009553z^{-4} - 0.001357z^{-5} + z^{-6}$$

$$C(z) = 1 + 0.1103z^{-1} + 0.0579z^{-2} - 0.2928z^{-3}$$

#### 4.5.2 Discussion of Fits in Model 3

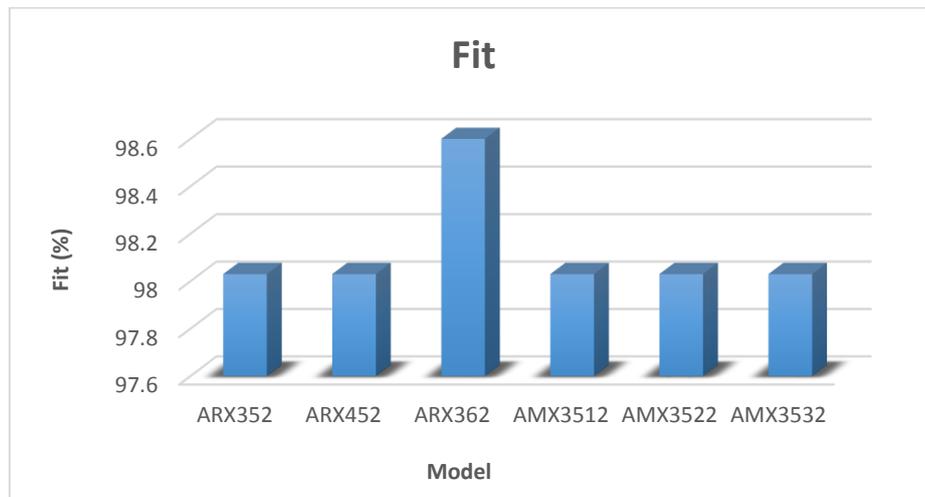


Figure 4.34 Fit Chart for Model 3

The models that need to be compared in terms of fit value consist of ARX 352, ARX 452, ARX 362, AMX 3512, AMX 3522 and AMX 3532. By referring with the true specification, ARX 362 prove that this model has the highest percentage fit value of 98.60%. In conclusion, the result can be interpreted as the best result when the value of best fit is precisely heading to 100 percent.

#### 4.5.3 Discussion of Final Prediction Error in Model 3

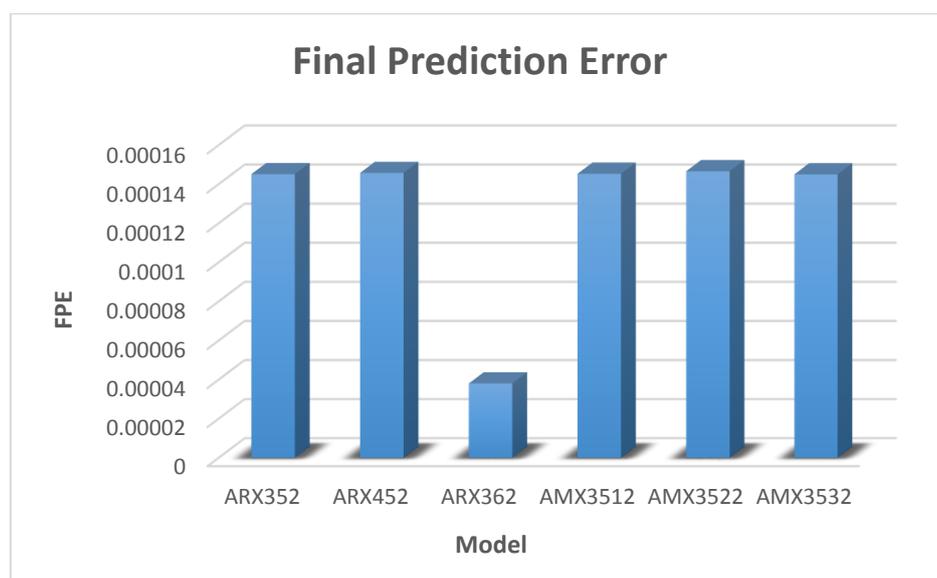


Figure 4.35 FPE Chart for Model 3

Based on FPE chart in Figure 4.35, it clearly show that FPE value of ARX 362 is the lowest which is 0.000038 followed by true model specification AMX 3532 is 0.0001450. The optimum error value can be estimated as there is more variable obtain in the system. Hence, the smallest value will be produced and indicate that the model provide better FPE results.

#### 4.5.4 Discussion of Mean Square Error in Model 3

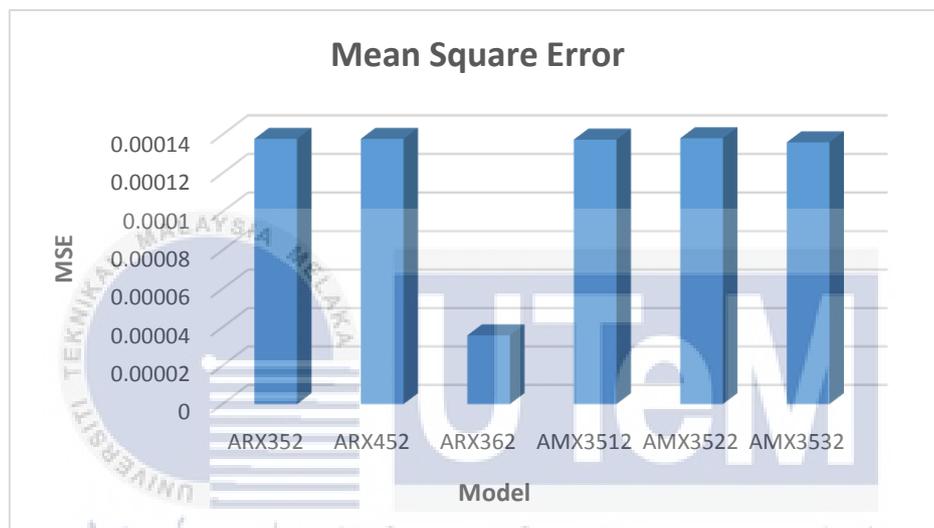


Figure 4.36 MSE Chart for Model 3

This bar chart as in Figure 4.36 represents the value of Mean Square Error (MSE) for six different types of model consist in Model 3. From the chart above proves that ARX 362 has the smallest value of MSE and there is a slight difference among the other models. To be precise, the best MSE value is ARX 362 model which has the lowest value of 0.00003579.

#### 4.5.5 Summary of Model Properties for Model 3

Table 4.3 below shows the summary of model properties for Model 3.

Table 4.3 Summary of ARX and ARMAX Model Properties for Model 3

Model	Fit (%)	FPE	MSE	Discrete-Time Coefficient
ARX 352	98.03	0.0001451	0.0001378	$A(z) = 1 - 0.2006z^{-1} - 0.0004166z^{-2} + 0.5993z^{-3}$ $B(z) = 0.5001z^{-2} - 0.001115z^{-3} - 0.0006585z^{-4} - 0.001463z^{-5} + z^{-6}$
ARX 452	98.03	0.0001457	0.0001378	$A(z) = 1 - 0.2004z^{-1} - 0.000416z^{-2} + 0.5993z^{-3} + 0.0002647z^{-4}$ $B(z) = 0.5001z^{-2} - 0.00103z^{-3} - 0.0006245z^{-4} - 0.001459z^{-5} + z^{-6}$
ARX 362	98.60	0.0000380	0.00003579	$A(z) = 1 - 0.2009z^{-1} - 0.0003195z^{-2} + 0.5993z^{-3}$ $B(z) = 0.5001z^{-2} - 0.001266z^{-3} - 0.0006194z^{-4} - 0.001474z^{-5} + z^{-6} + 0.000477z^{-7}$
AMX 3512	98.03	0.0001453	0.0001374	$A(z) = 1 - 0.2006z^{-1} - 0.0004126z^{-2} + 0.5992z^{-3}$ $B(z) = 0.5001z^{-2} - 0.001114z^{-3} - 0.0006523z^{-4} - 0.001473z^{-5} + z^{-6}$ $C(z) = 1 + 0.09753z^{-1}$
AMX 3522	98.03	0.0001466	0.0001381	$A(z) = 1 - 0.2006z^{-1} - 0.0004238z^{-2} + 0.5992z^{-3}$ $B(z) = 0.5001z^{-2} - 0.001038z^{-3} - 0.0006538z^{-4} - 0.001455z^{-5} + z^{-6}$ $C(z) = 1 + 0.1483z^{-1} + 0.09289z^{-2}$
AMX 3532	98.03	0.0001450	0.0001360	$A(z) = 1 - 0.2006z^{-1} - 0.0004903z^{-2} + 0.5993z^{-3}$ $B(z) = 0.5001z^{-2} - 0.0007981z^{-3} - 0.0009553z^{-4} - 0.001357z^{-5} + z^{-6}$ $C(z) = 1 + 0.1103z^{-1} + 0.0579z^{-2} - 0.2928z^{-3}$

#### 4.5.6 Analysis of Model 3

Table 4.3 shows that the final result of all six types of model that need to be analyze in Model 3. The result stated are the final value of best fit, FPE and MSE value obtain from a simulation process. It clearly describe that among three of ARX model which consist of model ARX 352, ARX 452 and ARX 362, prove that ARX 362 has the highest best fit value of 98.60%.

As for FPE result, it shows that ARX 362 gives the lowest value of 0.000038 compared to model ARX 352 and ARX 452. Based on Akaike's theory, the smaller the value of FPE will result in less error in the system. The less error could be obtain as there is more variable in the system. However, it can be one of the disadvantage as the more variable in the system, the system will be more complicated. For Mean Square Error (MSE), the lowest value among all the models was also given by ARX 362 model which is 0.00003579.

However, the result of simulated data of AMX 3512, AMX 3522 and AMX 3532 models shows the percentage fit achieved is less than the result of ARX 362. The percentage of fit obtain for AMX 3512, AMX 3522 and AMX 3532 are 98.03% which they have the same result as model ARX 352 and ARX 452. Although the percentage of fit gain for all model are same, the FPE and MSE value has a slight difference. The lowest value is given by ARMAX model is 0.0001450 for FPE and 0.0001360 for MSE. Both result are obtain from true model specification of AMX 3532 since the model gives the lowest value.

From the result obtain for all six different types of model, it precisely shows that ARX 362 present the highest fit value of 98.60% with a low FPE value of 0.000038 and low MSE value of 0.00003579 to meet the criteria needed. The result is due to additional variable in the model that is  $u(t - 7)$ . Due to random disturbance provided, ARMAX could not provide better fit.

By comparing with a true model specification which is AMX 3532, model ARX 362 also show that it has the largest number of percentage. As the fit value achieve an optimum amount which closer to 100%, it can be considered that the model structure provide better fit. Even though ARMAX is commonly known to capture noise model better but in this analysis, ARX prove that it has better performance when an extra input variable is added.

#### 4.6 Simulation using Real Data

The preprocess method has been carried out by removing means using the real data as shown in Figure 4.37. Real Data on dryer machine input and output data is used to compare the performance obtain from real data, Model 1, Model 2 and Model 3. The input data,  $u_2$  act as heating power and for the output data,  $y_2$  act as the temperature of the outflow air. The implementation of real data used two different model structure each for ARX and ARMAX model. The model structure used are ARX 692, ARX 6102, AMX 6922 and AMX 6942.

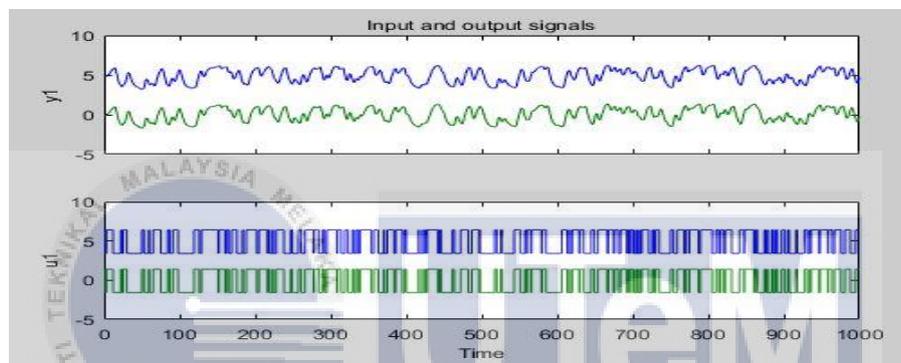


Figure 4.37 Time Plot

The simulation result of model output for time domain data by using the real data shows in Figure 4.38 consist of ARX 692, ARX 6102, AMX 6922 and AMX 6942. The model output graph shows the comparison of the simulated output. From the graph, it shows that the highest fit of 89.80% was produced by model AMX 6922 but in comparison with AMX 6942, there is slight different where AMX 6942 shows the 89.78% value of fit. As for model ARX 692 and ARX 6102, the fit is lower by about 1%.

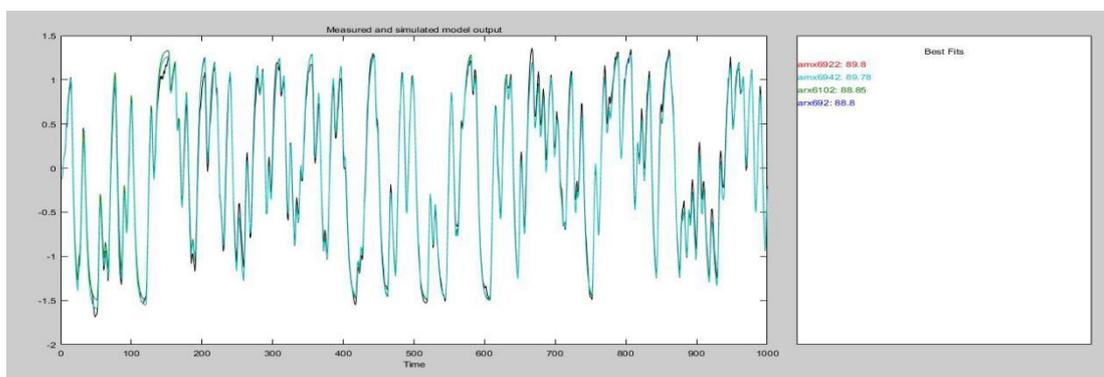


Figure 4.38 Model Output

## 4.6.1 Simulated Results

### 4.6.1.1 ARX 692

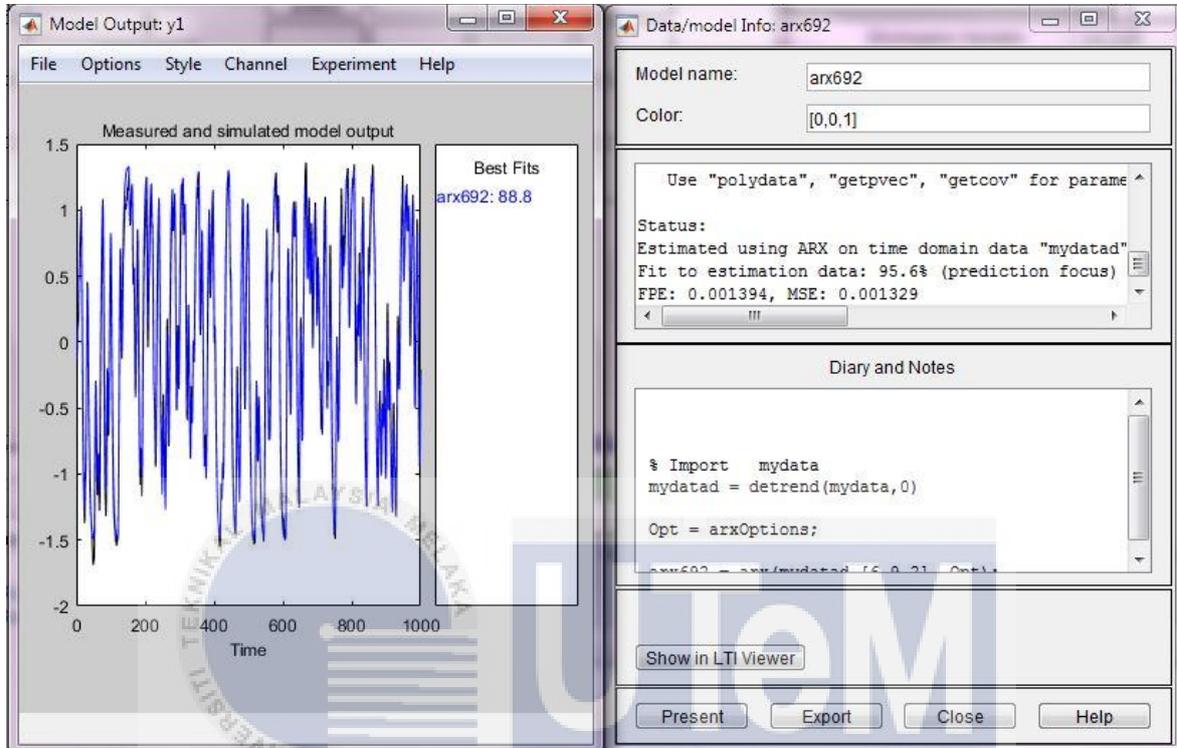


Figure 4.39 Model Output for ARX 692

In the ARX 692 model, the Final Prediction Error is 0.001394 and Mean Square Error is 0.001329.

Discrete-time IDPOLY of ARX 692 model is:

$$A(Z) = 1 - 0.9417z^{-1} + 0.003123z^{-2} - 0.04692z^{-3} + 0.08292z^{-4} - 0.01831z^{-5} + 0.03539z^{-6}$$

$$B(Z) = 0.005686z^{-2} + 0.06431z^{-3} + 0.0633z^{-4} + 0.02115z^{-5} - 0.004842z^{-6} - 0.01424z^{-7} - 0.01309z^{-8} - 0.008236z^{-9} - 0.005861z^{-10}$$

### 4.6.1.2 ARX 6102

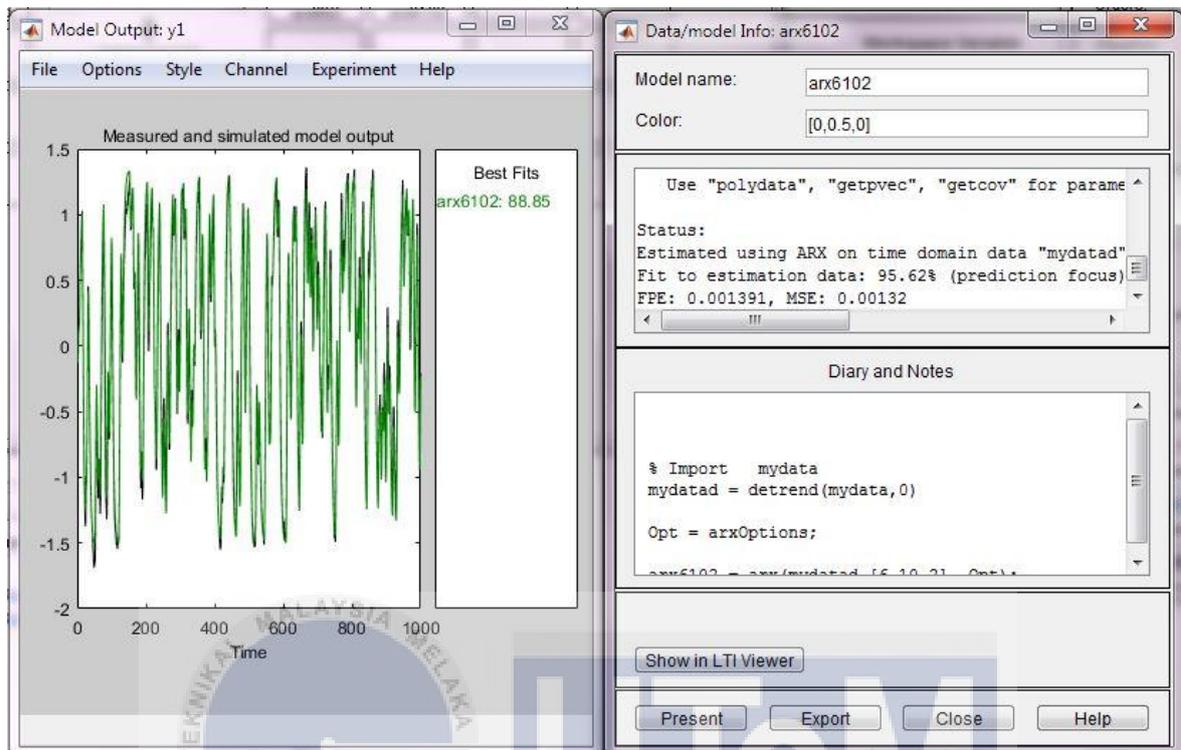


Figure 4.40 Model Output for ARX 6102

In the ARX 6102 model, the Final Prediction Error is 0.001391, and Mean Square Error is 0.001320.

Discrete-time IDPOLY of ARX 6102 model is:

$$A(Z) = 1 - 0.9338z^{-1} + 0.006018z^{-2} - 0.05206z^{-3} + 0.07261z^{-4} - 0.04662z^{-5} + 0.0603z^{-6}$$

$$B(Z) = 0.0056z^{-2} + 0.06438z^{-3} + 0.06392z^{-4} + 0.02247z^{-5} - 0.003829z^{-6} - 0.01434z^{-7} - 0.01569z^{-8} - 0.01137z^{-9} - 0.006711z^{-10} - 0.003285z^{-11}$$

### 4.6.1.3 AMX 6922

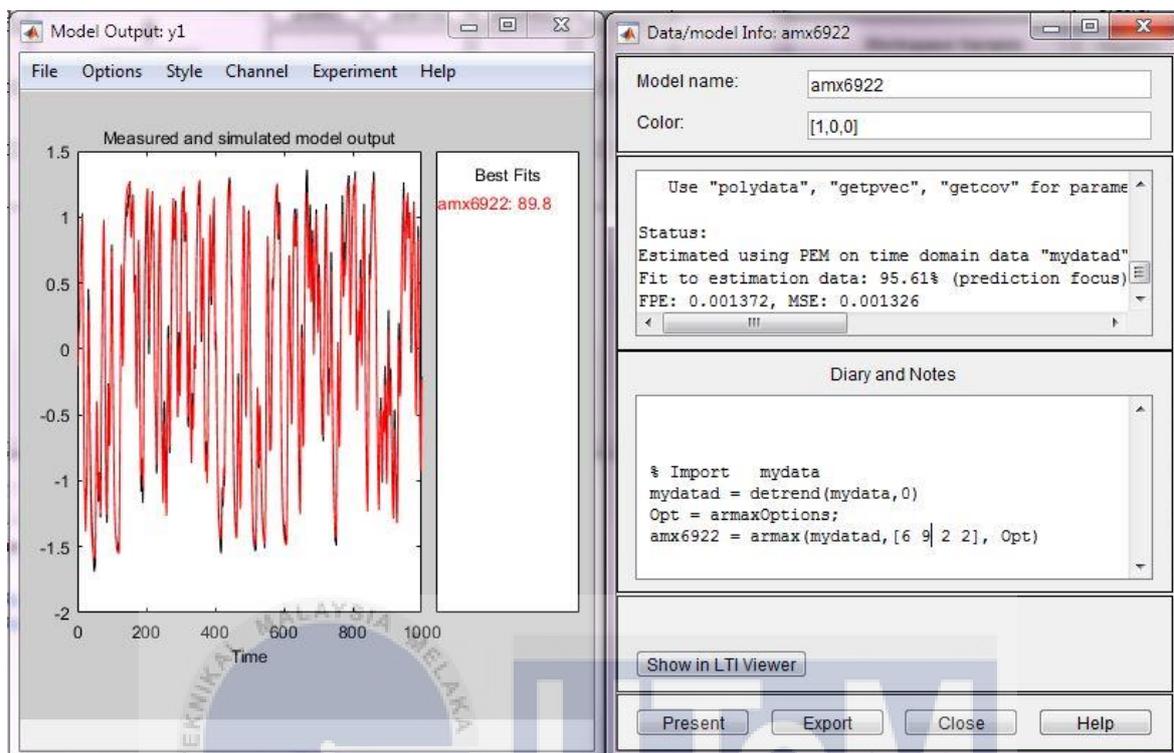


Figure 4.41 Model Output for AMX 6922

In the AMX 6922 model, the Final Prediction Error is 0.001372 and Mean Square Error is 0.001326.

Discrete-time IDPOLY of ARX 6102 model is:

$$A(Z) = 1 - 1.314z^{-1} - 0.1266z^{-2} + 0.418z^{-3} + 0.1005z^{-4} - 0.05769z^{-5} - 0.01712z^{-6}$$

$$B(Z) = 0.005337z^{-2} + 0.06218z^{-3} + 0.03672z^{-4} - 0.03308z^{-5} - 0.04212z^{-6} - 0.02095z^{-7} - 0.005691z^{-8} + 0.00001946z^{-9} + 0.0003697z^{-10}$$

$$C(Z) = 1 - 0.4115z^{-1} - 0.5088z^{-2}$$

#### 4.6.1.4 AMX 6942

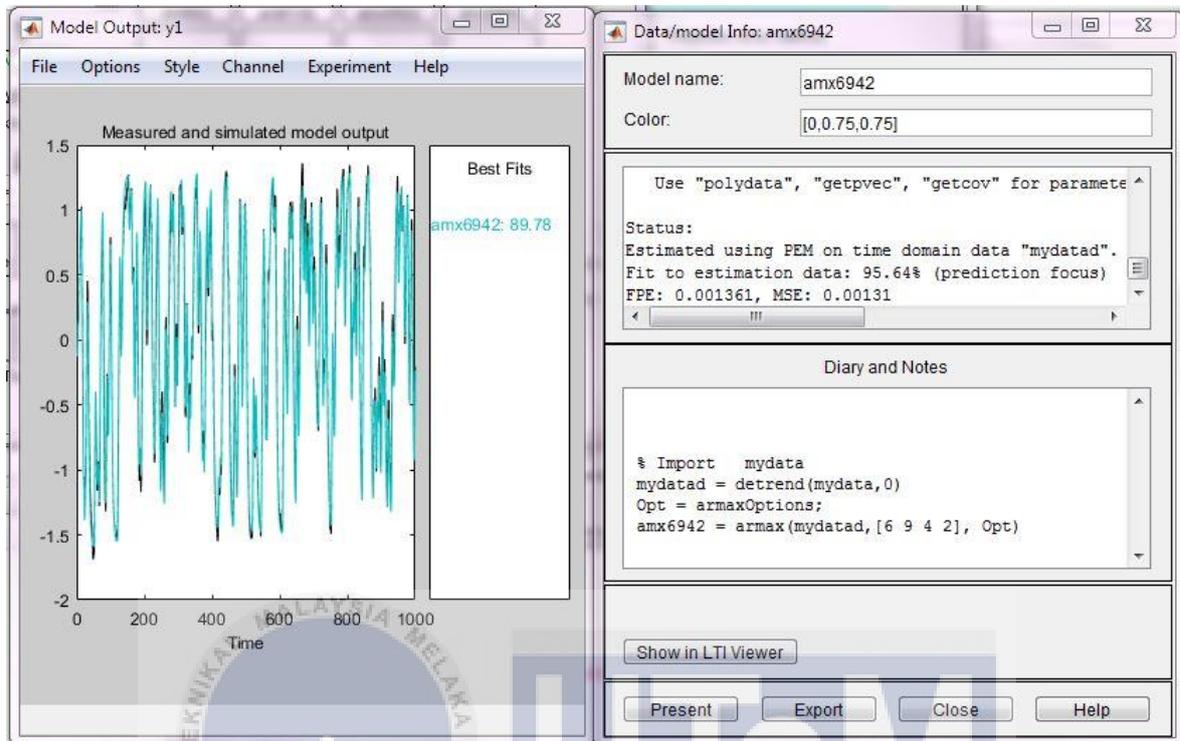


Figure 4.42 Model Output for AMX 6942

In the AMX 6924 model, the Final Prediction Error is 0.001361 and Mean Square Error is 0.001.

Discrete-time IDPOLY of ARX 6102 model is:

$$A(Z) = 1 - 0.2386z^{-1} - 1.656z^{-2} + 0.2635z^{-3} + 0.9324z^{-4} - 0.2371z^{-5} - 0.05859z^{-6}$$

$$B(Z) = 0.005404z^{-2} + 0.0677z^{-3} + 0.1031z^{-4} - 0.001557z^{-5} - 0.09177z^{-6} - 0.06035z^{-7} - 0.01671z^{-8} - 0.001607z^{-9} + 0.001751z^{-10}$$

$$C(Z) = 1 + 0.6791z^{-1} - 1.064z^{-2} - 0.716z^{-3} + 0.2582z^{-4}$$

#### 4.6.2 Discussion of Fit

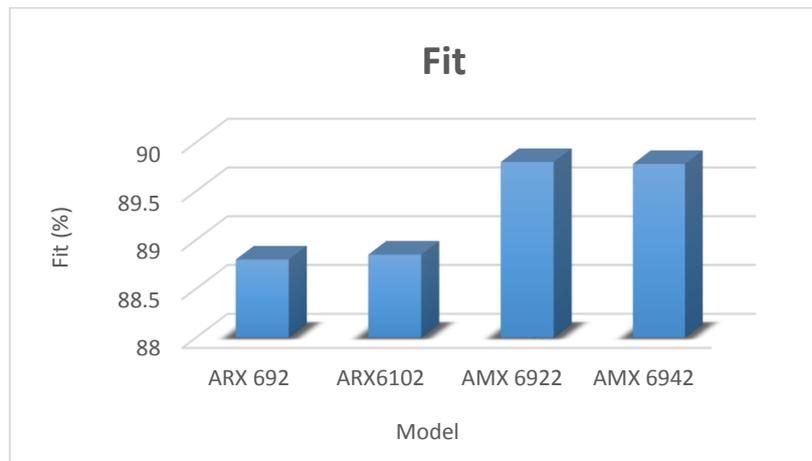


Figure 4.43 Fit Chart for Real Data

Figure 4.43 above shows that the fit value of ARX 692, ARX 6102, AMX 6922 and AMX 6942 by using the real data. From the simulated results, AMX 6922 gives the highest fit value of 89.80% but there is only a slight difference with AMX 6942 which has the fit value of 89.78%. By comparing with ARX 692 and ARX 6102, AMX 6922 proves that the fit value obtain for both model has lower value which are 88.80% and 88.85% respectively.

#### 4.6.3 Discussion of Final Prediction Error

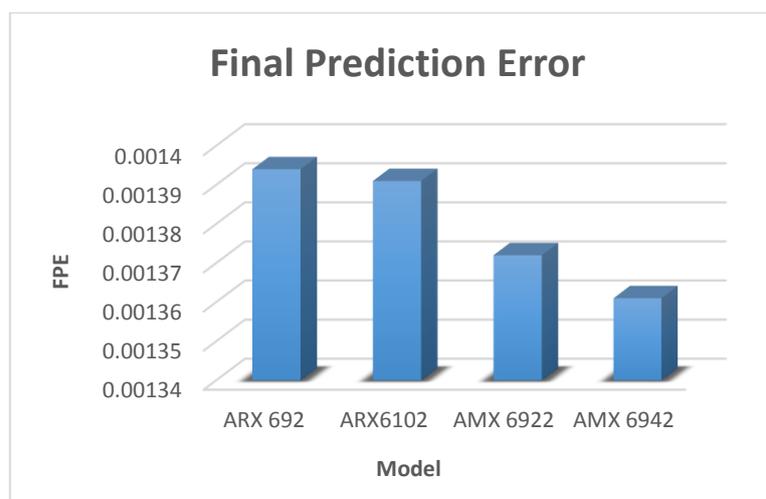


Figure 4.44 FPE Chart for Real Data

From the chart in Figure 4.44 explains that the value of Final Prediction Error (FPE) for ARX 692, ARX 6102, AMX 6922 and AMX 6942. The FPE value shows that AMX 6942 has the smallest value which is 0.001361 compared to AMX 6922 even though the fit value of AMX6922 is better. In terms of ARX 692 and ARX 6102, the simulated results also shows that the FPE value of AMX 6942 still produce the smallest value among the models.

#### 4.6.4 Discussion of Mean Square Error

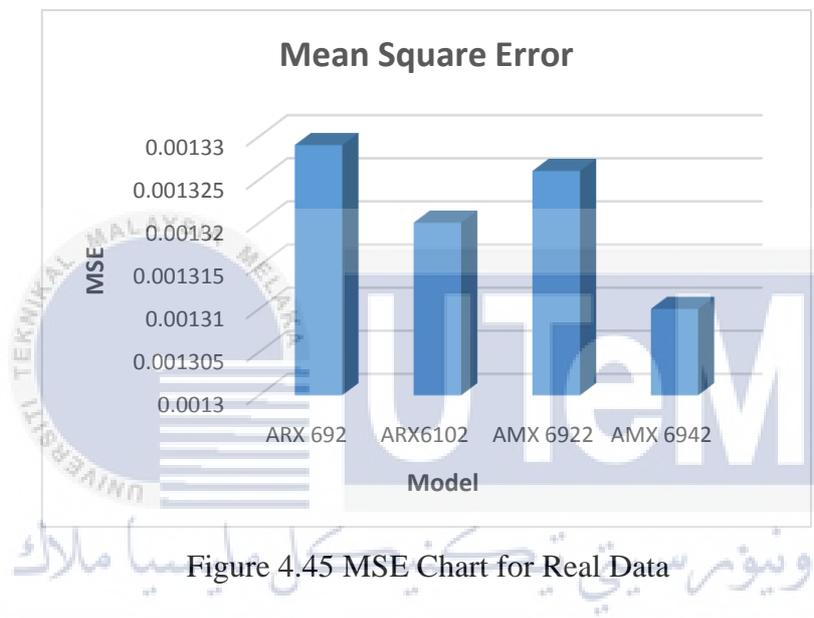


Figure 4.45 MSE Chart for Real Data

The bar chart as in Figure 4.45 represents the value of Mean Square Error (MSE) simulated by using the real data. From the chart above, AMX 6942 has the smallest value of MSE. As for the MSE value for the other models, it has small difference among the models. In conclusion, the best MSE value is AMX 6942 model which is 0.00131.

#### 4.6.5 Summary of Model Properties for Real Data

Table 4.4 as can be seen below shows the summary of model properties for Real Data.

Table 4.4 Summary of ARX and ARMAX Model Properties for Real Data

Model	Fit	FPE	MSE	Discrete-time Coefficient
ARX 692	88.80	0.001394	0.001329	$A(Z) = 1 - 0.9417z^{-1} + 0.003123z^{-2} - 0.04692z^{-3} + 0.08292z^{-4} - 0.01831z^{-5} + 0.03539z^{-6}$ $B(Z) = 0.005686z^{-2} + 0.06431z^{-3} + 0.0633z^{-4} + 0.02115z^{-5} - 0.004842z^{-6} - 0.01424z^{-7} - 0.01309z^{-8} - 0.008236z^{-9} - 0.005861z^{-10}$
ARX 6102	88.85	0.001391	0.001320	$A(Z) = 1 - 0.9338z^{-1} + 0.006018z^{-2} - 0.05206z^{-3} + 0.07261z^{-4} - 0.04662z^{-5} + 0.0603z^{-6}$ $B(Z) = 0.0056z^{-2} + 0.06438z^{-3} + 0.06392z^{-4} + 0.02247z^{-5} - 0.003829z^{-6} - 0.01434z^{-7} - 0.01569z^{-8} - 0.01137z^{-9} - 0.006711z^{-10} - 0.003285z^{-11}$
AMX 6922	89.80	0.001372	0.001326	$A(Z) = 1 - 1.314z^{-1} - 0.1266z^{-2} + 0.418z^{-3} + 0.1005z^{-4} - 0.05769z^{-5} - 0.01712z^{-6}$ $B(Z) = 0.005337z^{-2} + 0.06218z^{-3} + 0.03672z^{-4} - 0.03308z^{-5} - 0.04212z^{-6} - 0.02095z^{-7} - 0.005691z^{-8} + 0.00001946z^{-9} + 0.0003697z^{-10}$ $C(Z) = 1 - 0.4115z^{-1} - 0.5088z^{-2}$
AMX 6942	89.78	0.001361	0.00131	$A(Z) = 1 - 0.2386z^{-1} - 1.656z^{-2} + 0.2635z^{-3} + 0.9324z^{-4} - 0.2371z^{-5} - 0.05859z^{-6}$ $B(Z) = 0.005404z^{-2} + 0.0677z^{-3} + 0.1031z^{-4} - 0.001557z^{-5} - 0.09177z^{-6} - 0.06035z^{-7} - 0.01671z^{-8} - 0.001607z^{-9} + 0.001751z^{-10}$ $C(Z) = 1 + 0.6791z^{-1} - 1.064z^{-2} - 0.716z^{-3} + 0.2582z^{-4}$

#### 4.6.6 Analysis of Implementation of Real Data

The summary results of the implementation of real data is shown in Table 4.4. From the results, AMX model clearly shows that it gives better result in terms of fit, FPE and MSE value. Although, there is not much different of AMX 6922 and AMX 6942 regarding the fit value, but AMX 6942 results proves that the FPE and MSE value are smaller which are 0.001361 and 0.00131 respectively compared to AMX 6922. It is proven that better performance will obtain as there is an additional error term which are  $e(t-3)$  and  $e(t-4)$  that will indicate it has a better performance.

As for ARX model, the results can be concluded that extra input variable added will exhibit the better performance of the model. But theoretically, ARMAX model regularly provide much better performance as this model generally known to capture noise model better. ARX 6102 shows that with additional input variable provide a fit value of 88.85% which a bit higher than ARX 692 that gives 88.8% of percentage value of fit but as for ARMAX model shows it has the smallest error value. Hence, for real data implementation proves that ARMAX model has a better performance compared to ARX model but from previous analysis show that ARX provide better performance. In conclusion, the better performance of both ARX and ARMAX model still depending on the data distribution.

## CHAPTER 5

### CONCLUSION AND RECOMMENDATION

#### 5.1 Conclusions

Based on the project had been done, it can be conclude that the objectives of this study are successfully achieved. The goals of this project is to simulate modelling between ARX and ARMAX model using System Identification. System Identification toolbox was used in MATLAB R2015b software in order to simulate the parametric model given with a different model structure. From the simulation that had been performed, three equation with a true specification model each were compared.

Besides, this study investigate the comparison of the modelling performance of ARX and ARMAX model based on several selected performance indicators. The performance indicators used were fit, final prediction error and mean square error. From the analysis, it was found that the overall performance of the ARX model is better to the ARMAX model in terms of model fitness ability in response of measured data. It is proven that with increasing in input order number in model structure will results in less error obtain. But with the implementation of real data, the results showed that ARMAX model has better performance. Hence, it can be conclude that either ARX or ARMAX has better performance, it is still depending on the data distribution itself.

## 5.2 Recommendations

Since this study proved that with a larger input order number will results in less error could be obtained, different general model such as Box-Jenkins and Output-Error can be used by considering its input order number of the model structure in future studies. By using a different model from this analysis can proved that the findings of the results in this study are reliable. Besides, frequency response, transient response and model residual also can be used as a parameter to monitor the difference. By investigating different type of parameters and models, the outcome of the results will show its differences and it is easier to compare.



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