ACTIVE CONTROL OF QUARTER-CAR SUSPENSION SYSTEM USING LINEAR QUADRATIC REGULATOR

This report submitted in accordance with requirement of University Teknikal Malaysia Melaka (UTeM) for Bachelor of Electrical Engineering (Control, Instrumentation & Automation) With Honours

by

SYAHIRAH HANANI BINTI AHMAD HANAFIAH

B011310151

940224-06-5226

FACULTY OF ELECTRICAL ENGINEERING

2017

I hereby declare that I have read through this report entitled "Active Control of Quarter-Car Suspension System using Linear Quadratic Regulator" and found that it has comply the partial fulfillment for awarding the degree of Bachelor in Electrical Engineering (Control, Instrumentation and Automation) with Honors.

Signature : ……………………………………………………

Supervisor's Name : DR. MUHAMMAD NIZAM BIN KAMARUDIN.

Date :

ACTIVE CONTROL OF QUARTER-CAR SUSPENSION SYSTEM USING LINEAR QUADRATIC REGULATOR

SYAHIRAH HANANI BINTI AHMAD HANAFIAH

A report submitted in partial fulfillment of the requirement for the degree of Bachelor in Electrical Engineering (Control, Instrumentation and Automation) with Honors.

Faculty of Electrical Engineering

UNIVERSITI TEKNIKAL MALAYSIA MELAKA

2017

I hereby, declared this report entitled "Active Control of Quarter-Car Suspension System using Linear Quadratic Regulator" is the results of my own research except as cited in references. The report has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

Signature : ....

Name : SYAHIRAH HANANI BINTI AHMAD HANAFIAH.

Date :

To my beloved father, Ahmad Hanafiah bin Harun,

To my appreciated mother, Rosnani binti Mohammad,

My adored brothers and sister,

My supervisor and all my lecturers,

All my friends.

For giving me moral support, cooperation, encouragement and understandings.

Thank You So Much and Love You All Forever.

ACKNOWLEDGEMENT

Alhamdulillah, thanks to Allah SWT, with His willing for giving me the opportunity to complete this Final Year Project which is title Active Control of Quarter-Car Suspension System using Linear Quadratic Regulator. This final year report was prepared for Faculty of Electrical Engineering, University Teknikal Malaysia Melaka (UTeM), basically for student in final year to complete the undergraduate program that leads to the degree of Bachelor of Engineering in Electrical.

Firstly, I would like to express my deepest thanks to, Dr. Muhammad Nizam bin Kamarudin, a lecturer at Faculty of Electrical Engineering UTeM and assign, as my supervisor who had guided be a lot of task during two semester session 2016/2017. I also want to thanks the lecturers and staffs of Faculty of Electrical Engineering UTeM for their cooperation during I complete the final year project that had given valuable information, suggestion and guidance in the compilation and preparation this final year project.

Deepest thanks and appreciation to my parent, family and special mate of mine, and others for their cooperation, encouragement, constructive suggestion and full of support for the report completion, from the beginning till the end.

Last but not least, thanks to all my friends and everyone, that have been contributed supporting my work and help myself during the final year project progress till it is fully completed.

Thank you.

ABSTRACT

The aim of this project is to formulate an active control of quarter-car suspensions systems by using Linear Quadratic Regulator (LQR). Nowadays, car suspensions systems typically rated by its ability to maintaining good road handling and to improve passenger comfort. Active suspension poses the ability to give better performance of suspension by reducing traditional design compromise between handling and comfort by directly controlling the suspensions force actuators, which is close-loop system. In this project, a dynamic of active suspension system for a Quarter-Car model is derived. An LQR is designed to improve the ride comfort and handling, as well as to minimize vibration due to road roughness or uneven road profile. The transient performance such as settling time and overshoot is analysed in order to observe the performance of the proposed LQR-based active suspension control. The results are obtained via the simulation works using MATLAB with SIMULINK® toolbox.

ABSTRAK

Tujuan projek ini adalah untuk merangka kawalan aktif sistem penggantungan suku kereta dengan menggunakan Linear Quadratic Regulator (LQR). Pada masa kini, sistem penggantungan kereta biasanya dinilai oleh keupayaannya untuk mengekalkan pengendalian jalan raya yang baik dan untuk meningkatkan keselesaan penumpang. Gantungan aktif menimbulkan kemampuan untuk memberikan prestasi penggantungan yang lebih baik dengan mengurangkan reka bentuk tradisional berada diantara pengendalian dan keselesaan dengan mengawal terus daya penggerak sistem penggantungan kereta yang merupakan sistem gelung tertutup. Dalam projek ini, sistem gantungan aktif yang dinamik untuk model Quarter-Car diperolehi. LQR direka untuk meningkatkan keselesaan perjalanan dan pengendalian, serta untuk mengurangkan getaran disebabkan kekasaran jalan atau keadaan jalan yang tidak rata. Tindak balas semula jadi seperti masa penetap dan peratusan terlajak dianalisis untuk melihat prestasi kawalan gantungan aktif berdasarkan LQR yang dicadangkan. Keputusan diperoleh melalui kerja-kerja simulasi menggunakan MATLAB dan SIMULINK® toolbox.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

LQR	-	Linear Quadratic Regulator
PSS	-	Passive Suspensions Systems
ASS	-	Active Suspensions Systems
FYP	-	Final Year Project
DOF	-	Degree of Freedom
LHS	-	Left Hand Side
RHS	-	Right Hand Side
ANN	-	Artificial Neural Network
FLC	-	Fuzzy Logic Controller
GA	-	Genetic Algorithm

LIST OF SYMBOLS

M_1	-	mass of the wheel/unsprung mass (kg)
M_2	-	mass of the car body/sprung mass (kg)
r	-	road disturbance/road profile
X_w	-	wheel displacement (m)
X_s	-	car body displacement (m)
K_a	-	stiffness of car body spring (Nm/s)
K_t	-	stiffness of tire (N/m)
C_a	-	damper (Ns/m)
U_a	-	force actuator
$X_1 = \mathrm{X_s} - \mathrm{X_w}$	-	suspension travel
$\dot{\mathrm{X}}_\mathrm{s}$	-	car body velocity
$X_2 = \ddot{\mathrm{X}}_\mathrm{s}$	-	car body acceleration
$X_3 = \mathrm{X_w} - \mathrm{r}$	-	wheel deflection
$\dot{X}_w$	-	wheel velocity
$X_4 = \ddot{X}_w$	-	wheel acceleration

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Motivation

The need of suspension for vehicle is important due to safety aspect, ride comfort and good vehicle handling. Suspension system has the facility to minimize vibration due to road roughness or uneven road profile because this suspension system will seclude passenger from vibration and shocks arising from road roughness. Suspension comprises of the arrangement of springs, shock absorbers and linkages that link a vehicle to its wheels. The suspension system also can secure the vehicle itself and any payload or luggage from damage and wear. There are two types of suspension system which is passive suspension and active suspension. A passive suspension system store energy through spring and dissipate it by a damper because there is no energy supplied by the suspension component to the system. While, the ability of an active suspension system is to store, dissipate and to supply energy to the system because it has force actuator.

1.2 Problem Statement

An active suspension system is a regulation case where the attitude of vehicle need to be stabilized when perturbed by bumpy road, road roughness and uneven terrain. An active suspension system can be controlled by using state-feedback controller. In state-feedback controller, the feedback gain need to be designed. There are many choices in order to design the feedback gain, such as LQR controller, Fuzzy Logic controller, Robust H∞ control, Sliding Mode control and others. Thus, Linear quadratic regulator (LQR) was chosen in this project to design an active suspension controller because of its nature as a regulation case control.

1.3 Objective

This project embarks into following objectives:

1. To derive a mathematical model of active suspension system for a quarter-car model.
2. To design a Linear Quadratic Regulator (LQR) for active suspension control in order to stabilize the vehicle's attitude.
3. To analyse the transient performance (settling time and overshoot) of LQR-based active suspension control that reflect the improvement of ride comfort, car handling and vibration minimization due to road roughness or uneven road profile.

1.4 Scope

This project focuses on a quarter-car suspensions system. The proposed controller is a state-feedback type for regulation case where the states are assumed available for feedback. The feedback gain of the closed-loop control is obtained by using LQR. The results are obtained via the simulation works using MATLAB with SIMULINK® toolbox. Thus, this project requires no hardware part.

1.5 Thesis Outline

In general, this thesis is divided into five chapters:

Chapter 1: Introduction

Chapter 1 is an overview of the overall research project. This chapter describe about the motivation, problem statement, objective and the scope of the project.

Chapter 2: Literature Review

Chapter 2 presents the literature review and theory background. This chapter explain about the background of study which is related to the project.

Chapter 3: Research Methodology

Chapter 3 discusses about the methodology adopted for this research project. This chapter explain the overall process that has been conducted to complete this project.

Chapter 4: Results and Discussions

Chapter 4 shows the result obtained from data presented. The related parameters are arranged tidily using the aid of figures and tables. Hence, all the result is explained and compared. Then the comparison will be discussed.

Chapter 5: Conclusion and Recommendation

Chapter 5 explain the conclusion achieved in this project and some recommendation for future work.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter reviews related work on the project. This chapter consists of review on the system dynamics and models, review on the controllers and the conclusion of the research. The reviews are the most important parts because all the information from the research about the project is used as a reference to complete this project. All the review has been done by various source such as journal, book, website and others.

2.2 System Dynamics and Models

2.2.1 Vehicle model

Previous studies have reported that suspension is one of the important idea in order to improve the ride comfort, while maintaining the car handling characteristic and reduced the vibration due to different road profile [1]. The performance also can be improved by maintaining the relative position and movement between the vehicle body and wheels. Suspension system has been designed using three different model which is Quarter-car model, Half-Car model and Full-car model as shown in Figure 2.1, 2.2 and 2.3.

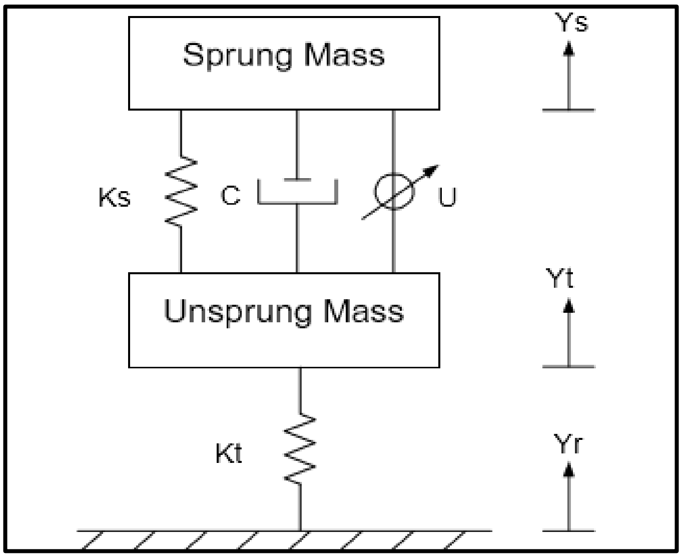


Figure 2.1: Quarter-Car model for an active suspension system.

2.2.1.1 Quarter-car model

The quarter-car model very often used for suspension analysis for an example in [1],[2],[3],[4],[5],[6] because it simple and can catch critical attributes of full model. The equation for the model movements are found by including vertical forces on the sprung and unsprung masses. From Figure 2.1, most of the quarter-car suspension model will represent the mass of the car body as the sprung mass, while the mass of the wheels as the unsprung mass. The quarter-car suspension model is represented as 2 DOF system. From the quarter car model, the design can be developed into Half-car model and Full car model and the models are represented in reference [7],[8],[9],[10],[11],[12].

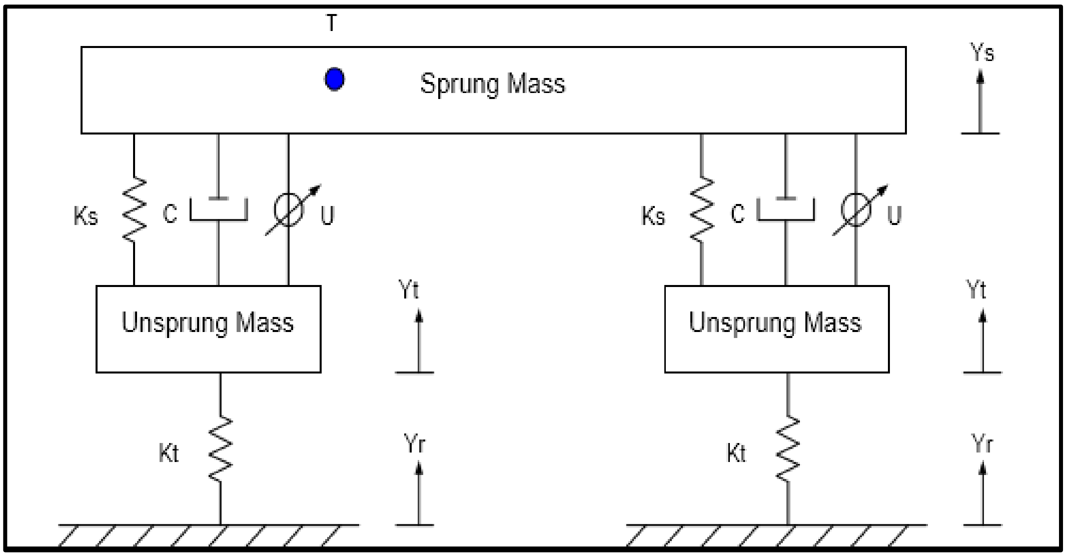


Figure 2.2: Half-Car model for an active suspension system.

1.1.1.1 Half-car model

The half-car suspension model is represented as a linear 4 DOF system. It comprises of a sprung mass (car body) connected to two unsprung masses (front and back wheels) at every corner. The sprung mass is free to haul and pitch, while the unsprung masses are free to bounce vertically regarding to the sprung mass. The suspensions between the sprung mass and unsprung masses are modelled as linear thick dampers and spring components, while the tires are modelled as simple linear springs without damping components [7].

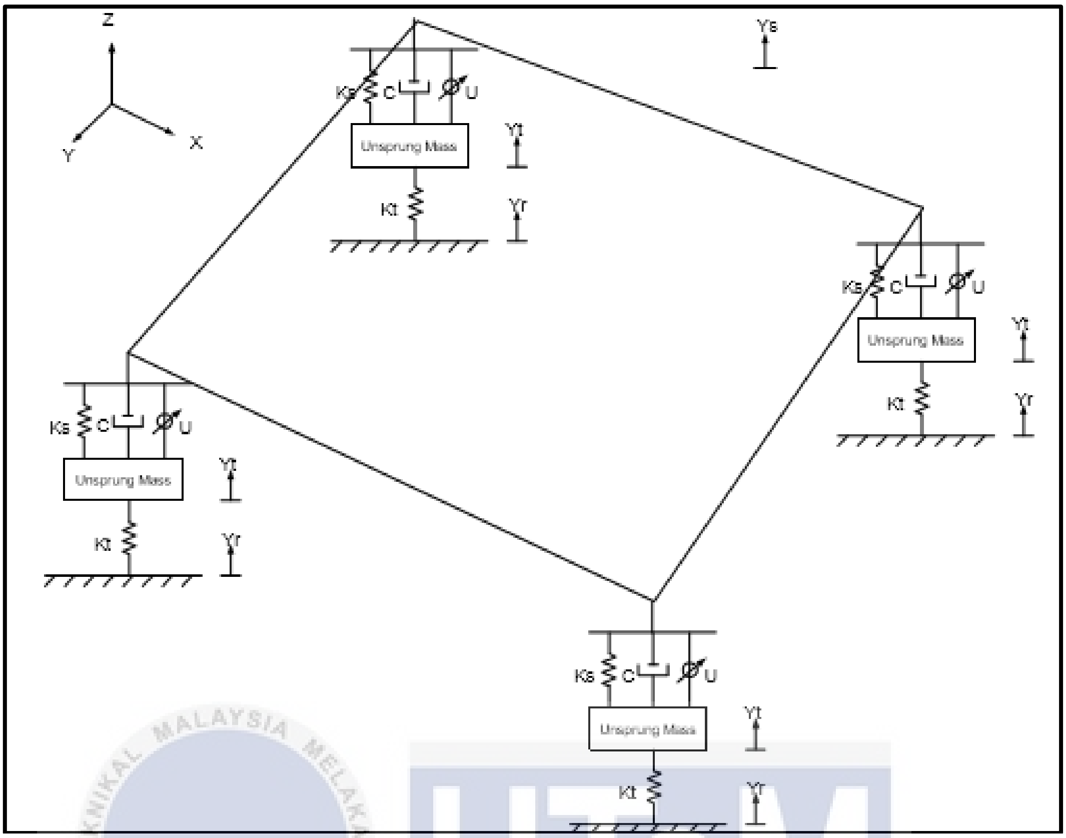


Figure 2.3: Full-Car model for an active suspension system.

2.2.1.3 Full-car model

The full car model can be designed by adding the link between the sprung mass to another four unsprung masses (body - front left and right, back left and right). Commonly, the link between sprung and unsprung masses will gave pitch and roll angle [11]. The basic modelling is still the same however there is extra thought about the bouncing, pitching and rolling need to be counted. The full-car suspension model represented as 7 DOF due to the haul, roll, and pitch movements of the sprung mass and the vertical movements of the unsprung masses, as depicted in Figure 2.3 [12].

2.2.2 Suspension system

In suspension system, it can be classified into six basic components which are maintaining the vehicle ride height, minimizing the factor of shock forces, making the wheel alignment in good condition, keeping the weight of the vehicle, maintaining the tires in contact with the road and controlling the direction of vehicle travel [13]. The suspension system normally divided into three categories which is passive suspension, semi-active suspension and active suspension. The main component use to designing the suspension system is a spring and parallel damper.

2.2.2.1 Passive suspension

A passive suspension system consists of springs and dampers. A passive suspension system can be found in controlling the elements of vertical movement of a vehicle. The advantage of passive suspension is economically and simple to design, while the main drawback is its limit of suppressing the vibration occurring due to road roughness. Besides that, another drawback of passive suspension system is it has fix characteristic [14]. Even though it does not apply energy to the system, but it controls the relative motion of the body to the wheel by using different types of damping or energy dissipating elements. Passive suspension has significant limitation in structural applications. Figure 2.4 shows the components of passive quarter-car model that consists of spring and damper [15].

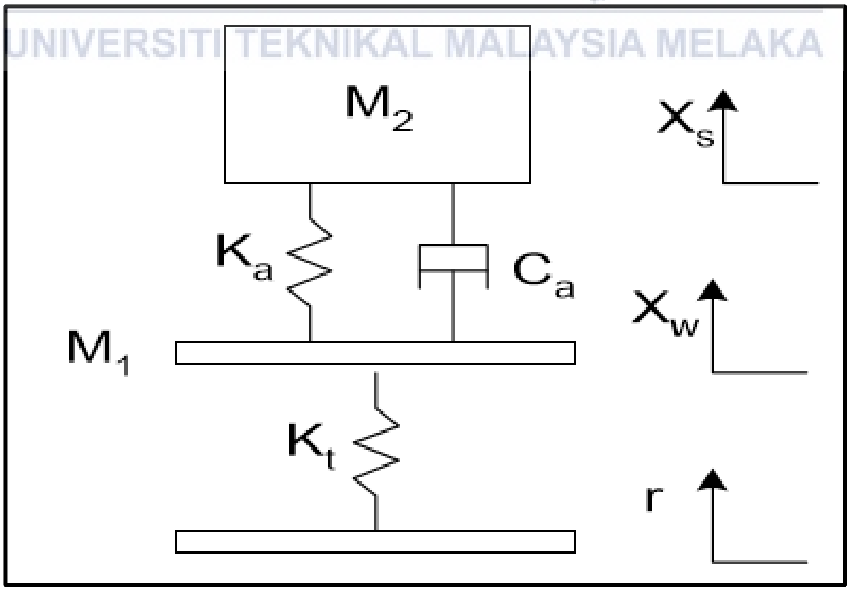


Figure 2.4: Passive suspension for quarter-car model.

2.2.2.2 Semi-active suspension

The semi-active suspension system utilizing a variable damper. Semi-active suspension system does not give any energy into suspension system before the damper is replaced by controllable damper. Semi-active gives a quick change in rate of springs damping coefficients. The semi-active system operated by changing the damping coefficient using electromagnetic valve which controlled inside the absorbers [16]. This type of suspension system used external power to operate. Sensors and actuator are added to distinguish the road profile for control input. The semi-active suspension system, gives freedom to vary the damper characteristics along the road [1]. The most generally semi-active suspension system is called skyhook damper. The component for semi-active suspension for quarter-car model is shows in Figure 2.5.

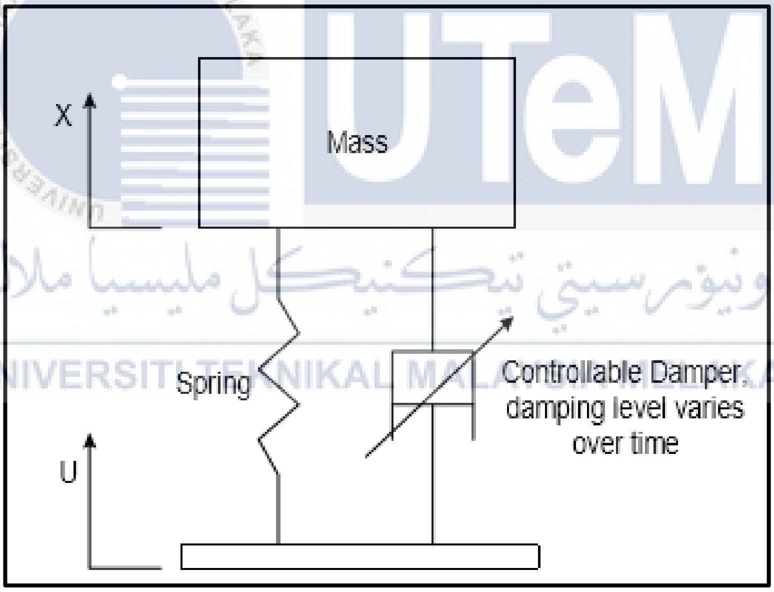


Figure 2.5: Semi-active suspension for quarter-car model.

2.2.2.3 Active Suspension

An active suspension system can react to the vertical changes in the road input. The damper or spring is mediating by the force actuator as shown in Figure 2.6. This force actuator has its own function which is to supply energy to the system or dissipate energy from the system. An active suspension system has many advantages as compared to passive suspension system, such that this system is a closed-loop control system that will correct the error and gave the output to the desired level. Furthermore, an active suspension also has the advantage of negative damping and can generate larger range of force at low velocities [17]. For an active suspension systems, the performance requirements include the aspects of ride comfort and car handling. For ride comfort, the main task for an active suspension is to design a controller which can improved in stabilizing the vertical and pitch motion of the car body and isolating the force transmitted to the passengers. Next, for good car handling, the dynamic tire load should not exceed the static ones for both front and rear wheels [10].

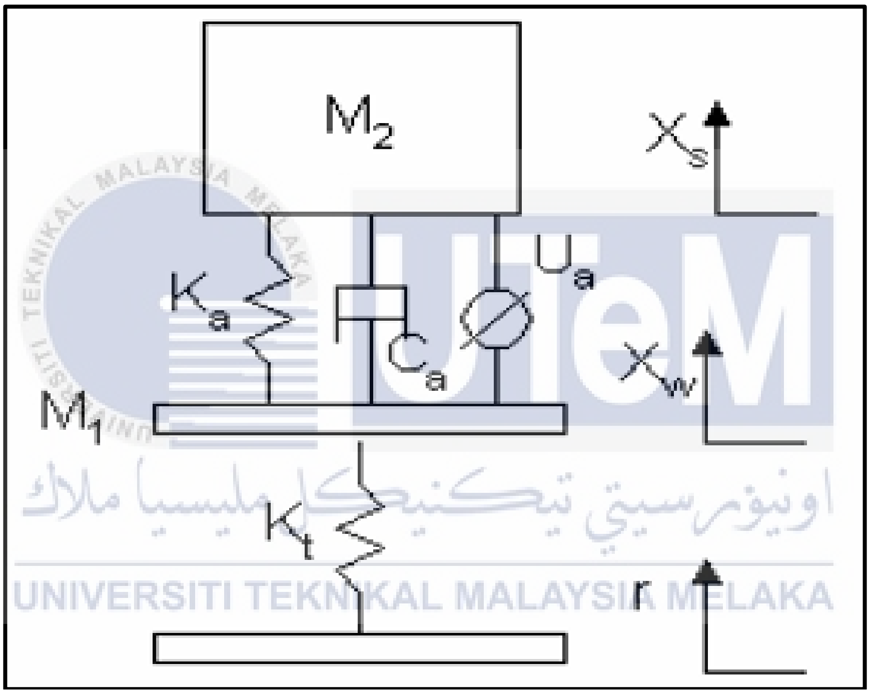


Figure 2.6: Active suspension for quarter-car model.

2.3 Controllers

Several researches have reported that there are many controllers can be utilized to observe the transient performance of an active suspension system such as LQR Controller, Fuzzy Logic Controller, Robust H∞ control, Sliding Mode Control. The controller creates forces to control the output parameters for an example suspension travel, car body acceleration and wheel deflection. In most of the research has been done, a quarter-car model is used for suspension system analysis. One of the reason is it can be derived easily and it can catch basic element of a real vehicle issue.

2.3.1 LQR controller

The LQR Controller is the controller that uses the least amount of control signal energy to take the output to zero. This controller also can be designed even though the optimizing energy is neglected.

The LQR approach of vehicle suspension control is comprehensively used as a piece of many studies in vehicle suspension control. It has been use in a simple quarter car model, half-car, and besides in full car model. The strength of LQR approach is it have best performance in term of body acceleration and body travel. With this type of controller, a perfect result can be obtained when all the performance index is considered [18]. Usage of the LQR controller to the active suspension system has been proposed in [19]. In this review, the LQR controller is used to improve the ride comfort and car handling for an active quarter car model.

LQR is one of the most popular control approaches normally been used by many researches in controlling the active suspension system. [20] has presented the LQR control approach in controlling a linear active suspension system. The study concluded that LQR control approach will result in better performance in terms of ride comfort.

The objective for state regulator problem is to transfer a system from initial state to the desired state with the minimum integral square error.

Consider a state variable feedback regulator for the system as:

$$u(t) = -Kx(t) \tag{2.3}$$

Where, K is the state feedback gain matrix.

The improvement technique comprises of deciding the control input U, which minimizes the performance index. The performance index J represents the performance characteristic requirement as well as the controller input limitation. The ideal controller of given system is characterized as controller design which minimizes the accompanying performance index.

$$J = \frac{1}{2}\int_0^t (x^t Q x + u^t R u) dt \qquad (2.4)$$

The matrix gain K is represented by:

$$K = R^{-1} B' P \qquad (2.6)$$

The matrix P must satisfy the reduced-matrix Riccati equation

$$A'P + PA - PBR^{-1}B'P + Q = 0 \qquad (2.7)$$

Then the feedback regulator U

$$\begin{aligned} u(t) &= -(R^{-1}B'P)x(t) \\ &= -Kx(t) \end{aligned} \qquad (2.8)$$

2.3.2 Fuzzy Logic controller

Fuzzy logic techniques can be utilized in different perspectives in controlled suspension systems. With reasonable cooperation functions and rule bases it thought to be insensitive to model. Fuzzy logic is a technique of controlling a system where all input conditions are not well defined. The use of fuzzy logic allows the usage of rule based control whereby the controller is defined by abstract that give them a fuzzy quality. The linguistic control strategy of the fuzzy algorithm serves as a fuzzy process model. Because of the linguistic statements from the rule base of the fuzzy logic controller, the control strategy takes after human thinking process. Usage of fuzzy logic techniques is presented [21].

Fuzzy Logic controller requires very less control force compared to others. For fuzzy logic controller, an appropriate input and output variable were decided. For instance, in active suspension system, utilizing numerical simulation techniques, the perfect choice for state variables was the acceleration and the velocity of the sprung mass. The output of the controller was force and represented with u (force of the actuator). The universe of talk for both the input and the output variables was isolated into three sections using the following linguistic variables, P (positive), Z (zero), and N (negative). The universe of talk for the input variable was found by subjecting the passive suspension to a couple of differing input conditions and review the maximum and minimum values for each specific input variable [22].

Triangular membership functions were initially chosen however in triangular membership function there is an issue due to the inborn sharpness in the triangular membership function's shape. The controller will act fast to the scarcest change in acceleration or velocity. The trapezoidal membership function can produce smoother control action due to the levelness at the highest point of the trapezoid shape [16].

2.3.3 Robust H∞ control

Robust H∞ control is designed to find the uncertainty and to predict the disturbance in model. Robust H∞ control have best suspension deflection and settling time. It confirms that Robust H∞ control of an active suspension system using the improvement of either a weighted single objective functional with hard propels or multi objective functional is an reasonable way to deal with the conflicting vehicle suspension performance issue [23].

The transfer function of H∞ control,

$$G(s) \,\epsilon\, C^{pq} \text{ is defined by } \|G(s)\|_{\infty} = \max \omega \,\epsilon\, R \|G(j\omega)\| \qquad (2.1)$$

Where, σ1 denotes the largest singular value of the complex matrix) $G(j\omega)$

Consider the system given by,

$$G(s) \,\epsilon\, C^{pq} \text{ is defined by } \|G(s)\|_{\infty} = \max \omega \,\epsilon\, R \|G(j\omega)\| \qquad (2.2)$$

State that, ω(t) is a disturbance acting on the system, while u(t) is the control action and z(t) is a performance index. There is also an issue with H ∞ control type which is, about finding the stabilizing control law u(t) = F(y(t)). The equations, stabilizes the system and minimizes the effect of the disturbance ω(t) on the performance index z. This goal can be achieved by minimizing the H ∞ control standard of the transfer function [23].

2.3.4 Sliding Mode control

Sliding mode control is designed to drive the system. The advantages of this controller are the switching function must be chosen that easy to be tailored to the dynamic system and the classes of uncertainty to be totally insensitive because of the closed loop response. Sliding mode characterized by a suite of feedback control laws and decision rule [24].

Sliding Mode control with disturbance observer can improved the ride comfort and road handling performances of an active quarter-car suspension system compared to passive suspension system. Relating Integral Sliding Mode control of a an active quarter-car suspension system is presented in [25]. Propose of this review is to show new approach in controlling an active suspension system. The approach utilized the relating integral sliding mode control contrive which asymptotic stability of the system in the midst of sliding mode is ensured [26].

2.4 Conclusion

This chapter discusses about the system that have been made by researchers in suspension systems. Based on the knowledge gained from the reviews, these three categories of suspension systems have its own advantages and disadvantages. However, researchers are focus on an active car suspension system and it is because the performance obtained is better than the other two types of suspension systems as mentioned before. By using the right controller techniques, the performances of ride comfort and car handling can be optimizing. This chapter also discusses the various control techniques used in an active suspension system. Therefore, in this project, a quarter-car model of an active suspension system will be controlled by using the LQR controller as shown in K-map in Appendix A.

CHAPTER 3

PROPOSED RESEARCH METHODOLOGY

3.1 Introduction

In this chapter, the methodology of the project including system modelling, mathematical modelling, controller design, overall process and result evaluation has been discussed. The flowchart shows how the project has been conducted to complete this project. This project will use MATLAB with SIMULINK® toolbox in order to design an active quarter-car suspension system.

3.2 System Modelling

The component of an active suspension system comprises of spring, damper, and force actuator as shown in Figure 3.1. An active suspension system can give better performances than passive suspension system by having force actuator. This is because, the force actuator has its own function which is to supply energy to the system or dissipate energy from the system.

The system has been modelled by deriving the active suspension system for a quarter-car model in order to obtain the mathematical model in time domain that will be used to design the LQR controller.

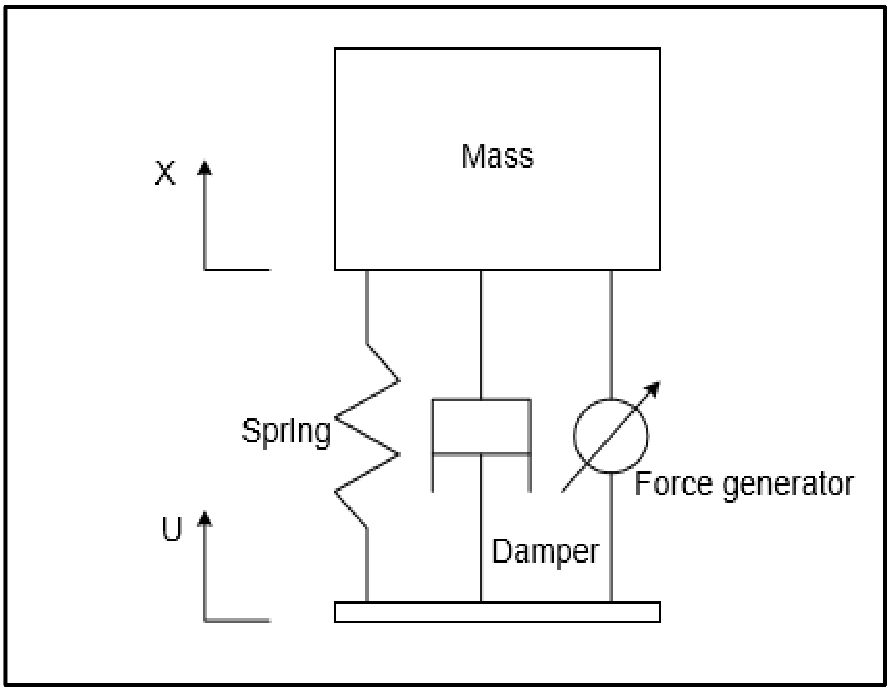


Figure 3.1: The component of an active suspension for quarter-car model.

3.3 Mathematical Modelling

Mathematical modelling of an active suspension system for a quarter-car model is derived based on Figure 3.2. The derivation for unsprung mass, M_1 and sprung mass M_2 is shown below:

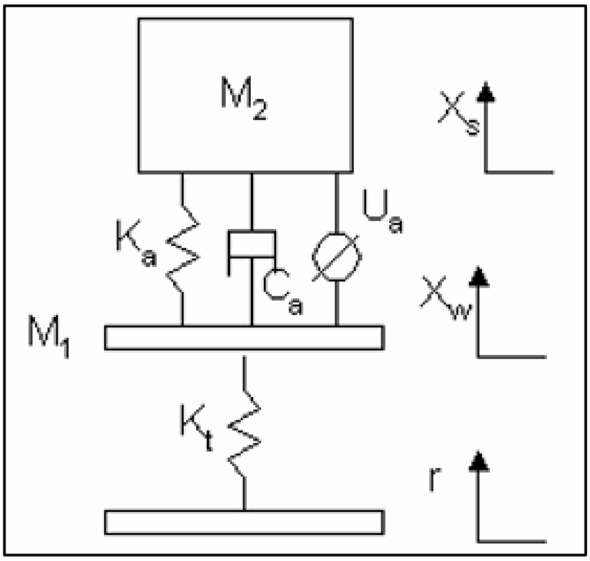


Figure 3.2: Quarter-car model for an active suspension.

<u>For Sprung Mass M_2:</u>

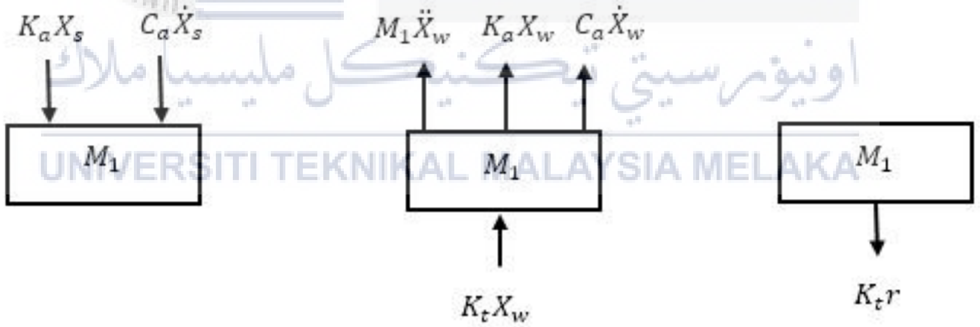
$$M_2\ddot{X}_s + C_a\dot{X}_s + K_aX_s = C_a\dot{X}_w + K_aX_w + U_a$$

$$M_2\ddot{X}_s = -C_a\dot{X}_s - K_aX_s + C_a\dot{X}_w + K_aX_w + U_a$$

$$\ddot{X}_s = \frac{-C_a(\dot{X}_s - \dot{X}_w) - K_a(X_s - X_w) + U_a}{M_2}$$

$$\ddot{X}_s = -\frac{C_a}{M_2}(\dot{X}_s - \dot{X}_w) - \frac{K_a}{M_2}(X_s - X_w) + \frac{U_a}{M_2} \qquad (3.1)$$

<u>For unsprung Mass M_1:</u>



$$M_1\ddot{X}_w + C_a\dot{X}_w + K_aX_w + K_tX_w + U_a = C_a\dot{X}_s + K_aX_s + K_tr$$

$$M_1\ddot{X}_w = C_a\dot{X}_s + K_aX_s - C_a\dot{X}_w - K_aX_w - K_tX_w + K_tr - U_a$$

$$\ddot{X}_s = \frac{C_a(\dot{X}_s - \dot{X}_w) + K_a(X_s - X_w) - K_t(X_w - r) - U_a}{M_1}$$

$$\ddot{X}_w = \frac{C_a}{M_1}(\dot{X}_s - \dot{X}_w) + \frac{K_a}{M_1}(X_s - X_w) - K_t(X_w - r) - \frac{U_a}{M_1} \qquad (3.2)$$

where;

M_1	=	mass of the wheel/unsprung mass (kg)
M_2	=	mass of the car body/sprung mass (kg)
r	=	road disturbance/road profile
X_w	=	wheel displacement (m)
X_s	=	car body displacement (m)
K_a	=	stiffness of car body spring (Nm/s)
K_t	=	stiffness of tire (N/m)
C_a	=	damper (Ns/m)
U_a	=	force actuator

Let the state variables are:

$$\begin{aligned} X_1 &= X_s - X_w \\ X_2 &= \dot{X}_s \\ X_3 &= X_w - r \\ X_4 &= \dot{X}_w \end{aligned} \tag{3.3}$$

Therefore, in state space equation, equation (4.3) can be written as:

$$\dot{X} = AX(t) + BU_a(t) + F\dot{r}(t) \tag{3.4}$$

so,

$$\begin{aligned} \dot{X}_1 &= \dot{X}_s - \dot{X}_w = X_2 - X_4 \\ \dot{X}_2 &= \ddot{X}_s \\ \dot{X}_3 &= \dot{X}_w - \dot{r} = X_4 - \dot{r} \\ \dot{X}_4 &= \ddot{X}_w \end{aligned} \tag{3.5}$$

where;

$X_1 = X_s - X_w$	=	suspension travel
$\dot{X}_s$	=	car body velocity
$X_2 = \ddot{X}_s$	=	car body acceleration
$X_3 = X_w - r$	=	wheel deflection
$\dot{X}_w$	=	wheel velocity
$X_4 = \ddot{X}_w$	=	wheel acceleration

From equation (3.1) and (3.2), rewrite the equation (3.4) in matrix form based on equation (3.3) and (3.5),

$$\dot{X} = AX(t) + BU_a(t) + F\dot{r}(t)$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{K_a}{M_2} & -\frac{C_a}{M_2} & 0 & \frac{C_a}{M_2} \\ 0 & 0 & 0 & 1 \\ \frac{K_a}{M_1} & \frac{C_a}{M_1} & -\frac{K_t}{M_1} & -\frac{C_a}{M_1} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M_2} \\ 0 \\ -\frac{1}{M_1} \end{bmatrix} U_a + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} \dot{r} \qquad (3.6)$$

Then, from equation (3.4), the block diagram for an active quarter-car suspension system without controller can be designed as depicted in Figure 3.3.

$$\dot{X} = AX(t) + BU_a(t) + F\dot{r}(t)$$

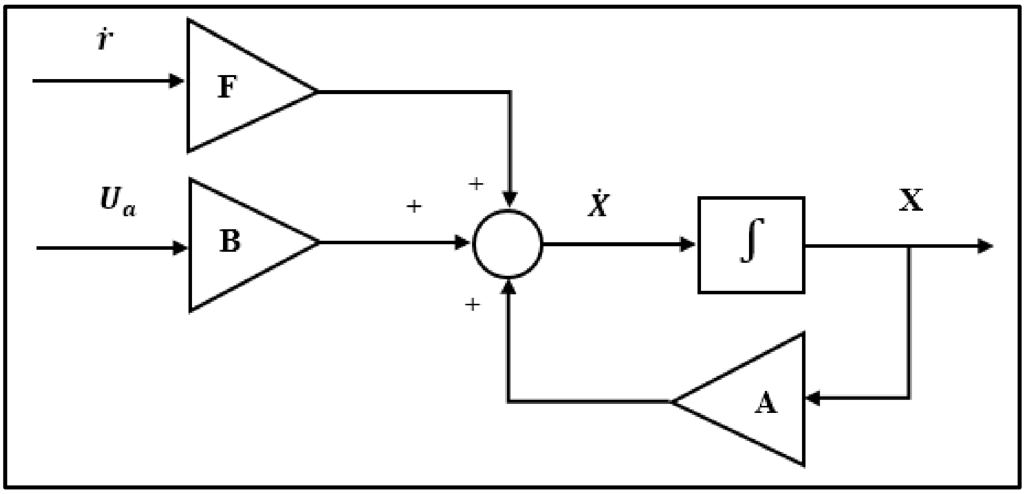


Figure 3.3: Block diagram for an active quarter-car suspension system without controller.

Next, to design controller for the system, equation (3.7) is substitute into the equation (3.4).

$$U_a(t) = -KX(t) \quad (3.7)$$

Substitute equation (3.7) into equation (3.4),

$$\dot{X} = AX(t) - BKX(t) + F\dot{r}(t)$$
$$\dot{X} = (A - BK)X(t) + F\dot{r}(t) \quad (3.8)$$

From equation (3.8), the block diagram for an active quarter-car suspension system with controller can be designed as depicted in Figure 3.4.

$$\dot{X} = (A - BK)X(t) + F\dot{r}(t)$$

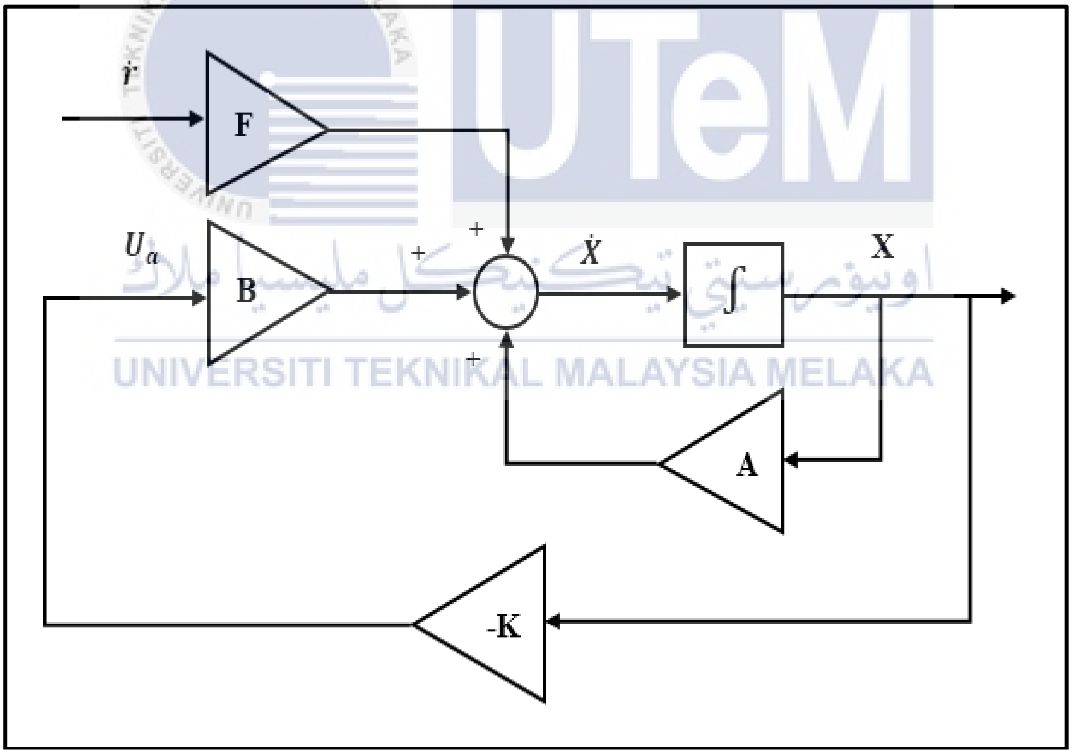


Figure 3.4: Block diagram for an active quarter-car suspension system with controller.

3.4 Controller Design

In this project, an active control of quarter-car suspension system will be controlled by using two method which is Pole Placement and LQR controller. The both controller was compared to show the control law performance of LQR controller better than Pole Placement. The step for controller design is depicted in Figure 3.5 while the Pole Placement and LQR controller design as depicted in Figure 3.6 and Figure 3.7 is the sub-step for controller design.

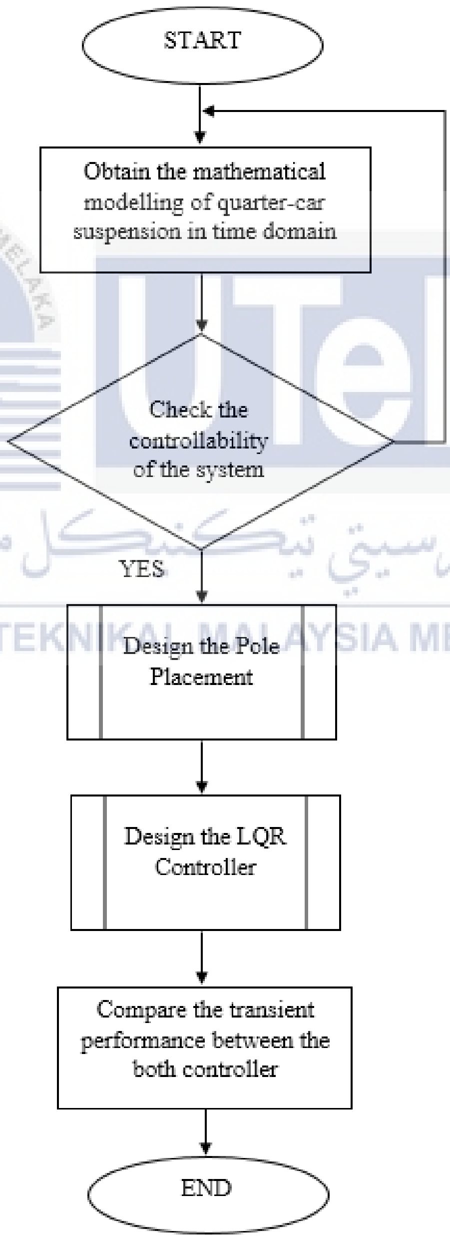


Figure 3.5: Flow chart for controller design.

3.4.1 Pole Placement Design

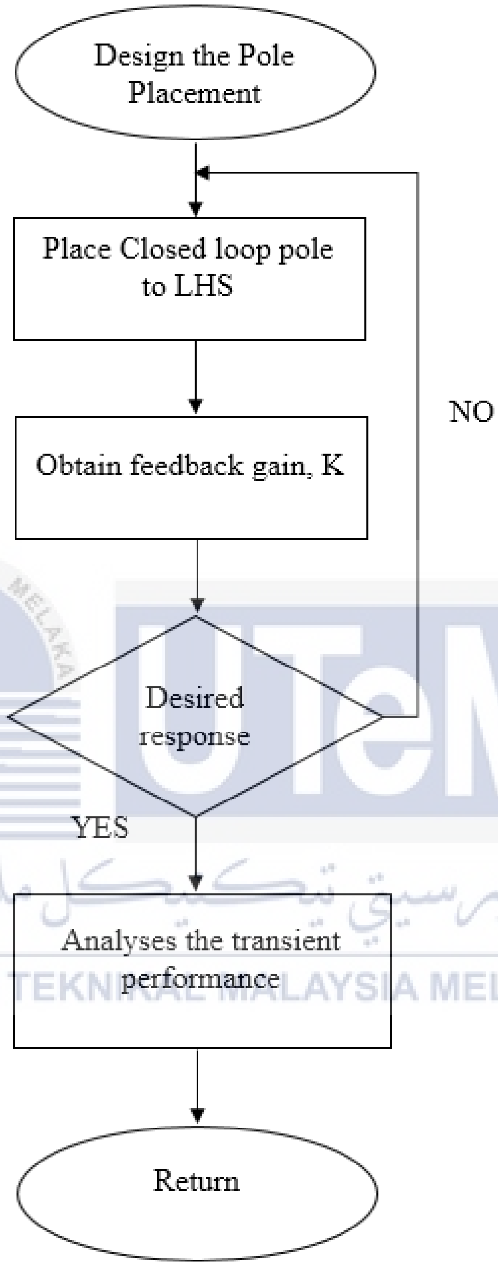


Figure 3.6 Flow chart for Pole Placement design.

The Pole Placement has been designed in simulation works using MATLAB with SIMULINK® toolbox. Derivation of Pole Placement design has been done to prove that the feedback gain K value for the simulation and calculation are similar. The derivation is start by finding the controllability and stability of the system and follow by designing the Pole Placement. The derivation of the Pole Placement design is shown below:

$$\dot{X} = AX(t) + BU_a(t)$$

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\\ \dot{X}_3\\ \dot{X}_4\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{K_a}{M_2} & -\frac{C_a}{M_2} & 0 & \frac{C_a}{M_2} \\ 0 & 0 & 0 & 1 \\ \frac{K_a}{M_1} & \frac{C_a}{M_1} & -\frac{K_t}{M_1} & -\frac{C_a}{M_1} \end{bmatrix} \begin{bmatrix} X_1\\ X_2\\ X_3\\ X_4\end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M_2} \\ 0 \\ -\frac{1}{M_1}\end{bmatrix} U_a$$

The parameter from [2] is substitute in above matrix,

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\\ \dot{X}_3\\ \dot{X}_4\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -57.97 & -3.448 & 0 & 3.448 \\ 0 & 0 & 0 & 1 \\ 284.9 & 16.95 & -3220 & -16.95 \end{bmatrix} \begin{bmatrix} x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix} 0 \\ 0.003448 \\ 0 \\ -0.01695\end{bmatrix}$$

First, check the controllability of the system,

$$AB = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -57.97 & -3.448 & 0 & 3.448 \\ 0 & 0 & 0 & 1 \\ 284.9 & 16.95 & -3220 & -16.95 \end{bmatrix} \begin{bmatrix} 0 \\ 0.003448 \\ 0 \\ -0.01695\end{bmatrix} = \begin{bmatrix} 0.0204 \\ -0.0703 \\ -0.0170 \\ 0.3457\end{bmatrix}$$

$$A^2B = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -57.97 & -3.448 & 0 & 3.448 \\ 0 & 0 & 0 & 1 \\ 284.9 & 16.95 & -3220 & -16.95 \end{bmatrix}^2 \begin{bmatrix} 0 \\ 0.003448 \\ 0 \\ -0.01695\end{bmatrix} = \begin{bmatrix} -0.0135 \\ -0.1605 \\ -0.0170 \\ -3.8792\end{bmatrix}$$

$$A^3B = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -57.97 & -3.448 & 0 & 3.448 \\ 0 & 0 & 0 & 1 \\ 284.9 & 16.95 & -3220 & -16.95 \end{bmatrix}^3 \begin{bmatrix} 0 \\ 0.003448 \\ 0 \\ -0.01695\end{bmatrix} = \begin{bmatrix} 0.0204 \\ -0.8302 \\ -0.0170 \\ 99.3337\end{bmatrix}$$

$$C_m = [B \quad AB \quad A^2B \quad A^3B]$$

$$C_m = \begin{bmatrix} 0 & 0.0204 & -0.0135 & 0.0204 \\ 0.0034 & -0.0702 & -0.1605 & -0.8362 \\ 0 & -0.0170 & -0.0170 & -0.0170 \\ -0.0170 & 0.3457 & -3.8792 & 99.3337 \end{bmatrix}$$

$$= 0 + (-1)(0.0034) \begin{vmatrix} 0.0204 & -0.0135 & 0.0204 \\ -0.0170 & -0.0170 & -0.0170 \\ 0.3457 & -3.8792 & 99.3337 \end{vmatrix} + 0 + (-1)(-0.0170)$$

X

$$= 0 + (-1)(0.0034) \begin{vmatrix} 0.0204 & -0.0135 & 0.0204 \\ -0.0170 & -0.0170 & -0.0170 \\ 0.3457 & -3.8792 & 99.3337 \end{vmatrix} + 0 + (-1)(-0.0170)$$

Y

$$\begin{vmatrix} 0.0204 & -0.0135 & 0.0204 \\ -0.0703 & -0.1605 & -0.8362 \\ -0.0170 & -0.0170 & -0.0170 \end{vmatrix}$$

$$|X| = (1)(0.0204) \begin{vmatrix} -0.0170 & -0.0170 \\ -3.8792 & 99.3337 \end{vmatrix} + (-1)(0.0135) \begin{vmatrix} -0.0170 & 0.0204 \\ 0.3457 & 99.3337 \end{vmatrix}$$
$$+(1)(0.0204) \begin{vmatrix} -0.0170 & -0.0170 \\ 0.3457 & -3.8792 \end{vmatrix}$$

$$= (1)(0.0204)(-1.7546) + (-1)(-0.0135)(-1.6957) + (1)(0.0204)(0.0718) = -0.0572$$

$$|Y| = (1)(0.0204) \begin{vmatrix} -0.1605 & -0.8362 \\ -0.0170 & -0.0170 \end{vmatrix} + (-1)(0.0135) \begin{vmatrix} -0.0703 & -0.8362 \\ -0.0170 & -0.0170 \end{vmatrix}$$
$$+(1)(0.0204) \begin{vmatrix} -0.0703 & -0.1605 \\ -0.0170 & -0.0170 \end{vmatrix}$$

$$= (1)(0.0204)(-0.0115) + (-1)(-0.0135)(-0.0130) + (1)(0.0204)(-0.0015) = -0.0004$$

$$= 0 + (-1)(0.0034)(-0.0572) + 0 + (-1)(-0.0170)(-0.0004) = 0.0002 \neq 0$$

rank (C_m) = 4 = order of system

So, the system is controllable.

Next, find the stability of the system,

$$|sI - A| = 0$$

$$\left| s\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 & -1 \\ -57.97 & -3.448 & 0 & 3.448 \\ 0 & 0 & 0 & 1 \\ 284.9 & 16.95 & -3220 & -16.95 \end{bmatrix} \right| = 0$$

$$\begin{bmatrix} s & -1 & 0 & 1 \\ 57.97 & s+3.448 & 0 & -3.448 \\ 0 & 0 & s & -1 \\ -284.9 & -16.95 & 3220 & s+16.95 \end{bmatrix} = 0$$

$$|sI - A| =$$

$$0 + 0 + (1)(s) \overset{X}{\begin{vmatrix} s & 1 \\ 57.97 & s+3.448 & -3.448 \\ -284.9 & -16.95 & s+16.95 \end{vmatrix}} + (-1)(-1) \overset{Y}{\begin{vmatrix} s & 0 \\ 57.97 & s+3.448 & 0 \\ -284.9 & -16.95 & 3220 \end{vmatrix}}$$

$$|X| = (1)(s) \begin{vmatrix} s+3.448 & -3.448 \\ -16.95 & s+16.95 \end{vmatrix} + (-1)(-1) + \begin{vmatrix} 57.97 & -3.448 \\ -284.9 & s+16.95 \end{vmatrix} + (1)(1) \begin{vmatrix} 57.97 & s+3.448 \\ -284.9 & -16.95 \end{vmatrix}$$

$$= (1)(s)[(s+3.448)(s+16.95) - (-3.448)(-16.95)] + (-1)(-1)[(57.97)(s+16.95) - (3.448)(-284.9)] + (1)(1)[(57.97)(-16.95) - (s+3.448)(-284.9)]$$

$$= (1)(s)\,[(s^2 + 16.95s + 3.448s + 58.4436) - 58.4436] + (-1)(-1)\,[(57.97s + 982.5915) - 982.3352] + (1)(1)\,[-982.5915 - (-284.9s - 982.3352)]$$

$$= (1)(s)(s^2 + 20.398s) + (-1)(-1)(57.97s + 0.2563) + (1)(1)(284.9s - 0.2563)$$

$$= s^3 + 20.398s^2 + 57.97s + 0.2563 + 284.9s - 0.2563$$
$$= s^3 + 20.398s^2 + 342.87s$$

$$|Y| = (1)(s) \begin{vmatrix} s+3.448 & 0 \\ -16.95 & 3220 \end{vmatrix} + (-1)(-1) + \begin{vmatrix} 57.97 & 0 \\ -284.9 & 3220 \end{vmatrix} + 0$$

$$= (1)(s)[(s + 3.448)(3220) - (0)(-16.95)] + (-1)(-1)\ [(57.97)(3220) - (0)(-284.9)] + 0$$

$$= (1)(s)[3220s + 11102.56] + (-1)(-1)[186663.4]$$
$$= 3220s^2 + 11102.56s + 186663.4$$

$$0 = 0 + 0 + (1)(s)(s + 20.398s^2 + 342.87s) + (-1)(-1)(32220s^2 + 11102.56s + 186663.4)$$
$$0 = s^4 + 20.398s^3 + 342.57s^2 + 3220s^2 + 11102.56s + 186663.4$$
$$0 = s^4 + 20.398s^3 + 3562.87A^2 + 11445.43s + 186663.4$$
$$0 = (s^2 + 17.4595s + 3457.58)(s^2 + 2.93847s + 53.9867)$$

So,

$$s = \begin{bmatrix} -8.7298 + j58.1496 \\ -8.7298 - j58.1496 \\ -1.4692j + 7.1992 \\ -1.469 - j7.19922 \end{bmatrix}$$

Hence, the system is stable because there are no poles located at RHS of s-plane.

Then, design Pole Placement by place the closed-loop pole to LHS,

$$A_{CL} = A - BK$$

$$A_{CL} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -57.97 & -3.448 & 0 & 3.448 \\ 0 & 0 & 0 & 1 \\ 284.9 & 16.95 & -3220 & -16.95 \end{bmatrix} - \begin{bmatrix} 0 \\ 0.003448 \\ 0 \\ -0.01695 \end{bmatrix} [K_1 \quad K_2 \quad K_3 \quad K_4]$$

$$= \begin{bmatrix} 0 & 1 & 0 & -1 \\ -57.97 & -3.448 & 0 & 3.448 \\ 0 & 0 & 0 & 1 \\ 284.9 & 16.95 & -3220 & -16.95 \end{bmatrix} -$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.003448K_1 & 0.003448K_2 & 0.003448K_3 & 0.003448K_4 \\ 0 & 0 & 0 & 0 \\ -0.01695K_1 & -0.01695K_2 & -0.01695K_3 & -0.01695K_4 \end{bmatrix}$$

$$|sI - A_{CL}| = 0$$

$$0 = sI - A_{CL}$$

$$0 = s\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} -$$

$$\begin{bmatrix} 0 & 1 & 0 & -1 \\ -57.97-0.003448K_1 & -3.448-0.003448K_2 & -0.003448K_3 & 3.448-0.003448K_4 \\ 0 & 0 & 0 & 1 \\ 284.9+0.01695K_1 & 16.95+0.01695K_2 & -3220+0.01695K_3 & -16.95+0.01695K_4 \end{bmatrix}$$

$$= \begin{bmatrix} s & -1 & 0 & 1 \\ 57.97+0.003448K_1 & s-(-3.448-0.003448K_2) & 0.003448K_3 & -3.448+0.003448K_4 \\ 0 & 0 & s & -1 \\ -284.9-0.01695K_1 & -16.95-0.01695K_2 & 3220-0.01695K_3 & s-(-16.95+0.01695K_4) \end{bmatrix}$$

$|sI - A_{CL}| =$

$$0+0+(1)(s)\begin{vmatrix} s & -1 & 1 \\ 57.97+0.003448K_1 & s+3.448+0.003448K_2 & -3.448+0.003448K_4 \\ -284.9-0.01695K_1 & -16.95-0.01695K_2 & s+16.95+0.01695K_4 \end{vmatrix}$$

Y

$$+(-1)(-1)\begin{vmatrix} s & -1 & 0 \\ 57.97+0.003448K_1 & s+3.448+0.003448K_2 & 0.003448K_3 \\ -284.9-0.01695K_1 & -16.95-0.01695K_2 & 3220-0.01695K_3 \end{vmatrix}$$

$$|X| = (1)(s)\begin{vmatrix} s+3.448+0.003448K_2 & -3.448+0.003448K_4 \\ -16.95-0.01695K_2 & s+16.95+0.01695K_4 \end{vmatrix} +$$

$$(-1)(-1)\begin{vmatrix} 57.97+0.003448K_1 & -3.448+0.003448K_4 \\ -284.9-0.01695K_1 & s+16.95+0.01695K_4 \end{vmatrix} +$$

$$(1)(1)\begin{vmatrix} 57.97+0.003448K_1 & s+3.448+0.003448K_2 \\ -284.9-0.01695K_1 & -16.95-0.01695K_2 \end{vmatrix}$$

$$= (1)(s)[(s+3.448+0.003448K_2)(s+16.95+0.01695K_4) - (-3.448+0.003448K_4)(-16.95-0.01695K_2)] + (-1)(-1)[(57.97+0.003448K_1)(s+16.95+0.01695K_4) - (-3.448+0.003448K_4)(-284.9-0.01695K_1)] + (1)(1)[(57.97+0.003448K_1)(-16.95-0.01695K_2) - (s+3.448+0.003448K_2)(-284.9-0.01695K_1)]$$

$$= (1)(s)\,[(s^2+16.95s-0.01695K_4s+3.448s+58.4436-0.0584K_4+0.003448K_2s + 0.0584K_2-0.00006K_2K_4) - (58.4436+0.0584K_2-0.0584K_4-0.00006K_2K_4)] + (-1)(-1)[(57.97s+982.5915-0.9823K_4+0.003448K_1s+0.0584K_1 - 0.00006K_1K_4) - (982.3352+0.0584K_1-0.00006K_1K_4)] + [(982.5915-0.9826K_2 - 0.0584K_1-0.00006K_1K_2) - (284.9s-0.01695K_1s-982.3352-0.0584K_1 - 0.9823K_2-0.00006K_1K_2)]$$

$$= (s)\,(s^2+20.398s-0.01695K_4s+3.448s+0.003448K_2s+(57.97s+0.2563- 0.0003K_4+0.003448K_1s)+(-0.2563-0.0003K_2+284.9s+0.01695K_1s)$$

$$= s^3+20.398s^2-0.01695K_4s^2+0.003448K_2s^2+57.97s-0.0003K_4 + 0.0204K_1s-0.0003K_2+284.9s$$

$$|Y| = (1)(s)\begin{vmatrix} s+3.448+0.003448K_2 & 0.003448K_3 \\ -16.95-0.01695K_2 & 3220-0.01695K_3 \end{vmatrix} +$$

$$(-1)(-1)\begin{vmatrix} 57.97+0.003448K_1 & 0.003448K_3 \\ -284.9-0.01695K_1 & 3220-0.01695K_3 \end{vmatrix} + 0$$

$$= (1)(s)[(s+3.448+0.003448K_2)(3220-0.01695K_3) - (0.003448K_3)(3220-0.01695K_3)] + (-1)(-1)[(57.97+0.003448K_1)(3220-0.01695K_3) - (0.003448K_3)(-284.9-0.01695K_1)] + 0$$

$$= (1)(s)\,[3220s - 0.01695K_3 s + 11102.56 - 0.0584K_3 + 11.1026K_2 - 0.00006K_2K_3) - (-0.0584K_3 - 0.00006K_2K_3)] + (-1)(-1)[(186663.4 - 0.9826K_3 + 11.1026K_1 - 0.00006K_1K_3) - (0.9823K_3 - 0.00006K_1K_3)]$$

$$= 3220s^2 + 0.01695K_3s^2 + 11102.56s - 11.1026K_2s + 186663.4 - 0.0003K_3 + 11.1026K_1$$

$$= 0 + 0 + (1)(s)(s^3 + 20.398s^2 + 0.01695K_4s^2 + 0.003448K_2s^2 + 57.97s - 0.0003K_4 + 0.0204K_1s - 0.0003K_2 + 284.9s)(3220s^2 + 0.01695K_3s^2 + 11102.56s + 11.1026K_2s + 186663.4 - 0.0003K_3$$

$$= s^4 + 20.398s^3 - 0.01695K_4s^3 + 0.003448K_2s^3 + 57.97s^2 - 0.0003K_4s + 0.0204K_1s^2 - 0.0003K_2s + 284.9s^2 + 3220s^2 + 0.01695K_3s^2 + 11102.56s + 11.1026K_2s + 186663.4 - 0.0003K_3 + 11.1026K_1$$

Place the closed-loop pole to LHS,

Let $s = (-10, -11, -12, -13)$

When $s = -10$,

$13.1426K_1 - 114.471K_2 - 1.6953K_3 + 16.953K_4 = -421526.8$………………....... 1

When $s = -11$,

$13.571K_1 - 126.714588K_2 - 2.05125K_3 + 22.5638K_4 = -483133.772$………… 2

When $s = -12$,

$$14.0402K_1 - 139.185744K_2 - 2.4411K_3 + 29.29323K_4 = -551974.216 \ldots\ldots\ldots\ldots \quad 3$$

When $s = -13$,

$$14.5502K_1 - 151.905156K_2 - 2.86485K_3 + 37.24305K_4 = -628201.744 \ldots\ldots\ldots\ldots \quad 4$$

Compare the four (4) equation above by using simultaneous method and find the value for $K_{1,}K_2, K_3$ and K_4.

So, $K = [-0.1526 \quad -0.0046 \quad 1.4516 \quad -0.0160]$

While, in simulation,

$K = [-0.1527 \quad -0.0046 \quad 1.4519 \quad -0.0160]$

Hence, the result for feedback gain K obtained in simulation works and calculation are similar.

3.4.2 LQR Controller Design

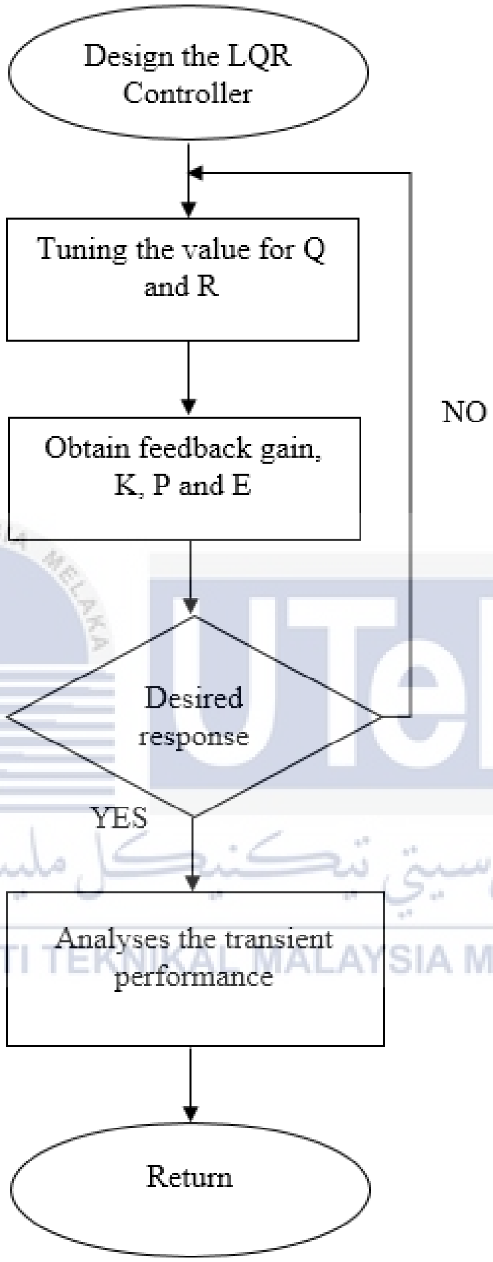


Figure 3.7: Flow chart for LQR controller design.

For this part, the LQR Controller has been designed in simulation works by tuning the value of matrix Q and weight factor R. The value of matrix Q and weight factor R that has been used in designing the LQR Controller as shown in equation 3.9 and 3.10. Hence, equation 3.11 shows the value of feedback gain K, optimum P and E that has been obtained in simulation works.

$$Q = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 1000 \end{bmatrix} \quad (3.9)$$

$$R = 0.0001 \quad (3.10)$$

$$K = \begin{bmatrix} 295 & 2578 & -30120.9 & -2220 \end{bmatrix}$$

$$P = \begin{bmatrix} 4439 & 90 & 612 & 17 \\ 90 & 93 & -979 & 4 \\ 612 & -979 & 55240 & -15 \\ 17 & 4 & 15 & 14 \end{bmatrix} \quad (3.11)$$

$$E = \begin{bmatrix} -28.1413 + j51.5046 \\ -28.1413 - j51.5046 \\ -5.3164 + j5.1849 \\ -5.3164 - j5.1849 \end{bmatrix}$$

3.5 Overall process for this project

The flowchart in Figure 3.8 shows the overall process that has been conducted to complete this project.

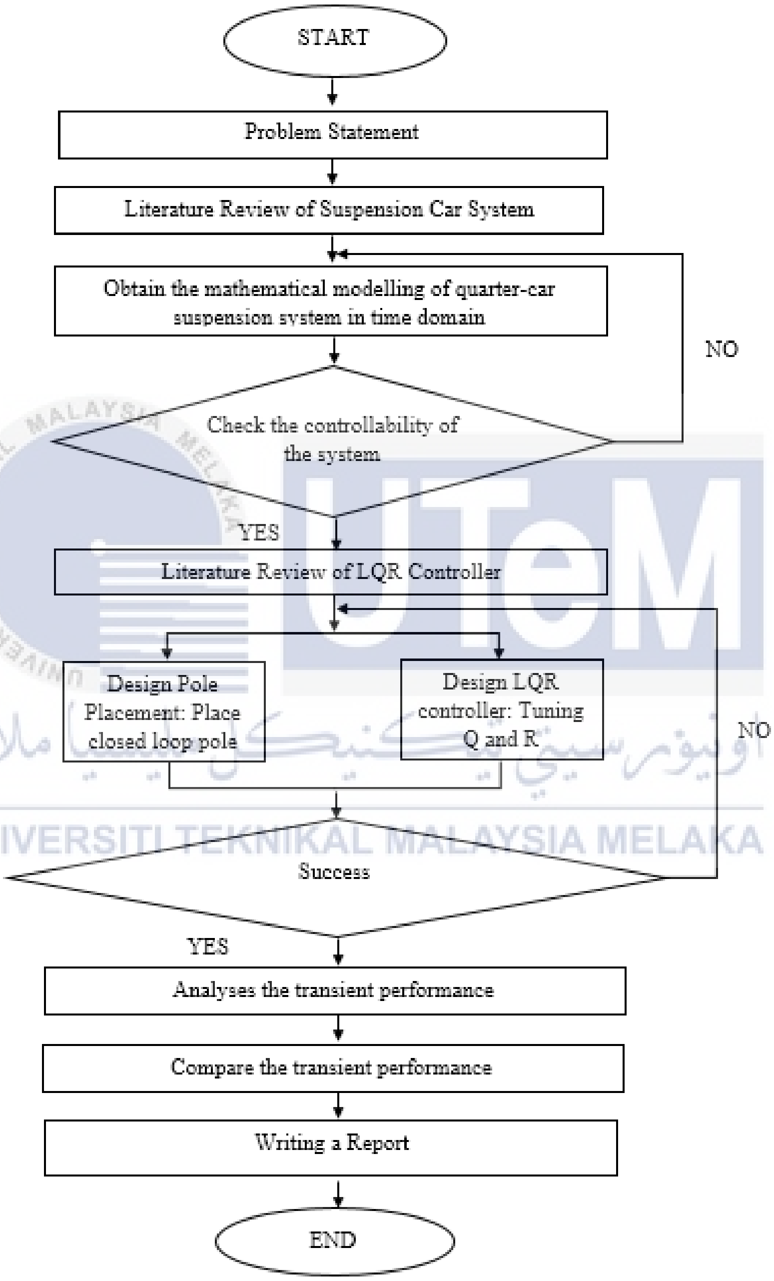


Figure 3.8: Flow chart for overall process of this project.

3.6 Result Evaluation

The result of this project will improve the ride comfort, car handling and passenger safety. The vibration of vehicle will be minimized due to road roughness. All the results will be evaluated via the simulation works using MATLAB with SIMULINK® toolbox.

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Introduction

This chapter shows the study of modelling of an active suspension system for a quarter-car model. Simulation results using MATLAB with SIMULINK® toolbox will be observed in this chapter. The performance criteria that have been analysed are the ride comfort and car handling. The force used in this system is directly transmitted to the passenger and this situation will be affected to the ride comfort of a car while the displacement of wheel characteristic will show the car handling achievement of an active suspension system.

4.2 Simulation Works

The parameters of the system are taken from [2] as shown in Table 4.1 used to design the active quarter-car suspension system in simulation works.

Table 4.1: Parameter of Quarter-Car

M_1 = 59 kg
M_2 = 290 kg
K_a = 16812 N/m
K_t = 190000 N/m
C_a = 1000 Ns/m

4.2.1 Disturbance Test

The system applied with step signal input as fixed disturbance to represent the road profile of the system in order to obtaining the controller parameter and the effect of the system performance. In this project, two different road profile has been used for quarter-car simulation. Figure 4.1 shows the system with disturbance construct in SIMULINK® toolbox.

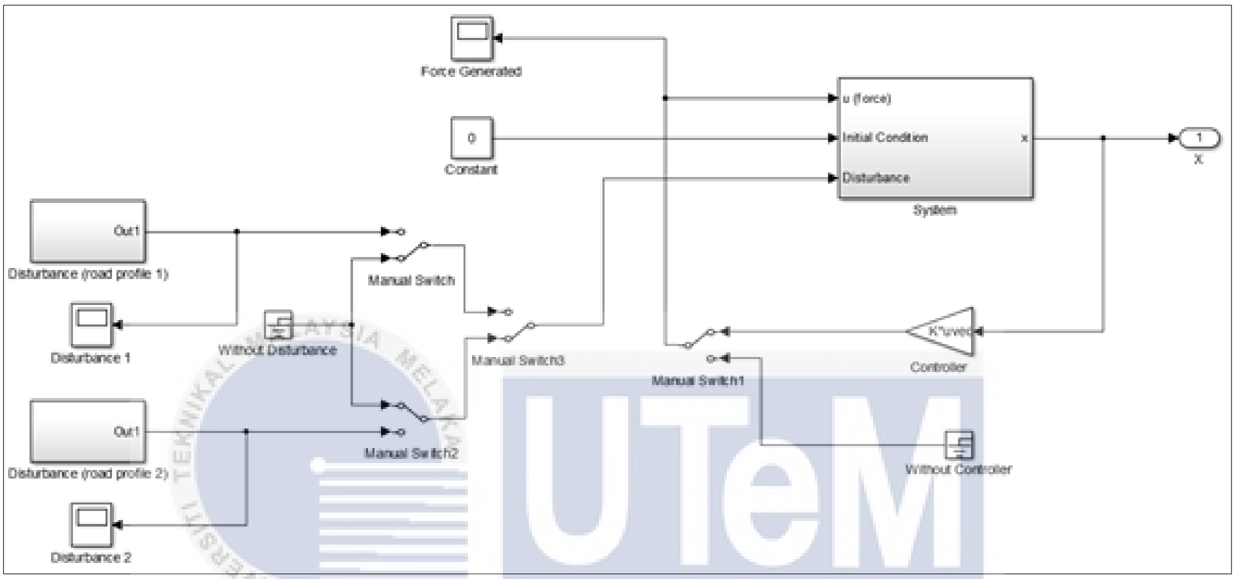


Figure 4.1: The system with disturbance.

4.2.1.1 Road profile 1

The road profile 1 is assumed to be a single bump. The step input signal equation for road profile 1 can be written as:

$$r(t) = 0.15u(t-9) - 0.15u(t-10)$$

The step input signal graph for road profile 1 has been designed in Simulink as shown in Figure 4.2.

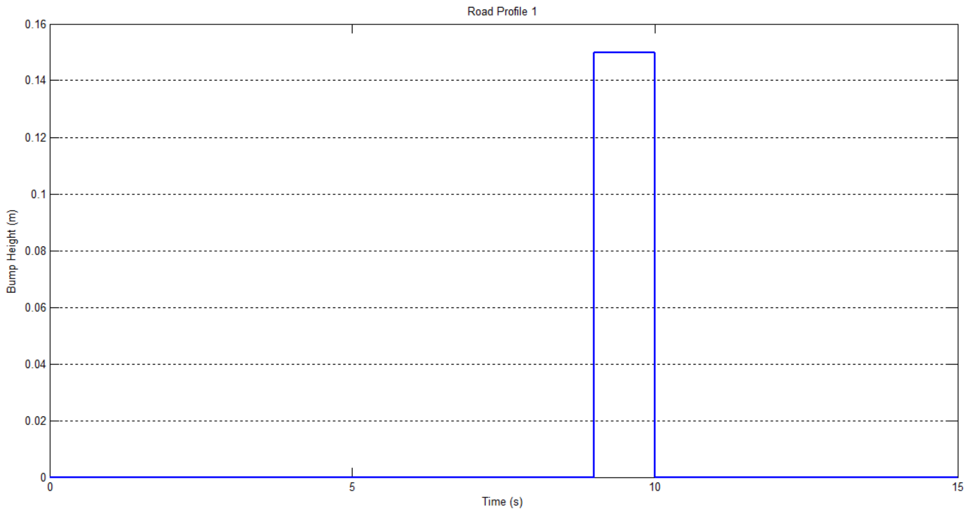


Figure 4.2: The step input signal graph for road profile 1.

4.2.1.2 Road profile 2

The road profile 2 is assumed have 2 bump. The step input signal equation for road profile 2 can be written as:

$$r(t) = \big(0.15u(t-2) - 0.15u(t-3)\big) + (0.1u(t-9) - 0.1u(t-10))$$

The step input signal graph for road profile 1 has been designed in Simulink as shown in Figure 4.3.

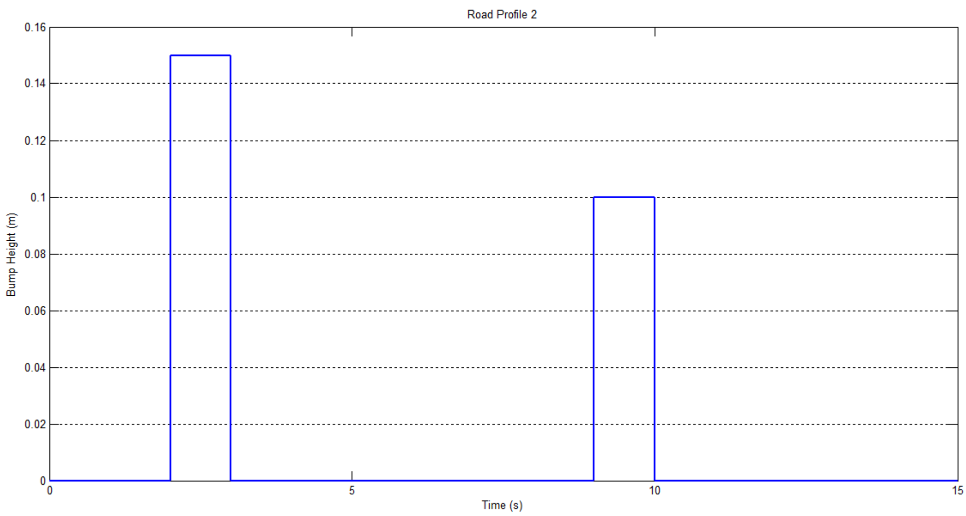


Figure 4.3: The step input signal graph for road profile 2.

4.3 Controller Performance

The feedback gain of the state-feedback controller can be designed using various type of controller, such as LQR controller, Fuzzy Logic controller, Robust Control, Sliding Mode control and others. In this project the Pole Placement has been designed as a controller to control an active suspension system. The purpose of that design is to compare the control law performance between the LQR controller with others controller.

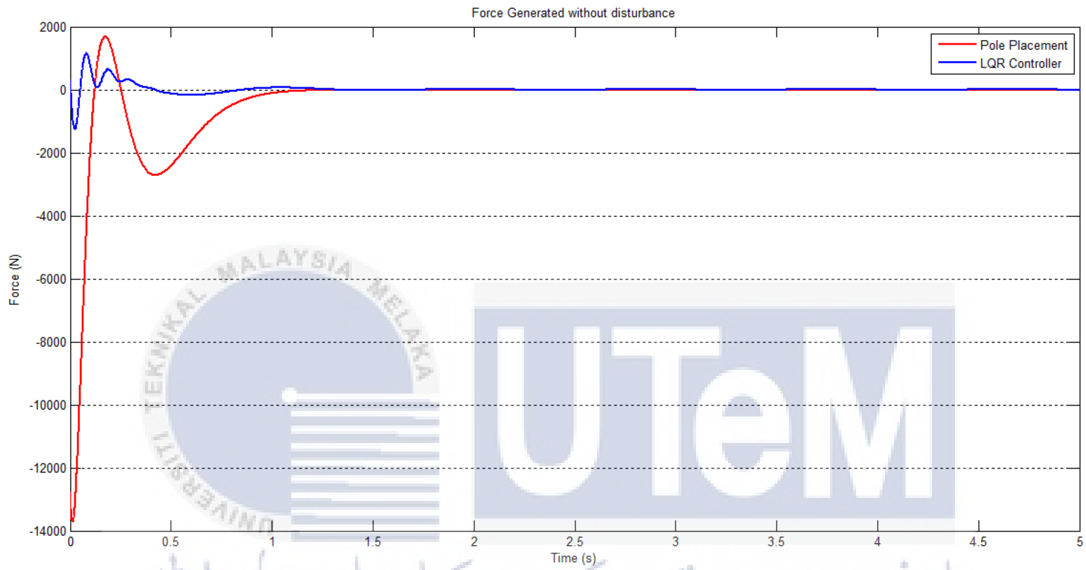


Figure 4.4: The force generated without disturbance.

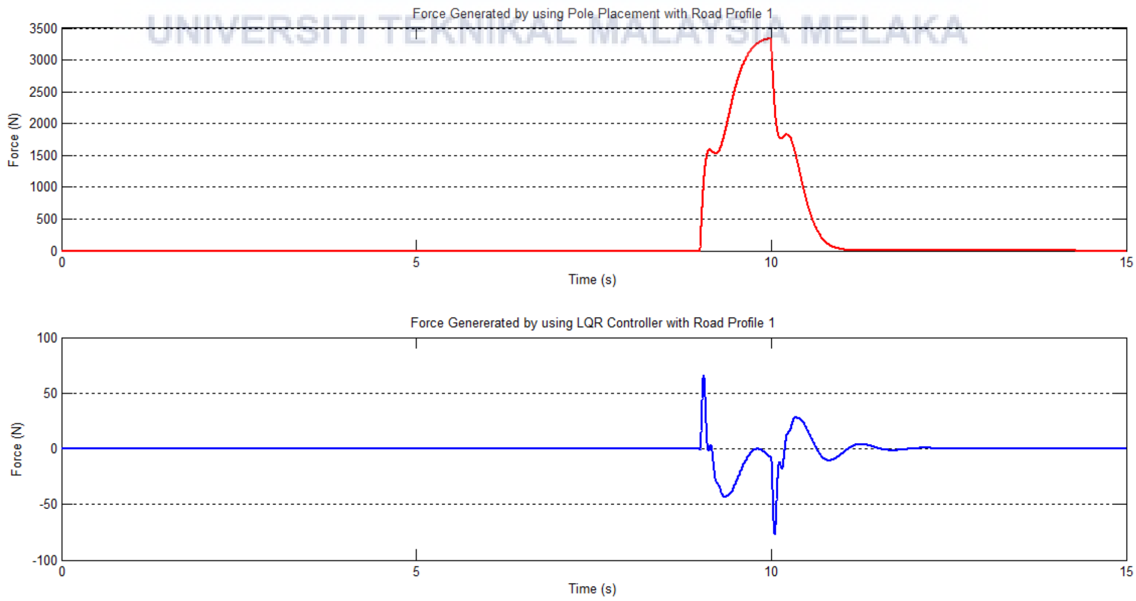


Figure 4.5: The force generated with road profile 1.

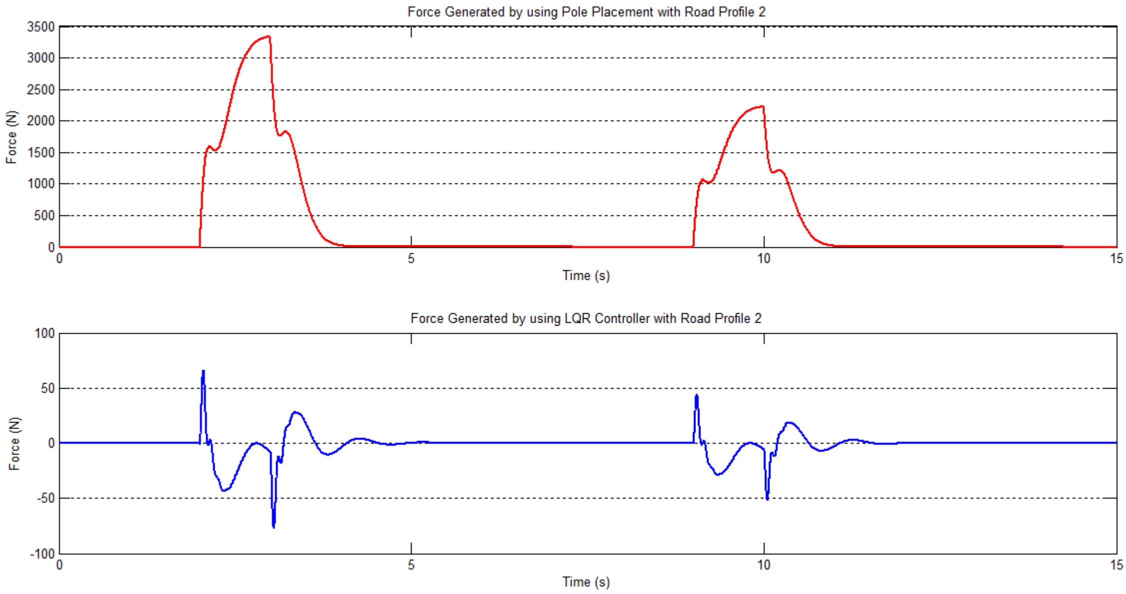


Figure 4.6: The force generated with road profile 2.

The Figure 4.4, Figure 4.5 and Figure 4.6 above shows that the comparison of control law performance between pole placement and LQR Controller. There are three comparison have been done to show the performance between the both controller. First, Figure 4.4 shows the performance of force generated without disturbance. Second, Figure 4.5 shows the performance of force generated with road profile 1 and Figure 4.6 shows the performance of force generated with road profile 2. From the three figure, the performance of control law for LQR Controller is better than Pole Placement. The transient performance of the both controller has been recorded in Table 4.2 below.

Table 4.2: Comparison of performance between Pole Placement and LQR Controller.

Elements		Maximum peak (N)		Settling Time (s)	
		Pole Placement	LQR Controller	Pole Placement	LQR Controller
Without Disturbance		1689	1162	2.588	2.172
Road Profile 1		3341	66.32	2.520	3.430
Road Profile 2	Bump1	3341	66.32	2.286	3.400
	Bump 2	2227	44.21	2.690	3.410

From the Table 4.2, the maximum peak value for Pole Placement is higher than LQR Controller. While the settling time of Pole Placement is lowest than LQR Controller. This is because the Pole Placement can stabilize the performance faster than LQR controller instead of higher maximum peak value. Otherwise, the LQR Controller can give better performance by reducing the maximum peak value with the affordable time taken for the system to stabilize. From the control law performance for both controller, it can conclude that LQR Controller is better in designing the controller for an active quarter-car suspension system.

4.4 Suspension system performance

The performance of the suspension system may be analysed through the suspension travel, car body acceleration and wheel deflection. The suspension travel represents the performance of the vibration due to road roughness. The car body acceleration represents the performance of ride comfort. The wheel deflection represents the performance of the car handling. In this part, all the results have been analysed and compared between the performance of suspension system without controller and with LQR controller.

4.4.1 Suspension system performance for road profile 1

This part shows the suspension system performance for road profile 1. In real-time road profile1 represent a single bump on the road. The performance shows the vehicle condition when perturbed by a single bumpy road.

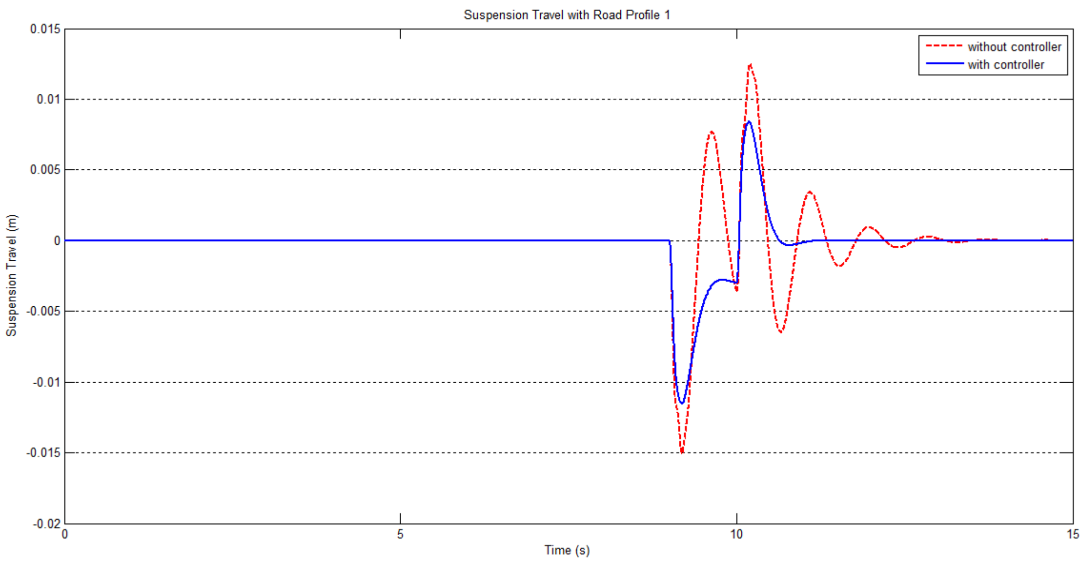


Figure 4.7: The suspension travel with road profile 1.

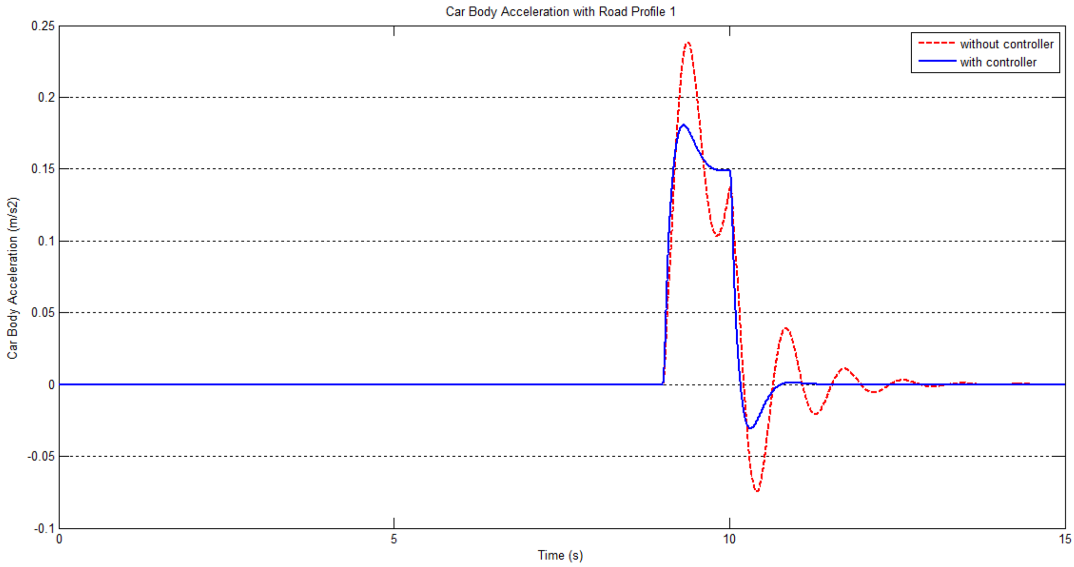


Figure 4.8: The car body acceleration with road profile 1.

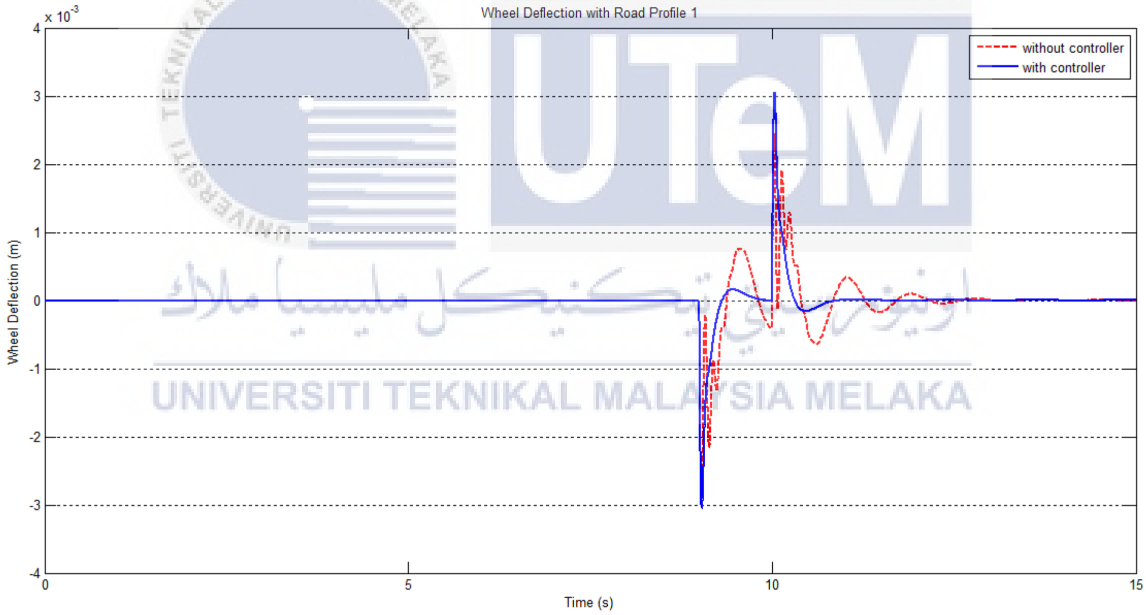


Figure 4.9: The wheel deflection with road profile 1.

The Figure 4.7, Figure 4.8 and Figure 4.9 above shows that the comparison of the performance of suspension system without controller and with LQR Controller for road profile 1. Figure 4.7 shows the performance of suspension travel, Figure 4.8 shows the performance of car body acceleration and Figure 4.9 shows the performance of wheel deflection due road profile 1. From the three figure, the maximum peak of the system without controller is higher than the system with LQR Controller. The Table 4.3 shows the result of the transient performance of the suspension system with road profile 1.

Table 4.3: Comparison of suspension system performance for road profile 1.

Elements	Maximum peak		Percentage Reduction (%)	Settling Time (s)	
	Without Controller	LQR Controller		Without Controller	LQR Controller
Suspension Travel (m)	0.0125	0.0084	32.8	5.819	2.199
Car Body Acceleration (m/s2)	0.2383	0.1805	24.3	5.980	2.360
Wheel Deflection (m)	0.0025	0.0031	19.4	6.140	2.520

4.4.2 Suspension system performance for road profile 2

This part shows the suspension system performance for road profile 2. In real-time road profile 2 represent 2 bump on the road. The performance shows the vehicle condition when perturbed by a 2 bumpy road.

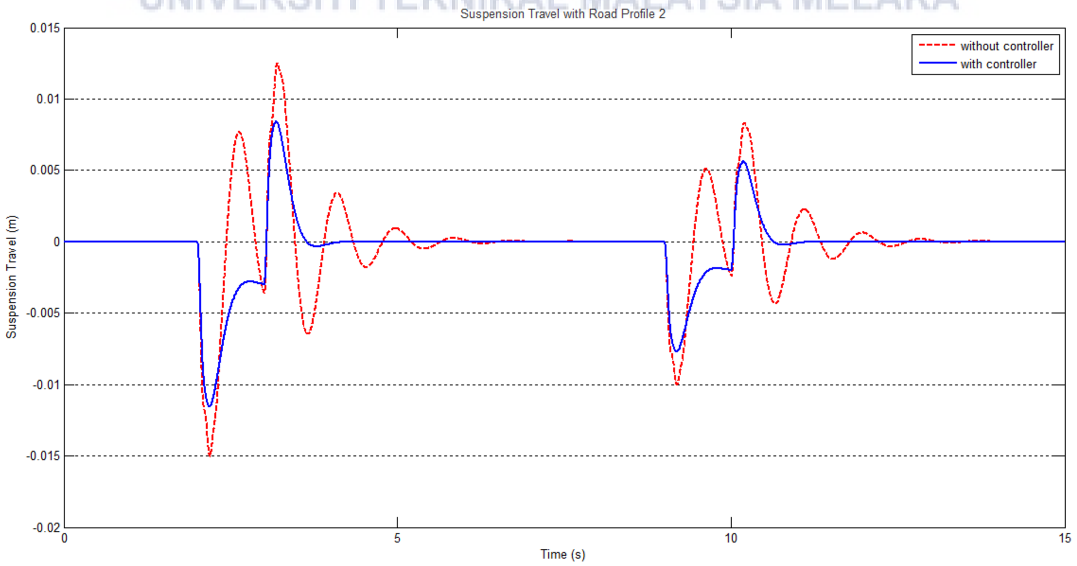


Figure 4.10: The suspension travel with road profile 2.

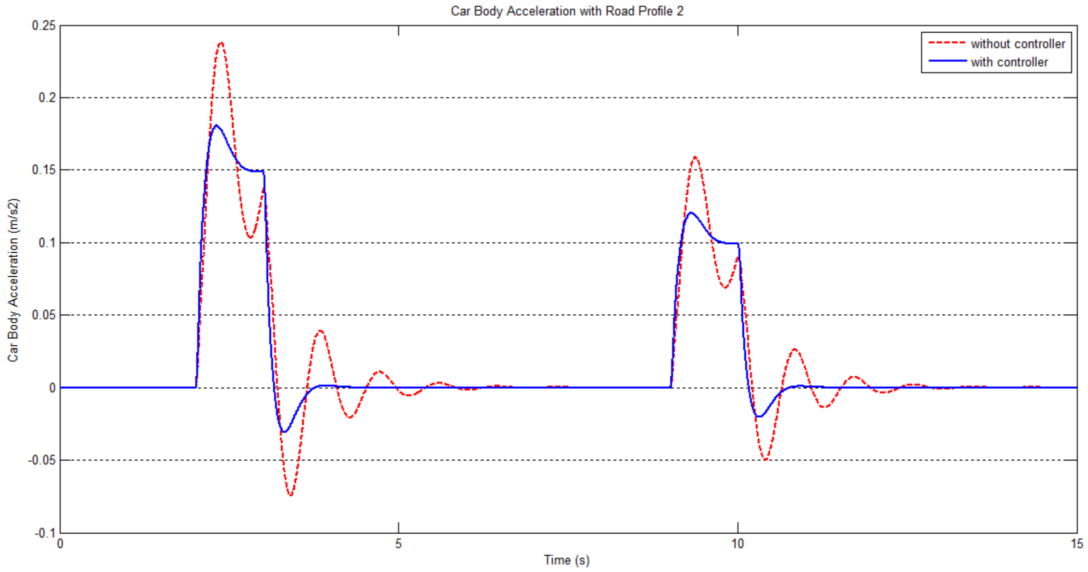


Figure 4.11: The car body acceleration with road profile 2.

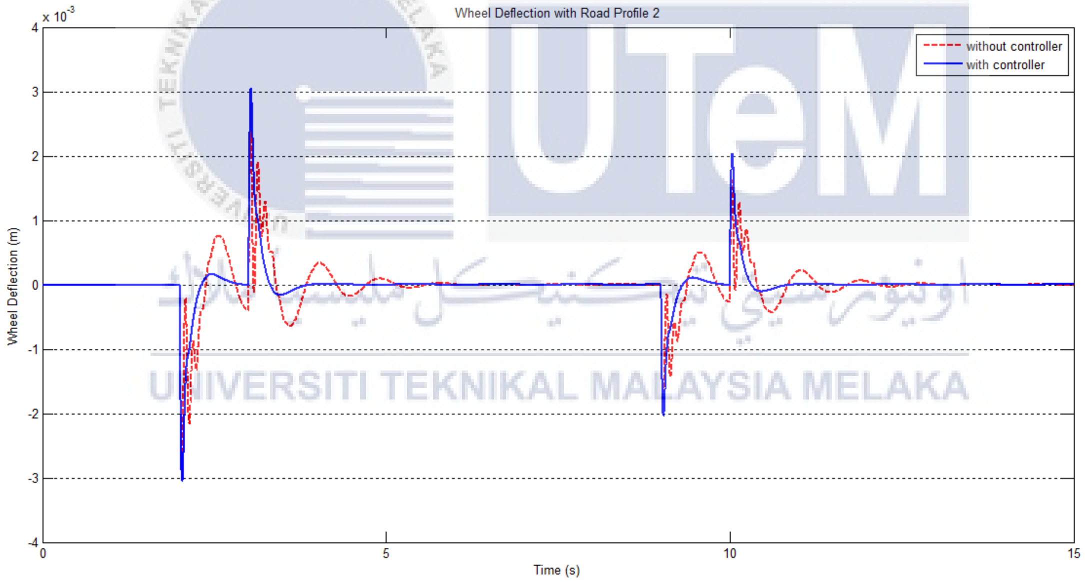


Figure 4.12: The wheel deflection with road profile 1.

Based on Figure 4.10, Figure 4.11 and Figure 4.12 above shows that the comparison of the performance of suspension system without controller and with LQR Controller for road profile 2. The performance of suspension system in this part is same as in road profile 1, where Figure 4.10 shows the performance of suspension travel, Figure 4.11 shows the performance of car body acceleration and Figure 4.12 shows the performance of wheel deflection due road profile 2. From the three figure, the maximum peak of the system without controller is also higher than the system with LQR Controller. The Table 4.4 shows the result of the transient performance of the suspension system with road profile 2.

Table 4.4: Comparison of suspension system performance for road profile 2.

Elements	Maximum peak				Percentage Reduction (%)		Settling Time (s)			
	Without Controller		LQR Controller				Without Controller		LQR Controller	
	Bump 1	Bump 2	Bump 1	Bump 2	Bump 1	Bump 2	Bump 1	Bump 2	Bump 1	Bump 2
Suspension Travel (m)	0.0125	0.0083	0.0084	0.0056	32.8	32.5	5.764	5.820	2.810	2.230
Car Body Acceleration (m/s2)	0.2383	0.1589	0.1805	0.1204	24.3	24.2	5.544	5.580	2.368	2.310
Wheel Deflection (m)	0.0025	0.0016	0.0031	0.0020	19.4	20	5.650	5.730	2.373	2.400

By comparing the performance of the system without controller and the system with LQR Controller in Table 4.3 and Table 4.4, it clearly shows that the LQR Controller performances shows lower amplitude and faster settling time compared with the system without controller. After implement the LQR Controller, the performance of suspension travel, car body acceleration and wheel deflection have been improved.

Suspension travel and car body acceleration performance for the system with LQR Controller for the two types of road profile can be reduce the amplitude and settling time compare to the system without controller. Wheel deflection performance also improve even though the amplitude is slightly higher compare with the system without controller however the performances shows the settling time is very fast.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

In this project, Linear Quadratic Regulator (LQR) is used as a controller to analyse the transient performances of the active quarter-car suspensions systems. At the beginning to achieve this objective, the mathematical model of an active suspension system for a quarter-car model has been derived. The result of the system clearly show the performance of an active suspension system without the controller and with the controller. The development suspension system without controller shows the increasing of amplitude at early oscillation is worst before the system become stable on the road. Therefore, in the next stage the LQR controller are introduced into the system to improve the performance of the suspension system.

Based on the result of the controller performance, it can be concluded that state-feedback design using pole placement and LQR controller can be stabilize the control law of the system. But, it was illustrated that the LQR controller gives a better achievement than pole placement. The capability to resolve the problem in term of to maintain the car handling and passenger's comfort for LQR controller is better than pole placement. Furthermore, the LQR controller proves that the reducing enforcement such as reducing the percent of overshoots and transient respond while driving on the uneven road profile. So, as a conclusion, all the objective of this project has been achieved

5.2 Recommendation

Work study that has been done, on an active control of quarter-car suspension system using Linear Quadratic Regulator. In this project, the LQR Controller has been designed as the state-feedback controller. Instead of using LQR Controller, there are others method can be used to tune the feedback gain. The feedback gain can be tune by using computational method or artificial intelligent techniques such as Artificial Neural Network (ANN), Fuzzy Logic Controller (FLC), Genetic Algorithm (GA) and others. The implement of other method to tune the feedback gain can improve robustness towards various road profile. Other than that, to get a better suspension system performance, the suspension system using quarter-car model can be changed with the full-car model.

REFERENCES

[1] Kallemullah; Faris; Hasbullah, "Design of Robust,Fuzzy and LQR Controller for Active Suspension of a Quarter Car Model," 2011.

[2] A. Agharkakli, G. S. Sabet, and A. Barouz, "Simulation and Analysis of Passive and Active Suspension System Using Quarter Car Model for Different Road Profile," *Int. J. Eng. Trends Technol.*, vol. 3, pp. 636–644, 2012.

[3] H. Li, X. Jing, H. Lam, and P. Shi, "Fuzzy Sampled-Data Control for Uncertain," *IEEE Trans. Cybern.*, vol. 44, no. 7, pp. 1111–1126, 2014.

[4] A. Tahir, J. Yasin, M. Daneshtalab, and J. Plosila, "Active Suspension System for Heavy Vehicles," *Fundam. Electr. Eng.*, 2014.

[5] J. Watton, K. M. Holford, and P. Surawattanawan, "Electrohydraulic e V ects on the modelling of a vehicle active suspension," *J. Automob. Eng.*, vol. 215, pp. 1077–1092, 2001.

[6] M. Miaomiao, C. Hong, and C. Yanfeng, "Backstepping Based Constrained Control of Nonlinear Hydraulic Active Suspensions *," *Proc. 26th Chinese Control Conf.*, 2007.

[7] C. Huang, J. Lin, and C. Chen, "Expert Systems with Applications Road-adaptive algorithm design of half-car active suspension system," *Expert Syst. Appl.*, vol. 37, pp. 4392–4402, 2010.

[8] I. Fialho and G. J. Balas, "Linear Parameter-Varying Gain-Scheduling," *IEEE Trans. Control Syst. Technol.*, vol. 10, no. 1, pp. 43–54, 2002.

[9] W. Sun, Z. Zhao, and H. Gao, "Active Suspension Systems," *IEEE Trans. Ind. Electron.*, vol. 60, no. 9, pp. 3889–3896, 2013.

[10] W. Sun, H. Gao, S. Member, and O. Kaynak, "Adaptive Backstepping Control for Active Suspension Systems With Hard Constraints," *IEEE/ASME Trans. MECHATRONICS*, vol. 18, no. 3, pp. 1072–1079, 2013.

[11] W. Sun, H. Gao, and B. Yao, "Adaptive Robust Vibration Control of Full-Car Active Suspensions With Electrohydraulic Actuators," *IEEE Trans. Control Syst. Technol.*, vol. 21, no. 6, 2013.

[12] C. Göhrle, A. Schindler, A. Wagner, and O. Sawodny, "Design and Vehicle Implementation of Preview Active Suspension Controllers," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 3, pp. 1135–1142, 2014.

[13] J.D.Robson, "Road Surface Description and vehicle Response," *Veh. Syst. Dyn.*, vol. 1, pp. 25–35, 1986.

[14] S. A. H. R.S.Sharp, "The Relative Performance Capabilities of Passive, Active, and Semi-Active Car Suspension System," *Proc.Inst, Mach.Enars*, 1986.

[15] A. Elmadang MM., El-Tamimi, "Subclass of Nonlinear Passive and Semi-active Damping for Vibration Isolation.," *Comput. Struct.*, vol. 36, pp. 23–45, 1990.

[16] A. Raukich, S., Titli, "Design of active and semi-active automotive suspension using fuzzy logic," *12th World Cogress 2 Sydney*, pp. 253–257, 1993.

[17] Lauwerys;C.;J. Swevers;P. Sas, "Design and experimental validation of a linear robust controller for an active suspension of a quarter car," 2004.

[18] Hespanhna J.P, *Undergraduate Lecture Note on LQG/LGR controller Design*. University of California Santa Barbara, 2007.

[19] N. Sam Y.M., Ghani M.R.A and Ahmad, "LQR Controller for Active Car Suspension.," *IEEE Control Syst.*, pp. 14441–1444, 2000.

[20] H. W. M. William D.L., "Active Suspension Control to Improve Vehicle Ride and Handling," 1997.

[21] & L. T. T. Wu S.J., Chiang H.H., Chen J.H., "Optimal Fuzzy Control Design for Half-Car Active Suspension Systems," 2004.

[22] J. Lin, Y-J., Lu, Y-Q., Padovan, "Fuzzy logic control for vehicle suspension systems," *Int. J. Veh. Des.*, vol. 14, pp. 457–470, 1993.

[23] T. H. D. & F. A. Shariati A., "Decentralized Robust H-∞ Controller Design for a Full Car Active Suspension System," 2004.

[24] M. D. H.Imine, L.Fridman, H. Shraim, "Sliding Mode Based Analysis and Identification Of Vehicle Dynamics," *Third Ed.Publishing Company, Inc*, 2011.

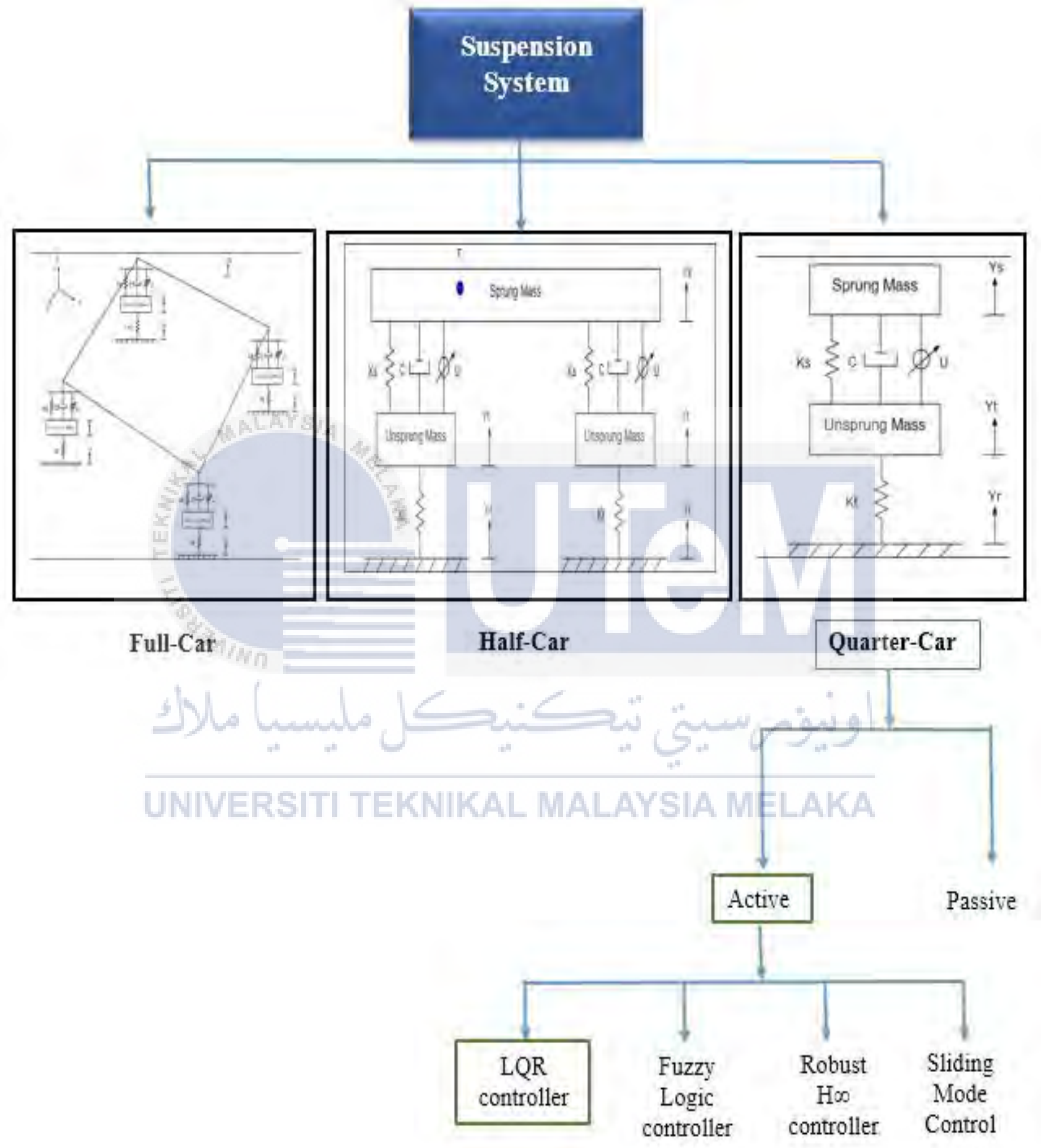
[25] Sam Y.M., Osman H.S.O., Ghani M.R.A., "A Class of Proportional-Integral sliding Mode Control with Application to Active Suspension System," 2004.

[26] F. A. Ansari and R. Taparia, "Modeling , Analysis and Control of Active Suspension System using Sliding Mode Control and Disturbance," *Int. J. Sci. Res. Publ.*, vol. 3, no. 1, pp. 1–6, 2013.

APPENDIX

APPENDIX A

K-map of the suspension system



APPENDIX B

Gantt Chart for FYP 1.

Item																
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
FYP TITLE																
Find Supervisor																
Title Confirmation																
Familiar with Topic																
Log Book Preparation																
CHAPTER 1																
Objective & Scope																
Problem Statement																
CHAPTER 2																
Find Sources																
Literature Review																
CHAPTER 3																
Methodology																
CHAPTER 4																
Preliminary Result																
CHAPTER 5																
Conclusion																
FYP Draft Report Submission																
FYP Presentation																
FYP Report Submission																

Gantt Chart for FYP 2.

Task																		
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
Design the suspension system using MATLAB with SIMUINK toolbox	■	■	■	■														
Design Pole Placement and LQR controller to control the system					■	■	■	■	■	■								
Analyse and compare the result								■	■	■	■	■						
Update the FYP report									■	■	■	■	■					
FYP Draft Report Submission													■	■				
FYP Presentation															■			
FYP Report Submission																■		
FYP Hardbound Submission																	■	■

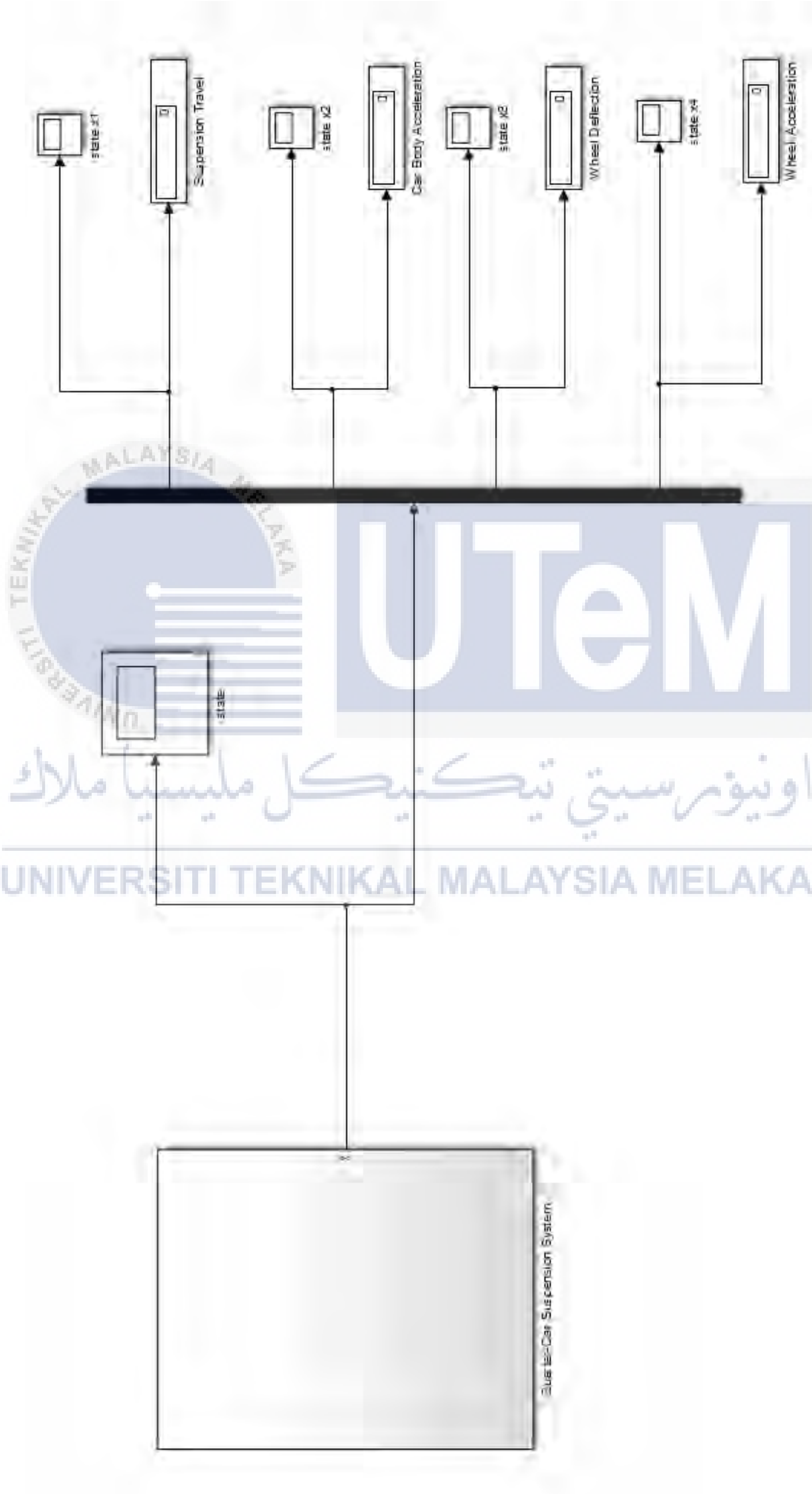
APPENDIX C

Coding

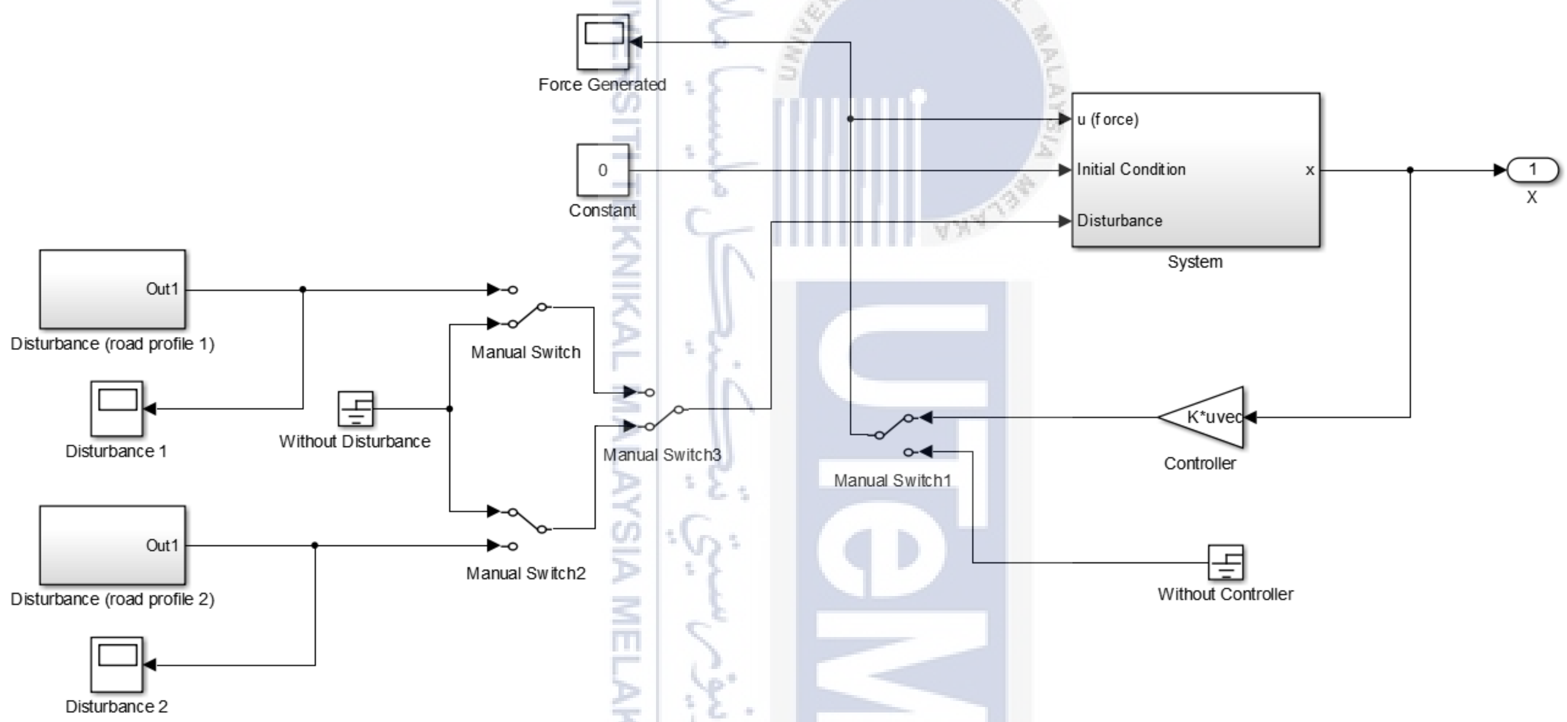
```
%create state-space model
m1=59; %unsprung mass(kg)
m2=290; %sprung mass (kg)
ka=16812; %stiffness of car body spring (N/m)
kt=190000; %stiffness of tire (N/m)
ca=1000; %damper (Ns/m)

A=[0 1 0 -1;-ka/m2 -ca/m2 0 ca/m2;0 0 0 1;ka/m1 ca/m1 -kt/m1 -ca/m1];
%system matrix
B=[0 1/m2 0 -1/m1]'; %input matrix
C=[0 0 0 0]; %output matrix
D=0;
system=ss(A,B,C,D)

%%
%create state-space model with disturbance
m1=59; %unsprung mass(kg)
m2=290; %sprung mass (kg)
ka=16812; %stiffness of car body spring (N/m)
kt=190000; %stiffness of tire (N/m)
ca=1000; %damper (Ns/m)

A=[0 1 0 -1;-ka/m2 -ca/m2 0 ca/m2;0 0 0 1;ka/m1 ca/m1 -kt/m1 -ca/m1];
%system matrix
B=[0 1/m2 0 -1/m1]'; %input matrix
C=[0 0 0 0]; %output matrix
D=0;
F=[0 0 -1 0]';
system=ss(A,B,C,D)
%%
%check poles OL system
eig(A) %Find eigenvalue of system A (OL poles)

% check controllability & observability
rank(ctrb(A,B))%if rank=orderof system = controlable
rank(obsv(A,C))%if rank=orderof system = observable

%%
%Design Controller

%Control using Pole Placement
K=place(A,B,[-10 -11 -12 -13])

%check poles CL system
eig(A-B*K)%Find eigenvalue of system A-Bk (CL poles)

%%
%Control using LQR controller
R=0.0001
Q=1000*eye(4)
[K,P,E]=lqr(system,Q,R)
```

APPENDIX D

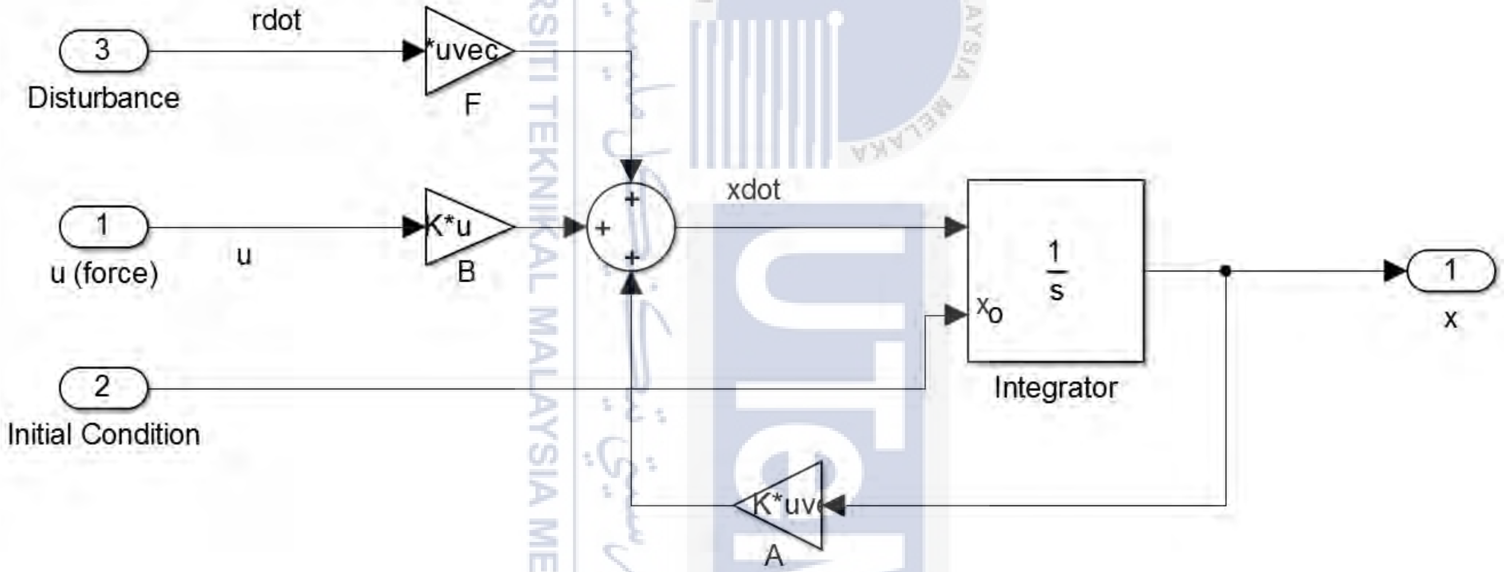
Block Diagram



Quarter-Car Suspension System.



System with disturbance and controller.



System without controller