DYNAMIC STABILITY
ON LATERAL DIRECTIONAL OF AN AIRCRAFT USING AILERON

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A report submitted
in fulfillment of the requirements for the degree of
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DECLARATION

I declare that this project report entitled "Dynamic Stability on Lateral Directional of An Aircraft Using Aileron" is the result of my own work except as cited in the references.

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Date : 6 JUNE 2018

APPROVAL

I hereby declare that I have read this project report and in my opinion this project report is sufficient in terms of scope and quality for the award of degree of Bachelor of Mechanical Engineering (with Honours).

Signature :

Name : PROF. MADYA AHMAD RIVAI

Date : 6 JUNE 2018

DEDICATION

Dedicated to my beloved father and mother,

my friends and family members.

For their encouragement and supports throughout the research.

ABSTRACT

The dynamic stability analysis of the aircraft can be evaluated in two dimensional modes: longitudinal stability analysis and lateral directional stability analysis. This present work focuses on the second mode which is lateral directional. The dynamic stability analysis is carried out by evaluating the behavior of three flight parameter due to aileron deflection. These three flight parameters are sideslip angle ß, roll angle Ø and yaw angle ψ. These three flight parameters describe the behavior of the aircraft in lateral directional and it can be obtained by solving the governing equation of lateral directional. In this present work, a computer code is developed in MATLAB software to solve the governing equation of flight motion using Laplace transformation. The validation was carried out by comparing the results for the lateral directional of aircraft model Learjet 24 with the results provided by Marcello R. Napolitano (2012). After that, the developed computer code in MATLAB is applied to all three types of aircraft models for flight dynamic analysis. These three types of aircraft models are Cessna 182, Cessna 310, and Cessna 620. The developed computer code in MATLAB provides four types of aileron deflection, namely single doublet impulse, multiple doublet impulse, single doublet and multiple doublet. The results show that each aircraft has different response due to the deflection of aileron and the developed computer code represents a useful tool for flight dynamic analysis.

ABSTRAK

Analisis kestabilan dinamik penerbangan dapat dilakukan melalui dua mod dimensi, iaitu analisis kestabilan arah bujur dan arah sisi. Kajian ini memberi tumpuan kepada mod yang kedua, iaitu analisis kestabilan arah sisi. Analisis kestabilan ini dilakukan untuk menilai kelakuan tiga parameter akibat daripada pesongan aileron. Ketiga-tiga parameter penerbangan ini adalah sudut sisi ß, sudut roll Ø dan sudut yaw ψ. Ketiga-tiga parameter penerbangan ini menggambarkan kelakuan sesebuah kapal terbang dari arah sisi dan ia dapat diperolehi dengan menyelesaikan persamaan bagi arah sisi. Dalam kajian ini, kod komputer dibangunkan dalam perisian MATLAB untuk menyelesaikan persamaan gerakan dengan penggunaan transformasi Laplace. Pengesahan telah dilakukan dengan membandingkan keputusan untuk arah model kapal terbang Learjet 24 dengan keputusan yang diperolehi daripada Marcello R. Napolitano (2012). Selepas itu, kod komputer yang dibangunkan di perisian MATLAB digunakan untuk membuat analisis kestabilan dinamik penerbangan bagi tiga jenis model kapal terbang. Ketiga-tiga jenis model kapal terbang ini adalah Cessna 182, Cessna 310, dan Cessna 620. Kod komputer di perisian MATLAB ini menyediakan empat jenis pesongan aileron, iaitu dorongan doublet tunggal, dorongan doublet berganda, doublet tunggal dan doublet berganda. Keputusan bagi kajian ini menunjukkan bahawa setiap kapal terbang mempunyai tindak balas yang berbeza terhadap pesongan aileron dan kod komputer yang dibangunkan merupakan alat yang amat berguna untuk analisis kestabilan dinamik penerbangan.

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I would like to take this opportunity to express my sincere appreciation to my project supervisor, Prof. Madya Ahmad Rivai from Faculty of Mechanical Engineering, Universiti Teknikal Malaysia Melaka (UTeM) for giving me this chance to work together, giving me encouragement, supports and his persistently guidance towards the completion of this project.

Besides that, I would like to thank my friends and colleagues who provided me with their support and assistance throughout my journey. Lastly, special thanks to all my family members for their endless encouragement and supports in completing this project. The product of this project work would not be possible without all of them.

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LIST OF ABBREVIATIONS

D_1	Denominator
AOA	Angle of Attack
CG	Center of Gravity
FLC	Fuzzy Logic Controller
LQR	Linear Quadratic Regulator
Num	Numerator
STFLC	Self-Tuning Fuzzy Logic Controller

LIST OF SYMBOLS

a	=	Aileron
AR	=	Aspect Ratio
b	=	Wingspan
c	=	Chord
C_{l_p}	=	The Variation of Rolling Moment Coefficient with Roll Rate.
C_{l_r}	=	The Variation of Rolling Moment Coefficient with Yaw Rate.
C_{l_β}	=	The Variation of Rolling Moment Coefficient with Sideslip Angle.
$C_{l_{\delta_A}}$	=	The Variation of Rolling Moment Coefficient with Aileron Deflection.
$C_{l_{\delta_R}}$	=	The Variation of Rolling Moment Coefficient with Rudder Deflection.
C_{n_p}	=	The Variation of Yawing Moment Coefficient with Roll Rate.
C_{n_r}	=	The Variation of Yawing Moment Coefficient with Yaw Rate.
C_{nT_β}	=	The Variation of Yawing Moment Coefficient with Thrust
C_{n_β}	=	The Variation of Yawing Moment Coefficient with Sideslip Angle.
$C_{n_{\delta_A}}$	=	The Variation of Yawing Moment Coefficient with Aileron Deflection.
$C_{n_{\delta_R}}$	=	The Variation of Yawing Moment Coefficient with Rudder Deflection.
C_{Y_p}	=	The Variation of Side Force Coefficient with Roll Rate.

C_{Y_r} = The Variation of Side Force Coefficient with Yaw Rate.

C_{Y_β} = The Variation of Side Force Coefficient with Sideslip Angle.

$C_{Y_{\delta_A}}$ = The Variation of Side Force Coefficient with Aileron Deflection.

$C_{Y_{\delta_R}}$ = The Variation of Side Force Coefficient with Rudder Deflection.

g = Acceleration of Gravity

h = Altitude

I = Characteristic Length

I_{XXB} = Moment of Inertia x-axis

I_{XZB} = Product of Inertia x and z-axis

I_{YYB} = Moment of Inertia y-axis

I_{ZZB} = Moment of Inertia z-axis

L = Sum of the Component of Moment in x axes

L_p = Rolling Moment with Roll Rate.

L_r = Rolling Moment with Yaw Rate.

L_β = Rolling Moment with Sideslip Angle.

$L_{\delta A}$ = Rolling Moment with Aileron Deflection.

$L_{\delta R}$ = Rolling Moment with Rudder Deflection.

M = Sum of the Component of Moment in y axes

M = Mach Number

m = Mass of Aircraft

N = Sum of the Component of Moment in z axes

N_r = Yawing Moment with Yaw Rate.

$N_{T\beta}$ = Yawing Moment with Thrust

N_{β}	=	Yawing Moment with Sideslip Angle.
$N_{\delta A}$	=	Yawing Moment with Aileron Deflection.
$N_{\delta R}$	=	Yawing Moment with Rudder Deflection.
p	=	Component of Rotational Velocity about x axes
q	=	Component of Rotational Velocity about y axes
Q	=	Dynamic Pressure
q_1	=	Dynamic Pressure
r	=	Component of Rotational Velocity about z axes
S	=	Reference Area
u	=	Instantaneous Linear Velocity in x axes
U	=	Axial Velocity
U_1	=	Air Speed
v	=	Instantaneous Linear Velocity in y axes
v	=	Sideslip Velocity
V	=	Total Velocity Vector
w	=	Instantaneous Linear Velocity in z axes
X	=	Sum of the Component of Forces in x axes
x_{cg}	=	Location of CG-%MAC
Y	=	Sum of the Component of Forces in y axes
y	=	Length from Longitudinal Axis
Y_p	=	Side Force with Roll Rate.
Y_r	=	Side Force with Yaw Rate.
Y_{β}	=	Side Force with Sideslip Angle.

$Y_{\delta A}$	=	Side Force with Aileron Deflection.
$Y_{\delta R}$	=	Side Force with Rudder Deflection.
Z	=	Sum of the Component of Forces in z axes
β	=	Sideslip Angle
Γ	=	Dihedral Angle
C_l	=	Coefficient of Rolling Moment
C_{Law}	=	Coefficient of Lift due to Wing
Cm	=	Coefficient of Yawing Moment
Cn	=	Coefficient of Pitching Moment
Cx	=	Coefficient of Force in x axes
Cy	=	Coefficient of Force in y axes
Cz	=	Coefficient of Force in z axes
L_F	=	Lift Generated due to Fin
N_p	=	Yawing Moment with Roll Rate.
α	=	Angle of Attack
δ	=	Deflection
δ_A	=	Deflection of Aileron
τ	=	Taper Ratio
ψ	=	Yawing Angle
ϕ	=	Roll Angle

CHAPTER 1

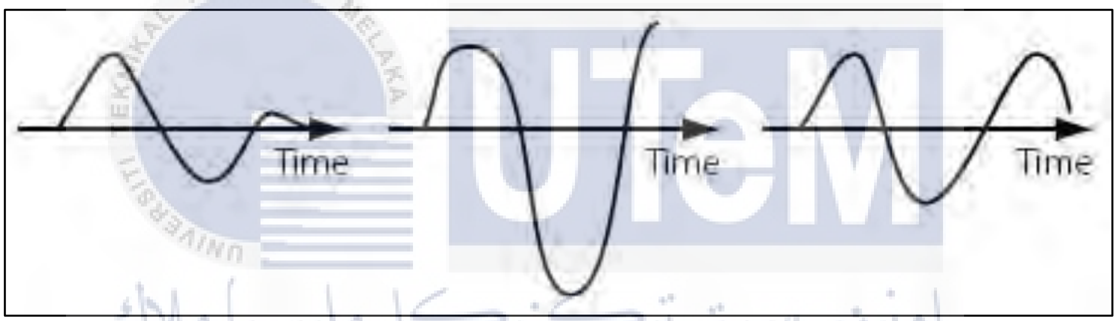
INTRODUCTION

1.1 Background of Study

After the first flight made by Wilbur and Orville Wright in 1903, there was a rapid development in aeronautical and the progress and study made in the following decade was impressive. However, the flying qualities of their aircraft that time were always less than satisfactory due to the problems of stability and control. Many researchers like Bryan and Lanchester were studying the problems of stability and control at that time and they managed to come out with mathematical method to describe the flight dynamic and the general equations of motion of an aircraft with six degrees of freedom to describe the aircraft motion which it is in the same form as they are known today. (Anderson & Eberhardt, 2001)

Stability and control are study on how well an aircraft flies and how easily the aircraft can be controlled. Stability is the ability of an aircraft to return to a previous condition if it is upset by disturbance. The disturbance can be generated by the pilot' s action or atmospheric phenomena such as turbulence or a gust of air. An aircraft must have a good stability so that the pilot does not become tired by keep having to control the aircraft in order to make it stable. Whereas control is the ability to command the aircraft to perform a specific maneuver or to maintain or change its conditions. To achieve equilibrium flight and perform maneuvers, the aircraft must have aerodynamic and propulsive controls. (Nelson, 1998)

Most of the aircrafts are built to be very stable but some like fighter jets are built to be unstable and fly without the help of computer controlled by fly-by-wire systems. Generally, there are two types of stability which are static stability and dynamic stability. Static stability is the tendency of an aircraft to return to its original flight attitude when it is disturbed. Dynamic stability deals with the motion caused by the disturbance changes with time. (Colin Cutler, 2015)



(c) Positive Static and Positive Dynamic Stability
(a) Positive Static and Negative Dynamic Stability
(b) Positive Static and Negative Dynamic Stability

Figure 1.1 Type of Dynamic Stability (Anderson & Eberhardt, 2001)

The stability and control of an aircraft is important in shaping its dynamic behavior. In order to obtain good flying qualities, it is necessary to achieve good stability characteristics. Nowadays, modern flight dynamics is not only focused with the dynamics, stability and control of the aircraft, it is also focused on complex interaction between the aircraft and the flight control system. The flight control system includes the motion sensors, computers in flight control applications and other control hardware. Therefore, it is convenient to treat the aircraft as a system component in the analysis of stability and control.

Figure 1.2 shows the relationship between control and response as a function of flight condition and the influence of atmospheric disturbances. It also describes about the basic input-output relationship on which the flying and handling of aircraft depend is significant. Central of this is a mathematical model of the aircraft which usually involves the equation of motion of the aircraft. This mathematical model describes about the complete response to control which is in terms of displacement, velocity and acceleration variables. For the flight condition, it provides the condition under the parameters such as altitude, Mach number, geometry of the aircraft, mass and trim state (Cook, 2013).

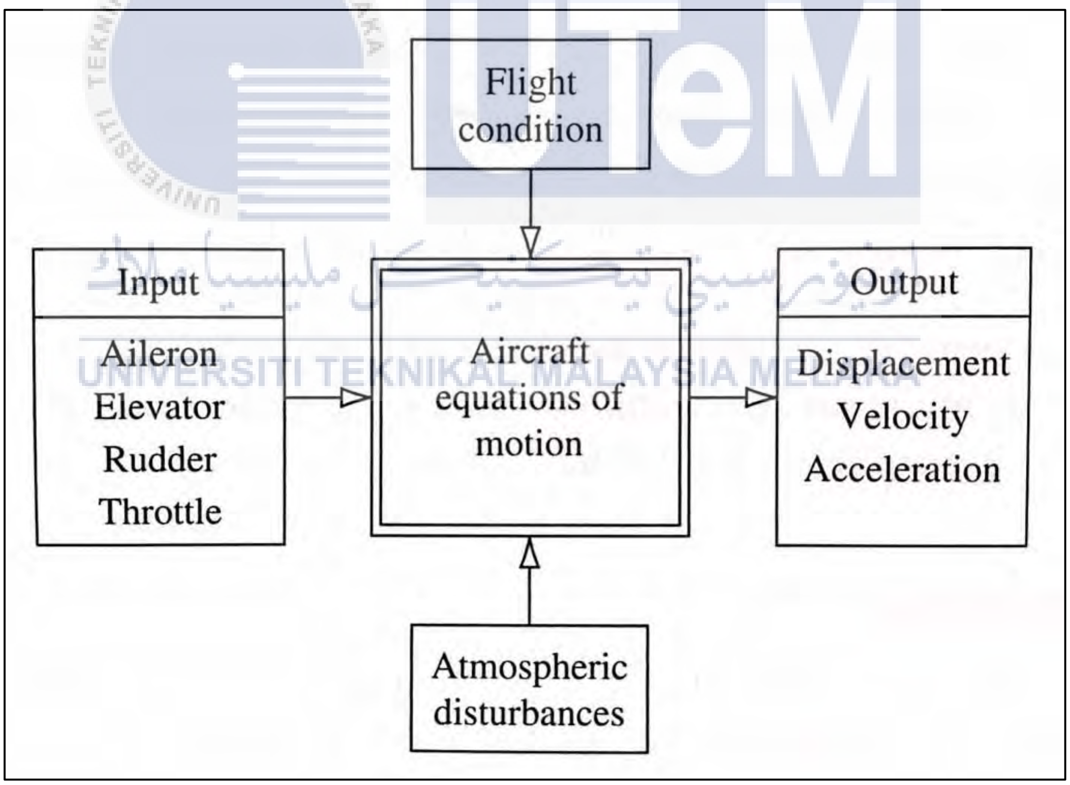


Figure 1.2 Basic Control-Response Relationship (Cook, 2013)

Through this project, a study on the dynamic stability behavior of aircraft, namely in predicting the aircraft response due to the aileron deflection in lateral directional will be carried out. A computer code of control system is developed in MATLAB using the control law and the aerodynamic data that obtained, and the flight dynamic analysis in lateral directional is studied with the function of deflection of ailerons.

1.2 Problem Statement

A poor stability and control flight behavior may lead to an aviation accident. According to Chen, Chen, Xie, & Yuan (2017), loss of control is the greatest contributors to aircraft accidents worldwide. This is the reason why stability and control are very important in aircraft and one of the biggest improvements in aircraft over the last few decades has been in the area of stability and control. Control surfaces such as rudder, elevator and aileron are the components that used to control the flight behavior. In this project, an analysis and study on the behavior of the dynamic stability on rolling due to the aileron deflection of an aircraft. In the context of this project, there are some questions included:

a) What is the corresponding rolling and yawing moment coefficient with respect to ailerons deflection?

b) What is the behavior of the dynamic stability on lateral directional due to the aileron deflection of an aircraft?

c) What is the relationship between the rolling and yawing moment and control law on an aircraft?

1.3 Objectives

The objectives of this project are as follows:

a) To determine the variation of rolling and yawing moment with respect to angle deflection of aileron.
b) To study the behavior of the dynamic stability on lateral directional due to the aileron deflection of an aircraft.

1.4 Scope of Project

In this project, a study on the dynamic stability behavior of aircraft, namely in predicting the aircraft response due to the aileron deflection in lateral directional will be carried out. By referring to Marcello R. Napolitano (2012), the necessary input data for the flight dynamic analysis is collected. Next, a computer code of control system is developed in MATLAB using the control law and the aerodynamic data that obtained, and the flight dynamic analysis in lateral directional is studied with the function of deflection of ailerons. The validation is carried out by comparing the results for the lateral directional of aircraft model Learjet 24 with the results provided by Marcello R. Napolitano (2012). Through this project, we will find out the behavior of the dynamic stability due to the aileron deflection of three types aircraft models which are Cessna 182, Cessna 310, and Cessna 620.

CHAPTER 2

THEORY

2.1 Basic Aerodynamics of Aircraft

Aerodynamics is the study of the dynamic properties of moving air and the interaction of air with moving bodies. Anything that moves through the air is related to aerodynamics. the field of aeronautics, the principles of aerodynamics is important and applies in designing and operating an aircraft that fly in the Earth's atmosphere. (Sandra May, 2015)

According to Vilnius (2017), there are four forces acting on an aircraft in flight which are gravity, lift, thrust and drag. These forces are in equilibrium when the aircraft is at constant level or speed.

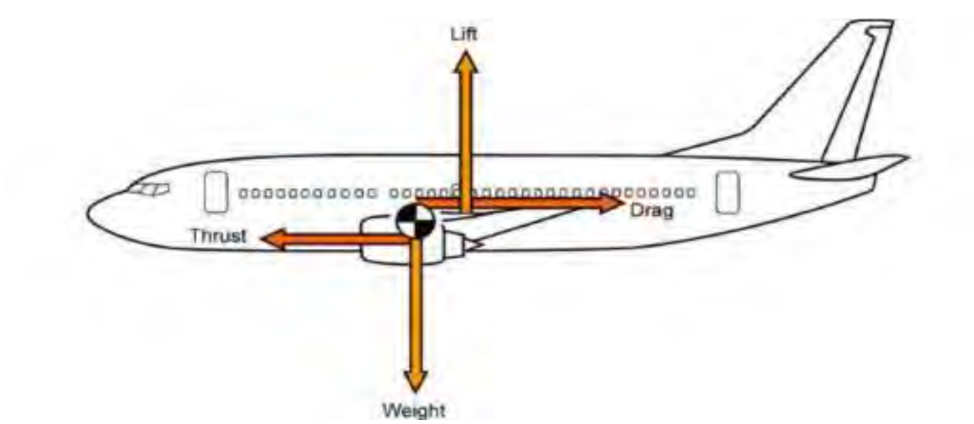


Figure 2.1 The Forces Acting in Flight (Vilnius, 2017)

Drag is the force that opposes an aircraft's motion through the air and tries to slow an aircraft down. It results from the difference in velocity between the aircraft and the air. In most cases such as automobile, drag is undesirable and to be minimized because the drag requires power to overcome it. Drag is depending on shape of aircraft, its speed, the surface roughness, the density of the air etc.

Trust is the force that moves the aircraft forward. To overcome drag forces of aircraft, trust must be generated by a motor- driven propeller or a jet engine. When the aircraft is flying with a constant speed, this shows that the force of the trust is just enough to encounter the aerodynamic drag.

The path over the upper wing's curve is longer than the path over lower wing's curve. As the air flows over the upper surface of the wing, its velocity increase and its pressure decrease according to the Bernoulli's Principle which stated that an increase in the velocity of any fluid will always accompanied by a decrease in pressure. Thus, an area with lower pressure is created. There is also an area on the lower surface of the wing with higher pressure. This higher pressure tends to push the wing upward from lower surface to upper surface. The difference in pressure between upper and lower surfaces of the wing is called Lift.

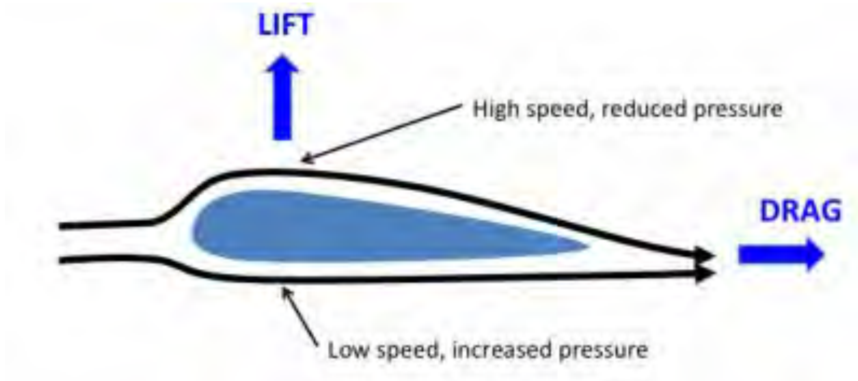


Figure 2.2 The Flow of Air Around the Airfoil (Jim Lucas, 2014)

Weight is a downward force that acts on an aircraft due to the interaction between the weight of the aircraft and the Earth gravity. The weight of an aircraft is the result of the force of gravity acting on the mass of aircraft. It is depending with number of passengers, cargo and the amount of fuel loaded on the aircraft.

2.2 Axes of an Aircraft

Before developing a mathematical model of the aircraft, it is necessary to comprise a mathematical framework in which the equation of motion can be built in a consistent way. An aircraft in flight is controlled by three axes of rotation. These axes of rotation are the lateral, vertical and longitudinal. All these axes of rotation meet at the center of gravity (CG) of the aircraft. The CG is the point where the aircraft would in balanced if the aircraft is lift by using an imaginary string that attached to that CG point.

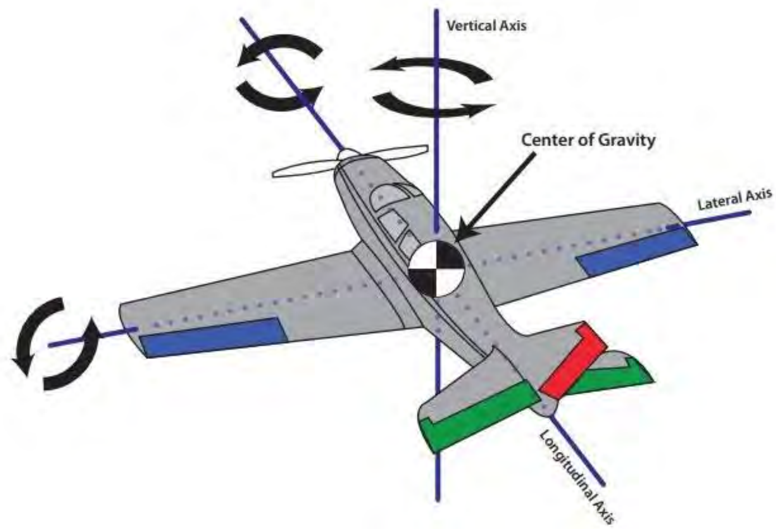


Figure 2.3 The Three Axes of an Aircraft (NASA, 2010)

The lateral axis of rotation is the axis that runs from wingtip to another wingtip and it stays fixed when the aircraft undergoes pitching moment whereas the vertical axis of rotation is the axis that runs vertically through the CG point. The vertical axis stays fixed when the aircraft yaws. The last one is the longitudinal axis which runs from nose to tail and it stays fixed when the aircraft rolls. (Andrew Hartley, 2013)

According to Caughey (2011), the standard notation for describing the aerodynamic forces and moments that acting on a aircraft in flight are shown in Figure 2.4.

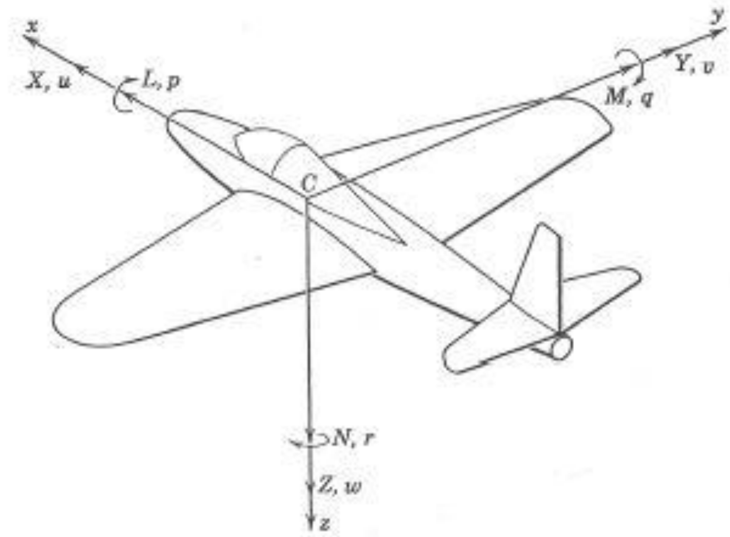


Figure 2.4 Standard Notation for Aerodynamic Forces and Moments of Aircraft (Caughey, 2011)

The variables x, y and z are the coordinates with the origin located at center of gravity of the aircraft. The x-axis is in the symmetry plane of the aircraft which points towards the nose of the aircraft. The z-axis also lies on the symmetry plane and it is perpendicular to the x-axis with pointing downwards. The y-axis is pointing out the right wing which completes a right-handed orthogonal system.

The variables X, Y, and Z stand for the sum of the components of aerodynamic, thrust and weight forces of the aircraft in the directions of the x, y and z axes.

The variables L, M, and N represent the sum of components of aerodynamic moment about x, y, and z axes respectively.

The variables p, q, and r stand for the instantaneous components of the rotational velocity about x, y, and z axes respectively.

The variables u, v and w represent the instantaneous linear velocity in the directions of the x, y and z axes.

The aerodynamic forces that are defined in terms of the flight dynamic pressure, Q and the reference area, S. The followings are the formula for those aerodynamic forces:

$$\text{Axial Force, } X = C_x QS \tag{2.1}$$

$$\text{Side Force, } Y = C_y QS \tag{2.2}$$

$$\text{Normal Force, } Z = C_z QS \tag{2.3}$$

The aerodynamic moment also defined in terms of the flight dynamic pressure, Q and the reference area, S as shown:

$$\text{Rolling Moment, } L = C_l QSl \tag{2.4}$$

$$\text{Pitching Moment, } M = C_m QSl \tag{2.5}$$

$$\text{Yawing Moment, } N = C_n QSl \tag{2.6}$$

where S is the reference area which is taken as the wing area;

l is the characteristic length which is taken as the wingspan for rolling and yawing moment and the mean chord for pitching moment;

$C_{x,}, C_y, C_z, C_l, C_m, and\ C_n$ are the aerodynamic coefficient of the aircraft.

2.3 Aerodynamic Control

An aircraft is designed with certain fixed and movable control surfaces for the purpose of stability and control during takeoff, flight and landing. Each of them is designed to conduct a specific function in the flight of the aircraft. It allows a pilot to control and adjust the aircraft's flight attitude. They are divided into two major groups which are primary and secondary control surfaces. The primary control surfaces of an aircraft include ailerons, elevators and rudder. The secondary control surfaces of an aircraft include trim tabs, spoilers, flaps and slats (Kroes & Rardon, 1988). The aircraft can be controlled by providing an incremental lift on the control surfaces. This incremental lift force is produced by deflecting the control surfaces and it creates a moment about the aircraft's center of gravity as the control surfaces are at some distance from the center of gravity. (Nelson, 1998)

Cook (2013) defines that a positive control action by the pilot gives rise to a positive reponse of the aircraft, whereas, a positive displacement of the control surface contribute to a negative reponse of the aircraft.

In this sub-topic, only primary control surfaces are discussed as it acts as the main control surfaces of an aircraft.

2.3.1 Ailerons

According to Senson & Ritter (2011), ailerons are movable small hinged surfaces attached to the trailing edge of both wing of an aircraft as shown in the Figure 2.5. It is used to perform a rolling motion for an aircraft during the flight. Rotation about longitudinal axis which is the axis in the direction of the flight from nose to tail is called roll.

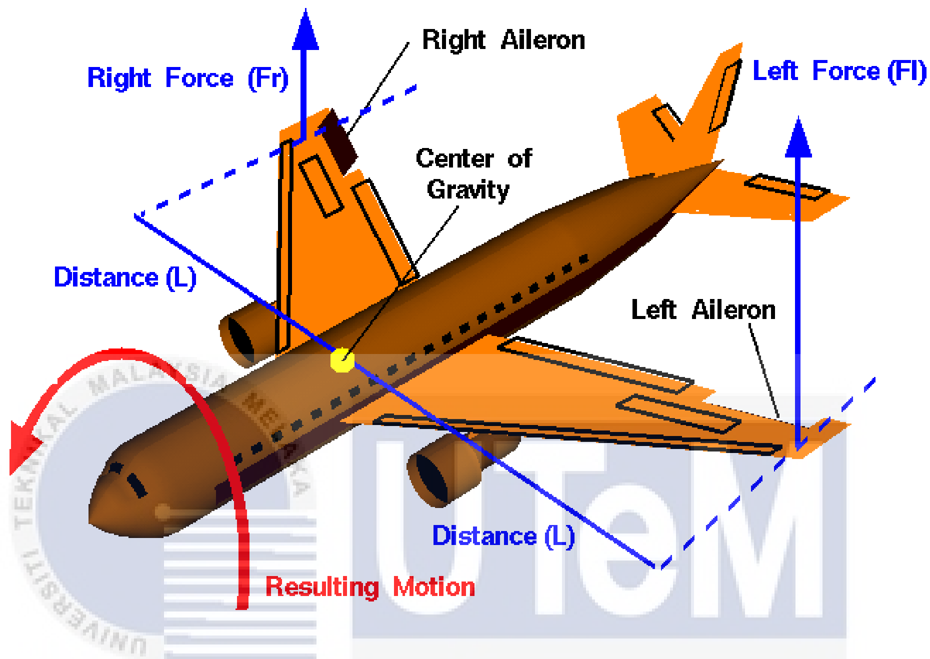


Figure 2.5 Location of the Ailerons (NASA, 2010)

Cook (2013) defines that a positive rolling motion is when the pilot push a right positive force on the stick and generate a positive stick displacement. This causes the right aileron to deflect upward whereas the left aileron deflects downward. Thus, the right wing is being pull down by the decreased lift on the right wing and a positive rolling is generated.

The ailerons move in opposite direction from each other to generate a roll motion. As the pilots turn the control yoke to left in the cockpit, the aileron on the left wing moves and deflects upwards, raising the trailing edge, decreasing the angle of attack (AOA) and the amount of lift produced by the left wing while the aileron on the right wing deflects downwards which increases the amount of lift produced on the right wing. The excess lift on the right wing tends to lift it upward while the decreased lift on the left wing allows gravity to pull it downward. This result a rotation about the longitudinal axis called roll. For both wings, the lift force produced by the ailerons is applied at the aerodynamic center of the aircraft which is a distance from the center of gravity of the aircraft. If the pilots reverse the ailerons deflection for both wings where the left aileron deflects downward, and right aileron deflects upward, the aircraft will roll in the opposite direction.

Aileron control in an aircraft during the flight is complicated by an effect called adverse yaw. Creating aerodynamic lift always creating drag as well. An aileron that deflects downward creates more drag than the aileron on the opposite wing that deflects upward the same degree. This results the turning of the aircraft toward the side on which the downward-deflecting aileron is located by the drag of the downward-deflecting aileron. To overcome the effect of adverse yaw, the ailerons of the aircraft is rigged for differential movement. The differential control makes the up-deflecting aileron to more a larger distance compared to down-deflecting aileron. This is to balance the drag between the ailerons and avoid the adverse yaw.

2.3.2 Elevator

According to Senson & Ritter (2011), elevator is a movable horizontal surface that connected to trailing edge of the horizontal stabilizer in most of the aircraft as shown in the Figure 2.6. It is used to perform a pitching motion for an aircraft during the flight. Rotation about lateral axis which is the axis that perpendicular to the direction of the flight from wingtip to wingtip is called pitch.

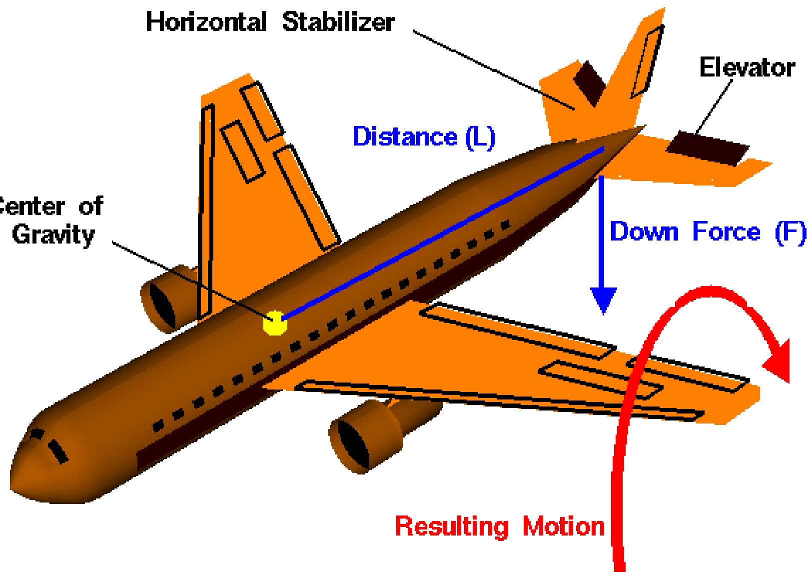


Figure 2.6 Location of the Elevator (NASA, 2010)

A positive pitching moment is such that the pilot pull force on the stick and make a positive aft stick displacement. This causes the elevator trailing edge to deflect upward and generate a downward lift force. Thus, the nose of the aircraft will be pitch upward. (Cook, 2013)

The elevator is designed as it is aligned with the horizontal stabilizer to minimize the drag created. In the cockpit, the pilots can push the control stick or yoke forward to make the elevator to deflect downward, effectively deflects the stabilizer's trailing edge downward, increasing angle of attack (AOA) and finally increase the lift produced. This will cause a rotation motion about the lateral axes which is the pitch motion.

The elevator is used to control the attitude of the nose of the aircraft and the angle of attack of the wing which causes the aircraft to climb or to dive. The downward deflection of elevator tends to pitch the nose of the aircraft downward. In the other hand, pulling the control stick or yoke backward will generate opposite result which is pitch the nose of the aircraft upward in the perspective of pilot.

2.3.3 Rudder

According to Senson & Ritter (2011), the rudder is a vertical movable surface that attached to the trailing edge of the vertical stabilizer by hinges as shown in Figure 2.7. As the rudder deflects, it varies the amount of side force generated by the vertical stabilizer and it is used to control the yawing motion of the aircraft. Rotation about the vertical axis which is the axis that perpendicular to the longitudinal and lateral axis is called yaw.

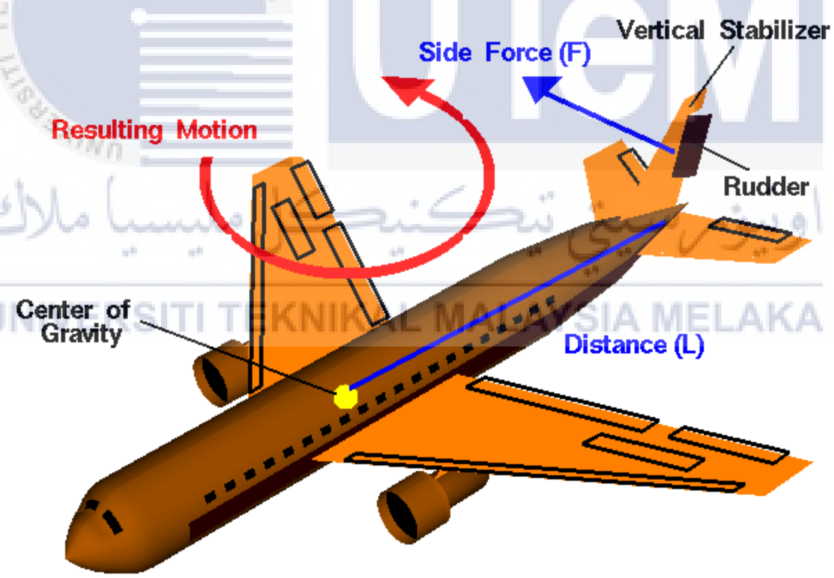


Figure 2.7 Location of the Rudder (NASA, 2010)

A positive yaw happens when the pilot push force on the right rudder pedal and generate a positive displacement of the rudder bar. This makes the rudder trailing edge to deflect to the right of the aircraft. Therefore, the nose of the aircraft will yaw to the right and generate a positive response. (Cook, 2013)

In the cockpit, pilots control the movement of the rudder by pressing the pedal by the feet of the pilots. When the right pedal is pressed, the rudder will swing to the right, and increases the dynamic air pressure on its right side. The increased in air pressure tends to push the tail of the aircraft to the left and causes the nose of the aircraft to turn to the right.

Although the function of the rudder is to turn the nose of the aircraft, but it must be pointed out that the aircraft cannot make a nice turn by using the rudder itself. Based on Newton's first law of motion, it states that a moving body tends to continue moving in a straight line unless some outside force changes its direction. When an aircraft is in flight and its rudder is deflected, the aircraft will turn or yaw, but it will continue to move in the same direction since there is no external force changes its direction. Therefore, the aircraft will experience sideslip. (Kroes & Rardon, 1988)

In reality, the ailerons or spoilers are used to turn the aircraft by banking the aircraft to one side in order to have nice turn without sideslip. The banking of the aircraft creates an unbalanced side force of the wing lift force which causes the aircraft to curve its flight path. The rudder is to ensure that the aircraft is properly aligned to the curved flight during the maneuver. (NASA, 2010)

2.4 Stability and Control

Flying and handling qualities are depending on each other substantially. They are usually described in terms of the stability and control of an aircraft. Stability is the natural tendency of an object to maintain and back to its original position. The desired position for an aircraft is straight and level flight. For recreational aircraft, it is designed to be inherently stable whereas for fighter aircraft, it is designed to be inherently unstable. (Senson & Ritter, 2011)

According to Caughey (2011), control is about dealing with the issue whether the aerodynamic controls are enough to trim the aircraft in order to produce an equilibrium state. It is related to flying qualities. The control must be in maintaining the aircraft in desired equilibrium states and does not make the pilot overly tire.

2.4.1 Static Stability

There are three types of static stability as shown in the Figure 2.8. The three types are statically stable, unstable and neutral.

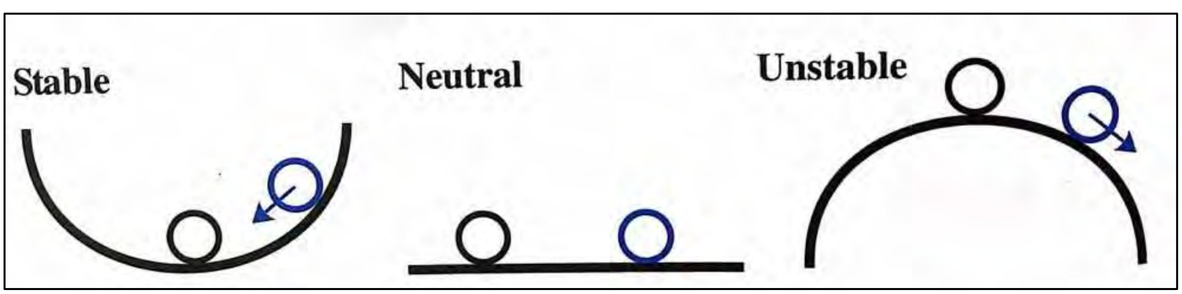


Figure 2.8 Type of Static Stability (Anderson & Eberhardt, 2001)

For an aircraft, static stability means that the tendency to return to its equilibrium conditions if there is some disturbance from equilibrium. If an aircraft has the tendency to back to equilibrium after being disturbed by a gust of air or some other perturbation, it is said to be statically stable. If the aircraft has a tendency to move in the direction of disturbance, it is unstable. If the aircraft has neither the tendency to back nor to continue in the direction of the disturbance after subjected to a disturbance, it is neutral in static stability. (Kroes & Rardon, 1988)

2.4.2 Dynamic Stability

Referring to Kroes & Rardon (1988), dynamic stability is dealing with the resulting motion with time which is different from static stability. When an aircraft is disturbed by a gust of air or some others disturbance from its equilibrium, the time history of the resulting motion represents the dynamic stability of the aircraft. It describes how the motion caused by a disturbance changes with time.

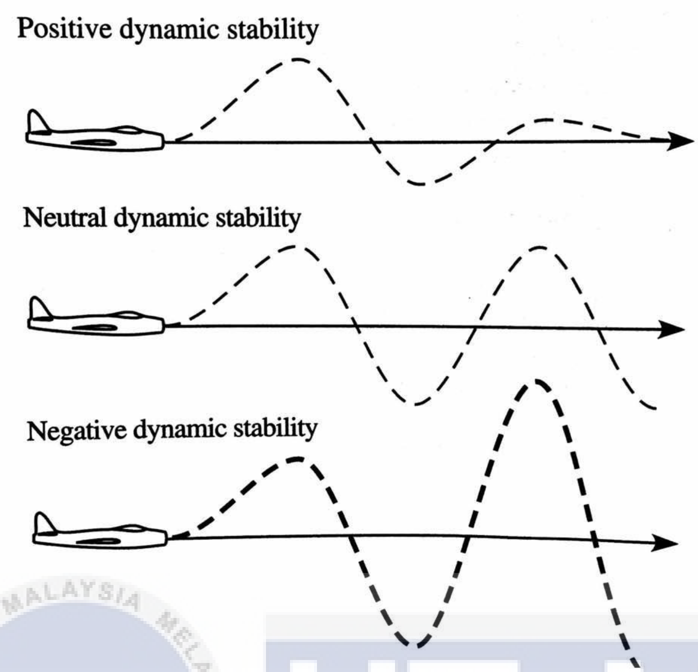


Figure 2.9 Type of Dynamic Stability (Anderson & Eberhardt, 2001)

There are three types of dynamic stability for longitudinal stability of an aircraft which shown in Figure 2.9. The first path shows a positive dynamic stability. When an aircraft is pitched up, there was a restoring force which help the aircraft to back to its equilibrium condition and it is called statically stable. The aircraft tends to oscillate about its original altitude and the oscillations become smaller and smaller in magnitude.

The second path is a neutral dynamic stability. For this case, the aircraft is said to be statically stable as there is a restoring force. However, the amplitude of the oscillation does not decrease with time.

The third path in the Figure 2.9 shows a negative dynamic stability. It is also statically stable because there is a restoring force. But for this path, the amplitude of the oscillations increases with time. (Anderson & Eberhardt, 2001)

An aircraft that has static stability does not necessarily guarantee it has dynamic stability. However, if an aircraft has positive dynamic stability, it implies that it has also static stability as well. (Kroes & Rardon, 1988)

2.4.3 Longitudinal Stability

The stability of an aircraft about the lateral axis is called longitudinal stability. It is the tendency for an aircraft at a specific pitch angle to back to its attitude when it is in equilibrium. If an aircraft does not have longitudinal stability, it will oscillate or pitch up and down about the lateral axis when affected by disturbance.

In determining the longitudinal stability of an aircraft, it is important to see the center of gravity with respect to the center of lift as shown in the Figure 2.10. If the center of life is directly coincidence with the center of gravity, it shows that the aircraft in flight has neutral longitudinal stability which will generate no inherent pitch moments around the center of gravity.

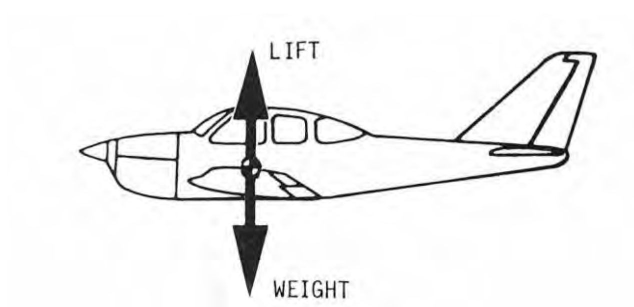


Figure 2.10 Neutral Longitudinal Stability (Kroes & Rardon, 1988)

If an aircraft with the center of lift located at in front of the center of gravity, it will negative stability and generate an undesirable pitching moment during the flight. This happens when the aircraft is loaded and causes the center of gravity shifted behind the center of lift. There is an increase in the amplitude of the pitching moment when get disturbed.

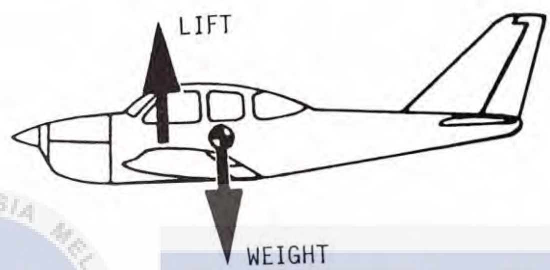


Figure 2.11 Negative Longitudinal Stability (Kroes & Rardon, 1988)

Figure 2.12 shows an aircraft with its center of lift located behind the center of gravity. The aircraft is no longer in balanced, but it is stable because there is a restoring torque that generated by the tail. Therefore, the aircraft will pitch back toward its balanced condition and continue straight and level flight.

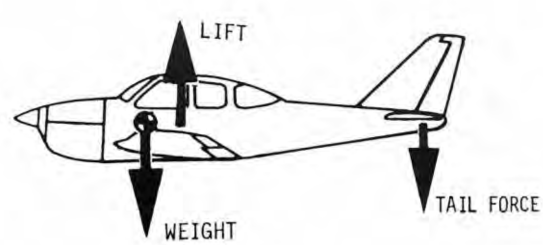


Figure 2.12 Positive Longitudinal Stability (Kroes & Rardon, 1988)

2.4.4 Lateral Stability

The stability of an aircraft around the longitudinal axis or the roll axis is the lateral stability. An aircraft with lateral stability will tends to back to a wing-level equilibrium in the roll sense after being disturbed from a level attitude by some disturbance such as turbulent air. (Kroes & Rardon, 1988)

Senson and Ritter (2011) stated that although there are many others parameter that will contribute to the lateral stability, but the wing dihedral is the most visible and easiest parameter to tune the degree of lateral stability to a desired level. Dihedral is the lateral angle of wing with respect to the horizontal plane as shown in the Figure 2.13.

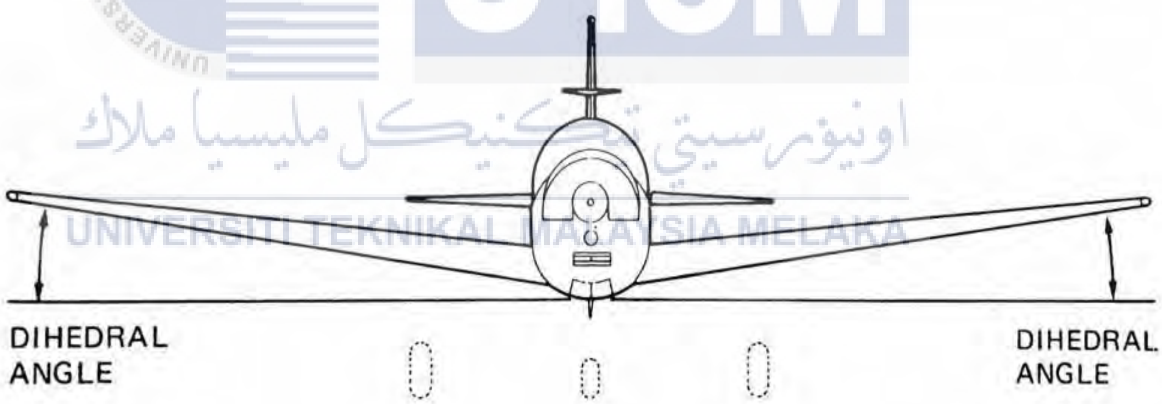


Figure 2.13 Dihedral of Aircraft Wings (Kroes & Rardon, 1988)

The purpose of positive dihedral of an aircraft is to provide lateral stability to the aircraft. The aircraft will start to slide slip with sideslip velocity, v after experience a small lateral disturbance in roll. This is when the stabilizing effect of the dihedral occurs as illustrated in

Figure 2.14. Consider an aircraft with dihedral angle of Γ. The sideslip velocity has a component v^1 that resolved perpendicular to the plane of the wing.

$$v^1 = v \sin \Gamma$$

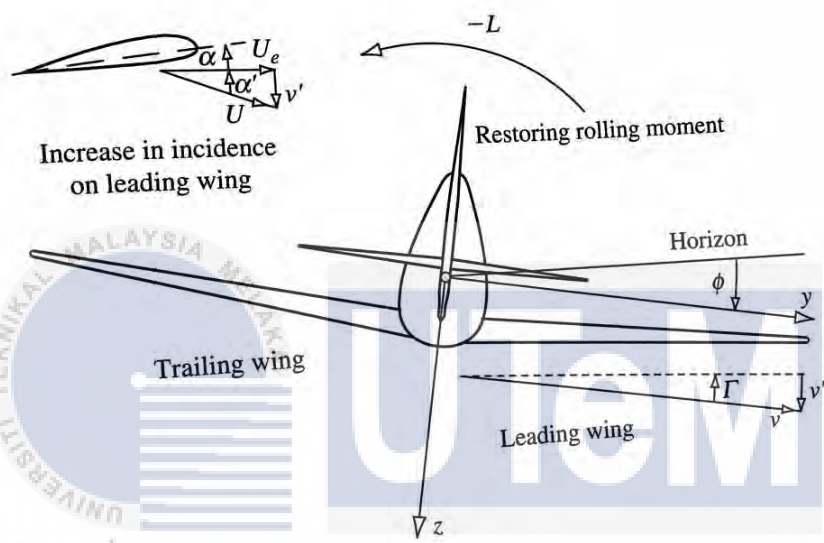


Figure 2.14 Dihedral Effect on Lateral Stability (Senson & Ritter, 2011)

The small component v^1 adds together with the axial velocity U^e to increase the angle of attack of the wing by α^1. If the aircraft roll to the right, the tip of the right wing moves downward, the increase in the angle of attack on the right wing will lead to an increase in lift. At the same time, the left-wing experiences decrease in lift because the angle with the horizontal plan increases. This causes more lift on the right wing and less lift from the left wing and generates a restoring rolling moment.

The dihedral effect is then to say that it creates a negative rolling moment as a restoring moment and make the aircraft to back to its zero-sideslip wing level attitude. Therefore, in order to be laterally stable, an aircraft needs to fulfill the condition that the rolling moment resulting from a positive disturbance in roll must be negative with its mathematical terms:

$$\frac{dC_l}{d\emptyset} < 0 \tag{2.7}$$

where C_l is the rolling moment coefficient. (Senson & Ritter, 2011)

2.4.5 Directional Stability

The stability of an aircraft around the vertical axis is called directional stability. The aircraft that is directionally stable will return to its straight flight path after yawed to one side or the other. (Kroes & Rardon, 1988)

The aircraft is required to fly with zero sideslip angle in the yaw sense. Although there are many others parameter which contribute to the directional stability, but the fin is the most visible component to the directional stability. Too much of directional stability will cause an aircraft hard to maneuver directionally.

If an aircraft is having a positive sideslip disturbance as shown in Figure 2.15, the sideslip velocity, v combines with the axial velocity component U and results a positive sideslip angle, β. This mean the nose of the aircraft swing to the left about the total velocity vector, V. A positive sideslip angle equivalent to a negative yaw angle.

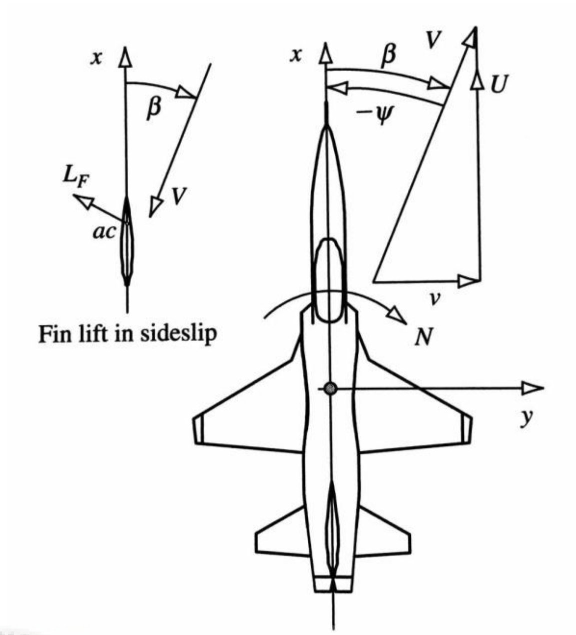


Figure 2.15 Directional Stability (Senson & Ritter, 2011)

In the disturbance, the fin on the upper portion of the tail section is at a non-zero angle of attack which is equivalent to the sideslip angle, β. The fin is then generated lift, L_F which direct to the left side of the aircraft and creating a positive yawing moment, N. This yawing moment generated is stabilizing the aircraft as it causes the aircraft to yaw to the right until the sideslip angle become zero. In order to be directionally stable for an aircraft, the condition of the aircraft is established as in the following mathematical terms:

$$\frac{dC_n}{d\psi} < 0 \tag{2.8}$$

$$or\ \frac{dC_n}{d\beta} < 0 \tag{2.9}$$

where C_n is the yawing moment coefficient. (Senson & Ritter, 2011)

CHAPTER 3

LITERATURE REVIEW

In this chapter, journal papers which are related to this field of study are reviewed and studied. The relevant journals are selected based on the scope of this study which covers stability analysis and control system of an aircraft.

3.1 Stability Analysis and Controller Design for the Roll Angle Control of an Aircraft by Mbaocha C.C, Obiora Valentine. T (2013)

This study investigates the stability of a general Navion aircraft using PID toolkit of MATLAB 2014 software. In this study, a compensator was developed to correct the problem of inversion problem earlier encountered. The results obtained proved that the percentage required for the system to go over its steady state is 8.1672%. The time taken for the system response to reach its target value from initial value of zero is 0.2426 sec. The time taken for the system to reach steady state after initial rise is 3.0384 secs.

3.1.1 Methodology

In this study, it begins with the modeling phase with the derivation of required mathematical model to describe the lateral directional motion of the general Navion aircraft. State space analysis is used to model the general Navion aircraft.

There is a few assumption and approximation need to be considered before obtaining the equation of motion. For lateral dynamic of an aircraft, it is assumed that the aircraft is in steady., cruise with altitude and velocity are constants. Also, the speed of aircraft and flight conditions are constants. By using the data from the general Navion aircraft, its state space modelling equation is obtained.

After that, the MATLAB software is created, and it is used to determine the stability of the aircraft system. A PID controller is then being designed to correct the inversion problem that earlier encountered. By using the PID tuner application in MATLAB, the controller model is tuned until close to the values of overshoot, rise time and settling time are achieved.

3.1.2 Results and Discussion

Based on the simulation result in MATLAB, the step response of roll angle model of Navion aircraft is obtained and shown in Figure 3.1. It is seen that the dynamical characteristics of the Navion aircraft is not accepted as there is an inversion in the loop. Therefore, the overshoot, rise and settling time must be modified using feedback control.

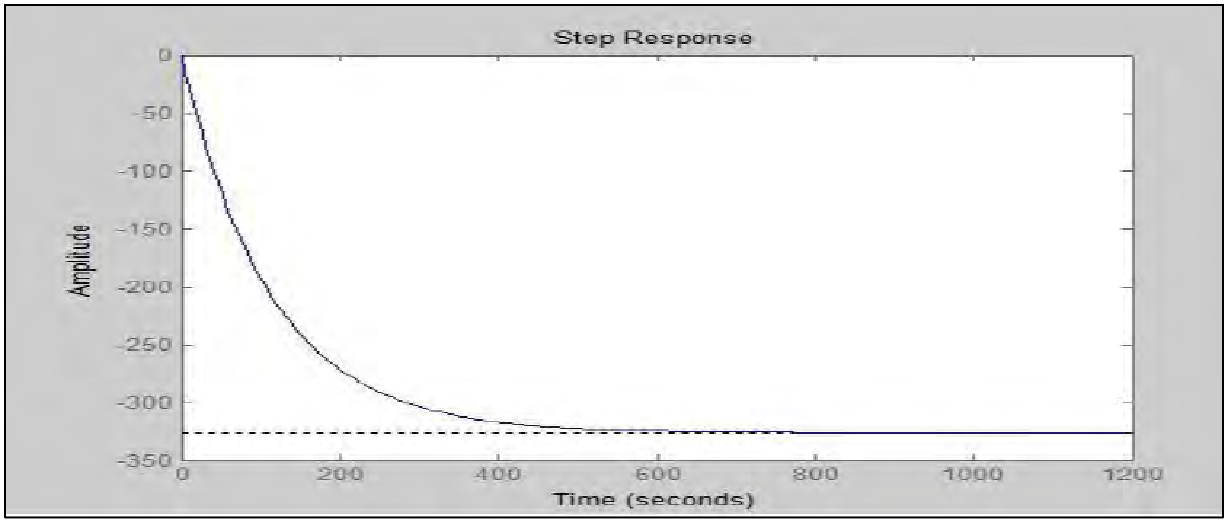


Figure 3.1 Step Response of Roll Angle Model of Navion Aircraft

Figure 3.2 shows the PID controller with its simple feedback system. By using the PID tuner application in MATLAB, the controller model is tuned until close to the values of overshoot, rise time and settling time are achieved. The values for the PIC controller in this design are $K_p = -2.1413, K_i = -1.2626, K_j = -0.123$.

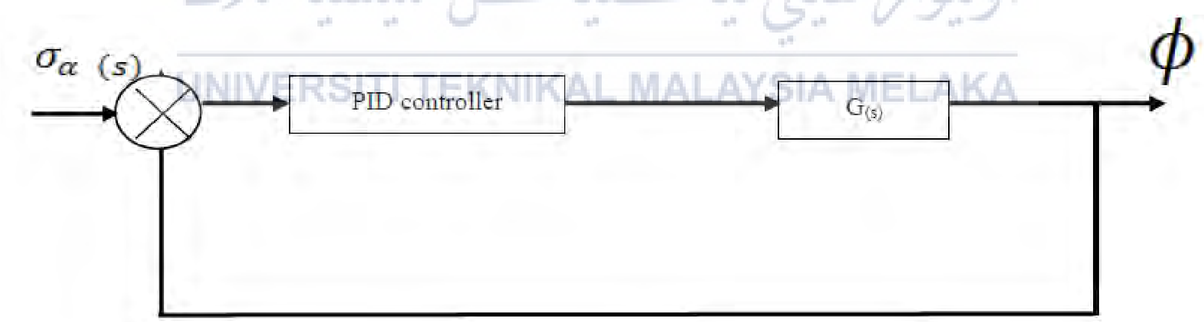


Figure 3.2 PID Controller with its Simple Feedback System

The step response of the simple feedback system is shown in Figure 3.3. From the step plot, it shows that the PID controller compensated for the inversion problem that earlier encountered. The overshoot was roughly 8% and zero steady state error was achieved.

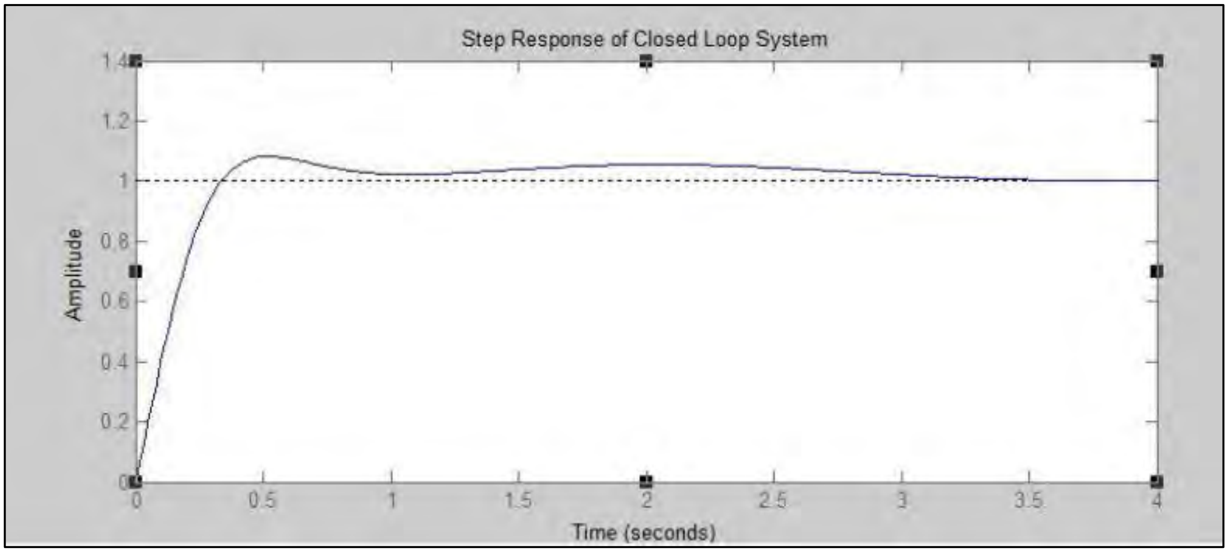


Figure 3.3 Step Response of the Closed Loop Feedback System

3.1.3 Conclusion

In this study, the PID controller that designed is able to compensate the inversion problem. An overshoot of 8.1672%, a rise time of 0.2426 sec, a settling time of 3.0384 secs are achieved. Therefore, the designed controller has successfully achieved the system stability.

3.2 A Self-Tuning Fuzzy Logic Controller for Aircraft Roll Control System by Akyazi, Ali Usta, & Sefa Akpinar (2013)

In this paper, an aircraft roll control system based on autopilot operating conditions is developed and simulated using MATLAB software. This work begins with the derivation of aircraft mathematical model to describe the lateral directional motion of an aircraft. There are three controllers applied for the controlling of roll angle of the modeled aircraft system. There are Linear Quadratic Regulator (LQR), Fuzzy Logic Controller (FLC), and Self-Tuning Fuzzy Logic Controller (STFLC). The simulation results of roll controllers are shown in time domain and the performance analysis is discussed and presented to determine which controller gives better performance in terms of roll angle.

3.2.1 Methodology

In this study, it begins with the derivation of aircraft mathematical model to describe the lateral directional motion of an aircraft.

There is a few assumption and approximation need to be considered before obtaining the equation of motion. The aircraft is assumed to be in steady-cruise at constant altitude and velocity, the thrust and drag cancel out the lift and the weight balance out each other. Also, the aircraft is assumed that change in pitch angle does not change the speed of an aircraft.

For this roll control system, the input will be the aileron deflection and the output will be the roll angle. The data from General Aviation Airplane: Navion is used in this system analysis and modeling. Besides, with the lateral directional derivatives stability parameters of this aircraft given, the transfer function of roll angle due to the deflection of aileron is obtained.

In the design process of the proposed controller, Linear Quadratic Regulator (LQR), Fuzzy Logic Controller (FLC), and Self-Tuning Fuzzy Logic Controller (STFLC) are proposed in roll control system. Figure 3.4, 3.5, and 3.6 show the basic structure of LQR, FLC, and STFLC respectively.

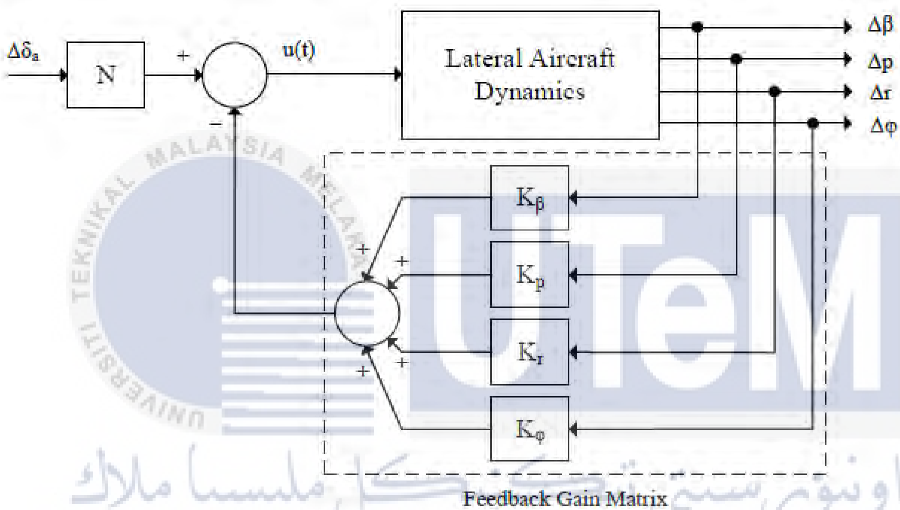


Figure 3.4 The Basic Structure of Linear Quadratic Regulator

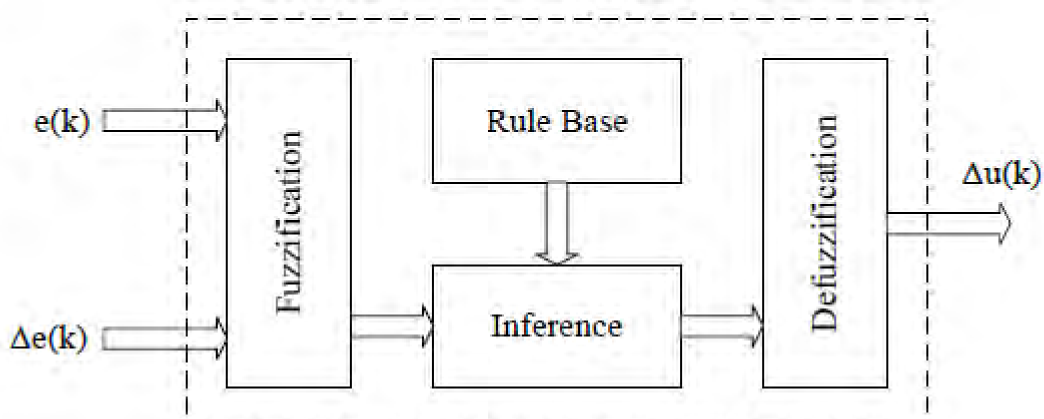


Figure 3.5 The Basic Structure of Fuzzy Logic Controller

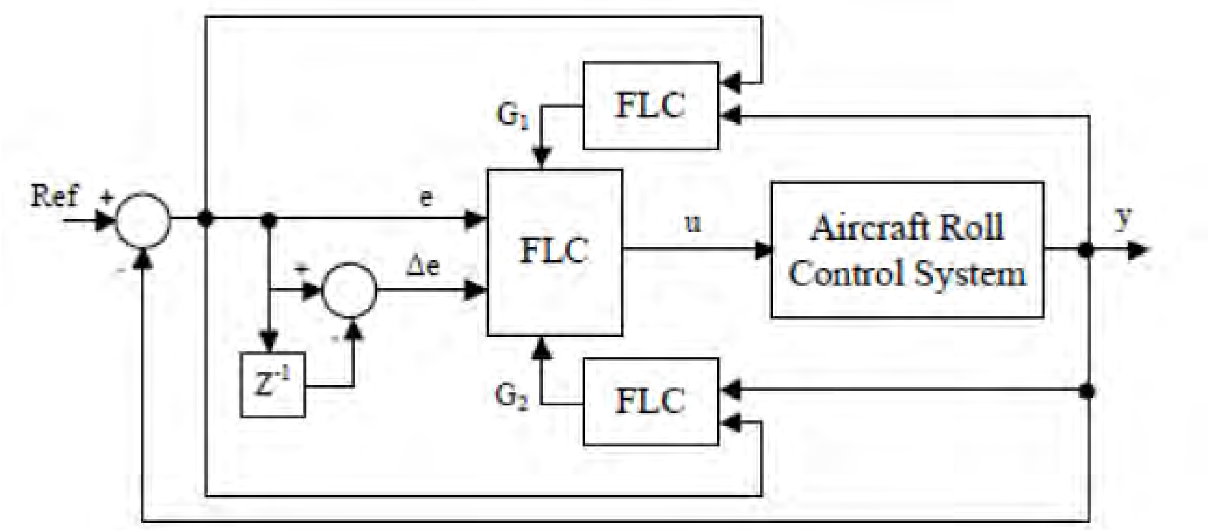


Figure 3.6 The Basic Structure of Self-Tuning Fuzzy Logic Controller

3.2.2 Results and Discussion

The aircraft roll control system is simulated in MATLAB software using Linear Quadratic Regulator (LQR), Fuzzy Logic Controller (FLC), and Self-Tuning Fuzzy Logic Controller (STFLC). For all the simulation, the reference value is set as 0.15 radian.

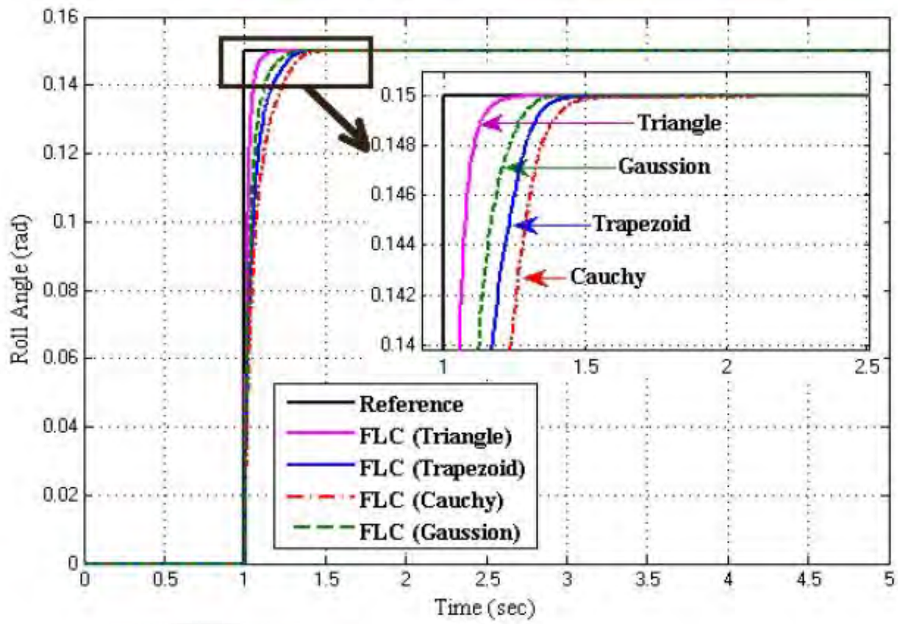


Figure 3.7 The System Response with Various Membership Functions

Figure 3.7 shows that the FLC-triangle gives the best response as compared to others. After determining the membership function, the rule tables of FLC is examined to see which fuzzy rule gives the better response.

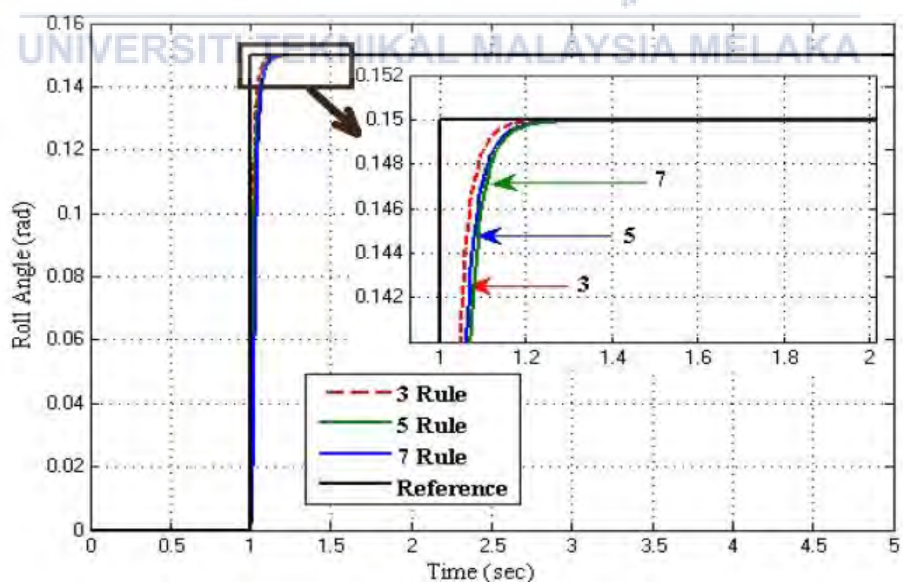


Figure 3.8 The System Response with Fuzzy Rule Tables

Figure 3.8 shows the FLC with nine rules gives the better response compared to others. Then, the system response of Linear Quadratic Regulator (LQR), Fuzzy Logic Controller (FLC), and Self-Tuning Fuzzy Logic Controller (STFLC) are plotted in the same graph for comparison purpose.

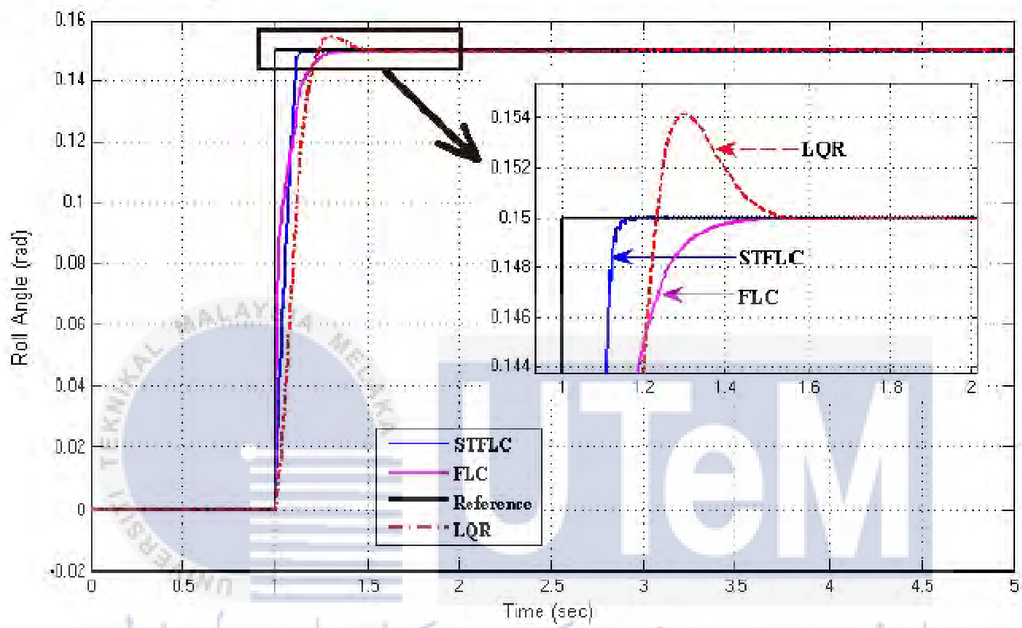


Figure 3.9 The Performance Analysis for Three Proposed Controller

Figure 3.9 indicates that the STFL controller gives the faster response compared to LQR and FLC in terms of rising time. But, the LQR controller is occurred overshoot more than FLC and STFLC.

3.2.3 Conclusion

In this paper, LQR, FLC, and STFLC are successfully designed. As the results, STFLC gives the better performance in terms of rising time, settling time, steady state error and overshoot.

3.3 Modeling of an Adaptive Controller for an Aircraft Roll Control System using PID, Fuzzy-PID and Genetic Algorithm by Gouthami & Rani (2016)

This paper is about the modeling of controllers for an aircraft roll control system and simulating using MATLAB software. The control system considered in this roll control system is PID controller. In this work, PID controller with different possible combinations of P, I, D parameters for roll transfer function is mathematically modeled and simulated. Besides, Fuzzy controller and Fuzzy integrated PID are proposed also in order to achieve the required criteria.

3.3.1 Methodology

In this study, it begins with the derivation of aircraft mathematical model to describe the lateral directional motion of an aircraft.

For this roll control system, the input will be the aileron deflection and the output will be the roll angle. The data from General Aviation Airplane: Navion is used in this system analysis and modeling. Besides, with the lateral directional derivatives stability parameters of this aircraft given, the transfer function of roll angle due to the deflection of aileron is obtained.

The modeling of the controllers is carried out in MATLAB software for the roll action transfer function.

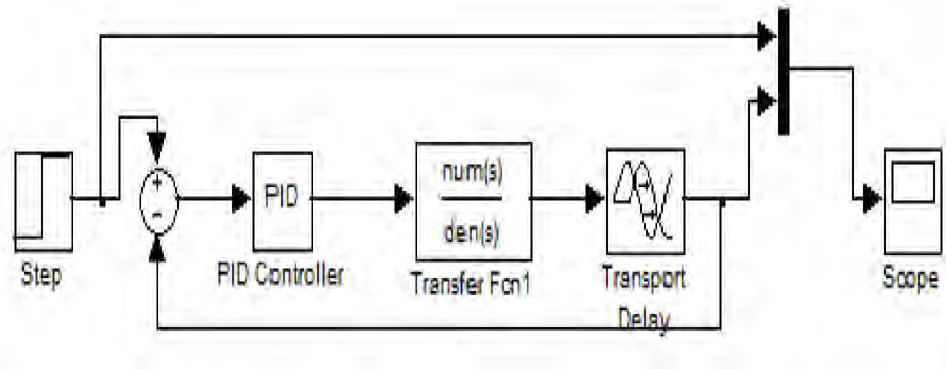


Figure 3.10 Simulink Model of PID controller

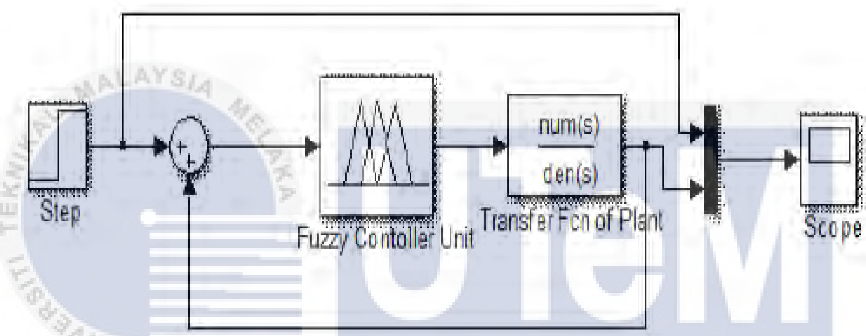


Figure 3.11 Simulink Model of Fuzzy Logic Controller

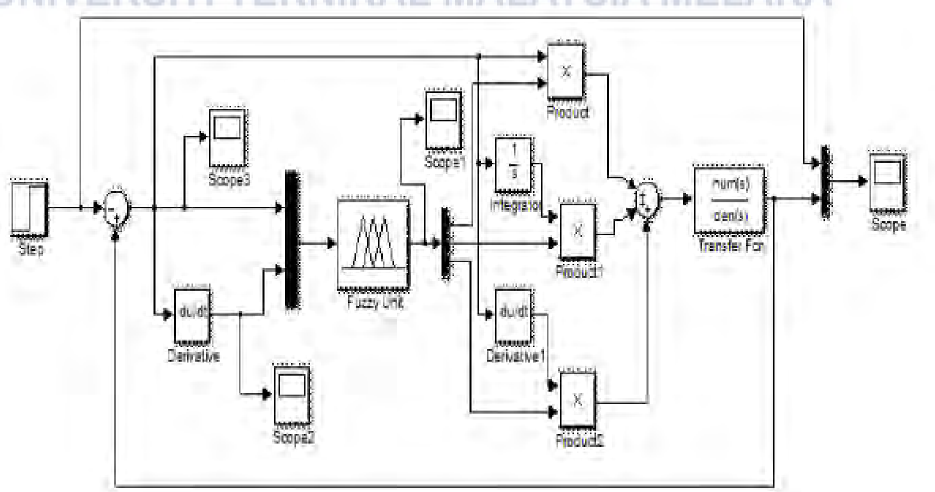


Figure 3.12 Simulink Model of Fuzzy-PID Controller

3.3.2 Results and Discussion

All the comparisons of parameters like rise time, settling time and overshoot for all methods are presented in the Table 3.1.

Table 3.1 Comparison of Different Parameters

Algorithm	Rise time (sec)	Setting time (sec)	Peak Overshoot (%)
PI	140	477	12. 2
PD	1. 36	3. 2 3	2. 6 2
PID(Automatic)	9	39.8	9
PID(Robust)	0.194	0.965	16
PID-ZN	16. 5	85. 4	17. 1
PID-GA	***0.0108***	***0.9***	***0***
IMC	1.7	13	70
FLC	0. 557	5. 5	12
Fuzzy-PID	0. 5	6	4
LQR	0.29	0.365	2.8
NEM(Non-linear Energy Method)	20	30	0

Based on Table 3.1, it is observed that the rise time, settling time and overshoot are optimum for PID controller with genetic algorithm optimization. The response is achieved with the minimum steady state error compared to others.

3.3.3 Conclusion

In this paper, the proposed adaptive controller for an aircraft roll control system was designed in MATLAB software. The obtained simulation results show that PID controller with Genetic Algorithm Optimization has fastest response (only 1m sec) in comparison with other controllers.

CHAPTER 4

METHODOLOGY

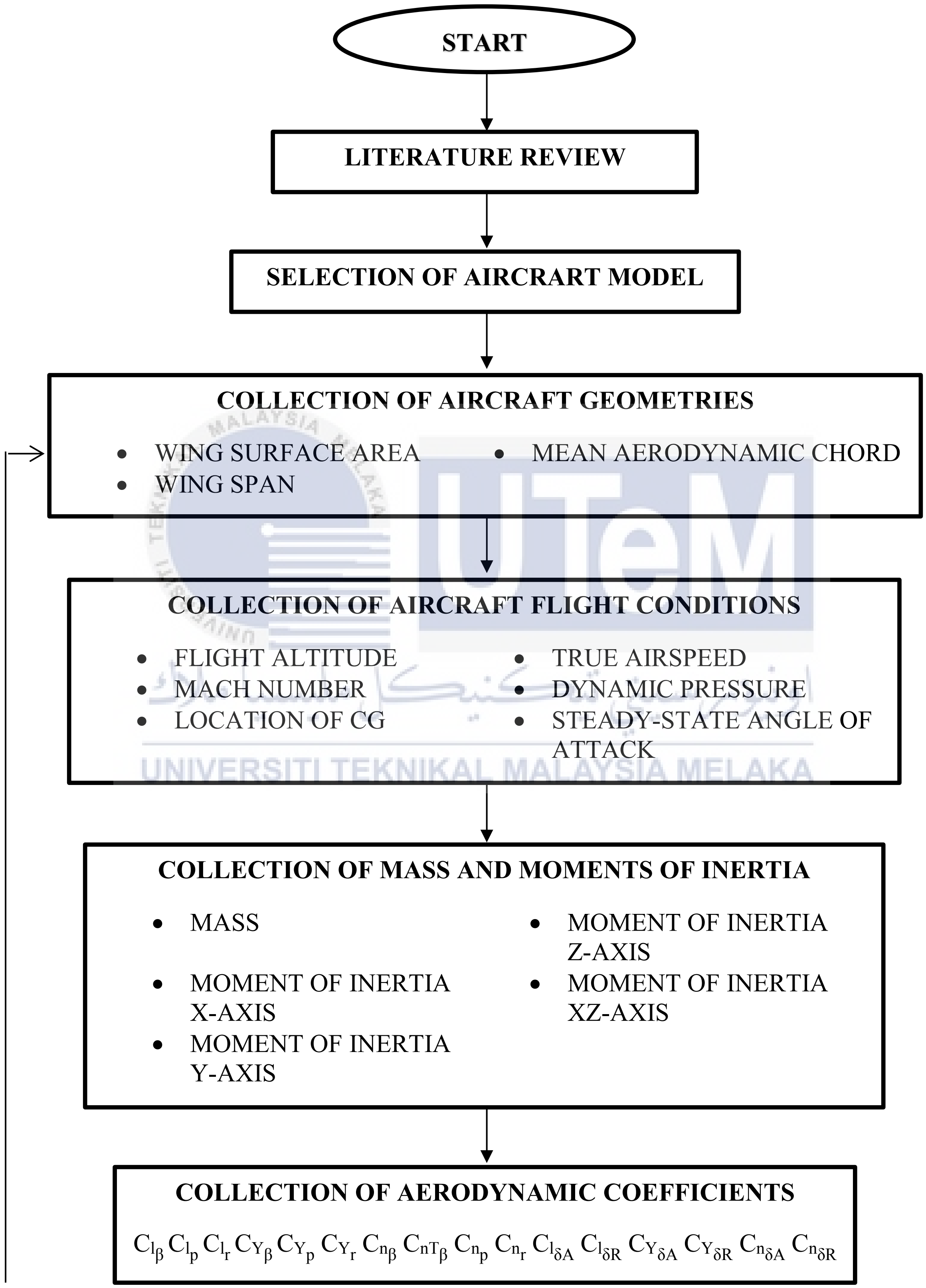
4.1 Introduction

This chapter describes about the methodology used in this project to study the dynamic stability behavior of aircraft, namely in predicting the aircraft response due to the aileron deflection in lateral directional. The flow chart of this project is shown in the Figure 4.1.

This project starts with study the theory of aircraft dynamics for better understanding in flight. Next, three type of low speed aircraft model are selected as the model of this project which are Cessna 182, Cessna 310, and Cessna 620.

After that, the necessary input data for the flight dynamic analysis are collected. These input data are aircraft geometries, mass and moment of inertia properties, flight conditions data and lateral directional aerodynamic stability and control derivatives coefficients.

A computer code of control system is developed in MATLAB using the control law and the aerodynamic data that obtained, and the flight dynamic analysis in lateral directional is studied with the function of deflection of ailerons. The validation is carried out by comparing the results for the lateral directional of aircraft model Learjet 24 with the results provided by Marcello R. Napolitano (2012). The developed computer code in MATLAB is applied to all three type of aircraft models. The results are being analyzed and the behavior of the flight dynamic stability due to the aileron deflection of an aircraft is discussed.



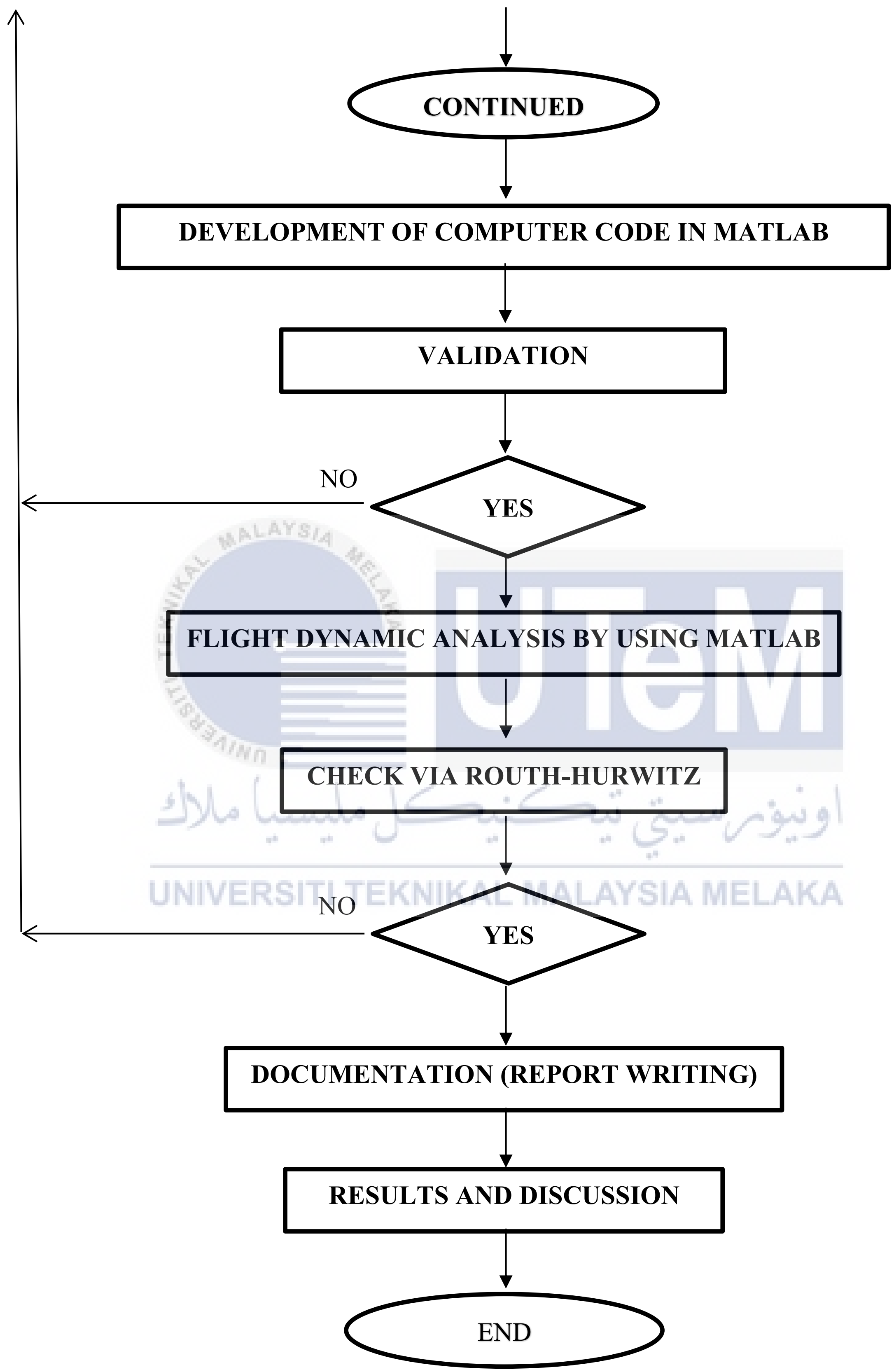


Figure 4.1 Methodology Flow Char

4.2 Literature Study and Data Mining

At the initial stage, journals, articles or any source of materials regarding to this project is studied. Planning in PSM 1 and 2 is done by constructing Gantt Chart after a few discussions with lecturer. Basic information regarding to control law which related to the lateral directional is obtained. The data mining includes the study of aerodynamic coefficient due to the deflection of aileron, stability and control and others directly or indirectly that would contribute to the success of this project.

4.3 Selection of Aircraft Model

In this present work, three type of low speed aircraft model are selected as the model of this project which are Cessna 182, Cessna 310, and Cessna 620.

Cessna 182 is a four seats and single piston engine aircraft whereas Cessna 310 is a four to six seats and twin piston engine aircraft. Cessna 620 is a business airplane that powered by four piston engines. 620 in its model name indicated is twice the aircraft that Cessna 310 is.

Figure 4.2, 4.3 and 4.4 show the Cessna 182, Cessna 310, and Cessna 620 aircraft model respectively.

Figure 4.2 Cessna 182 Aircraft Model (Steve Ells, 2016)

Figure 4.3 Cessna 310 Aircraft Model (Dave Sandidge, 2016)

Figure 4.4 Cessna 620 Aircraft Model (Mike Jerram, 2010)

4.4 Collection of Aircraft Data

The input data of all three types of aircraft models for the developed computer code in MATLAB is collected. These input data are aircraft geometries, mass and moment of inertia properties, flight conditions data and lateral directional aerodynamic stability and control derivatives coefficients.

For the aerodynamic coefficients, there are numerous ways can be used to obtain it. The most accurate way to obtain the aerodynamic coefficients is by using wind tunnel as it uses real aircraft or prototype as model. Unfortunately, the use of wind tunnel in calculating the aerodynamic coefficients is very expensive and time consuming. This method requires long time and cost a lot of money to build a prototype of an aircraft. The other method is by using Computational Fluid Dynamics software. But this method makes the computer code complicated and it requires high computer memory to calculate the aerodynamic coefficients. Therefore, the method to collect the aerodynamic coefficients that is suitable in this present work is by referring to Marcello R. Napolitano (2012).

The input data of all three types of aircraft models are presented in the following tables:

a. Table 4.1 shows the aircraft's geometric data.
b. Table 4.2 shows the aircraft's flight condition data.
c. Table 4.3 shows the aircraft's mass and inertial data.
d. Table 4.4 shows the lateral directional aerodynamic stability and control derivatives coefficients.

Table 4.1 Aircraft Geometric Data

Geometric	Symbol	Cessna 182	Cessna 310	Cessna 620
Wing Surface (ft^2)	S	174.00	175.00	340.00
Mean Aerodynamic Chord (ft)	c	4.90	4.79	6.58
Wing Span (ft)	b	36.00	36.90	55.10

Table 4.2 Aircraft Flight Conditions Data

Flight Conditions Data	Symbol	Cessna 182	Cessna 310	Cessna 620
Altitude (ft)	h	5000.000	8000.000	18000.000
Air Speed (ft/s)	U_1	220.100	312.500	366.800
Mach Number	M	0.201	0.288	0.351
Angle of Attack (deg)	α	0.000	0.000	0.000
Dynamic Pressure (Ibs/ ft^2)	q_1	49.600	91.200	91.100
Acceleration of Gravity (ft/s^2)	g	32.000	32.000	32.000
Location of CG-%MAC (ft)	x_{cg}	0.264	0.330	0.250

Table 4.3 Aircraft Mass and Inertial Data

Mass and Inertia	Symbol	Cessna 182	Cessna 310	Cessna 620
Mass (Ibs)	m	2650	4600	15000
Moment of Inertia x-axis (slug ft^2)	I_{XXB}	948	8884	64811
Moment of Inertia y-axis (slug ft^2)	I_{YYB}	1346	1939	17300
Moment of Inertia z-axis (slug ft^2)	I_{ZZB}	1967	11001	64543
Product of Inertia x and z-axis (slug ft^2)	I_{XZB}	0	0	0

Table 4.4 Lateral Directional Aerodynamic Coefficients

Coefficients	Cessna 182	Cessna 310	Cessna 620
Stability Derivatives			
C_{l_β}	-0.0923	-0.1096	-0.1381
C_{l_p}	-0.4840	-0.5510	-0.5660
C_{l_r}	0.0798	0.0729	0.1166
C_{Y_β}	-0.3930	-0.6980	-0.8830
C_{Y_p}	-0.0750	-0.1410	-0.2270
C_{Y_r}	0.2140	0.3550	0.4480
C_{n_β}	0.0587	0.1444	0.1739
C_{nT_β}	0.0000	0.0000	0.0000
C_{n_p}	-0.0278	-0.0257	-0.0501
C_{n_r}	-0.0937	-0.1495	-0.2000
Control Derivatives			
$C_{l_{\delta_A}}$	0.2290	0.1720	0.1776
$C_{l_{\delta_R}}$	0.0147	0.0192	0.0200
$C_{Y_{\delta_A}}$	0.0000	0.0000	0.0000
$C_{Y_{\delta_R}}$	0.1870	0.2300	0.2000
$C_{n_{\delta_A}}$	-0.0216	-0.0168	-0.0194
$C_{n_{\delta_R}}$	-0.0645	-0.1152	-0.1054

4.5 Development of Computer Code in MATLAB

Before the development of computer code in MATLAB, the solution of aircraft equations of motion in lateral directional based on Laplace Transformations and transfer functions are studied. After that, all the lateral directional equations of motion will be converted into computer code written in MATLAB programming language for flight dynamic analysis. The validation is carried out by comparing the results for the lateral directional of aircraft model Learjet 24 with the results provided by Marcello R. Napolitano (2012).

4.5.1 The Lateral Directional Aerodynamic Stability and Control Derivatives

The lateral directional aerodynamic stability and control derivatives which are the combination of aerodynamic coefficients along with aircraft geometry (b, S), inertial parameters (I_{XX}, I_{YY}, I_{XZ}) and flight conditions (q_1) are presented in Table 4.5.

Table 4.5 Formula of Lateral Directional Stability and Control Derivatives

DERIVATIVES	UNIT	DERIVATIVES	UNIT
$Y_\beta = \dfrac{q_1 S C_{Y_\beta}}{m}$	(ft sec^{-2})	$Y_p = \dfrac{q_1 S C_{Y_p}}{m}\dfrac{b}{2U_1}$	(ft sec^{-1})
$Y_r = \dfrac{q_1 S C_{Y_r}}{m}\dfrac{b}{2U_1}$	(ft sec^{-1})	$Y_{\delta_A} = \dfrac{q_1 S C_{Y_{\delta A}}}{m}$	(ft sec^{-2})
$Y_{\delta_R} = \dfrac{q_1 S C_{Y_{\delta R}}}{m}$	(ft sec^{-2})	$L_p = \dfrac{q_1 S b C_{l_p}}{I_{xx}}\dfrac{b}{2U_1}$	(sec^{-1})
$L_\beta = \dfrac{q_1 S C_{l_\beta} \mathrm{b}}{I_{xx}}$	(sec^{-2})	$L_{\delta_A} = \dfrac{q_1 S C_{l_{\delta A}} \mathrm{b}}{I_{xx}}$	(sec^{-2})

$L_r = \dfrac{q_1 S b C_{l_r}}{I_{xx}} \dfrac{b}{2U_1}$	(sec^{-1})	$L_{\delta R} = \dfrac{q_1 S C_{l_{\delta R}} b}{I_{xx}}$	(sec^{-2})
$N_\beta = \dfrac{q_1 S C_{n_\beta} b}{I_{zz}}$	(sec^{-2})	$N_{T\beta} = \dfrac{q_1 S C_{n_{T\beta}} b}{I_{zz}}$	(sec^{-2})
$N_p = \dfrac{q_1 S b C_{n_p}}{I_{zz}} \dfrac{b}{2U_1}$	(sec^{-1})	$N_r = \dfrac{q_1 S b C_{n_r}}{I_{zz}} \dfrac{b}{2U_1}$	(sec^{-1})
$N_{\delta_R} = \dfrac{q_1 S C_{l_{\delta R}} b}{I_{zz}}$	(sec^{-2})	$N_{\delta_A} = \dfrac{q_1 S C_{n_{\delta A}} b}{I_{zz}}$	(sec^{-2})

4.5.2 The Application of Laplace Transformations to the Lateral Directional Equations of Motions

The simulation is based on the transfer functions, and these transfer functions are obtained by applying Laplace Transform to the differential equations that model the dynamic behavior of an aircraft.

By using the coefficients in Table 4.5, the small perturbation lateral directional equation in terms of stability and control derivatives are in the form:

$$(V_{P_1}\dot{\beta} + V_{P_1}\psi) = g\phi + Y_\beta \beta + Y_{\dot{\phi}}\dot{\phi} + Y_{\dot{\psi}}\dot{\psi} + Y_{\delta_A}\delta_A + Y_{\delta_R}\delta_R \tag{4.1}$$

$$\ddot{\phi} - \frac{I_{XZ}}{I_{XX}}\ddot{\psi} = L_\beta \beta + L_{\dot{\phi}}\dot{\phi} + L_{\dot{\psi}}\dot{\psi} + L_{\delta_A}\delta_A + L_{\delta_R}\delta_R \tag{4.2}$$

$$\ddot{\psi} - \frac{I_{XZ}}{I_{ZZ}}\ddot{\phi} = N_\beta \beta + N_{\dot{\phi}}\dot{\phi} + N_{\dot{\psi}}\dot{\psi} + N_{\delta_A}\delta_A + N_{\delta_R}\delta_R \tag{4.3}$$

Where $Y_{\dot{\phi}} = Y_P, Y_{\dot{\psi}} = Y_r, L_{\dot{\phi}} = L_P, L_{\dot{\psi}} = L_r, N_{\dot{\phi}} = N_P, N_{\dot{\psi}} = N_r$.

After that, Laplace transformations are applied to both sides of the equations, assuming zero at initial state due to steady-state condition:

$$L(\delta_A) = \delta_A(s), L(\delta_R) = \delta_R(s) \tag{4.6}$$

$$L(\beta) = \beta(s), L(\dot{\beta}) = s\beta(s) \tag{4.7}$$

$$L(\phi) = \phi(s), L(\dot{\phi}) = L(p) = s\phi(s),, L(\ddot{\phi}) = L(\dot{p}) = s^2\phi(s), \tag{4.8}$$

$$L(\psi) = \psi(s), L(\dot{\psi}) = L(r) = s\psi(s),, L(\ddot{\psi}) = L(\dot{r}) = s^2\psi(s), \tag{4.9}$$

In the lateral direction equations, $\delta\,(s)$ represents the deflection of control surfaces. In this present work, only deflection of aileron will be involved. Therefore, the input in this system is the deflection of aileron, δ_A. Next, the coefficients of the equations are grouped in terms of $(\beta(s), \phi(s), \psi(s))$. Assuming ratio $I_1 = \frac{I_{XZ}}{I_{XX}}, I_2 = \frac{I_{XZ}}{I_{ZZ}}$, the small perturbation lateral direction equations leading to:

$$(sV_{P_1} - Y_\beta)\beta(s) - (sY_p + g\cos\Theta_1)\phi(s) + s(V_{P_1} - Y_r)\,\psi(s) = Y_{\delta_A}\delta_A\,(s) \tag{4.10}$$

$$-L_\beta\beta(s) + s(s - L_p)\phi(s) - s(sI_1 + L_r)\,\psi(s) = L_{\delta_A}\delta_A\,(s) \tag{4.11}$$

$$-N_\beta\beta(s) - s(sI_2 + N_P)\,\phi(s) + s(s - N_r)\psi(s) = N_{\delta_A}\delta_A\,(s) \tag{4.12}$$

In a transfer function, given a known input, u(t) and its Laplace transform, $\mathrm{U(s)} = \mathrm{L(u(t))}$, the output y(t) of a generic system in the time domain can be derived as inverse Laplace transform $\mathrm{y(t)} = L^{-1}(\mathrm{Y(s)}) = L^{-1}(\frac{Y(s)}{U(s)}\,\mathrm{x}\,\mathrm{U(s)})$ where $\frac{Y(s)}{U(s)}$ is the transfer function of the system. The concept of transfer function is shown in the Figure 4.5.

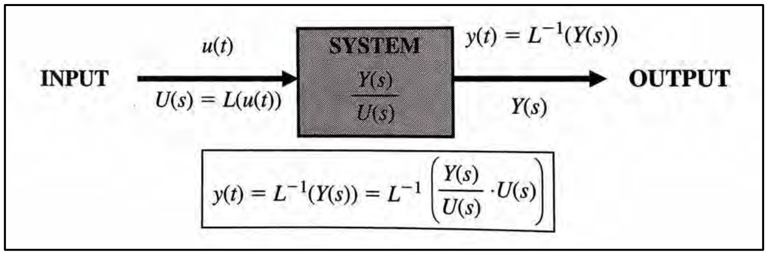


Figure 4.5 The Concept of Transfer Function (Marcello R. Napolitano, 2012)

For the lateral directional, the input is the deflection of aileron, δ_A whereas the outputs are represented by variables $(\beta(t), \phi(t), \psi(t))$. Therefore, by using the transfer functions $(\frac{\beta(s)}{\delta_A(s)}, \frac{\phi(s)}{\delta_A(s)}, \frac{\psi(s)}{\delta_A(s)})$, the preceding equations can be presented in the following matrix format:

$$\begin{bmatrix} (sV_{P_1} - Y_\beta) & -(sY_p + g) & s(V_{P_1} - Y_r) \\ -L_\beta & s(s - L_p) & -s(sI_1 + L_r) \\ -N_\beta & -s(sI_2 + N_P) & s(s - N_r) \end{bmatrix} \begin{bmatrix} \frac{\beta(s)}{\delta_A(s)} \\ \frac{\phi(s)}{\delta_A(s)} \\ \frac{\psi(s)}{\delta_A(s)} \end{bmatrix} = \begin{bmatrix} Y_{\delta_A} \\ L_{\delta_A} \\ N_{\delta_A} \end{bmatrix} \tag{4.13}$$

The application of Cramer's rule to the Equation 4.13 in the matrix format leads to following solutions for the preceding transfer functions:

$$\frac{\beta(s)}{\delta_A(s)} = \frac{\begin{vmatrix} Y_{\delta_A} & -(sY_p + g\cos\Theta_1)) & s(V_{P_1} - Y_r) \\ L_{\delta_A} & s(s - L_p) & -s(sI_1 + L_r) \\ N_{\delta_A} & -s(sI_2 + N_P) & s(s - N_r) \end{vmatrix}}{\begin{vmatrix} (sV_{P_1} - Y_\beta) & -(sY_p + g\cos\Theta_1) & s(V_{P_1} - Y_r) \\ -L_\beta & s(s - L_p) & -s(sI_1 + L_r) \\ -N_\beta & -s(sI_2 + N_P) & s(s - N_r) \end{vmatrix}} = \frac{Num_{\beta A}(s)}{D_1(s)} \tag{4.14}$$

$$\frac{\phi(s)}{\delta_A(s)}=\frac{\begin{vmatrix}(sV_{P_1}-Y_\beta) & Y_{\delta_A} & s(V_{P_1}-Y_r)\\ -L_\beta & L_{\delta_A} & -\mathrm{s}(sI_1+L_r)\\ -(N_\beta+N_{T\beta}) & N_{\delta_A} & s(s-N_r)\end{vmatrix}}{\begin{vmatrix}(sV_{P_1}-Y_\beta) & -(sY_p+g\cos\Theta_1) & s(V_{P_1}-Y_r)\\ -L_\beta & s(s-L_p) & -\mathrm{s}(sI_1+L_r)\\ -N_\beta & -\mathrm{s}(sI_2+N_P) & s(s-N_r)\end{vmatrix}}=\frac{Num_{\phi A}(s)}{D_1(s)} \tag{4.15}$$

$$\frac{\psi(s)}{\delta_A(s)}=\frac{\begin{vmatrix}(sV_{P_1}-Y_\beta) & -(sY_p+g\cos\Theta_1) & Y_{\delta_A}\\ -L_\beta & s(s-L_p) & L_{\delta_A}\\ -(N_\beta+N_{T\beta}) & -\mathrm{s}(sI_2+N_P) & N_{\delta_A}\end{vmatrix}}{\begin{vmatrix}(sV_{P_1}-Y_\beta) & -(sY_p+g\cos\Theta_1) & s(V_{P_1}-Y_r)\\ -L_\beta & s(s-L_p) & -\mathrm{s}(sI_1+L_r)\\ -N_\beta & -\mathrm{s}(sI_2+N_P) & s(s-N_r)\end{vmatrix}}=\frac{Num_{\psi A}(s)}{D_1(s)} \tag{4.16}$$

The time response of the aircraft system requires an analysis of the coefficients of the polynomials at the numerator and denominator of the preceding transfer functions. These coefficients are derived from grouping terms with the same order from the calculation of preceding determinants. The expressions for numerator polynomials $(Num_{\beta A}(s), Num_{\phi A}(s), Num_{\psi A}(s))$ is shown below.

$$Num_{\beta A}(s)=s(A_\beta s^3+B_\beta s^2+C_\beta s+D_\beta) \tag{4.17}$$

Where

$$A_\beta=Y_{\delta_A}(1-I_1I_2) \tag{4.18}$$

$$B_\beta=-Y_{\delta_A}(L_p+N_r+I_1L_r)+Y_p(L_{\delta_A}+I_1N_{\delta_A})+Y_r(L_{\delta_A}I_2+N_{\delta_A})$$
$$-V_{P_1}(L_{\delta_A}I_2+N_{\delta_A}) \tag{4.19}$$

$$C_\beta=Y_{\delta_A}(L_pN_r-N_pL_r)+Y_p(L_rN_{\delta_A}-N_rL_{\delta_A})+g(L_{\delta_A}+I_1N_{\delta_A})$$
$$+Y_r(L_{\delta_A}N_p-N_{\delta_A}L_p)-V_{P_1}(L_{\delta_A}N_p-N_{\delta_A}L_p) \tag{4.20}$$

$$D_\beta = g(L_r N_{\delta_A} - N_r L_{\delta_A}) \tag{4.21}$$

$$Num_{\phi A}(s) = s(A_\phi s^2 + B_\phi s + C_\phi) \tag{4.22}$$

Where

$$A_\phi = V_{P_1}\left(L_{\delta_A} + I_1 N_{\delta_A}\right) \tag{4.23}$$

$$B_\phi = V_{P_1}\left(L_r N_{\delta_A} - N_r L_{\delta_A}\right) - Y_\beta\left(L_{\delta_A} + I_1 N_{\delta_A}\right) + Y_{\delta_A}\left(L_\beta + I_1 N_\beta\right) \tag{4.24}$$

$$C_\phi = -Y_\beta\left(L_r N_{\delta_A} - N_r L_{\delta_A}\right) + Y_{\delta_A}\left(L_r N_\beta - L_\beta N_r\right) \tag{4.25}$$

$$+V_{P_1}\left(L_{\delta_A} N_\beta - L_\beta N_{\delta_A}\right) - Y_r\left(L_{\delta_A} N_\beta - L_\beta N_{\delta_A}\right)$$

$$Num_{\psi A}(s) = s(A_\psi s^3 + B_\psi s^2 + C_\psi s + D_\psi) \tag{4.26}$$

Where

$$A_\psi = V_{P_1}\left(N_{\delta_A} + I_2 L_{\delta_A}\right) \tag{4.27}$$

$$B_\psi = V_{P_1}\left(L_{\delta_A} N_p - N_{\delta_A} L_p\right) - Y_\beta\left(N_{\delta_A} + I_2 L_{\delta_A}\right) + Y_{\delta_A}\left(L_\beta I_2 + N_\beta\right) \tag{4.28}$$

$$C_\psi = -Y_\beta\left(L_{\delta_A} N_P - N_{\delta_A} L_P\right) + Y_p\left(L_{\delta_A} N_\beta - L_\beta N_{\delta_A}\right) \tag{4.29}$$

$$+Y_{\delta_A}\left(L_\beta N_P - L_P N_\beta\right)$$

$$D_\psi = g(L_{\delta_A} N_\beta - L_\beta N_{\delta_A}) \tag{4.30}$$

All the lateral directional transfer functions share the same denominator:

$$D_1 = s\ (A_1 s^4 + B_1 s^3 + C_1 s^2 + D_1 s + E_1) \tag{4.26}$$

Where

$$A_1 = V_{P_1}(1 - I_1 I_2) \tag{4.27}$$

$$B_1 = -Y_\beta(1 - I_1 I_2) - V_{P_1}(L_P + N_r + I_1 N_p + I_2 L_r) \tag{4.28}$$

$$C_1 = V_{P_1}\left(L_P N_r - N_p L_r\right) + Y_\beta\left(L_P + N_r + I_1 N_p + I_2 L_r\right) \tag{4.29}$$

$$-Y_p\left(L_\beta + I_1 N_\beta\right) + V_{P_1}\left(L_\beta I_2 + N_\beta\right) - Y_r\left(L_\beta I_2 + N_\beta\right)$$

$$D_1 = -Y_\beta\left(L_p N_r - N_p L_r\right) + Y_p\left(L_\beta N_r - L_r N_\beta\right) - g(L_\beta + I_1 N_\beta) \tag{4.30}$$

$$+V_{P_1}\left(L_\beta N_p - L_p N_\beta\right) - Y_r\left(L_\beta N_P - L_P N_\beta\right)$$

$$E_1 = g(L_\beta N_r - L_r N_\beta) \tag{4.31}$$

4.5.3 Routh-Hurwitz Analysis of the Lateral Directional Stability

The assessment of the dynamic stability of an aircraft system can be done by applying Routh-Hurwitz stability criteria. Given that the denominator of the lateral directional $D_1 = s\,(A_1 s^4 + B_1 s^3 + C_1 s^2 + D_1 s + E_1)$, is a fifth order polynomial. However, the root at the origin (s=0) can be excluded. Therefore, the Routh-Hurwitz criteria leads the denominator of the lateral directional to:

s^4	A_1	C_1	E_1
s^3	B_1	D_1	-
s^2	k_{11}	k_{12}	-
s^1	k_{21}	-	-
s^0	E_1	-	-

Where

$$k_{11} = \frac{B_1C_1 - A_1D_1}{B_1} \tag{4.32}$$

$$k_{12} = \frac{B_1E_1}{B_1} = E_1 \tag{4.33}$$

$$k_{21} = \frac{D_1k_{11} - B_1k_{12}}{k_{11}} = \frac{D_1(\frac{B_1C_1 - A_1D_1}{B_1}) - B_1E_1}{\frac{B_1C_1 - A_1D_1}{B_1}} = \frac{D_1(B_1C_1 - A_1D_1) - B_1^2E_1}{(B_1C_1 - A_1D_1)} \tag{4.34}$$

After that, the analysis focuses on the first column of the preceding array.

s^4	A_1	C_1	E_1
s^3	B_1	D_1	-
s^2	k_{11}	k_{12}	-
s^1	k_{21}	-	-
s^0	E_1	-	-

According to the Routh-Hurwitz stability criterion, a dynamic system is dynamically stable if the coefficients in the first column of the Routh-Hurwitz array have the same sign. Therefore, the following criteria need to be fulfilled to have lateral directional stability.

$$A_1 > 0, \qquad B_1 > 0, \qquad B_1C_1 - A_1D_1 > 0,$$

$$D_1\,(B_1C_1 - A_1D_1) - B_1^2E_1 > 0, \qquad E_1 > 0$$

If by inspection to be dynamically stability, the denominators of the transfer functions need to fulfill the following:

a. The equation has no missing terms.

b. All the coefficients are all same sign.

CHAPTER 5

RESULTS AND DISCUSSION

5.1 Introduction

The present work focuses on the development of a computer code using MATLAB software to study on the lateral and directional flight dynamic stability behavior due to aileron deflection of an aircraft. The computer code that is developed in such a way that for a given aircraft data and flight condition, this computer code will generate the following results in term of three main sub chapter:

a. Sub chapter 5.2: Validation by Comparing Preliminary Results and Results from Marcello R. Napolitano (2012).
b. Sub chapter 5.3: The Results of Lateral Directional Stability and Control Derivatives
c. Sub chapter 5.4: The Solution of Flight Equation of Motion using Lateral Directional Transfer Function
d. Sub chapter 5.5: Time History and Stability Behavior in the Lateral Directional

In this present work, the MATLAB computer code carries the results of three types of aircraft models which are Cessna 182, Cessna 310, and Cessna 620. It is created with a start menu as shown in Figure 5.1.

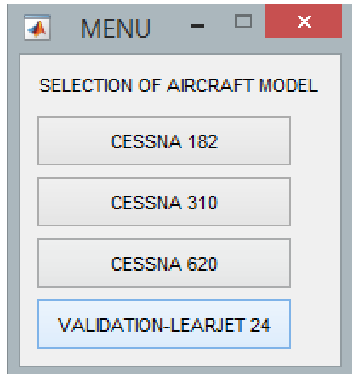


Figure 5.1: MATLAB Start Menu for the Selection of Aircraft Model

After this step, the user can select the method of the deflection of aileron as shown in Figure 5.2. There are four types of aileron deflection as below:

a. Single Doublet Impulse

b. Multiple Doublets Impulse

c. Singlet Doublet

d. Multiple Doublets

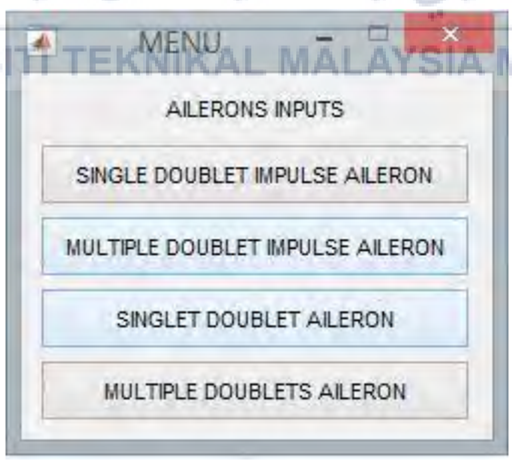


Figure 5.2 MATLAB Start Menu for the Selection of the Type of Aileron Deflection

5.2 Validation by Comparing Preliminary Results and Results from Marcello R. Napolitano (2012).

5.2.1 Preliminary Results

Before study the flight dynamic analysis of Cessna 182, Cessna 310, and Cessna 620, Learjet 24 is chosen to be studied in order to compare with the results that provided Marcello R. Napolitano (2012) for validation purpose.

5.2.1.1 Data of Learjet 24

This section provides a database for Learjet 24 aircraft. The database includes geometry data, flight conditions, inertial characteristics, and dimensionless aerodynamic coefficients.

Table 5.1 Geometric Data for the Learjet 24 Aircraft

Geometric	Symbol	Dimensions
Wing Surface (ft^2)	S	230
Mean Aerodynamic Chord (ft)	c	7.0
Wing Span (ft)	b	34

Table 5.2 Flight Conditions Data for the Learjet 24 Aircraft at Cruise

Flight Conditions Data	Symbol	Dimensions
Air Speed (ft/s)	U_1	677
Mach Number	M	0.7
Angle of Attack (deg)	α	1.5
Dynamic Pressure (Ibs/ ft^2)	q_1	134.6
Acceleration of Gravity (ft/s^2)	g	32
Location of CG-%MAC (ft)	x_{cg}	0.32

Table 5.3 Mass and Inertial Data for the Learjet 24 Aircraft at Cruise

Mass and Inertia	Symbol	Dimensions
Weight (Ibs)	W	9000
Moment of Inertia x-axis (slug ft^2)	I_{XXB}	6000
Moment of Inertia y-axis (slug ft^2)	I_{YYB}	17800
Moment of Inertia z-axis (slug ft^2)	I_{ZZB}	25000
Product of Inertia x-axis (slug ft^2)	I_{XZB}	1400

Table 5.4 Lateral Directional Aerodynamic Coefficients for the Learjet 24 Aircraft at Cruise

Coefficients	Dimensions
Stability Derivatives	
C_{l_β}	-0.100
C_{l_p}	-0.450
C_{l_r}	0.140
C_{Y_β}	-0.730
C_{Y_p}	0
C_{Y_r}	0.400
C_{n_β}	0.124
C_{nT_β}	0
C_{n_p}	-0.022
C_{n_r}	-0.200
Control Derivatives	
$C_{l_{\delta A}}$	0.178
$C_{l_{\delta R}}$	0.021
$C_{Y_{\delta A}}$	0
$C_{Y_{\delta R}}$	0.140
$C_{n_{\delta A}}$	-0.020
$C_{n_{\delta R}}$	-0.074

5.2.1.2 Results of Lateral Directional Responses for Learjet 24

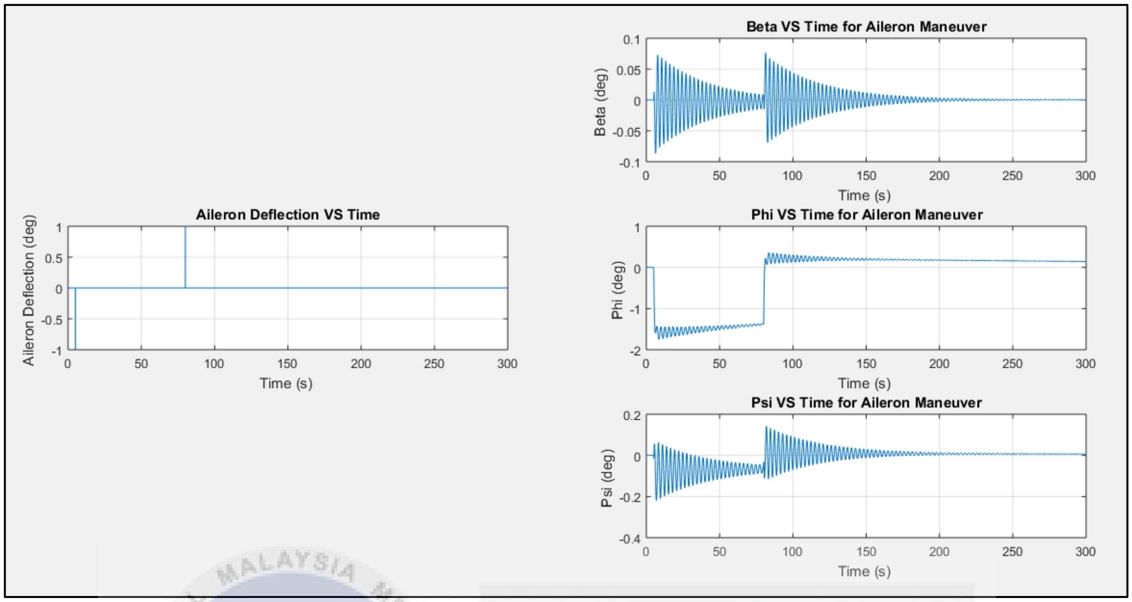


Figure 5.3 Lateral Directional Responses by Aileron Maneuver Obtained by Developed Computer Code in MATLAB

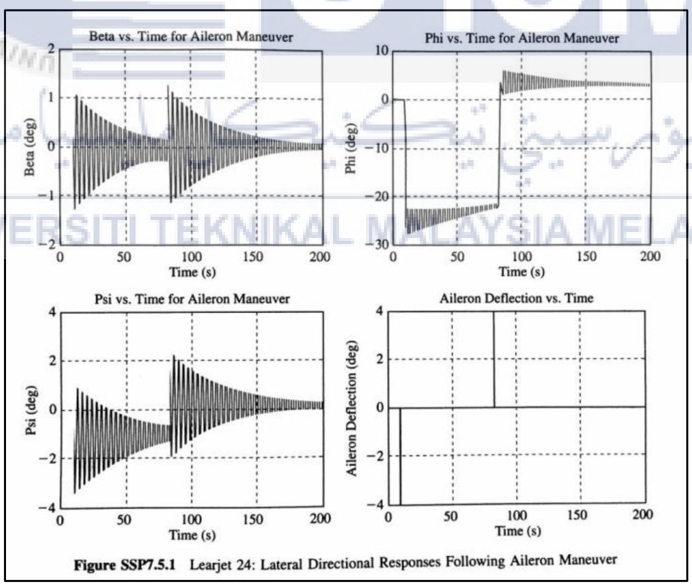


Figure 5.4 Lateral Directional Responses by Aileron Maneuver that Provided by Marcello R. Napolitano (2012)

The comparison result shows these two cases have good agreement. The stability behavior is similar for both results.

5.3 The Results of Lateral Directional Stability and Control Derivatives

The lateral directional stability derivative equations are listed in Table 4.5 Chapter 4. Based on MATLAB computer code that is created, the calculation for the lateral directional stability derivatives is done by solving the equations using the data such as lateral directional aerodynamic coefficients that listed in Table 4.4 Chapter 4. The results of lateral directional stability and control derivatives are shown is Table 5.5.

Table 5.5: The Lateral Directional Stability and Control Derivatives

	Cessna 182	**Cessna 310**	**Cessna 620**
Stability Derivatives			
L_β	-30.2501	-7.2654	-3.6366
L_p	-12.9725	-2.1565	-1.1195
L_r	2.1389	0.2853	0.2306
Y_β	-41.2129	-77.9806	-58.7114
Y_p	-0.6432	-0.9300	-1.1337
Y_r	1.8353	2.3416	2.2373
N_β	9.2719	7.7303	4.5983
N_{T_β}	0.0000	0.0000	0.0000
N_p	-0.3591	-0.0812	-0.0995
N_r	-1.2104	-0.4725	-0.3972
Control Derivatives			
$L_{\delta A}$	75.0517	11.4020	4.6767
$L_{\delta R}$	4.8177	1.2728	0.5267
$Y_{\delta A}$	0.0000	0.0000	0.0000
$Y_{\delta R}$	19.6102	25.6956	13.2982
$N_{\delta A}$	-3.4118	-0.8994	-0.5130
$N_{\delta R}$	-10.1880	-6.1671	-2.7870

5.4 The Solution of Flight Equation of Motion using Lateral Directional Transfer Function

The lateral directional transfer functions are listed in Chapter 4 (Equation 4.14-4.16). Based on MATLAB computer code that is created, the calculation for the coefficients of polynomials at the numerator and denominator of the transfer functions are done by solving the equations that listed in Chapter 4 (Equation 4.17-4.31). The results of the coefficients of polynomials at the numerator of the transfer functions are shown is Table 5.6 (a), 5.7 (a) and 5.8 (a). After that, the numerator of the lateral directional transfer functions in terms of β (s), Ø (s) and ψ (s) due to aileron deflections for three types of aircraft can be obtained by substituting the coefficients of polynomials at the numerator and presented in Table 5.6 (b), 5.7 (b) and 5.8 (b).

Table 5.6 (a): The Coefficients of Polynomials at the Numerator of β- Transfer Function

β- Transfer Function			
	Cessna 182	**Cessna 310**	**Cessna 620**
A_β	0.0000	0.0000	0.0000
B_β	-51.1241	-11.8107	-5.9365
C_β	17906.0000	1251.2000	527.6203
D_β	2690.1000	165.2175	56.0072

Table 5.6 (b): The Numerator of the Lateral Directional Transfer Functions in terms of β (s)

β- Transfer Function
$$\frac{\beta\ (s)}{\delta_A\ (s)} = \frac{Num_{\beta A}}{D_1} = \frac{s\ (A_\beta s^3 + B_\beta s^2 + C_\beta s + D_\beta)}{s\ (A_1 s^4 + B_1 s^3 + C_1 s^2 + D_1 s + E_1)}$$
$$Num_{\beta A} = \begin{bmatrix} Cessna\ 182 \\ Cessna\ 310 \\ Cessna\ 620 \end{bmatrix} = \begin{bmatrix} s\ (-51.1241s^2 + 17906s + 2690.1) \\ s\ (-11.8107s^2 + 1251.2s + 165.2175) \\ s\ (-5.9365s^2 + 527.6203s + 56.0072) \end{bmatrix}$$

Table 5.7 (a): The Coefficients of Polynomials at the Numerator of Ø- Transfer Function

Ø- Transfer Function			
	Cessna 182	**Cessna 310**	**Cessna 620**
Aø	16519.0000	3563.1000	1715.4000
Bø	21481.0000	2492.6000	912.5728
Cø	132800.0000	25711.0000	7262.0000

Table 5.7 (b): The Numerator of the Lateral Directional Transfer Functions in terms of Ø (s)

Ø- Transfer Function
$$\frac{\emptyset\ (s)}{\delta_A\ (s)} = \frac{Num_{\emptyset A}}{D_1} = \frac{s\ (A_\emptyset s^2 + B_\emptyset s + C_\emptyset)}{s\ (A_1 s^4 + B_1 s^3 + C_1 s^2 + D_1 s + E_1)}$$
$$Num_{\emptyset A} = \begin{bmatrix} Cessna\ 182 \\ Cessna\ 310 \\ Cessna\ 620 \end{bmatrix} = \begin{bmatrix} s\ (16519s^2 + 21481s + 132800) \\ s\ (3562.1s^2 + 2492.6s + 25711) \\ s\ (1715.4s^2 + 912.5728s + 7262) \end{bmatrix}$$

Table 5.8 (a): The Coefficients of Polynomials at the Numerator of ψ- Transfer Function

ψ- Transfer Function			
	Cessna 182	**Cessna 310**	**Cessna 620**
A_ψ	-750.9359	-281.0518	-188.1615
B_ψ	-15814.0000	-965.6449	-411.4450
C_ψ	-3316.0000	-299.3598	-83.3012
D_ψ	19084.0000	2627.7000	632.3967

Table 5.8 (b): The Numerator of the Lateral Directional Transfer Functions in terms of ψ (s)

ψ- Transfer Function
$$\frac{\psi\ (s)}{\delta_A\ (s)} = \frac{Num_{\psi A}}{D_1} = \frac{A_\psi s^3 + B_\psi s^2 + C_\psi s + D_\psi}{s\ (A_1 s^4 + B_1 s^3 + C_1 s^2 + D_1 s + E_1)}$$
$$Num_{\psi A} = \begin{bmatrix} Cessna\ 182 \\ Cessna\ 310 \\ Cessna\ 620 \end{bmatrix} = \begin{bmatrix} -750.9359s^3 - 15814s^2 - 3316s + 19084 \\ -281.0518s^3 - 965.6449s^2 - 299.3598s + 2627.7 \\ -188.1615s^3 - 411.445s^2 - 83.3012s + 632.3967 \end{bmatrix}$$

The results of the coefficients of polynomials at the denominator of the transfer functions are shown is Table 5.9 (a). After that, the denominator of the lateral directional transfer functions in terms of β (s), Ø (s) and ψ (s) due to aileron deflections for three types of aircraft models can be defined and presented in Table 5.9 (b).

Table 5.9 (a): The Coefficients of Polynomials at the Denominator of Lateral Directional Transfer Function

Denominator of Lateral Directional Transfer Function			
	Cessna 182	**Cessna 310**	**Cessna 620**
A_1	220.1000	312.5000	366.8000
B_1	3162.9000	899.5462	615.0269
C_1	6213.8000	2921.5000	1932.8000
D_1	30266.0000	5667.5000	2152.7000
E_1	540.4102	39.5241	12.3662

Table 5.9 (b): The Denominator of the Lateral Directional Transfer Functions

Denominator of Lateral Directional Transfer Function
$$\boldsymbol{D_1 = s\,(A_1 s^4 + B_1 s^3 + C_1 s^2 + D_1 s + E_1)}$$
$$\boldsymbol{D_1} = \begin{bmatrix} Cessna\ 182 \\ Cessna\ 310 \\ Cessna\ 620 \end{bmatrix} = \begin{bmatrix} s\,(220.1s^4 + 3162.9s^3 + 6213.8s^2 + 30266s + 540.4102) \\ s\,(312.5s^4 + 899.5462s^3 + 2921.5s^2 + 5667.5s + 39.5241) \\ s\,(366.8s^4 + 615.0269s^3 + 1932.8s^2 + 2152.7s + 12.3662) \end{bmatrix}$$

Based on the results of the denominator of the lateral directional transfer functions in terms of β (s), Ø (s) and ψ (s) due to aileron deflections, the system for three types of aircraft models is said to be dynamically stable. This is because the denominators of the transfer functions for all three types of aircraft models satisfies the Routh-Hurwitz criterion by inspection and fulfilling the following:

a. The equation has no missing terms.

b. All the coefficients are all same sign.

Table 5.10, 5.11 and 5.12 show the lateral directional transfer functions in terms of β (s), Ø (s) and ψ (s) respectively due to aileron deflections for three types of aircraft models.

Table 5.10 The Lateral Directional Transfer Functions in terms of β (s)

Type of Aircraft	β- Transfer Function
Cessna 182	$\frac{\beta\,(s)}{\delta_A\,(s)} = \frac{s\,(-51.1241s^2 + 17906s + 2690.1)}{s\,(220.1s^4 + 3162.9s^3 + 6213.8s^2 + 30266s + 540.4102)}$
Cessna 310	$\frac{\beta\,(s)}{\delta_A\,(s)} = \frac{s\,(-11.8107s^2 + 1251.2s + 165.2175)}{s\,(312.5s^4 + 899.5462s^3 + 2921.5s^2 + 5667.5s + 39.5241)}$
Cessna 620	$\frac{\beta\,(s)}{\delta_A\,(s)} = \frac{s\,(-5.9365s^2 + 527.6203s + 56.0072)}{s\,(366.8s^4 + 615.0269s^3 + 1932.8s^2 + 2152.7s + 12.3662)}$

Table 5.11 The Lateral Directional Transfer Functions in terms of Ø (s)

Type of Aircraft	Ø- Transfer Function
Cessna 182	$\frac{\emptyset\,(s)}{\delta_A\,(s)} = \frac{s\,(16519s^2 + 21481s + 132800)}{s\,(220.1s^4 + 3162.9s^3 + 6213.8s^2 + 30266s + 540.4102)}$
Cessna 310	$\frac{\emptyset\,(s)}{\delta_A\,(s)} = \frac{s\,(3562.1s^2 + 2492.6s + 25711)}{s\,(312.5s^4 + 899.5462s^3 + 2921.5s^2 + 5667.5s + 39.5241)}$
Cessna 620	$\frac{\emptyset\,(s)}{\delta_A\,(s)} = \frac{s\,(1715.4s^2 + 912.5728s + 7262}{s\,(366.8s^4 + 615.0269s^3 + 1932.8s^2 + 2152.7s + 12.3662)}$

Table 5.12 The Lateral Directional Transfer Functions in terms of ψ (s)

Type of Aircraft	ψ- Transfer Function
Cessna 182	$\frac{\psi\,(s)}{\delta_A\,(s)} = \frac{-750.9359s^3 - 15814s^2 - 3316s + 19084}{s\,(220.1s^4 + 3162.9s^3 + 6213.8s^2 + 30266s + 540.4102)}$
Cessna 310	$\frac{\psi\,(s)}{\delta_A\,(s)} = \frac{-281.0518s^3 - 965.6449s^2 - 299.3598s + 2627.7}{s\,(312.5s^4 + 899.5462s^3 + 2921.5s^2 + 5667.5s + 39.5241)}$
Cessna 620	$\frac{\psi\,(s)}{\delta_A\,(s)} = \frac{-188.1615s^3 - 411.445s^2 - 83.3012s + 632.3967}{s\,(366.8s^4 + 615.0269s^3 + 1932.8s^2 + 2152.7s + 12.3662)}$

5.5 Time History and Stability Behavior in the Lateral Directional

In this sub chapter, the time history and stability behavior in lateral directional for three types of aircraft models are discussed and the plots using the developed MATLAB computer code is presented to show the relationship between the lateral directional variables and the aileron deflections with respect to time. The lateral directional variables are sideslip angle ß, roll angle Ø and yaw angle ψ. The deflection of aileron is fixed as 1.0 degree, either up or down as input to obtain the stability behavior of the three types of aircraft models in the lateral directional.

In the results, the differences between the three types of aircraft models is the maximum and minimum value of sideslip angle ß, roll angle Ø and yaw angle ψ and also the time taken for the aircraft to return to its steady state behavior. These results are presented in Table 5.13 and 5.14.

5.5.1 Aileron Single Doublet Impulse

For single doublet impulse of aileron deflection, the aileron is set to be deflected 1 degree from (0^o, -1^o, 0^o) and from (0^o, 1^o, 0^o) as shown in Figure 5.5. The plots that show the stability behavior of sideslip angle ß, roll angle Ø and yaw angle ψ with respect to time due to single doublet impulse for three types of aircraft models are presented n Figure 5.5, 5.6 and 5.7 respectively.

The maximum and minimum value of sideslip angle ß, roll angle Ø and yaw angle ψ and the time taken for the aircraft to return to its steady state behavior for each aircraft model from Figure 5.5, 5.6 and 5.7 are presented in Table 5.13 and 5.14. The number (1) in Table 5.13 and 5.14 indicates the single doublet impulse deflection.

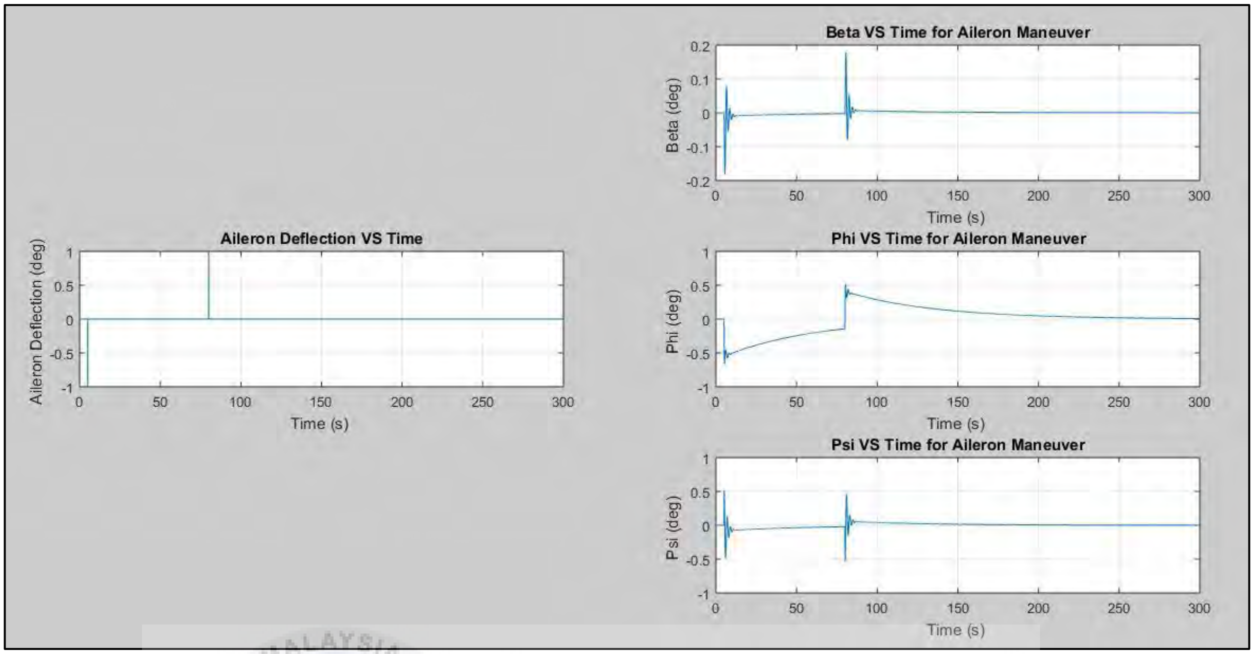


Figure 5.5 (a) Lateral Directional Responses Following Single Doublet Impulse Aileron Deflections of Cessna 182

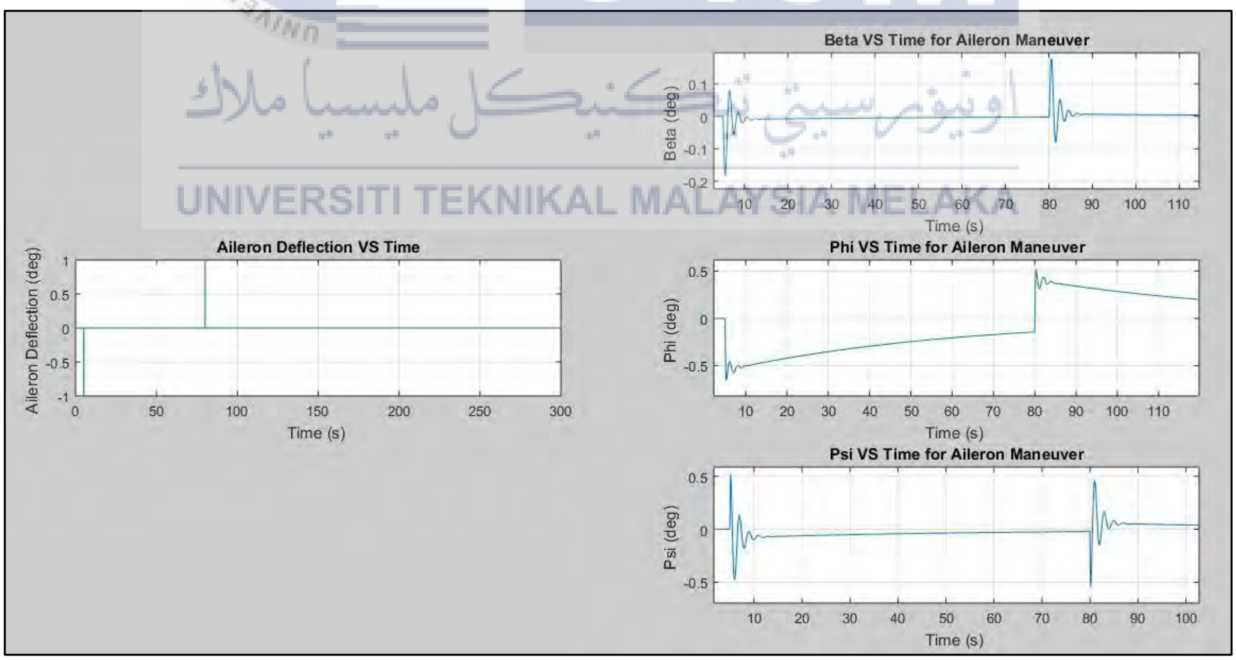


Figure 5.5 (b) Lateral Directional Responses Following Single Doublet Impulse Aileron Deflections of Cessna 182 (Zoomed)

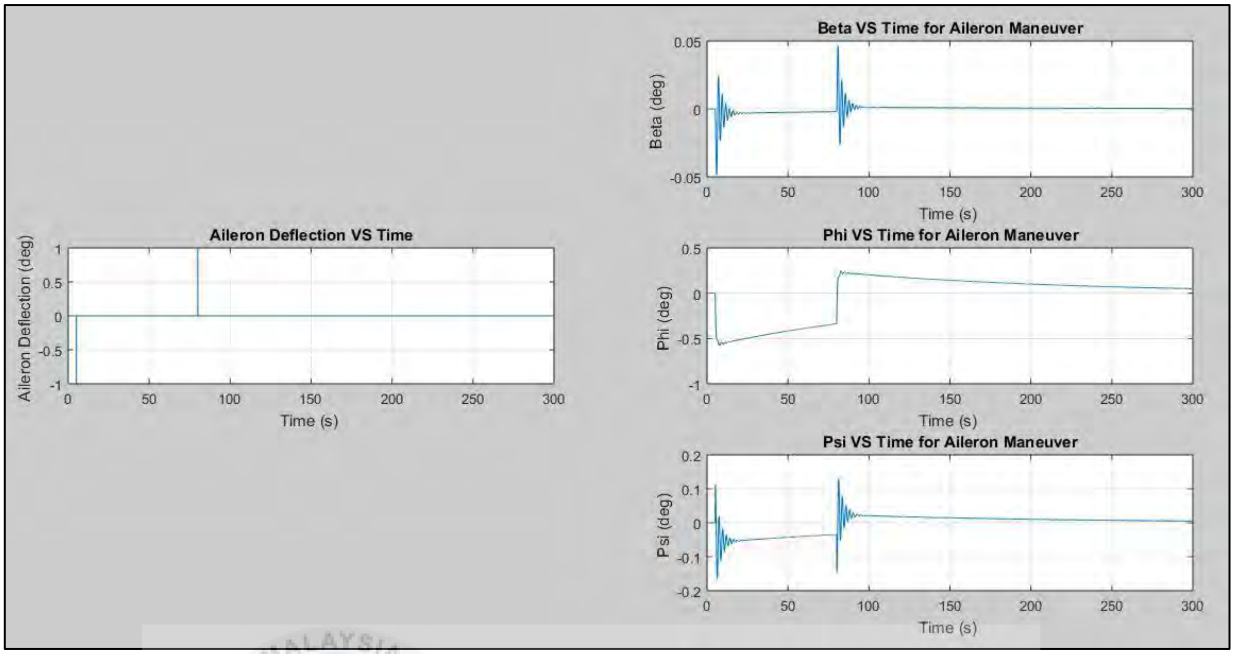


Figure 5.6 (a) Lateral Directional Responses Following Single Doublet Impulse Aileron Deflections of Cessna 310

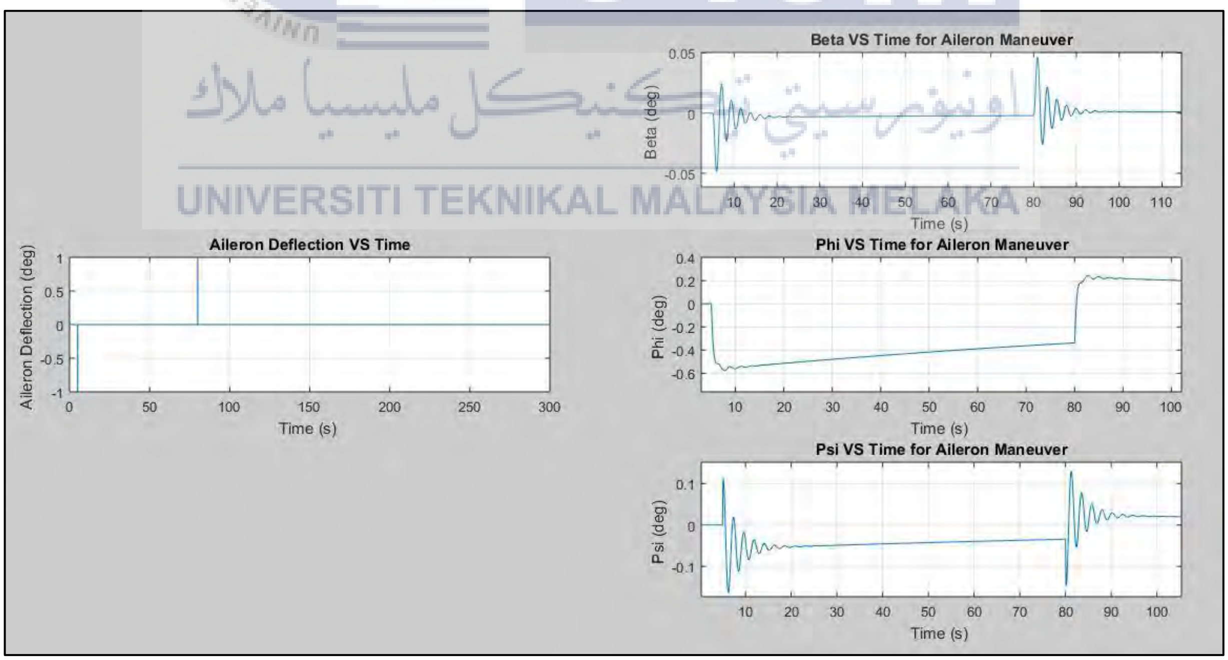


Figure 5.6 (b) Lateral Directional Responses Following Single Doublet Impulse Aileron Deflections of Cessna 310 (Zoomed)

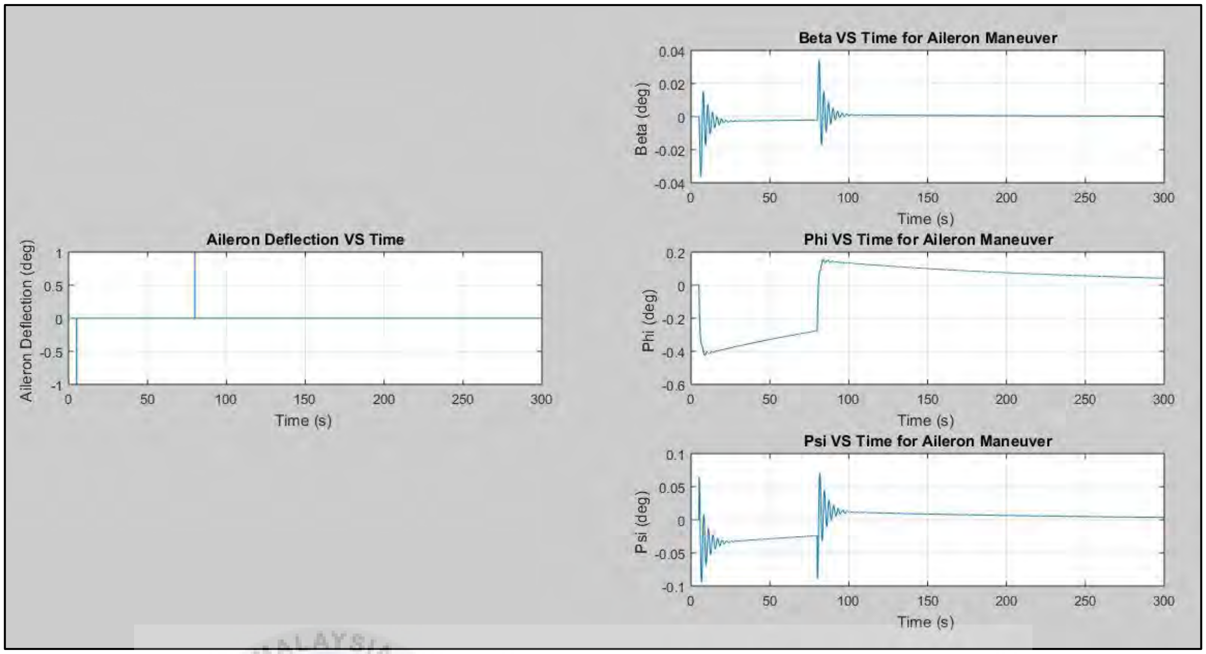


Figure 5.7 (a) Lateral Directional Responses Following Single Doublet Impulse Aileron Deflections of Cessna 620

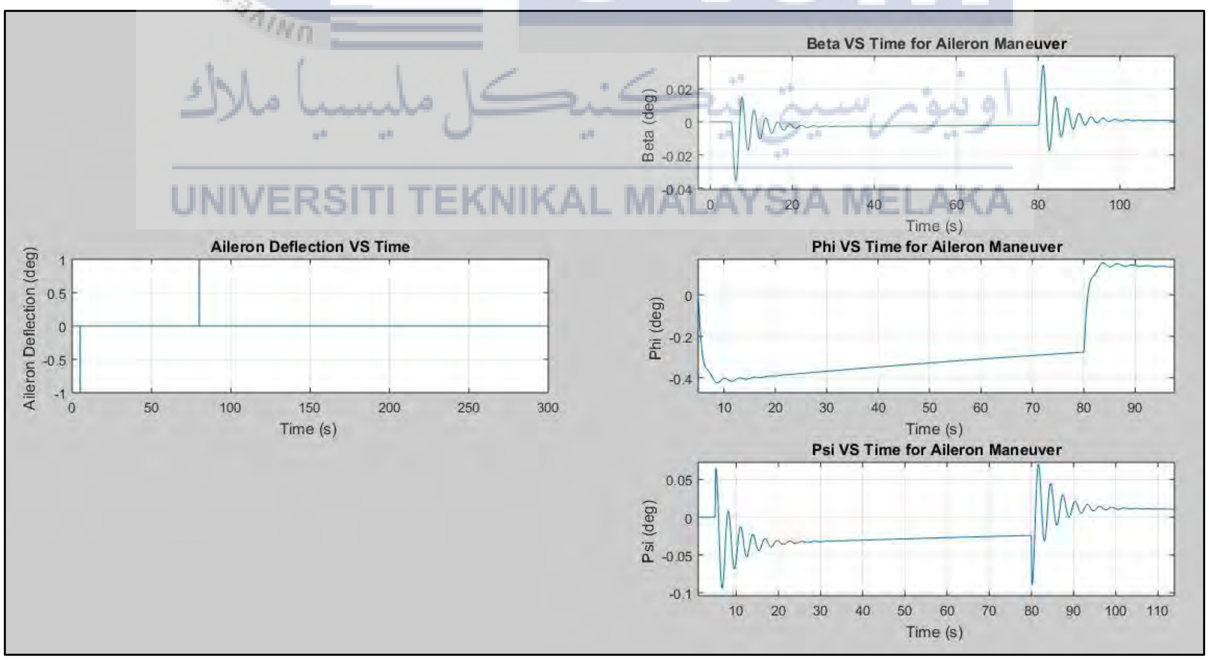


Figure 5.7 (b) Lateral Directional Responses Following Single Doublet Impulse Aileron Deflections of Cessna 620 (Zoomed)

5.5.2 Aileron Multiple Doublet Impulse

For multiple doublet impulse of aileron deflection, the aileron is set to be deflected at 1 degree from (0^o, -1^o, 0^o) and from (0^o, 1^o, 0^o) twice each as shown in Figure 5.8. The plots that show the stability behavior of sideslip angle ß, roll angle Ø and yaw angle ψ with respect to time due to multiple doublet impulse of aileron deflections for three types of aircraft models are presented n Figure 5.8, 5.9 and 5.10 respectively.

The maximum and minimum value of sideslip angle ß, roll angle Ø and yaw angle ψ and the time taken for the aircraft to return to its steady state behavior for each aircraft model from Figure 5.8, 5.9 and 5.10 are presented in Table 5.13 and 5.14. The number (2) in Table 5.13 and 5.14 indicates the multiple doublet impulse deflection.

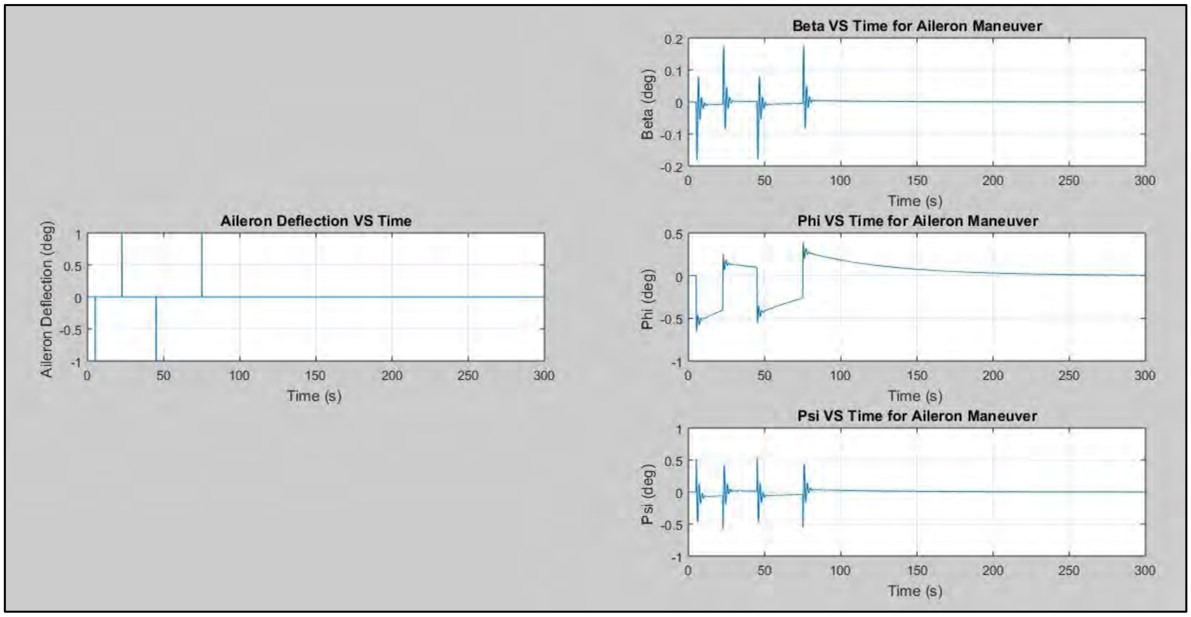


Figure 5.8 (a) Lateral Directional Responses Following Multiple Doublet Impulse Aileron Deflections of Cessna 182

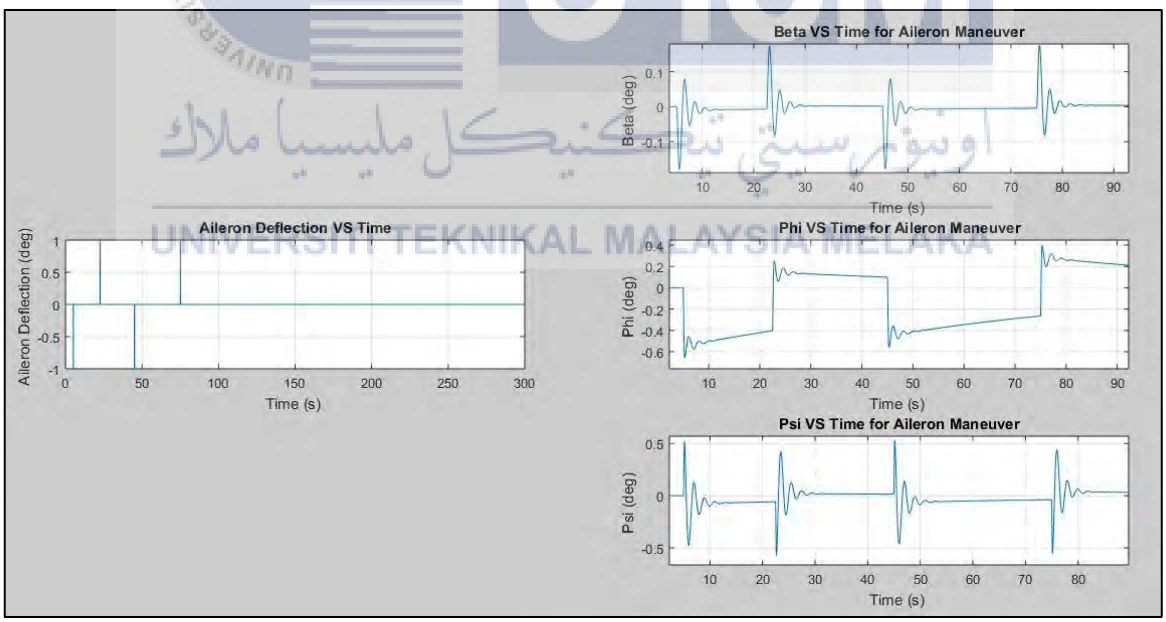


Figure 5.8 (b) Lateral Directional Responses Following Multiple Doublet Impulse Aileron Deflections of Cessna 182 (Zoomed)

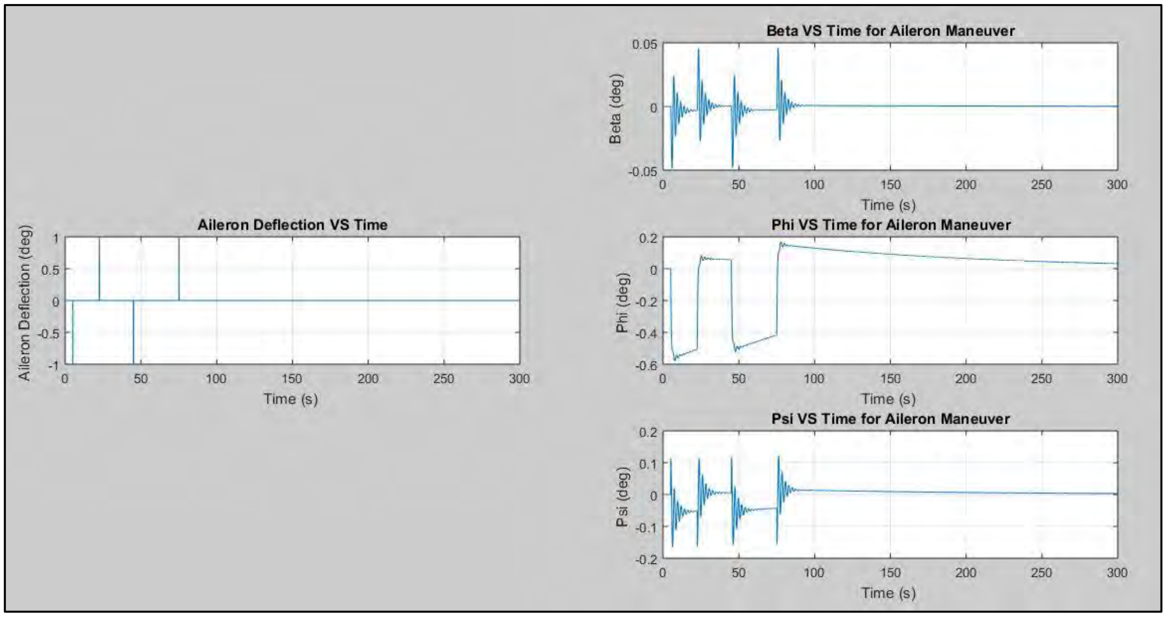


Figure 5.9 (a) Lateral Directional Responses Following Multiple Doublet Impulse Aileron Deflections of Cessna 310

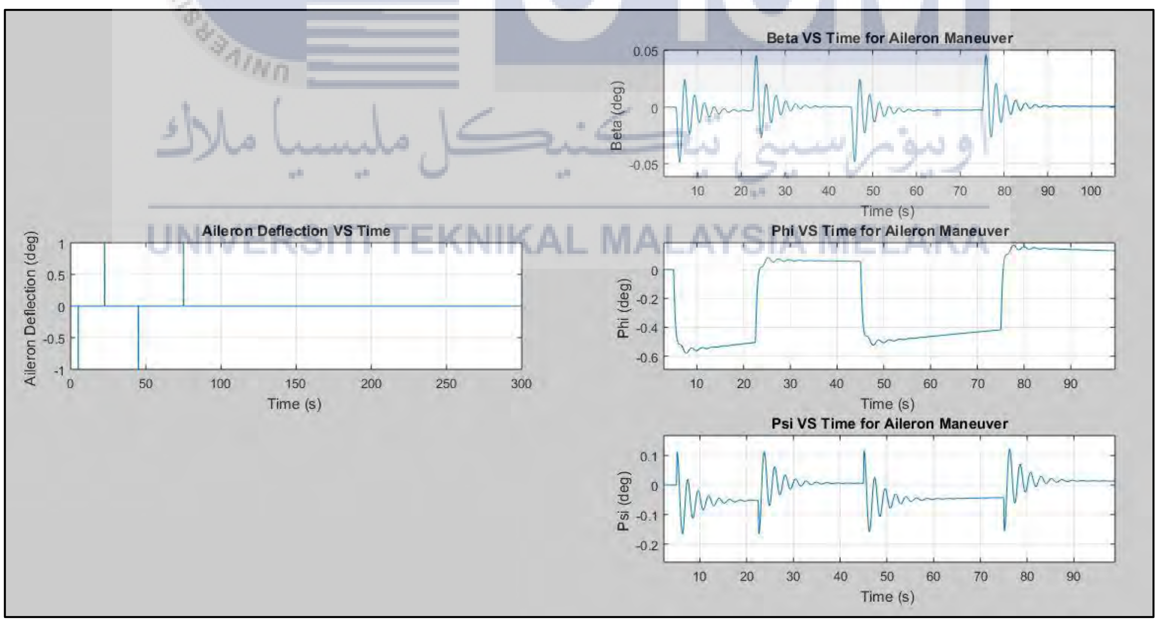


Figure 5.9 (b) Lateral Directional Responses Following Multiple Doublet Impulse Aileron Deflections of Cessna 310 (Zoomed)

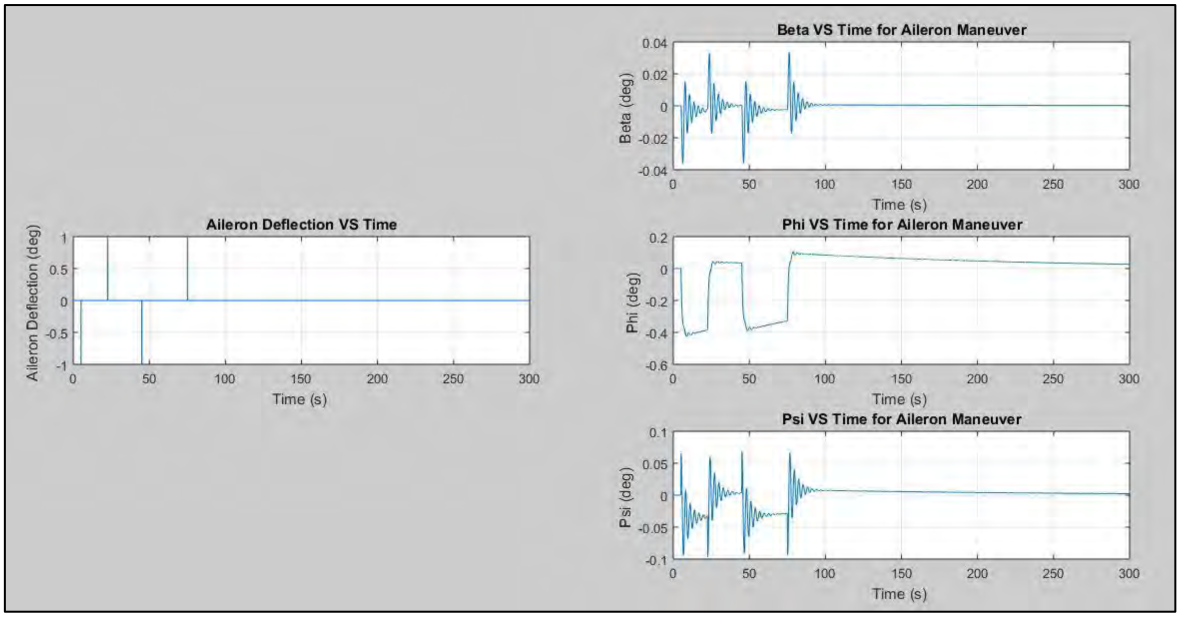


Figure 5.10 (a) Lateral Directional Responses Following Multiple Doublet Impulse Aileron Deflections of Cessna 620

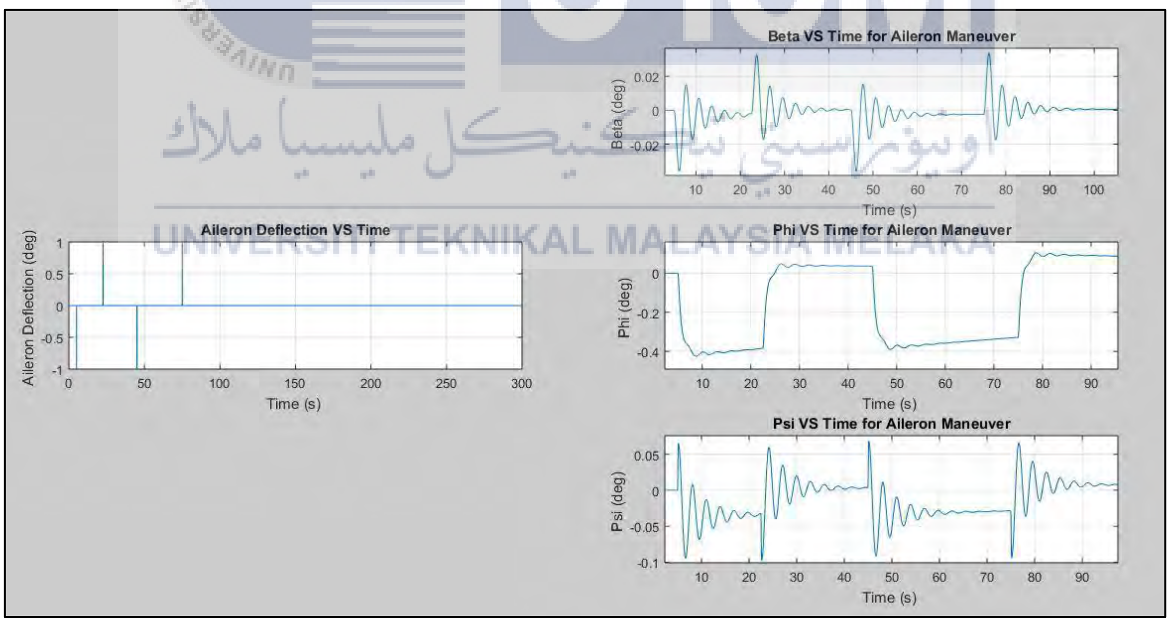


Figure 5.10 (b) Lateral Directional Responses Following Multiple Doublet Impulse Aileron Deflections of Cessna 620 (Zoomed)

5.5.3 Aileron Single Doublet

For single doublet of aileron deflection, the aileron is set to be deflected at 1 degree and holds for 0.5 second before deflected back to 0^o as shown in Figure 5.11. Same manner happens from (0^o, 1^o, 0^o). The plots that show the stability behavior of sideslip angle ß, roll angle Ø and yaw angle ψ with respect to time due to single doublet of aileron deflections for three types of aircraft models are presented n Figure 5.11, 5.12 and 5.13 respectively.

The maximum and minimum value of sideslip angle ß, roll angle Ø and yaw angle ψ and the time taken for the aircraft to return to its steady state behavior for each aircraft model from Figure 5.11, 5.12 and 5.13 are presented in Table 5.13 and 5.14. The number (3) in Table 5.13 and 5.14 indicates the single doublet deflection.

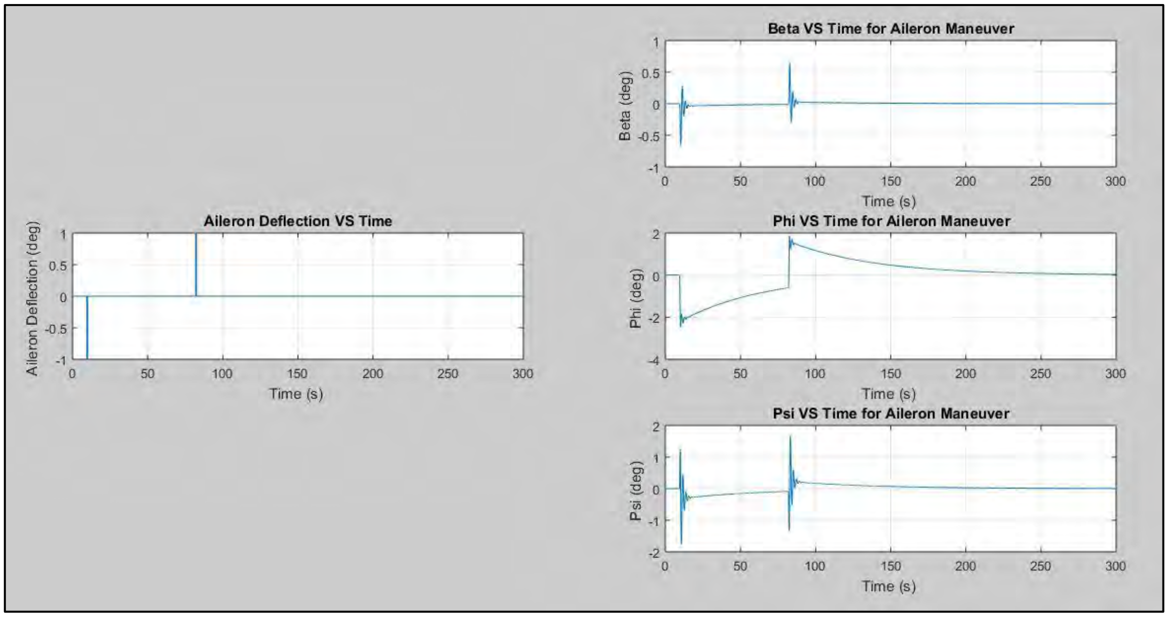


Figure 5.11 (a) Lateral Directional Responses Following Single Doublet Aileron Deflections of Cessna 182

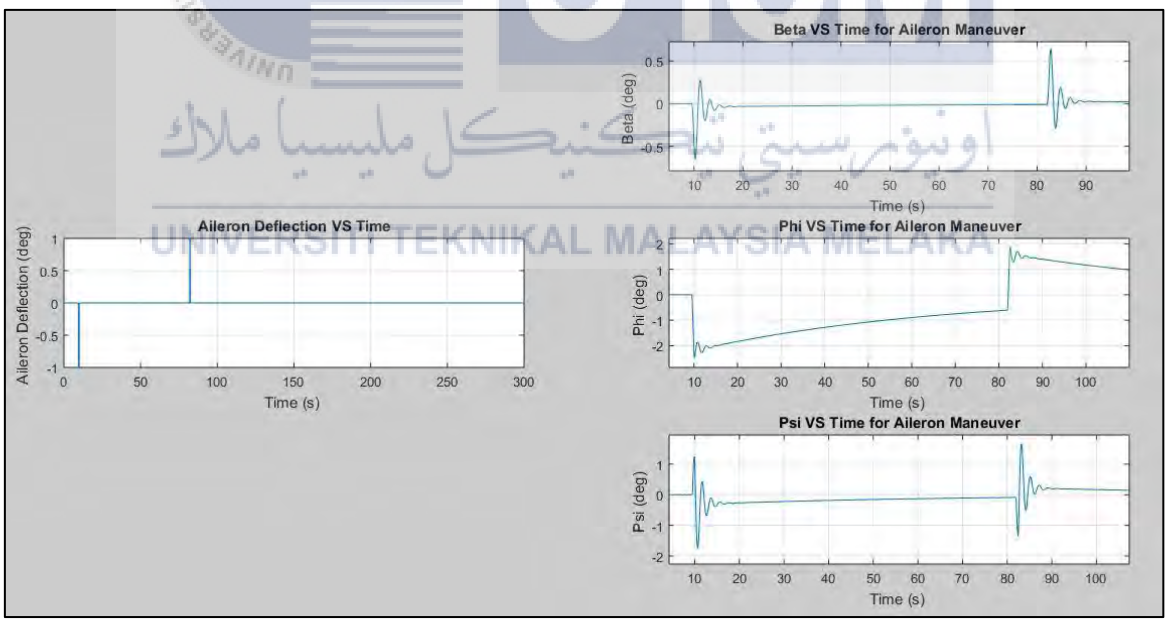


Figure 5.11 (b) Lateral Directional Responses Following Single Doublet Aileron Deflections of Cessna 182 (Zoomed)

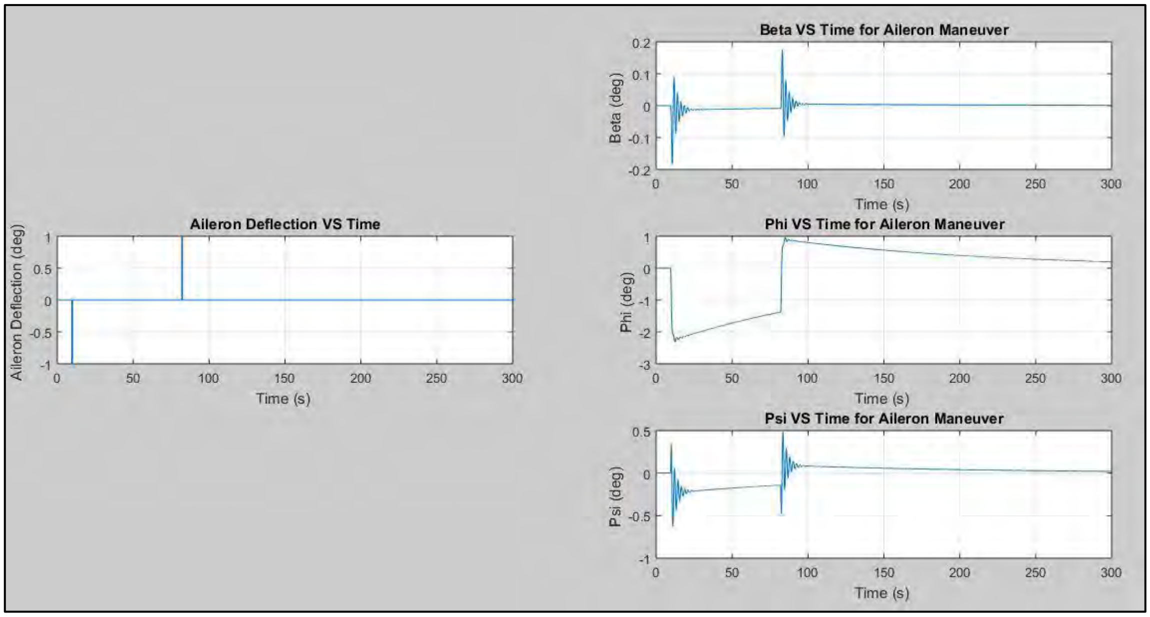


Figure 5.12 (a) Lateral Directional Responses Following Single Doublet Aileron Deflections of Cessna 310

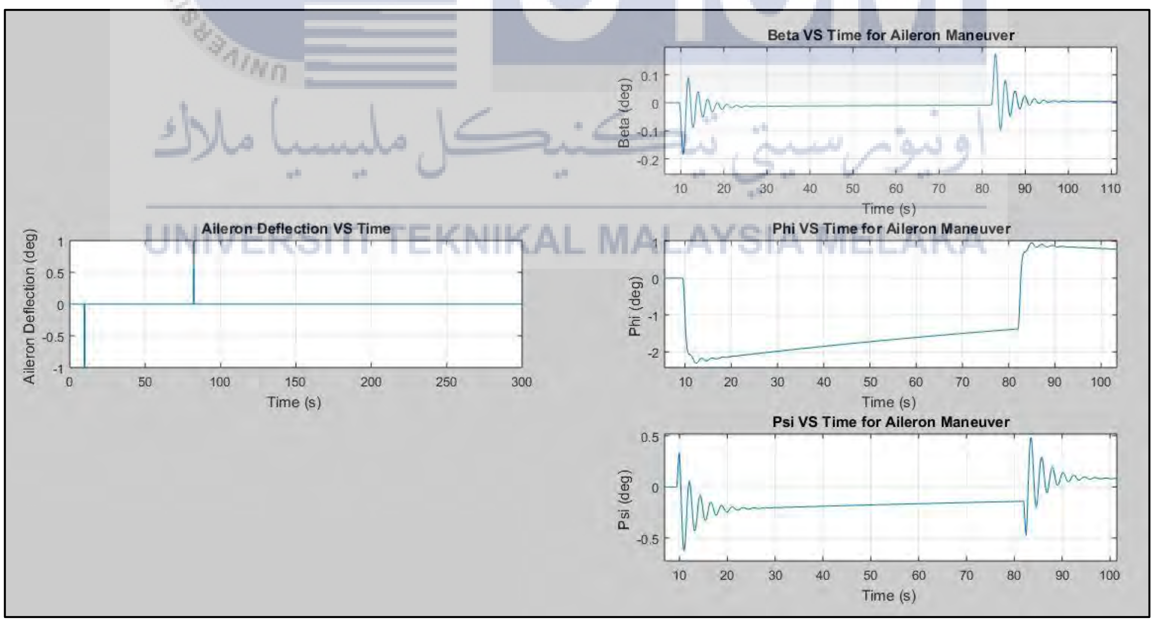


Figure 5.12 (b) Lateral Directional Responses Following Single Doublet Aileron Deflections of Cessna 310 (Zoomed)

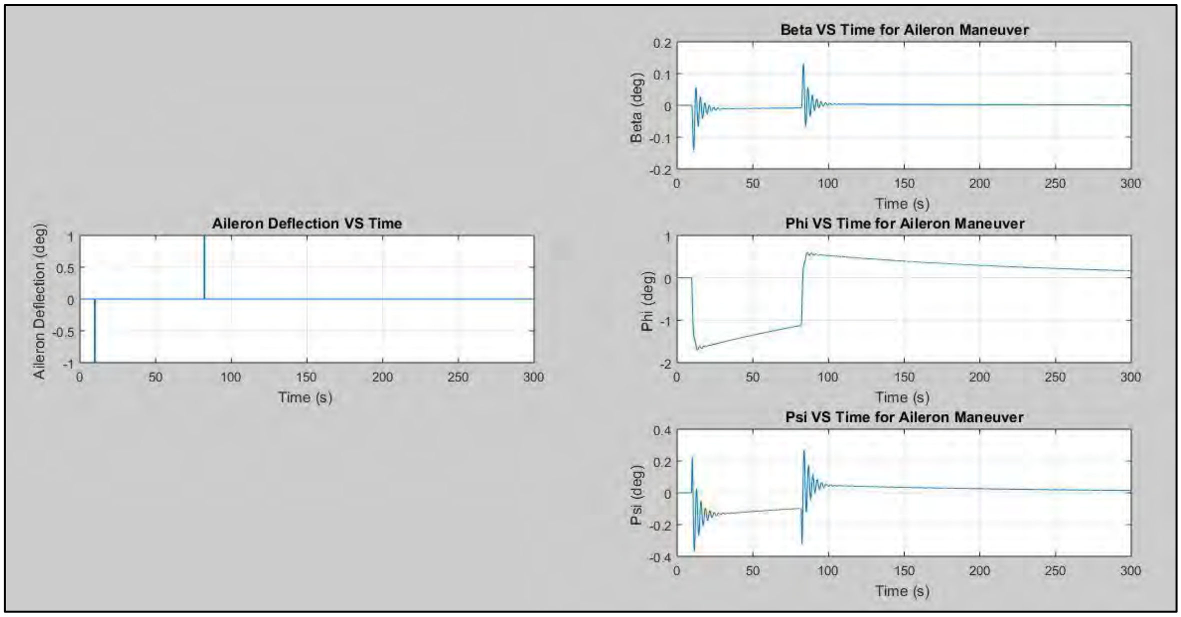


Figure 5.13 (a) Lateral Directional Responses Following Single Doublet Aileron Deflections of Cessna 620

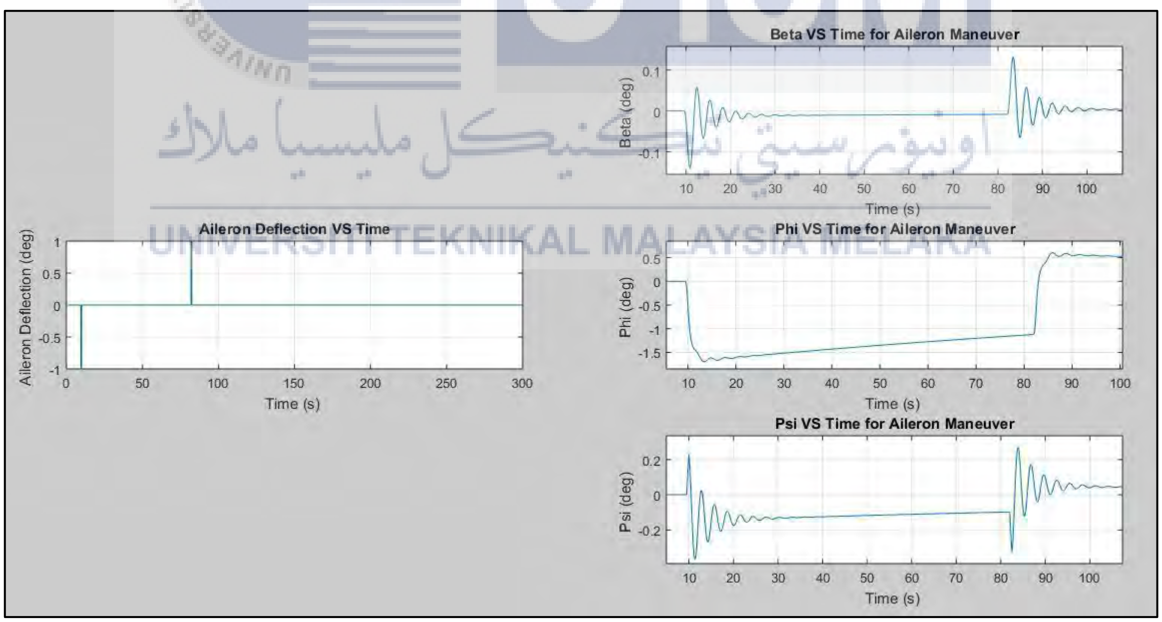


Figure 5.13 (b) Lateral Directional Responses Following Single Doublet Aileron Deflections of Cessna 620 (Zoomed)

5.5.4 Aileron Multiple Doublet

For multiple doublet of aileron deflection, the aileron is set to be deflected at 1 degree from (0^o, -1^o, 0^o) and from (0^o, 1^o, 0^o) twice each as shown in Figure 5.14. The deflection of aileron will be hold for 0.5s before deflected back to 0^o. The plots that show the stability behavior of sideslip angle ß, roll angle Ø and yaw angle ψ with respect to time due to multiple doublet of aileron deflections for three types of aircraft models are presented n Figure 5.14, 5.15 and 5.16 respectively.

The maximum and minimum value of sideslip angle ß, roll angle Ø and yaw angle ψ and the time taken for the aircraft to return to its steady state behavior for each aircraft model from Figure 5.14, 5.15 and 5.16 are presented in Table 5.13 and 5.14. The number (3) in Table 5.13 and 5.14 indicates the multiple doublet deflection.

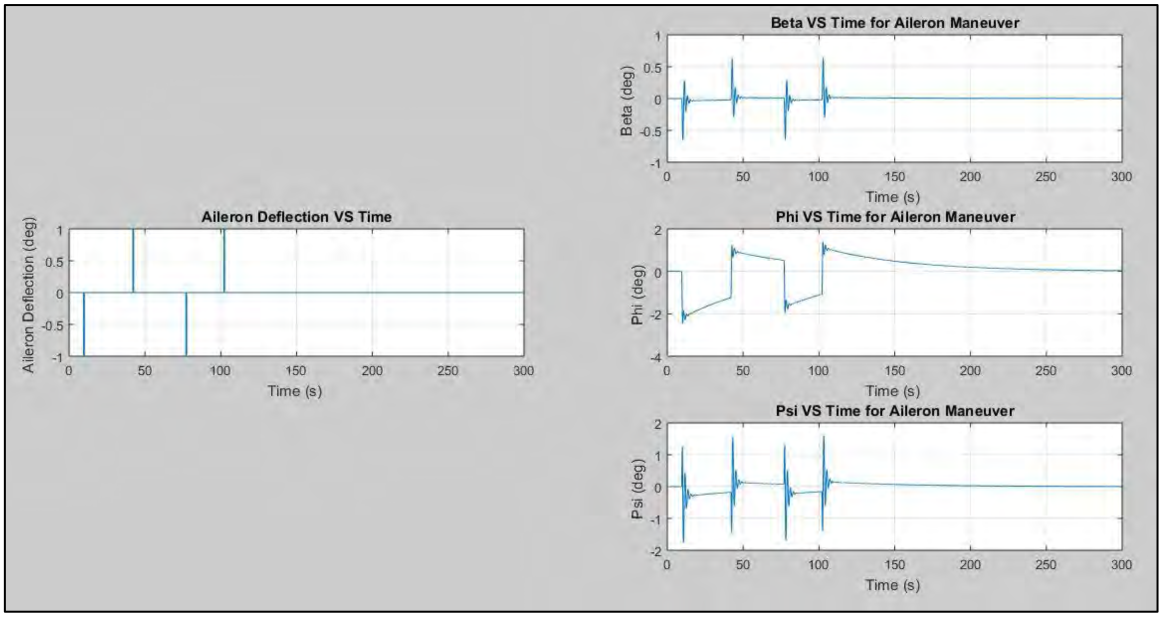


Figure 5.14 (a) Lateral Directional Responses Following Multiple Doublet Aileron Deflections of Cessna 182

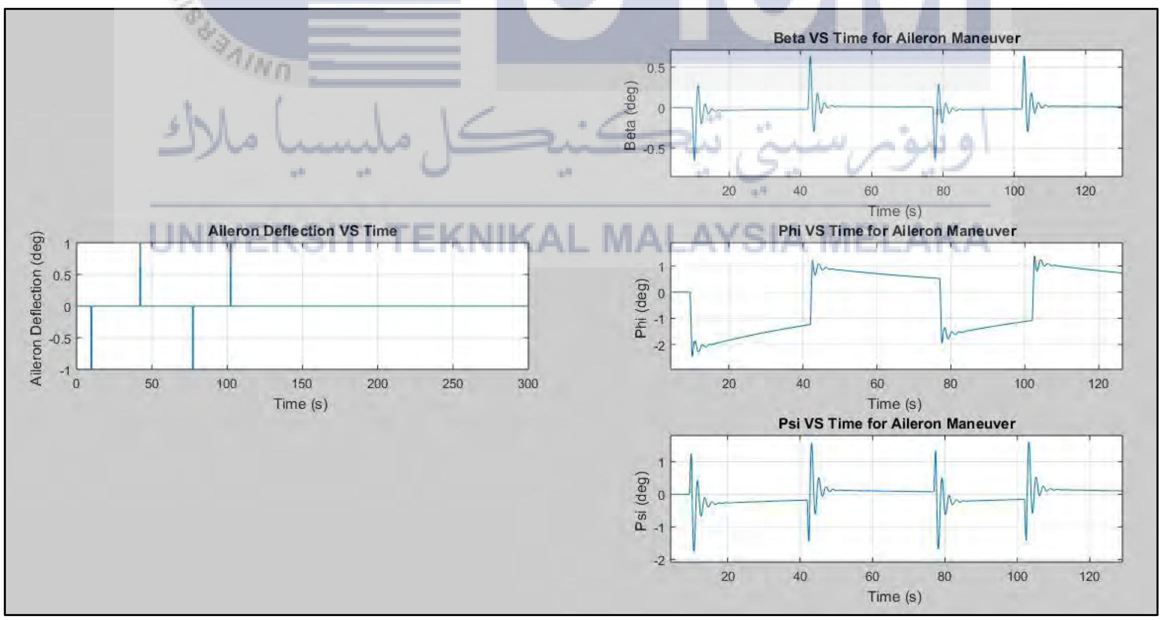


Figure 5.14 (b) Lateral Directional Responses Following Multiple Doublet Aileron Deflections of Cessna 182 (Zoomed)

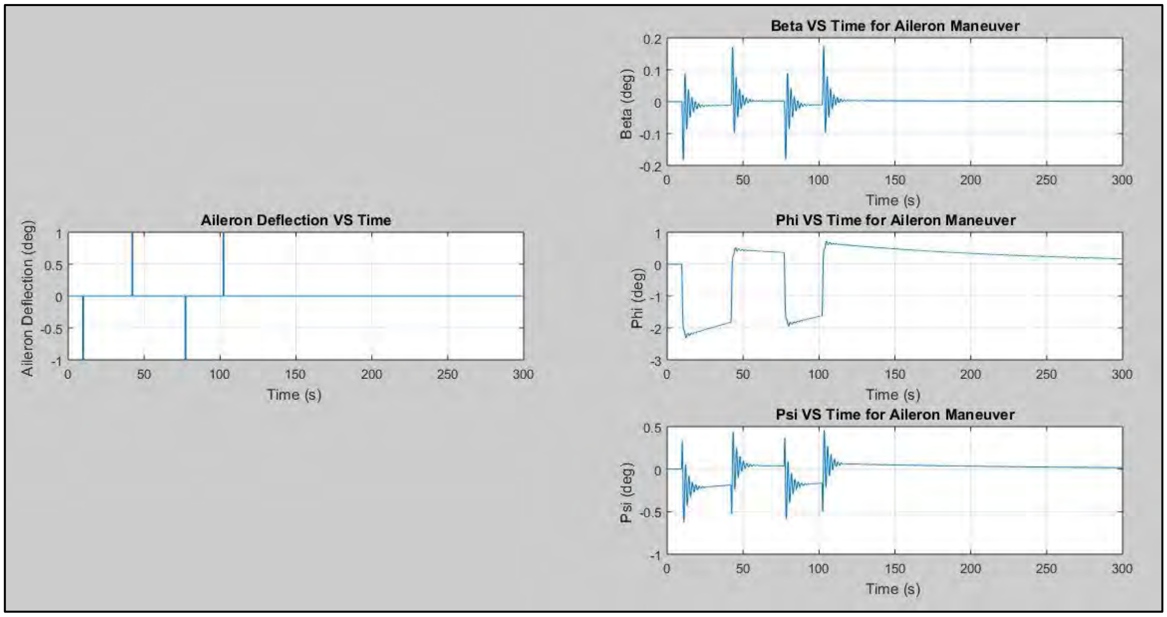


Figure 5.15 (a) Lateral Directional Responses Following Multiple Doublet Aileron Deflections of Cessna 310

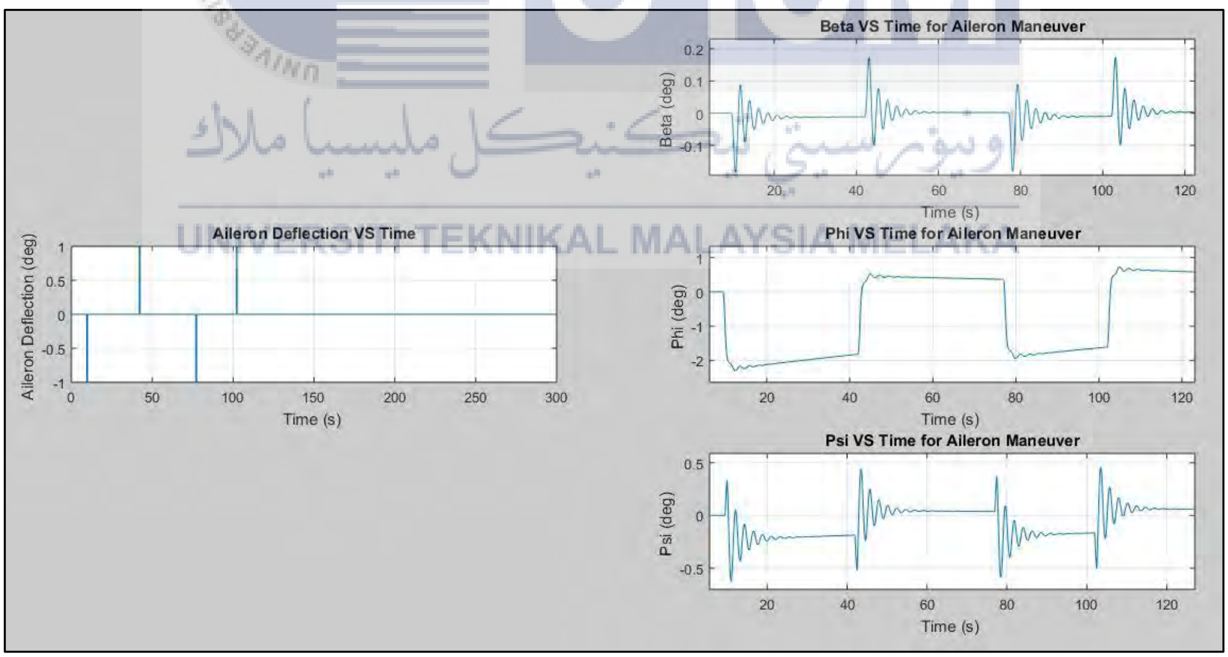


Figure 5.15 (b) Lateral Directional Responses Following Multiple Doublet Aileron Deflections of Cessna 310 (Zoomed)

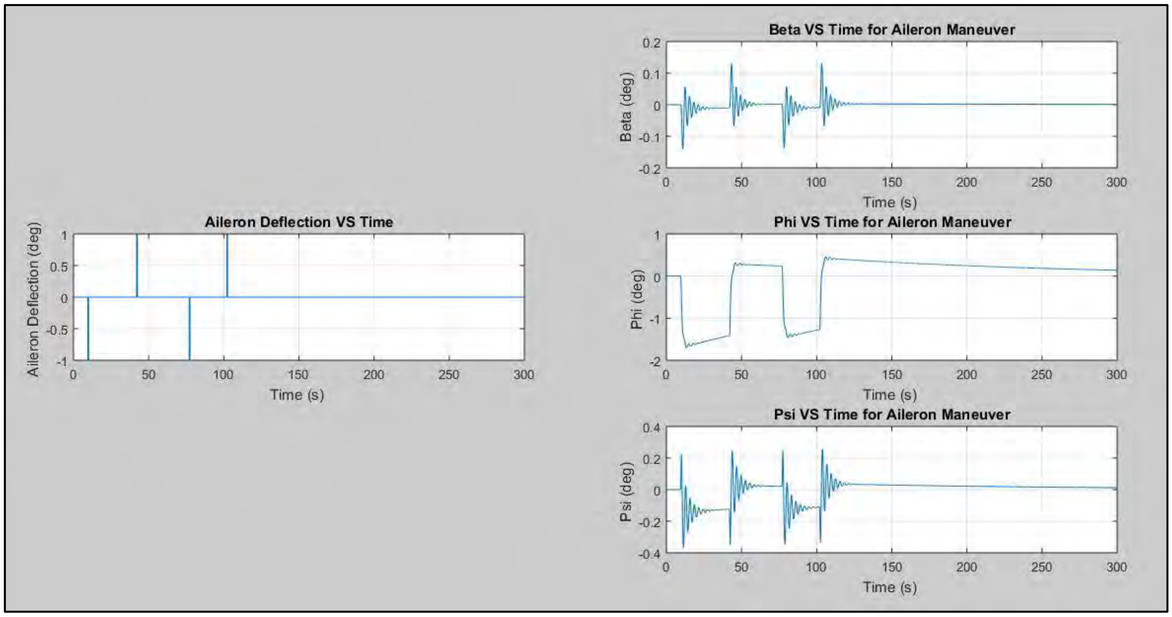


Figure 5.16 (a) Lateral Directional Responses Following Multiple Doublet Aileron Deflections of Cessna 620

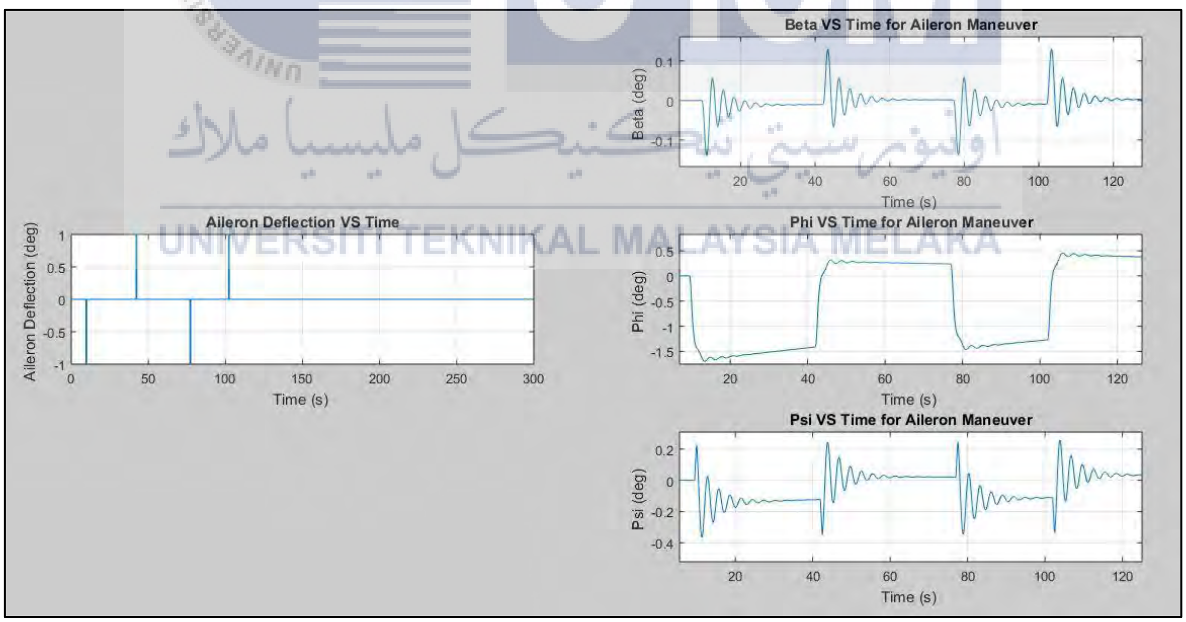


Figure 5.16 (b) Lateral Directional Responses Following Multiple Doublet Aileron Deflections of Cessna 620 (Zoomed)

Table 5.13 shows the maximum and minimum value of sideslip angle ß, roll angle Ø and yaw angle ψ due to aileron deflections for three types of aircraft models. Whereas, Table 5.14 shows the time taken for the aircraft to return to its steady state behavior after deflection of aileron for each aircraft model.

The number (1, 2, 3, and 4) in Table 5.13 and 5.14 indicate the type of aileron deflection which are single doublet impulse, multiple doublet impulse, single doublet and multiple doublet deflection.

Table 5.13: The Maximum and Minimum Values of β, ϕ, and ψ due to Aileron Deflection

Aircraft Model		**Sideslip angle β (Deg)**		**Yaw angle ψ (Deg)**		**Roll angle ϕ (Deg)**	
		Max	**Min**	**Max**	**Min**	**Max**	**Min**
Cessna 182	1	0.18	-0.18	0.50	-0.65	0.50	-0.55
	2	0.18	-0.18	0.39	-0.66	0.53	-0.58
	3	0.64	-0.66	1.87	-2.47	1.67	-1.75
	4	0.64	-0.66	1.40	-2.47	1.60	-1.75
Cessna 310	1	0.05	-0.05	0.25	-0.58	0.13	-0.17
	2	0.05	-0.05	0.17	-0.58	0.12	-0.16
	3	0.18	-0.18	0.96	-2.31	0.48	-0.62
	4	0.17	-0.18	0.72	-2.31	0.46	-0.62
Cessna 620	1	0.03	-0.04	0.15	-0.43	0.07	-0.09
	2	0.03	-0.04	0.11	-0.43	0.07	-0.10
	3	0.13	-0.14	0.60	-1.70	0.27	-0.37
	4	0.13	-0.14	0.46	-1.70	0.26	-0.37

Table 5.14: The Time Taken for the Aircraft to Return to its Steady State Behavior After Deflection of Aileron

Aircraft Model		**Sideslip angle β**	**Yaw angle ψ**	**Roll angle ϕ**
Cessna 182	1	140	300	200
	2	150	250	180
	3	180	300	260
	4	200	300	280
Cessna 310	1	120	300.0 at 0.1 degrees	300.0 at 0.01 degrees
	2	200	300.0 at 0.1 degrees	300.0 at 0.01 degrees
	3	240	300.0 at 0.2 degrees	300.0 at 0.02 degrees
	4	240	300.0 at 0.2 degrees	300.0 at 0.02 degrees
Cessna 620	1	240	300.0 at 0.1degrees	300.0 at 0.01 degrees
	2	250	300.0 at 0.1degrees	300.0 at 0.01 degrees
	3	260	300.0 at 0.2 degrees	300.0 at 0.01 degrees
	4	300	300.0 at 0.2 degrees	300.0 at 0.01 degrees

According to Aerospace Engineering desk reference, (2009), if the aircraft response time to control too long, it will become too sluggish. If the aircraft response time to control too short, it is more unstable, the rate of divergence becomes faster with increasing the workload of the pilot and reduce the passenger comfort.

Based on the lateral directional figures of all three types of aircraft models, all aircrafts tent to produce the similar stability behavior. Each aircraft tends to return to its steady state after the deflections of aileron. The aircrafts response immediately when the ailerons are deflected.

From those three type of aircraft models, it is found that the Cessna 182 has the biggest response to control and shortest time for it to return to its steady state after the deflection of aileron. It can be said that Cessna 182 is more sensitive to change compared to Cessna 310 and Cessna 620. Its sideslip angle ß, roll angle Ø and yaw angle ψ undergo damped immediately as the aileron deflects back to zero. This is because Cessna 182 has the highest value in stability and control derivatives relative to aileron deflection among all three types of aircraft models.

For Cessna 310 and Cessna 620, sideslip angle ß damped as the aileron deflects back to zero but roll angle Ø and yaw angle ψ damped as well in a slower manner than others and doesn't reach their steady state value at $t = 300s$.

With the same amount of input aileron deflection, all the three aircraft models show different response in terms of sideslip angle ß, roll angle Ø and yaw angle ψ. Therefore, it can be concluded that every aircraft will have its own aerodynamic characteristics, mass and inertia and its own stability behavior.

CHAPTER 6

CONCLUSION AND RECOMMENDATION

6.1 Conclusion

The input data of all three types of aircraft models for the developed computer code in MATLAB is collected from Marcello R. Napolitano (2012) . These input data are aircraft geometries, mass and moment of inertia properties, flight conditions data and lateral directional aerodynamic stability and control derivatives coefficients.

After that, control law is designed for lateral directional dimensional. All the lateral directional equations of motion are converted into computer code written in MATLAB programming language for flight dynamic analysis. The lateral directional flight analysis is carried out in the view of aileron only. Based on the computer code created, rolling and yawing moments with respect to aileron deflection for all three aircraft models are determined.

The validation is carried out by comparing the results for the lateral directional of aircraft model Learjet 24 with the results provided by Marcello R. Napolitano (2012). The comparison result shows these two cases have good agreement. The stability behavior is similar for both results. This shows that the computer code that developed in acceptable and reliable.

Based on the results of flight dynamic analysis that had been applied to three types of aircraft models, it can be concluded the present work has successfully developed a flight dynamic computer code in MATLAB software which allows user to evaluate and predict the aircraft response in lateral directional due to deflection of aileron. All three types of aircraft model have similar stability behavior. Each aircraft tends to return to its steady state after the aileron deflected back to zero. The aircrafts response immediately when the ailerons are deflected. It is found that the Cessna 182 has the biggest response to control and shortest time for it to return to its steady state.

All the three aircraft models show different response in terms of sideslip angle ß, roll angle Ø and yaw angle ψ. It means that each aircraft has its own dynamic stability behavior.

6.2 Recommendation

In this present work, the developed flight dynamic computer code in MATLAB software is created to allow user to evaluate and predict the aircraft response in lateral directional due to deflection of aileron. For the future work, it is suggested to include the aircraft response due to rudder deflection as well and to explode the aircraft response in longitudinal dimensional. In addition to this, it is possible to apply the developed computer code in MATLAB software for other aircraft, or to carry out dynamic stability due to different manner on how the control surface deflected to predict the aircraft response.

The comparison with results from other sources are necessary for further improvement in this developed MATLAB computer code to provide a more accurate result. It is suggested to find more available flight dynamic analysis for the purpose of comparison.

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APPENDIX A

Developed Computer Code in MATLAB Software

```
%Selection of Aircraft
Type_of_Aircraft  = menu ('SELECTION OF AIRCRAFT MODEL','CESSNA
182','CESSNA 310','CESSNA 620','VALIDATION-LEARJET 24');

if Type_of_Aircraft == 1

    %Cessna 182 Aircraft
    %Flight Condition: Cruise at altitude of 5,000

    %REFERENCE GEOMETRY
    S = 174;                %Wing Surface Area (ft^2)
    cbar = 4.9;             %Mean Aerodynamic Chord (ft)
    b = 36;                 %Wing Span (ft)
    xcg_bar = 0.264;        %Location of C.G. from Mean Aerodynamic Chord
(ft)

    %FLIGHT CONDITION DATA
    U1 = 220.1;             %True Airspeed (ft/s)
    M = 0.201;              %Mach Number
    alpha1 = 0/57.3;        %Angle of Attack (rad)
    theta1 = alpha1;
    q1 = 49.6;              %Dynamic Pressure (Ibs/ft^2)
    g = 32.2;               %Gravity Acceleration (ft/s^2)
    rad2deg = 57.3;
    deg2rad = 1/57.3;

    %MASS AND INERTIAL DATA
    W = 2650;               %Weight (Ibs)
    m = W/g;                %Mass (slug)
    IxxB = 948;             %Moment of Inertia x-axis along body axes
(slug*ft^2)
    IyyB = 1346;            %Moment of Inertia y-axis along body axes
(slug*ft^2)
    IzzB = 1967;            %Moment of Inertia z-axis along body axes
(slug*ft^2)
    IxzB = 0;               %Product of Inertia xz-plan along body axes
(slug*ft^2)

    %LATERAL-DIRECTIONAL STABILITY DERIVATIVES (DIMENSIONLESS ALONG
STABILITY
    %AXES)
    Clbeta = -0.0923;
    Clp = -0.484;
    Clr = 0.0798;
    Cybeta = -0.393;
    Cyp = -0.075;
    Cyr = 0.214;
    Cnbeta = 0.0587;
```

```
    CnTbeta = 0;
    Cnp = -0.0278;
    Cnr = -0.0937;

    %LATERAL-DIRECTIONAL CONTROL DERIVATIVES (DIMENSIONLESS ALONG
STABILITY
    %AXES)
    Cldeltaa = 0.229;
    Cldeltar = 0.0147;
    Cydeltaa = 0;
    Cydeltar = 0.187;
    Cndeltaa = -0.0216;
    Cndeltar = -0.0645;

end;

if Type_of_Aircraft == 2

    %Cessna 310 Aircraft
    %Flight Condition: Cruise at altitude of 8,000

    %REFERENCE GEOMETRY
    S = 175;                %Wing Surface Area (ft^2)
    cbar = 4.79;            %Mean Aerodynamic Chord (ft)
    b = 36.9;               %Wing Span (ft)
    xcg_bar = 0.33;         %Location of C.G. from Mean Aerodynamic Chord
(ft)

    %FLIGHT CONDITION DATA
    U1 = 312.5;             %True Airspeed (ft/s)
    M = 0.288;              %Mach Number
    alpha1 = 0/57.3;        %Angle of Attack (rad)
    theta1 = alpha1;
    q1 = 91.2;              %Dynamic Pressure (Ibs/ft^2)
    g = 32.2;               %Gravity Acceleration (ft/s^2)
    rad2deg = 57.3;
    deg2rad = 1/57.3;

    %MASS AND INERTIAL DATA
    W = 4600;               %Weight (Ibs)
    m = W/g;                %Mass (slug)
    IxxB = 8884;            %Moment of Inertia x-axis along body axes
(slug*ft^2)
    IyyB = 1939;            %Moment of Inertia y-axis along body axes
(slug*ft^2)
    IzzB = 11001;           %Moment of Inertia z-axis along body axes
(slug*ft^2)
    IxzB = 0;               %Product of Inertia xz-plan along body axes
(slug*ft^2)

    %LATERAL-DIRECTIONAL STABILITY DERIVATIVES (DIMENSIONLESS ALONG
STABILITY
    %AXES)
    Clbeta = -0.1096;
    Clp = -0.551;
    Clr = 0.0729;
    Cybeta = -0.698;
    Cyp = -0.1410;
    Cyr = 0.355;
    Cnbeta = 0.1444;
```

```
    CnTbeta = 0;
    Cnp = -0.0257;
    Cnr = -0.1495;

    %LATERAL-DIRECTIONAL CONTROL DERIVATIVES (DIMENSIONLESS ALONG
STABILITY
    %AXES)
    Cldeltaa = 0.172;
    Cldeltar = 0.0192;
    Cydeltaa = 0;
    Cydeltar = 0.230;
    Cndeltaa = -0.0168;
    Cndeltar = -0.1152;

end;

if Type_of_Aircraft == 3

    %Cessna 620 Aircraft
    %Flight Condition: Cruise at altitude of 18,000

    %REFERENCE GEOMETRY
    S = 340;                %Wing Surface Area (ft^2)
    cbar = 6.58;            %Mean Aerodynamic Chord (ft)
    b = 55.1;               %Wing Span (ft)
    xcg_bar = 0.25;         %Location of C.G. from Mean Aerodynamic Chord
(ft)

    %FLIGHT CONDITION DATA
    U1 = 366.8;             %True Airspeed (ft/s)
    M = 0.351;              %Mach Number
    alpha1 = 0/57.3;        %Angle of Attack (rad)
    theta1 = alpha1;
    q1 = 91.1;              %Dynamic Pressure (Ibs/ft^2)
    g = 32.2;               %Gravity Acceleration (ft/s^2)
    rad2deg = 57.3;
    deg2rad = 1/57.3;

    %MASS AND INERTIAL DATA
    W = 15000;              %Weight (Ibs)
    m = W/g;                %Mass (slug)
    IxxB = 64811;           %Moment of Inertia x-axis along body axes
(slug*ft^2)
    IyyB = 17300;           %Moment of Inertia y-axis along body axes
(slug*ft^2)
    IzzB = 64543;           %Moment of Inertia z-axis along body axes
(slug*ft^2)
    IxzB = 0;               %Product of Inertia xz-plan along body axes
(slug*ft^2)

    %LATERAL-DIRECTIONAL STABILITY DERIVATIVES (DIMENSIONLESS ALONG
STABILITY
    %AXES)
    Clbeta = -0.1381;
    Clp = -0.566;
    Clr = 0.1166;
    Cybeta = -0.883;
    Cyp = -0.227;
    Cyr = 0.448;
    Cnbeta = 0.1739;
```

```
    CnTbeta = 0;
    Cnp = -0.0501;
    Cnr = -0.2;

    %LATERAL-DIRECTIONAL CONTROL DERIVATIVES (DIMENSIONLESS ALONG
STABILITY
    %AXES)
    Cldeltaa = 0.1776;
    Cldeltar = 0.02;
    Cydeltaa = 0;
    Cydeltar = 0.2;
    Cndeltaa = -0.0194;
    Cndeltar = -0.1054;

end;

if Type_of_Aircraft == 4

    %Learjet 24 Aircraft
    %Flight Condition: Cruise at altitude of 40,000

    %REFERENCE GEOMETRY
    S = 230;                %Wing Surface Area (ft^2)
    cbar = 7.00;            %Mean Aerodynamic Chord (ft)
    b = 34.0;               %Wing Span (ft)
    xcg_bar = 0.32;         %Location of C.G. from Mean Aerodynamic Chord
(ft)

    %FLIGHT CONDITION DATA
    U1 = 677.0;             %True Airspeed (ft/s)
    M = 0.700;              %Mach Number
    alpha1 = 1.5/57.3;      %Angle of Attack (rad)
    theta1 = alpha1;
    q1 = 134.6;             %Dynamic Pressure (Ibs/ft^2)
    g = 32.2;               %Gravity Acceleration (ft/s^2)
    rad2deg = 57.3;
    deg2rad = 1/57.3;

    %MASS AND INERTIAL DATA
    W = 9000;               %Weight (Ibs)
    m = W/g;                %Mass (slug)
    IxxB = 6000;            %Moment of Inertia x-axis along body axes
(slug*ft^2)
    IyyB = 17800;           %Moment of Inertia y-axis along body axes
(slug*ft^2)
    IzzB = 25000;           %Moment of Inertia z-axis along body axes
(slug*ft^2)
    IxzB = 1400;            %Product of Inertia xz-plan along body axes
(slug*ft^2)

    %LATERAL-DIRECTIONAL STABILITY DERIVATIVES (DIMENSIONLESS ALONG
STABILITY
    %AXES)
    Clbeta = -0.100;
    Clp = -0.450;
    Clr = 0.140;
    Cybeta = -0.730;
    Cyp = 0;
    Cyr = 0.400;
    Cnbeta = 0.124;
```

```
    CnTbeta = 0;
    Cnp = -0.022;
    Cnr = -0.2;

    %LATERAL-DIRECTIONAL CONTROL DERIVATIVES (DIMENSIONLESS ALONG
STABILITY
    %AXES)
    Cldeltaa = 0.178;
    Cldeltar = 0.021;
    Cydeltaa = 0;
    Cydeltar = 0.140;
    Cndeltaa = -0.020;
    Cndeltar = -0.074;

end;

%TRANSFORMATION OF MOMENT AND PRODUCT OF INERTIAL FROM BODY TO STABILITY
%AXIS
A = [(cos(alpha1))^2, (sin(alpha1))^2, -sin(2*alpha1);
    (sin(alpha1))^2, (cos(alpha1))^2, sin(2*alpha1);
    (0.5*sin(2*alpha1)), (-0.5*sin(2*alpha1)), cos(2*alpha1)];
ib = [IxxB, IzzB, IxzB]';
c = A*ib;

Ixx = c(1,1);
Izz = c(2,1);
Ixz = c(3,1);

Abar1 = (Ixz/Ixx);
Bbar1 = (Ixz/Izz);

%LATERAL-DIRECTIONAL DIMENSIONAL STABILITY DERIVATIVES
Ybeta = (q1*S*Cybeta)/m;
Yp = (q1*S*b*Cyp)/(2*m*U1);
Yr = (q1*S*b*Cyr)/(2*m*U1);

Lbeta = (q1*S*b*Clbeta)/Ixx;
Lp = (q1*S*(b^2)*Clp)/(2*Ixx*U1);
Lr = (q1*S*(b^2)*Clr)/(2*Ixx*U1);

Nbeta = (q1*S*b*Cnbeta)/Izz;
NTbeta = (q1*S*b*CnTbeta)/Izz;
Np = (q1*S*(b^2)*Cnp)/(2*Izz*U1);
Nr = (q1*S*(b^2)*Cnr)/(2*Izz*U1);

%LATERAL-DIRECTIONAL DIMENSIONAL CONTROL DERIVATIVES
Ydeltaa = (q1*S*Cydeltaa)/m;
Ydeltar = (q1*S*Cydeltar)/m;

Ldeltaa = (q1*S*b*Cldeltaa)/Ixx;
Ldeltar = (q1*S*b*Cldeltar)/Ixx;

Ndeltaa = (q1*S*b*Cndeltaa)/Izz;
Ndeltar = (q1*S*b*Cndeltar)/Izz;

%COEFFICIENTS OF THE NUMERATOR, NUM(s)OF BETA-TRANSFER FUNCTION
%AILERON INPUT
AbetaI = Ydeltaa*(1-(Abar1*Bbar1));
```

```
BbetaI = -
Ydeltaa*(Nr+Lp+(Abar1*Np)+(Bbar1*Lr))+(Yp*(Ldeltaa+(Ndeltaa*Abar1)))+(Yr*
((Ldeltaa*Bbar1)+Ndeltaa))-((U1*(Ldeltaa*Bbar1)+Ndeltaa));
CbetaI = Ydeltaa*((Lp*Nr)-(Np*Lr))+(Yp*((Ndeltaa*Lr)-
(Ldeltaa*Nr)))+(g*cos(theta1)*(Ldeltaa+(Ndeltaa*Abar1)))+(Yr*((Ldeltaa*Np
)-(Ndeltaa*Lp)))-(U1*((Ldeltaa*Np)-(Ndeltaa*Lp)));
DbetaI = g*cos(theta1)*((Ndeltaa*Lr)-(Ldeltaa*Nr));

NbetaI = [AbetaI, BbetaI, CbetaI, DbetaI];

%COEFFICIENTS OF THE NUMERATOR, NUM(s)OF PHI-TRANSFER FUNCTION
%AILERON INPUT
AphiI = U1*(Ldeltaa+(Ndeltaa*Abar1));
BphiI = U1*((Ndeltaa*Lr)-(Ldeltaa*Nr))-
(Ybeta*(Ldeltaa+Ndeltaa*Abar1))+(Ydeltaa*(Lbeta+(Nbeta*Abar1)+(NTbeta*Aba
r1)));
CphiI = -Ybeta*((Ndeltaa*Lr)-
(Ldeltaa*Nr))+(Ydeltaa*((Lr*Nbeta)+(Lr*NTbeta)-(Nr*Lbeta)))+((U1-
Yr)*((Nbeta*Ldeltaa)+(NTbeta*Ldeltaa)-(Lbeta*Ndeltaa)));

NphiI = [AphiI, BphiI, CphiI];

%COEFFICIENTS OF THE NUMERATOR, NUM(s)OF PSI-TRANSFER FUNCTION
%AILERON INPUT
ApsiI = U1*(Ndeltaa+(Ldeltaa*Bbar1));
BpsiI = U1*((Ldeltaa*Np)-(Ndeltaa*Lp))-
(Ybeta*(Ndeltaa+(Ldeltaa*Bbar1)))+(Ydeltaa*((Lbeta*Bbar1)+Nbeta+NTbeta));
CpsiI = -Ybeta*((Ldeltaa*Np)-
(Ndeltaa*Lp))+(Yp*((Nbeta*Ldeltaa)+(NTbeta*Ldeltaa)-
(Lbeta*Ndeltaa)))+(Ydeltaa*((Lbeta*Np)-(Nbeta*Lp)-(NTbeta*Lp)));
DpsiI = g*cos(theta1)*((Nbeta*Ldeltaa)+(NTbeta*Ldeltaa)-(Lbeta*Ndeltaa));

NpsiI = [ApsiI, BpsiI, CpsiI, DpsiI];

%COEFFICIENTS DENOMINATOR FOR THE LATERAL-DIRECTIONAL CHARACTERISTIC
EQUATION
A2 = U1*(1-(Abar1*Bbar1));
B2 = -Ybeta*(1-(Abar1*Bbar1))-(U1*(Lp+Nr+(Abar1*Np)+(Bbar1*Lr)));
C2 = U1*((Lp*Nr)-(Lr*Np))+(Ybeta*(Nr+Lp+(Abar1*Np)+(Bbar1*Lr)))-
(Yp*(Lbeta+(Nbeta*Abar1)+(NTbeta*Abar1)))+(U1*((Lbeta*Bbar1)+Nbeta+NTbeta
))-(Yr*((Lbeta*Bbar1)+Nbeta+NTbeta));
D2 = -Ybeta*((Lp*Nr)-(Lr*Np))+(Yp*((Lbeta*Nr)-(Nbeta*Lr)-(NTbeta*Lr)))-
(g*cos(theta1)*(Lbeta+(Nbeta*Abar1)+(NTbeta*Abar1)))+(U1*((Lbeta*Np)-
(Nbeta*Lp)-(NTbeta*Lp)))-(Yr*((Lbeta*Np)-(Nbeta*Lp)-(NTbeta*Lp)));
E2 = g*cos(theta1)*((Lbeta*Nr)-(Nbeta*Lr)-(NTbeta*Lr));

Dbar2 = [A2, B2, C2, D2, E2];

%CHECKING OF DYNAMIC STABILITY VIA ROUTH-HURWITZ STABILITY CRITERIA

ROUTH = D2*((B2*C2)-(A2*D2))-((B2^2)*E2);

if A2 <= 0
    printf ('LATERAL DIRECTIONAL DYNAMIC STABILITY NOT SATISFIED!')

elseif B2 <=0
    printf ('LATERAL DIRECTIONAL DYNAMIC STABILITY NOT SATISFIED!')

elseif C2 <=0
```

```
    printf ('LATERAL DIRECTIONAL DYNAMIC STABILITY NOT SATISFIED!')

elseif D2 <=0
    printf ('LATERAL DIRECTIONAL DYNAMIC STABILITY NOT SATISFIED!')

elseif E2 <=0
    printf ('LATERAL DIRECTIONAL DYNAMIC STABILITY NOT SATISFIED!')

elseif ROUTH <=0
    printf ('LATERAL DIRECTIONAL DYNAMIC STABILITY NOT SATISFIED!')

end;

%TIME COLUMN VECTOR FOR SIMULATION
t = [0:0.025:300]';

%LIBRARY OF PILOT AILERON MANEUVERS
Library  = menu ('AILERONS INPUTS','SINGLE DOUBLET IMPULSE
AILERON','MULTIPLE DOUBLET IMPULSE AILERON','SINGLET DOUBLET
AILERON','MULTIPLE DOUBLETS AILERON');


%SINGLE DOUBLET IMPULSE AILERON
if Library == 1
    for i = 1:200
        da(i,1) = 0.0/57.3;
    end;

    for i = 201:205
        da(i,1) = -1.0/57.3;
    end;

    for i = 206:3200
        da(i,1) = 0.0/57.3;
    end;

    for i = 3201:3205
        da(i,1) = 1.0/57.3;
    end;

    for i = 3206:12001
        da(i,1) = 0.0/57.3;
    end;
end;

%MULTIPLE DOUBLET IMPULSE AILERON
if Library == 2
    for i = 1:200
        da(i,1) = 0.0/57.3;
    end;

    for i = 201:205
        da(i,1) = -1.0/57.3;
    end;

    for i = 206:900
        da(i,1) = 0.0/57.3;
    end;
```

```
    for i = 901:905
        da(i,1) = 1.0/57.3;
    end;

    for i = 906:1800
        da(i,1) = 0.0/57.3;
    end;

    for i = 1801:1805
        da(i,1) = -1.0/57.3;
    end;

    for i = 1806:3000
        da(i,1) = 0.0/57.3;
    end;

    for i = 3001:3005
        da(i,1) = 1.0/57.3;
    end;

    for i = 3006:12001
        da(i,1) = 0.0/57.3;
    end;
end;

%SINGLE DOUBLET AILERON
if Library == 3
    for i = 1:380
        da(i,1) = 0.0/57.3;
    end;

    for i = 381:400
        da(i,1) = -1.0/57.3;
    end;

    for i = 401:3280
        da(i,1) = 0.0/57.3;
    end;

    for i = 3281:3300
        da(i,1) = 1.0/57.3;
    end;

    for i = 3301:12001
        da(i,1) = 0.0/57.3;
    end;
end;

%MULTIPLE DOUBLET AILERON
if Library == 4
    for i = 1:380
        da(i,1) = 0.0/57.3;
    end;

    for i = 381:400
        da(i,1) = -1.0/57.3;
    end;
```

```
    for i = 401:1608
        da(i,1) = 0.0/57.3;
    end;

    for i = 1681:1700
        da(i,1) = 1.0/57.3;
    end;

    for i = 1701:3080
        da(i,1) = 0.0/57.3;
    end;

    for i = 3081:3100
        da(i,1) = -1.0/57.3;
    end;

    for i = 3101:4080
        da(i,1) = 0.0/57.3;
    end;

    for i = 4081:4100
        da(i,1) = 1.0/57.3;
    end;

    for i = 4101:12001
        da(i,1) = 0.0/57.3;
    end;
end;

%SIMULATION
sys_1 = tf(NbetaI, Dbar2);
betaI = lsim(sys_1, da, t);

sys_2 = tf(NphiI, Dbar2);
phiI = lsim(sys_2, da, t);

sys_3 = tf(NpsiI, Dbar2);
psiI = lsim(sys_3, da, t);

%PLOT RESULTS
betaI_deg = betaI*rad2deg;
phiI_deg = phiI*rad2deg;
psiI_deg = psiI*rad2deg;
da_deg = da*rad2deg;

subplot(3,2,2);
plot(t, betaI_deg);
title('Beta VS Time for Aileron Maneuver');
xlabel('Time (s) ');
ylabel('Beta (deg) ');
grid on;

subplot(3,2,4);
plot(t, phiI_deg);
title('Phi VS Time for Aileron Maneuver');
xlabel('Time (s) ');
ylabel('Phi (deg) ');
grid on;
```

```
subplot(3,2,6);
plot(t, psiI_deg);
title('Psi VS Time for Aileron Maneuver');
xlabel('Time (s) ');
ylabel('Psi (deg) ');
grid on;

subplot(3,2,3);
plot(t, da_deg);
title('Aileron Deflection VS Time');
xlabel('Time (s) ');
ylabel('Aileron Deflection (deg) ');
grid on;
```