



FACULTY OF ELECTRICAL ENGINEERING

**CONTROLLER DESIGN FOR NON LINEAR MOTORIZED
PROSTHETIC HAND SYSTEM**

Nur Awanis Binti Abdul Razak

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ABSTRACT

This paper described the controller design for nonlinear motorized prosthetic finger system. It can be used as a human assistive device to the amputee. Since the prosthetic device is wear by human, the accuracy of the system is crucially important to avoid unnecessary hazard to the user. In addition, the mathematical modelling of the system need to be find appropriately to ensure the accuracy of the system later on. The main objective of this project is to design the controller for the system. There are many type of controller that can be used in order to design the stable system of nonlinear actuated finger such as Proportional Integral (PI), Proportional Integral and Derivative (PID) and Fuzzy Logic controller. In this project, the Proportional Integral and Derivative (PID) controller will be use. The tuning of PID control parameter is for the position reference design and the position feedback control of the motor. Gradient Descent or Auto tune method can be used to improve the transient response performance of the motor by tuning the parameter of PID control. From this project, it is expected that prosthetic finger will move according to the desired position and smoothly with the presence of PID controller. The value of rise time, settling time, overshoot and steady state error can also be improved.

ABSTRAK

Penggerak selari jari merupakan jari palsu yang digerakkan secara selari oleh motor. Ianya boleh digunakan sebagai alat bantuan bagi individu yang tidak mempunyai jari ataupun kudung. Memandangkan jari tiruan ini dipakai oleh individu, ketepatan sistem sangat penting bagi mengelakkan sebarang masalah yang tidak diingini kepada pengguna. Sebagai tambahan, persamaan matematik juga haruslah dicari secara teratur bagi memastikan sistem tersebut dapat beroperasi dengan tepat. Tujuan utama projek ini dijalankan adalah untuk mencipta satu unit kawalan bagi jari palsu tersebut. Terdapat pelbagai jenis unit kawalan yang boleh digunakan bagi memastikan kestabilan sesuatu sistem seperti unit kawalan perkadaran bersepadu (PI), unit kawalan perkadaran bersepadu dan terbitan mutlak (PID) dan unit kawalan logik kabur. Dalam projek ini, unit kawalan perkadaran bersepadu dan terbitan mutlak (PID) akan digunakan. Penyelarasan unit kawalan PID adalah untuk kedudukan rujukan dan tindak balas kedudukan kawalan bagi motor. “Gradient Descent” atau penyelarasan automatic boleh digunakan bagi menambahbaikkan kepada tindak balas transisi bagi motor apabila unit kawalan PID diselaras. Keputusan yang dijangkakan dari projek ini ialah jari palsu tersebut akan bergerak pada kedudukan yang dikehendaki dan kajian juga membuktikan bahawa jari palsu tersebut boleh bergerak dengan lancar dengan bantuan penyelaras parameter PID yang dapat mengurangkan masa kemuncak, masa menaik, ralat keadaan kukuh dan masa penenangan.

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CHAPTER 1

INTRODUCTION

1.1 Motivation

The designing of fully functioning prosthetic finger or hand with smooth movement and strength like a real finger is the ultimate goal in motorized prosthetic finger. Unfortunately, current technologies that exist in designing and material are still long way to go in making this goal a successful. There are major factor that limiting the process to make the prosthetic finger model which is the power, weight and size of that model. In addition, other difficulty that faces in order to develop the model of prosthetic finger is to find the sufficient number of appropriate control source to control the required number of degree of freedom. Based on the research made, the prosthetic finger is dominated by the consideration of control. However, the importance of better multifunctional mechanism and better actuator cannot be ignored. The control will become useless if the effective finger mechanism is not available.

Human hand is perplexing that comprises of 27 bones with the huge number of muscle and ligaments to give countless of flexibility (DOF) in development. Moreover, each hand has a cluster more than 17000 material sensors. In any case, it is practically difficult to supplant something that is comparable with the genuine finger particularly when it is identified with strength and unwavering quality by utilizing the current innovation. The principle inspiration in directing this venture is to defeat the aggravation, grinding and commotion that happen in plant or process. As an expansion, the issue in outlining the controller is the instability about the subject and the nonlinear dynamic. Consequently, keeping in mind the end goal to get the soundness and unwavering quality for the framework, the most appropriate controller must pick.

1.2 Research Background

For a few recent decades, in manufacturing industry robot was widely used to assist human work. These robots have different type of speciality that suit the different purpose of manipulation. The normally used of robot machine in industries are suction cups and gripper but it have less flexibility compared to the fingered end effectors. Other than that, by using a multi-fingered robotic hand it can deliver an accurate and precise motion of the position. However, the most vital aspect in development of multi-fingered robotic hand is stability, cost and reliability. In order to produce a prosthetic hand that can perform like human hand, the system required control and stability. Thus, it is very important to build a prosthetic hand with accurate motion control [1].

Figure 1.1 shows the real-time control of a 7-DOF robotic hand for object grasping. The structure of the prosthetic hand consists of three 2-DOF fingers that have two joints and two links for each finger. The motor that is located inside of the robot link will actuated each of the robot joints. The motor is then joint together with encoder and interfaces with Motion Manager application for control via a motion controller [1].

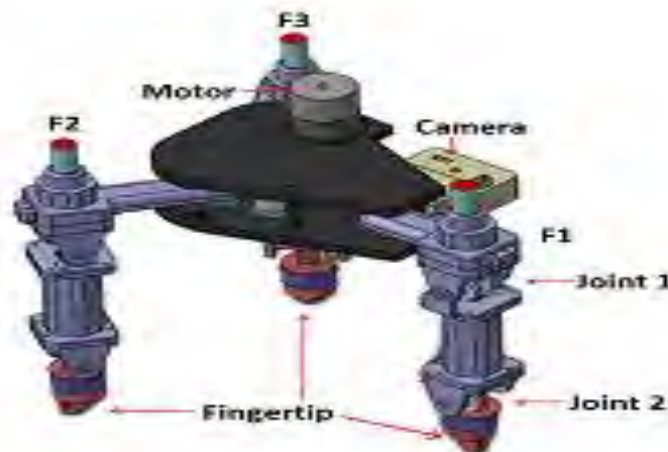


Figure 1.1 Real-time Control of a 7-DOF Robotic Hand for Object Grasping

During the last three decades, the development of robotic gripper for grasping many type of object has develop from using a simple mechanism to multiple degree of freedom of design. Most robotic grippers were designed specifically just to grasp on certain object form only. It is a difficult task to design a gripper finger because it required many considerations such as geometry of gripper, task requirement and the complexity of mechanism.

According to traditional way, it is necessary to have a physical prototype in order to test the ability of the hand to perform a variety of task. However, it is quite costly because many designs are required to develop the prototype. Thus, the simulation technique will provide another tool that enables the modern designer to simulate kinematic and dynamic of physical system and to investigate the performance of the design quickly. This technique will reduce the cost of prototype. Simulation is one of the powerful tools that exist in supporting the design, planning and analysis of dynamics performance of robotic grippers [2].

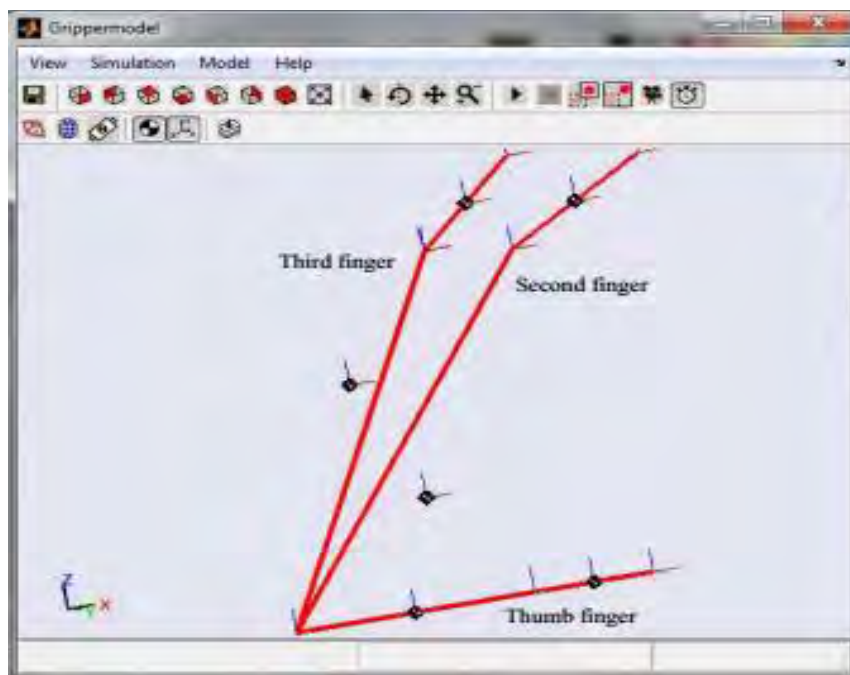


Figure 1.2 Gripper Position of Robotic Finger

Human hands are one of the body parts that are capable in manipulation task and grasping many type of object. As an example we can learn to play the musical instrument with our hand, use chopsticks and perform daily routine such as writing and cooking with a proper training. Unfortunately, the robotic hand that existing are not capable to demonstrate human level of dexterity. Dexterity of movements in human hand is achieved due to the neuromuscular control and biomechanics of the hand. The human level of dexterity must be analyse and understand in order to mimic the neuromuscular control and the biomechanics to develop the robotic hand [3].

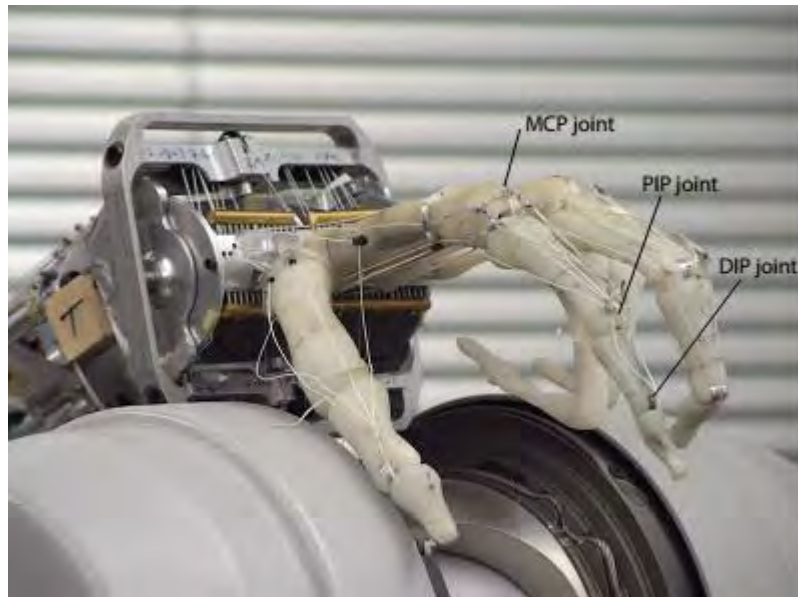


Figure 1.3 Skeleton Structure of Robotic Finger



Figure 1.4 Structure of Human Finger and Flexion of Angel of One Finger

Figure 1.4 illustrates the structure of human finger that consist of three parts which is distal phalanx, middle phalanx and proximal phalanx. In between of the phalanx there is distal inter-phalangeal joint and proximal inter-phalangeal joint. The finger's movement with flexion of angel of one finger can be described with one degree of freedom model (1 DOF).

1.3 Problem Statement

The problem statement for this project is a pure motion controller usually gives poor performance and can even cause instability. It also has a non-linearity problem along with the inaccuracy of the system. Thus, to overcome this problem the best controller that have high accuracy for the nonlinear actuated finger must be selected. Accuracy and control are important to produce an actuated finger that can perform as the real human finger. In addition, the simulation with PID controller was needed to be tuned in order to get more stable result for the system.

1.4 Objective

This project embarks into following objectives:

1. To model the prosthetic hand system by using Lagrange's equation.
2. To design the controller using PID based controller.
3. To verified and validate the performance of the controller via simulation.

1.5 Scope

The scope of this study is firstly, mathematical modelling have to be developed before the controller was designed to the system. In this paper the Lagrange's equation was used as an approach to the nonlinear motorized prosthetic finger system. Then, the controller of Proportional Integral and Derivative (PID) will be applied in order to gain the stable system. As for the tuning of PID, Gradient Descent method can be used to improve the transient response performance of the motor. This method was then compared to auto tuning method in order to prove it will give the most accurate result for the tuning of PID.

1.6 Expected Project Outcome

The expected project outcome is to obtain the most accurate result after the PID controller was design to the system. Then, the tuning method for PID controller will be proposed. The result from the tuning method which is Gradient Descent will be compared with the result obtain from the auto tune method. The best closed loop characteristics then can be determined from this comparison.

1.7 Proposal Outline

Chapter 1 discussed about the overall proposal of the project such as motivation, research background that related to the project, problem statement, objectives, scope and expected result that will be obtain later on. This project was conducted based on the problem statement and objective that has been listed.

Chapter 2 discussed about the literature review that related to this project. There are few papers that have been going through in order to gather some information about prosthetic finger system, controller applied to the system and it tuning method. Other than that, system modelling was also been discuss in this chapter.

Chapter 3 discussed about the methodology involves while doing this project. First of all, the prosthetic finger system was model by using the Lagrange's equation. Then, the controller has been design for the system along with it tuning method. In this chapter the steps for tuning method of PID controller will be discuss and describe. The Gradient Descent was used as the tuning method for the controller. Flow of the project can be seen in this chapter with some explanation for each step.

Chapter 4 discussed about the most important part for this project which is the result of the simulation. The result before and after designing of PID controller was observed in this chapter. It is obviously shown that the best result only can be obtained when the controller was installed in the system. Next, to determine the most accurate result, the tuning method was used for the system. This tuning method was then compared with other method in order to obtain the best trade off result.

Chapter 5 discuss about the conclusion for the whole project and also some recommendation about the future work plan.

CHAPTER 2

LITERATURE REVIEW

2.1 PID Controller

Past reviews have announced that Proportional Integral and Derivative (PID) is a procedure of referee the controller parameter to decide the coveted yield of control framework. The mistake between the procedure variable and its set point can be limited by utilizing the controller. In this manner, experimentation strategy has been utilized as a part of request to recognize the PID control parameter [1].

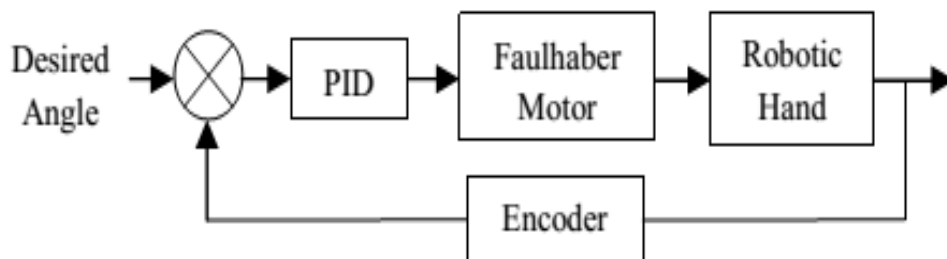
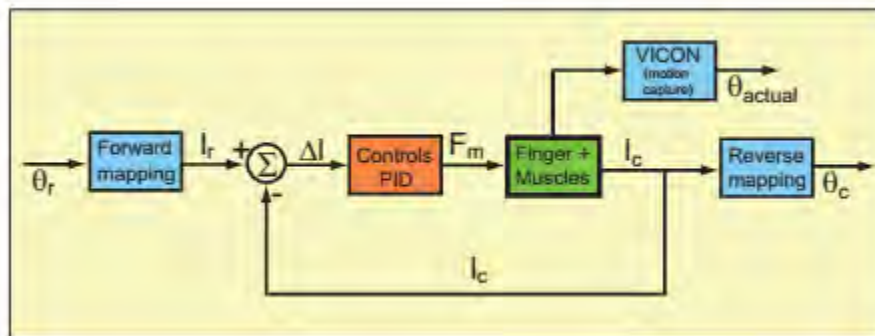


Figure 2.1 The control system of robot control

This paper has presents a coordinated plan handle for planning the five fingered gripper that is appropriate for smooth movement by utilizing analysis and reproduction of the model. Next, multi-shut circle with the powerful control of PID was connected to the framework for controlling both dynamic and kinematics movement of the five fingered gripper framework. The joint controller is a criticism controller comprises of two terms which is relative to speed and position blunders, it additionally presenting the subordinate and corresponding activities known as PID control. The kinematics movement of the position plots for each finger was controlled by utilizing the propel PID control with auto tuning [2]. A discrete Proportional Integral Derivative (PID) control procedure would significantly decrease the cost and size of the controller since it can supplant the complex electronic hardware [3].

Late confirmation demonstrates that with a specific end goal to acquire the precise situating of automated hand for getting a handle on, the tuning of PID control parameter was imperative to enhance the transient reaction. Prior to the tuning of PID is executed, position has somewhat vibrated because of the unfaltering state blunder and overshoot delivered by the engine [1]. The appropriate increases of differential (Kd), indispensable (Ki) and relative (Kp) esteem are controlled by utilizing auto tuning technique to accomplish quick reaction of consistent state (setting tune) without extreme overshoot. By tuning these three appropriate estimations of steady picks up in the PID controller calculation, the controller was found to give the essential control activity to particular process prerequisites [2]. At the given time interim which is test period (T), a discrete PID controller will peruses the blunder flag, figure it yield and control the information supply to the engine. In this way, to get the coveted outcome, the specimen time should dependably be not as much as the most brief time consistent in the framework [3]. The PID controller is set up for each muscle to track the coveted in light of the mistake of the framework [4].



Figures 2.2 The difference from the current muscle length, L_c is fed into the PID controller to determine the force that directly control the muscle

2.2 Fuzzy logic controller

From the current proof, it can be seen that to restrict the development of the coveted heading and to keep up a steady compel along the moving course, a position or drive controller with fuzzy logic must be created [5]. To exhibit on how simple the fuzzy sets enrolment can influence the execution of the framework, the examination is partitioned into two unique parts. The initial segment will just consider the question shape as the information

and the edge introduction as it yield while for the second part the execution was investigate just when two data sources are nourished into the framework. In fuzzy logic step, the framework can comprehend the execution of various fuzzy set enrolment with a specific end goal to choose the best technique for the getting a handle on framework.

In spite of the fact that the fuzzy tenets appear like extremely basic, despite everything it can give a superior outcome particularly in considering the best getting a handle on framework since it can give a general picture just by picking the fitting enrollment work [6]. Various specialists likewise have announced that simulated control, for example, Fuzzy logic, hereditary calculation, neural system and neurofuzzy control have been connected in numerous applications. Its application zone is wide in light of the fact that the fundamental shape for all summon sorts of controller comprises of info fuzzy govern base, surmising, fuzzification and yield defuzzification [7].

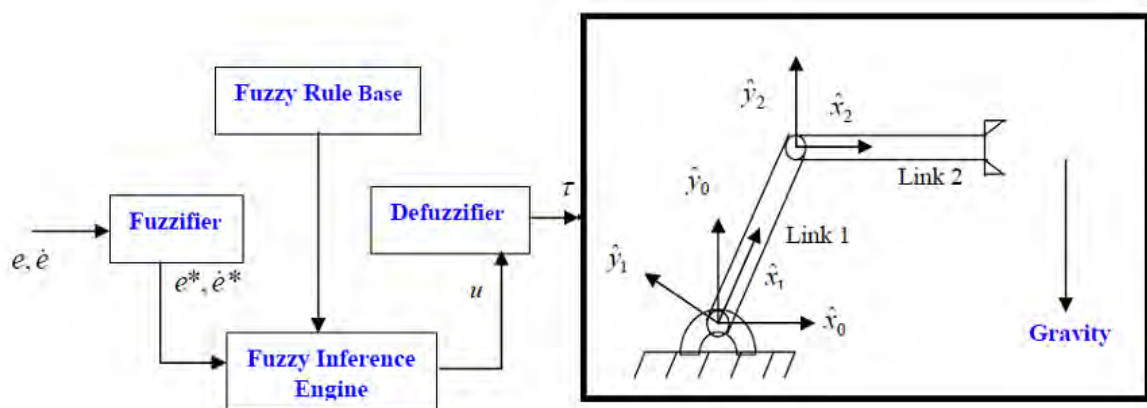


Figure in 2.3 Block diagram of fuzzy controller

The past review expressed that to decide the ideal control parameter precisely, a regular straight corresponding basic (PI) controller and fuzzy PI tuner ought to be joined as the constrain controller. On the off chance that this two things are not joined it is difficult to get the exact ideal control parameter in light of the fact that fuzzy PI tuner is working as an adjusted for the non-straight elements of the robot and the obscure aggravating power from the subject [5]. The fuzzy logic control can have preferred control over straight PI or PID control however it just can be accomplish if all the fundamental data about nonlinear dynamic plan framework and the parameter of robot controller can be decide. At that point, the technique for figured torque control can work great [7].

2.3 Conclusion

Overview that led by Shauri, Salleh, Hadi [1] demonstrated that the PID tuning of finger joints was effectively created to decrease the transient reaction, for example, rise time, overshoot, settling time, crest time and enduring state mistake. Engine will moved easily and it position was exactly to the objective position when the mistake and the overshoot have been disposed of from the framework. At that point, the reviews expressed that each finger of the gripper can be controlled by utilizing the vigorous control of PID plan since it is viably used to control the situating of the fingers [2]. The PID controller is working fine yet at some point it is important to make the controller more powerful (utmost runaway/flood) in a few applications. The exactness for either subsidiary (D) or vital (I) variable will end up noticeably poor if the example time is significantly littler or bigger than 1 second [3].

As a conclusion for the fuzzy logic controller, the controller was steady in the application scope of development and strengths. The underlying outcome demonstrated that the robot was succesfull to control the subject through straight and round developments by utilizing the typical and stroke subjects. At that point, amid the development the robot could keep up the steady drive regarding the matter [5]. A fuzzy logic based robot getting a handle on framework that was outline with three fingers which comprises of two typical fingers and a thumb is utilized for assessments. The triangular and Gaussian participation are two sorts of enrollment that were mostly examined [6]. The past reviews have demonstrated that the fuzzy tuning can tackle the issue with respect to the induction of blunder and change of mistake in light of the web based tuning coefficients. In light of reenactment and diagram strategy, the overshoot and swaying in nearness of the instability and outer aggravation can lessen [7].

Past work is directed to investigate the steadiness of the plan in the event of defective remuneration of the gravity term and potentially fall back on an adaption component on the framework satiate. In this way, the controller comprises of PD is plan on the position circle while a PI is created on the drive circle alongside the gravity pay and wanted contact constrain feedforward [8].

2.4 Tuning Method for PID Controller

There are many tuning method exists in order to tune the PID controller. The Ziegler Nichols is generally used for tuning the PID. This method will shows a successful result but it required an effort and takes a long time in order to obtain more satisfactory response. Ziegler–Nichols method depends on the parameter gain from the step response plant [10]. In the previous paper, the optimization of PID controller by using Gradient Descent method has been discussed. By applying the optimization technique from Gradient Descent method the performance of positioning tracking shows a significant improvement [11].

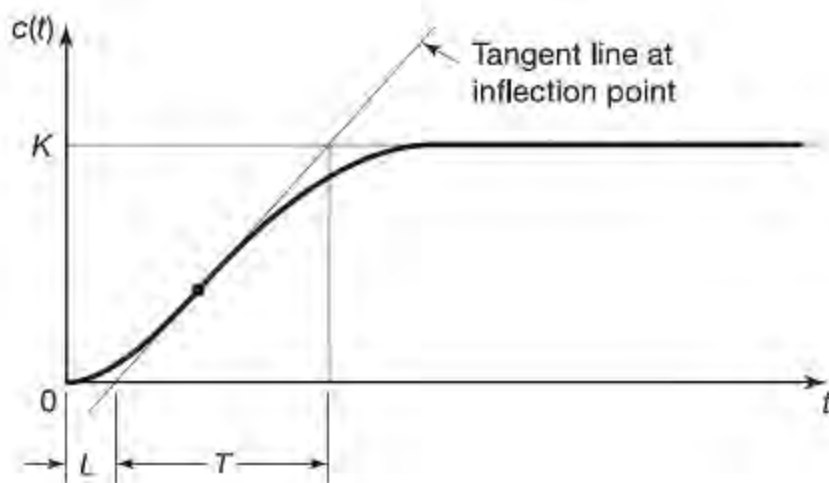


Figure 2.4 Ziegler Nichols Tuning Method

Type of controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9 \left(\frac{T}{L} \right)$	$\frac{L}{0.3}$	0
PID	$1.2 \left(\frac{T}{L} \right)$	$2L$	$0.5L$

Table 2.1 Parameter Obtained for Ziegler Nichols tuning method

The result shows that the tuned of PID controller give a feedback system of good disturbance rejection. However, in general the compensated system response to a step signal has a high control signal and high percent of overshoot which may lead the saturation of actuator [10]. The evaluation of PID controller performance by using Gradient Descent technique that was applied to the controller has been done. The result from the numerical simulation prove that an optimization techniques will produced more precise position trajectory tracking and significant improvement to the controller [11].

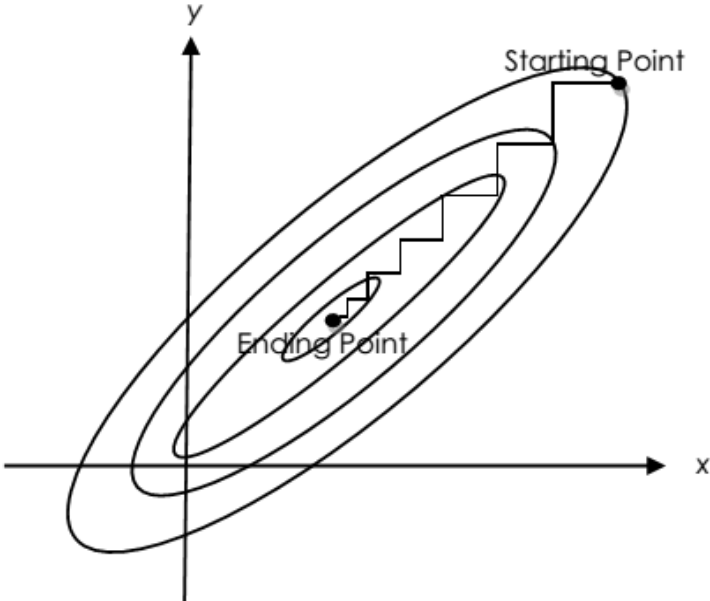


Figure 2.5 Directions to Reaching Local Minimum

CHAPTER 3

PROPOSED RESEARCH METHODOLOGY

This section covers the methodology part with the detail clarification to guarantee that this venture is finished and functioning admirably. There are numerous diaries that have been made in light of this venture can be utilized to energize some essential point and can be enhance as the up and coming reviews. The technique is use to decide the goal of the venture with a specific end goal to acquire the ideal and smooth outcome. Keeping in mind the end goal to assess this venture, the strategy comprises of five stages which are:

1. Data collection from literature review
2. System modelling of Prosthetic Finger system
3. Controller design (PID)
4. Tuning method of the controller
5. Result evaluation

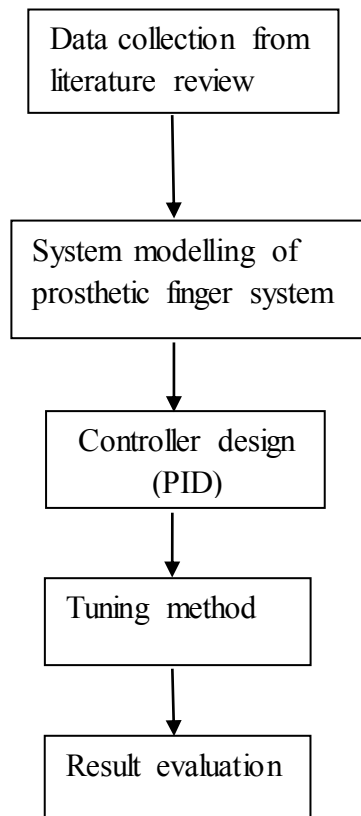


Figure 3.1: The flowchart of proposed research methodology

3.1 Literature review

In this part, the project was firstly started by finding the related journal and collects the important data from it. All of the journals are finding from the library and internet. There are many previous related journal was generated based on the title of prosthetic finger system. Thus, I have classified the data into four parts which is PID controller, Fuzzy logic controller, the comparison between these two controllers and method for tuning the controller which is Ziegler Nichols and Gradient Descent method. From the journal, I also can find further information about the controller and the advantages of each controller which is very useful for me to complete this project successfully. Other than that, by doing further research from the journal, I can listed the tuning method that exists and which method is commonly used for tuning controller. Then, I can make the comparison which tuning method is better for tuning the controller. In addition, I can understand more about the modelling, controller, tuning method and the moving position of the prosthetic fingers.

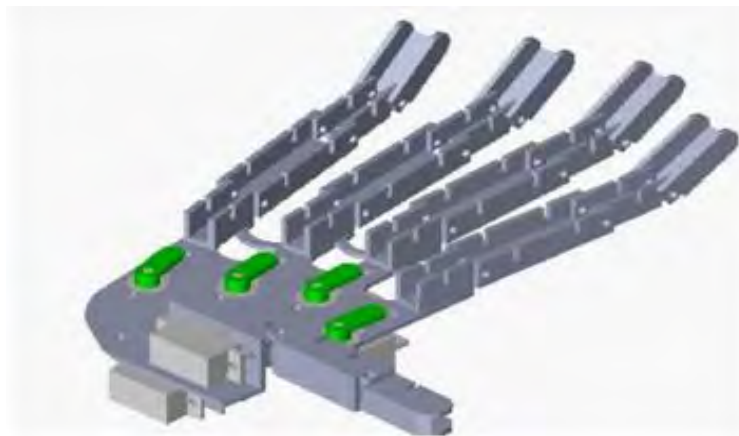


Figure 3.2: The design of five robotic fingers

3.2 Modelling of Prosthetic Finger

3.2.1 Mathematical modelling of Prosthetic finger

From the previous study, it shows that Lagrange Equation was used frequently in order to derive the model of prosthetic finger. Thus, as for the mathematical modelling Lagrangian Equation has been chosen.

Symbol	Parameter
l	Length
m	Mass
v	Linear velocity
ω	Angular velocity
g	Gravity (9.81 ms ⁻¹)

Table 3.1 Dynamic Parameter of Prosthetic Finger

The Lagrangian, $L = T - V$

(1)

T = Kinetic energy

V = Potential energy

Referring equation forward kinematic above, the angular velocity is computed using Euler Lagrange formula.

$$\omega = \frac{d\theta}{dt}$$

The angular velocity, $\omega = \dot{\theta}$

(2)

The Kinetic energy, T

$$T = \frac{1}{2} \sum (mV + l\omega^2)$$

(3)

$$T = \frac{1}{2} l \dot{\theta}^2 \left(\frac{1}{4} ml + 1 \right)$$

(4)