CHARACTERISATION OF WIDEBAND NONLINEAR VIBRATION ABSORBER USING ADJUSTABLE MAGNETIC STIFFNESS

ESWARAN A/L MANAKOR

UNIVERSITI TEKNIKAL MALAYSIA MELAKA

CHARACTERISATION OF WIDEBAND NONLINEAR VIBRATION ABSORBER USING ADJUSTABLE MAGNETIC STIFFNESS

ESWARAN A/L MANAKOR

This report is submitted
in fulfillment of the requirement for the degree of
Bachelor of Mechanical Engineering (Plant & Maintenance)

Faculty of Mechanical Engineering

UNIVERSITI TEKNIKAL MALAYSIA MELAKA

JUNE 2017

DECLARATION

I declare that this project report entitled “Characterisation of Wideband Nonlinear Vibration Absorber Using Adjustable Magnetic Stiffness” is the result of my own work except as cited in the references

Signature : ....

Name : ....

Date : ....

APPROVAL

I hereby declare that I have read this project report and in my opinion this report is sufficient in terms of scope and quality for the award of the degree of Bachelor of Mechanical Engineering (Plant & Maintenance).

Signature :. ....

Name of Supervisor : ....

Date : ....

DEDICATION

To my beloved father and mother

ABSTRACT

Linear dynamic vibration absorber can only suppress vibration of primary structure at one frequency. At other frequency it will amplify the vibration of primary structure. Nonlinear dynamic vibration absorber was introduced to increase the operational bandwidth of absorber. In this project, nonlinear dynamic vibration absorber was characterized using adjustable magnetic stiffness. Thisproject starts with identification first vibration mode of primary structure. Then Operating Deflection Shape (ODS) for the first vibration mode was constructed. Then, nonlinear dynamic vibration absorber was characterized using three different variables. They are effect of gap between magnets, effect of input level from shaker and effect of length of absorber beam. Lastly, performance of nonlinear dynamic vibration absorber on primary structure was investigated at three different points and three different gap levels.

ABSTRAK

Penyerap getaran linear hanya boleh menyekat getaran struktur utama pada satu frekuensi sahaja. Pada frekuensi lain ia akan menguatkan getaran struktur utama. Penyerap getaran tak linear telah diperkenalkan untuk meningkatkan jalur lebar operasi penyerap. Dalam projek ini, penyerap getaran tak linear dicirikan menggunakan kekukuhan magnet laras. Projek ini bermula dengan pengenalan mod getaran pertama struktur utama. Kemudian 'Operating Deflection Shape' (ODS) untuk mod getaran yang pertama telah dibina. Kemudian, penyerap getaran tak linear dicirikan menggunakan tiga pembolehubah yang berbeza. Mereka adalah kesan sela antara magnet, kesan tahap input dari penggoncang dan kesan panjang rasuk penyerap. Akhir sekali, prestasi penyerap getaran tak linear pada struktur utama telah disiasat di tiga tempat yang berbeza dan tiga tahap sela yang berbeza.

ACKNOWLEDGEMENT

I would like to convey my sincere gratitude to my supervisor Dr. Roszaidi Ramlan for giving me this opportunity to do this final year project under his supervision. He always encourages and gives me confidence to finish this project successfully. Whenever I encountered problem, he never hesitated to give me advice and guidance.

Secondly, I would like to thank Assistant Engineer Mr Johardi for his kindness in giving me time I needed to use the VibroAcoustics laboratory to complete my project. Also, I would like to thank a senior master's degree student Lim Kah Hei for giving me guidance in conducting experiment and sharing his knowledge in the field of vibration.

Finally, I would like to thank my parents and friends who give me encouragement and be patient with me during the past 1 year.

TABLE OF CONTENT

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

DVA	Dynamic vibration absorber
TVA	Tuned vibration absorber
ATVA	Adaptive tuned vibration absorber
MRE	Magnetorheological elastomer
MR	Magnetorheological
FRF	Frequency response function
PZT	Zicronite titanate
LVDT	Linear variable differential transformer
DR	Delayed resonator
FENE	Finite extensibility elastic
NDVA	Nonlinear dynamic vibration absorber
ODS	Operating deflection shape
SDOF	Single degree of freedom
MDOF	Multiple degree of freedom

LIST OF SYMBOLS

x_1	Deflection of primary structure
r	Ratio of primary structure natural frequency over absorber natural frequency
m_1	Mass of primary structure
K_1	Stiffness of primary structure
K_2	Stiffness of absorber
m_2	Mass of absorber
x_2	Deflection of absorber
F(t)	Force applied to primary structure
C	Damping coefficient of absorber
Z	Relative motion of absorber
β	Optimum tuning parameter
ζ	Optimum damping ratio
μ	Mass ratio
K_3	Nonlinear stiffness constant
x^3	Cubic stiffness of nonlinearity
E	Young's modulus
I	Second moment of inertia
m	Mass per unit length
L	Length

F_m	Magnet restoring force
ρ	Density

CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

Vibration in structure is an unwanted situation because it can potentially cause damage to the structure. When the structure excited by external force at its natural frequency, a phenomenon called resonance will occur. At this frequency, damping element of structure will be at minimum and energy of structure will be at maximum. A small auxiliary mass-spring system known as dynamic vibration absorber (DVA) is normally used to mitigate the vibration of the structure. This DVA needed to be tuned to match the natural frequency of primary structure to effectively suppress the unwanted vibration of primary structure (Ji, 2011). Linear DVA can be further divided into undamped and damped DVA. In undamped DVA, there will only be auxiliary mass and spring element. In damped DVA damper will be added in parallel to the spring in the setup of DVA.

Damped DVA initially developed by Den Hartog and Ormondroyd (1928). There is a term, called optimum tuned damped DVA in damped DVA section. In this optimum tuning method, several parameter such mass, stiffness and optimum damping ration must be considered (Bekdas.G & Nigdeli.S.M, 2011). Advantage of doing this optimum tuning is that, it will provide larger suppression of resonant vibration amplitude of the primary system and will generating wider safe operating frequencies range of the primary system. These damped and undamped DVA are called passive dynamic vibration absorber.

Then there are, semi-active and active type of dynamic vibration absorbers to effectively mitigate unwanted vibration at in primary structure. Semi-active type absorbers use smart materials control the absorbers. System parameters such as stiffness and damping element will be changed to change the dynamic response of the absorber. The system will be stable because it is constructed based on the passive DVA (Toshihiko.K, et al., 2016). In active type DVA, an actuator will be connected to the absorber. This actuator functions as control element to vary the stiffness and damper element of the absorber and adds flexibility to incorporate control theory to provide cancellation forces. (Chatterjee.S, 2010)

Although the absorber mention above which is categorized as a passive absorber is a famous and familiar device in mitigating unwanted vibration in mechanical structure, its only effective when it is precisely tuned to the frequency of a vibration mode and active and semi-active type DVA can be very complex in its development. Furthermore, the first study of non-linear absorber by Roberson, Pipes and Arnold have attract an attention in many literature and after realizing the limitations of linear absorbers, the non-linear vibration absorber was developed for their performance ability to widen the suppression bandwidth (Kerschen.G & Viguie.R, 2010).

In this project, magnets will be used in the characterisation of wideband nonlinear vibration absorber. One magnet will be attached to absorber beam and two more magnets will be attached to magnet wall. All the magnets will be setup facing different polarity, so that they will be attracted to each other. Three types of characterisation method will be investigated in this project. They are effect of gap between magnets, effect of level of input displacement and effect of length of absorber beam. Then, performance of absorber in mitigating vibration of primary structure will be investigated for first vibration mode of beam modelled as primary structure.

1.2 PROBLEM STATEMENT

Dynamic Vibration Absorber is an effective device used to suppress vibration in primary structures. Traditional DVA usually consist of mass and spring. DVA counteracts the motion of primary system by absorbing primary structure's vibration.

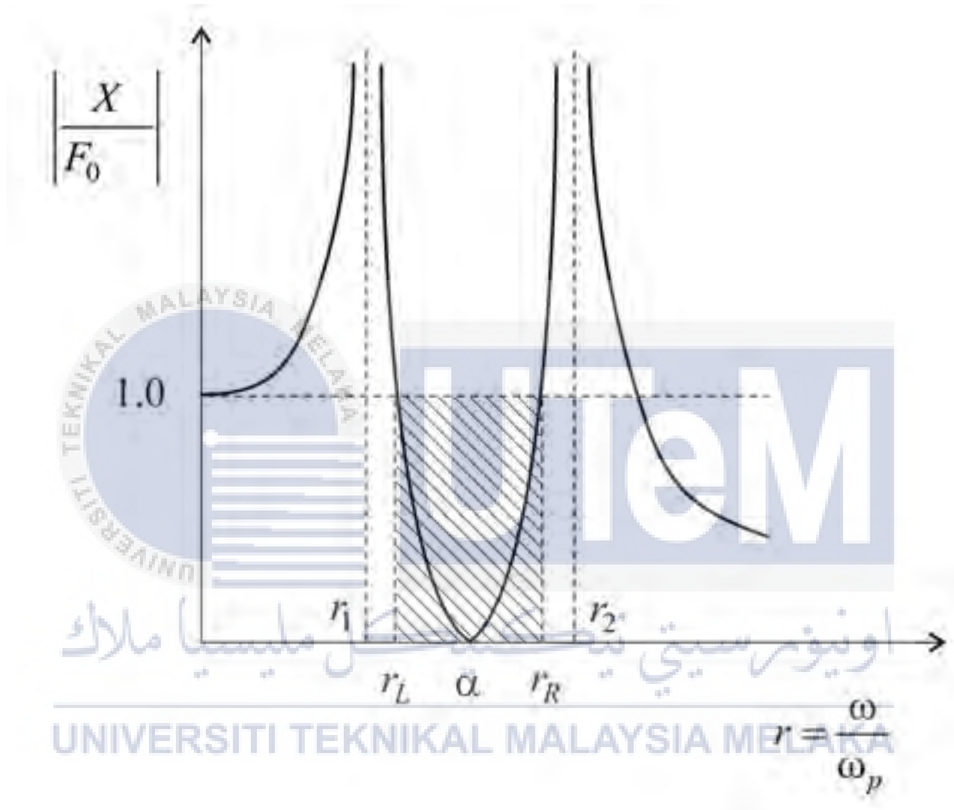


Figure 1.1: FRF for dynamic vibration absorber.

The problem with traditional undamped DVA is that it has a narrow bandwidth of operation as shown in Figure 1.1. Vibration of primary structure will be completely suppressed if the natural frequency of the absorber is perfectly tuned to the excitation frequency. In this case, it is assumed that receptance is below 1.0 is acceptable. As seen in the figure 1.1, absorber is only effective if r is between r_L and r_R because response of the system will be below 1.0 in this region. Other than the shaded region, transmissibility will be above 1.0 and it is not desired. If ω is excited at either r_1 or r_2 system's response will be infinite. This will be the case during startup and when system frequency varied during operation.

1.3 OBJECTIVE

The objectives of research studies are:

1. To study the properties of a non-linear single degree of freedom system.
2. To characterize the dynamic properties of the proposed absorber.
3. To investigate the performance of the nonlinear absorber when put on vibrating structure.

1.4 SCOPE OF PROJECT

The scope of this project are :

1. Only nonlinear characteristic of vibration absorber is investigated.
2. Performance of absorber only tested for first vibration mode of primary structure.

CHAPTER 2

LITERTURE REVIEW

2.1 Passive Dynamic Vibration Absorber

Dynamic vibration absorber (DVA) was first patented by Frahm in 1911 (Frahm, 1911) . The device was simple mass-spring system that can mitigate vibration of primary system consist of mass and spring. This device is simple, effective and easy to install and operate. This type of DVA suppress vibration at specific fixed designated frequency. When the excitation frequency varies from intended frequency, response of the system will be even larger than the primary system alone.

This narrow bandwidth of operation present problems during startup or random varying excitation frequency. Theoretically DVA completely cancels out vibration response at selected frequency but response of the system amplifies if the excitation frequency matches two new natural frequencies. To counter this problem, Den Hartog and Ormondroyd (1928) presented mathematical theory of damped DVA consist of mass-spring-damper. Tuning parameter and damping ratio is the main design parameters of damped DVA. Den Hartog first tackled the optimum solution of the damped DVA attached to primary system consist of only mass and spring. He studied on ‘fixed-point’ features where, response amplitude of primary mass is independent of absorber damping. This solution allows DVA to suppress vibration at wider range but it does not completely cancel out vibration at any point. Rest of this section explains some more recent literature review on DVA.

Wong (2008) propose to use DVA with setup different from that proposed by Den Hartog and Ormondroyd (1928). This new setup has damper directly connected to ground rather than connected to primary system in traditional DVA. Performance of new DVA was compared to the traditional DVA excited by ground motion. When both DVA was tuned optimally proposed new DVA has better performance in suppressing vibration compared to traditional DVA.

A traditional DVA with different setup was analyzed by Shen et al. (2016). Traditional DVA used by Den Hartog and Ormondroyd (1928) is called Voigt type DVA. A negative stiffness and damper directly connected to ground in this new proposed setup like in Figure 2.1. Optimal tuning condition was used to tune frequency ratio and damping ratio. A method was derived from fixed point theory to determine optimum negative stiffness ratio. Performance of proposed DVA was tested and compared to Voigt type DVA. For control and random excitation proposed DVA exhibits better performance compared to Voigt type DVA.

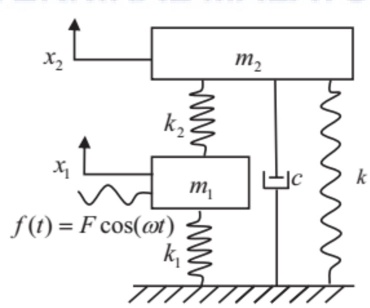


Figure 2.1: Setup of absorber with negative stiffness [Source: (Shen.Y, et al., 2016)].

Cheung et al. (2015) propose to add subsystem to DVA to a Multiple Degree of Freedom (MDOF) primary structure. They tested on seven different case studies. All seven case studies exhibit different properties to manipulated DVA setup. The common thing all the DVA setup exhibit was it has very minimum stiffness and damping effects to the primary structure at frequencies lower than the tuned natural frequency. This means that, all the absorbers can only be used at lower or first frequency modes only.

Usage of electric grass trimmer continuously will result in hand-arm vibration syndrome. This condition is caused by rotational unbalance of single string construction of nylon string. Ko et al. (2011) came up with the idea of using DVA to suppress the vibration produced by the electric grass trimmer and prevent hand-arm vibration syndrome. For this purpose, a DVA was attached to handle of electric grass trimmer. The problem was to determine the optimum location of absorber in the handle. Absorber was placed at several locations along the handle and tested for its vibration suppression ability as shown in Figure 2.2. Modal analysis and operating defection shape analysis of electric grass trimmer was carried out for cutting and no cutting situation. From the result, it was known that DVA has best performance level at 0.025L at the handle. It reduces vibration level in x and z axis, but increase vibration level in y axis. From this experiment, it is known that DVA can reduce vibration level of electric grass trimmer transmit to hand of user thus, prevent hand-arm vibration syndrome.

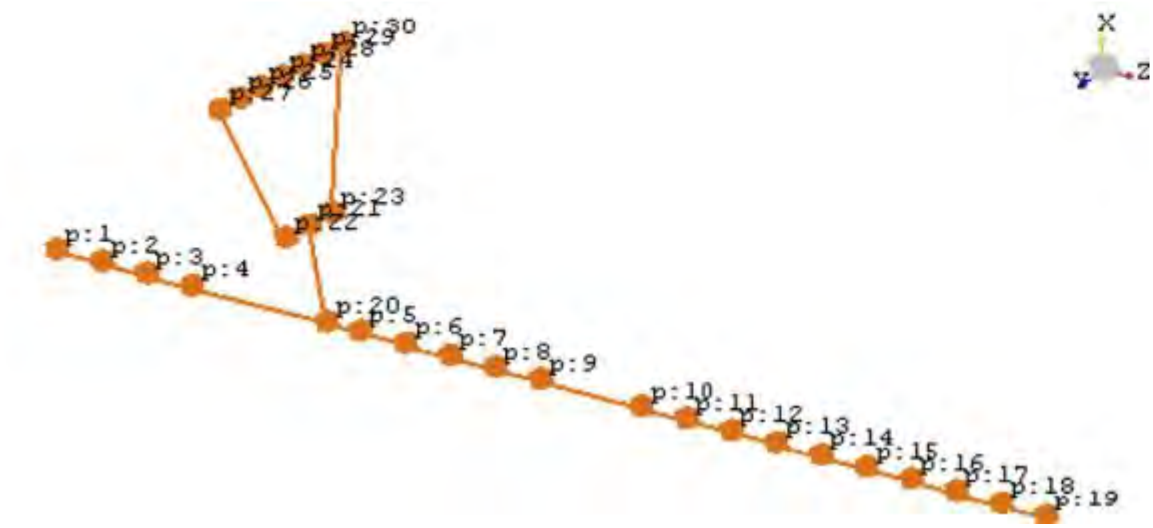


Figure 2.2: Positions where absorber attached in electric grass trimmer [Source (Ko.Y.H, et al., 2011)].

In Ji and Zhang (2011) research, primary system of weakly nonlinear oscillator having cubic nonlinearity was considered. When excited at one-third of its linearized natural frequency this system will experience super-harmonic resonance, where saddle-node bifurcations, jump and hysteresis phenomena are seen. To counter this problem authors, use linear DVA consist of small mass, spring and damper. When the primary system excited without absorber super harmonic amplitude is very high. This is where saddle-node bifurcation happens. There also exists unstable region when DVA is not attached. At this place jump-up phenomenon occurs. All this problem solved when linear DVA attached to main nonlinear system. Super harmonic resonance decreases from 0.18 to 0.002 when DVA is attached to main system. From this it is found out that there is no optimal value for damping for super harmonic resonance vibration just like passive DVA have.

2.2 Semi-active Type Dynamic Vibration Absorber.

Koo et al. (2008) address problem of narrow bandwidth in vibration absorber by using semi-active Tuned Vibration Absorber (TVA) instead of traditional passive DVA. Semi active TVA contains auxiliary mass, spring, and controllable damping element such as magnetorheological damper. Controllable damping element is controlled by displacement-based, on-off ground hook (on-off DBG) which adjusts damping level based on product of primary systems' displacement and the relative velocity across the damper. For passive DVA, the transmissibility of the primary structure when the damping ratio 0.0 is the complete isolation at valley and two infinity peaks. As the damping ratio increases, value of two resonant peaks decreases and the valley between two peaks widens. Further increase in damping ratio results in convergence of two peaks into a single peak. This coupling of two peaks means that two masses function as a single mass and making the DVA useless. In the case of semi-active DVA as the damping ratio increases two peaks decreases but notably the valley remains at the same position. This prevents the system couple and act as single mass system. The valley remains at the same position because on-off DBG makes sure that minimum damping ratio at the valley independent of the damping ratio of on-state. This prevents both peaks from coupling.

Adaptive tuned vibration absorber (ATVA) with magnetorheological elastomer (MRE) is proposed by Deng and Gong (2008) to overcome shortcomings of passive DVA. Magnetorheological (MR) are smart material with MR characteristics and have unique properties with magnetic field. Shear modulus of MRE depends on field strength. Coil current and field strength determines the equivalent stiffness of ATVA. Natural frequency of ATVA can be tuned to excitation frequency by tuning the coil current. Experimental data shows that relative frequency of ATVA can be changed as high as 145%. Theoretical and experimental data show that ATVA mitigate vibration at broad range of frequency compared to passive DVA.

Magnetorheological (MR) is a new type of semi-active tuned mass damper. Weber (2014) propose a new system of vibration absorber where MR-SVA is used to suppress vibration at wide bandwidth of excitation frequency. MR-SVA is made of desired stiffness force and desired friction force as shown in Figure 2.3. Positive or negative force augments or diminishes the passive spring stiffness and realize the real-time frequency tuning. Adaptive nonlinear damping control algorithm make sure amplitude of MR-SVA does not exceed its maximum value. From the FRF response it can be known that semi-active DVA is more effective than passive DVA at all voltage levels. Passive DVA and semi-active DVA at 0.9V reduction of vibration level is 12.4% when the internal damping cannot be reduced. At 0.6V of semi-active DVA the vibration reduction is almost 60% when internal damping can be reduced as seen in Figure 2.4.

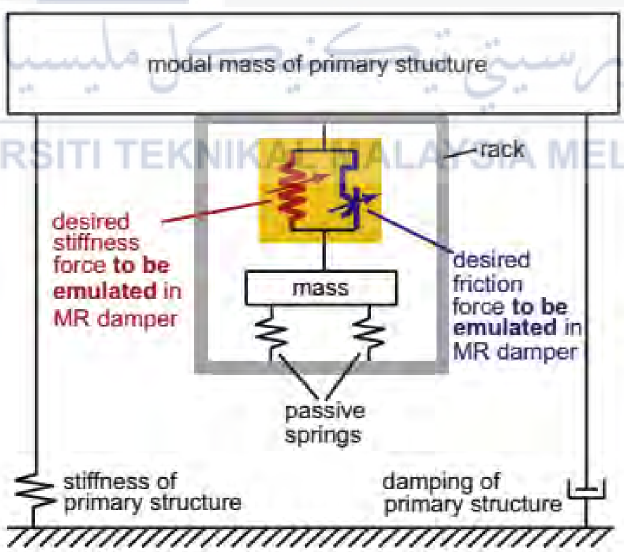


Figure 2.3: Design of MR-SVA [Source: (Weber.F, 2014)].

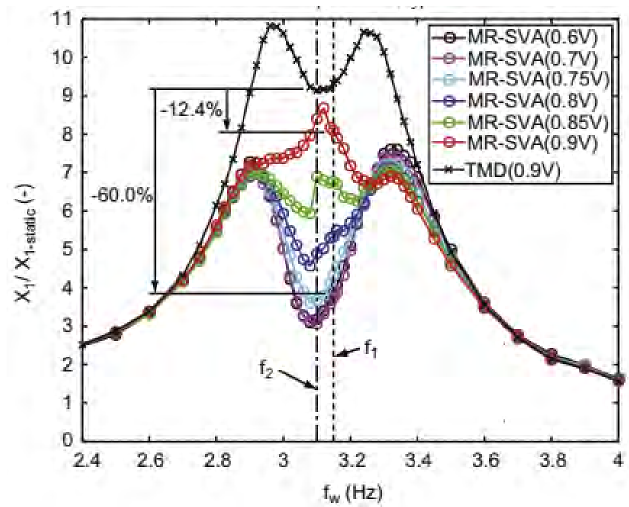


Figure 2.4: Comparison of semi-active and passive absorbers [Source: (Weber.F, 2014)].

Toshihiko et al. (2016) use semi-active type vibration absorber whose natural frequency can be tuned by applying magnetic field strength using stiffness-controllable elastomer MRE. In this type of absorber magnetization property of MRE is the primary source for the magnetic field generation. Vibration of multi-degree-freedom-structure is suppressed by using real-time stiffness-switching algorithm as in Figure 2.5. In frequency shift measurement, it is showed that nominal frequency of the absorber can be extended to other frequencies. It is also found out that damping value of absorber decreases when magnetic field increased.

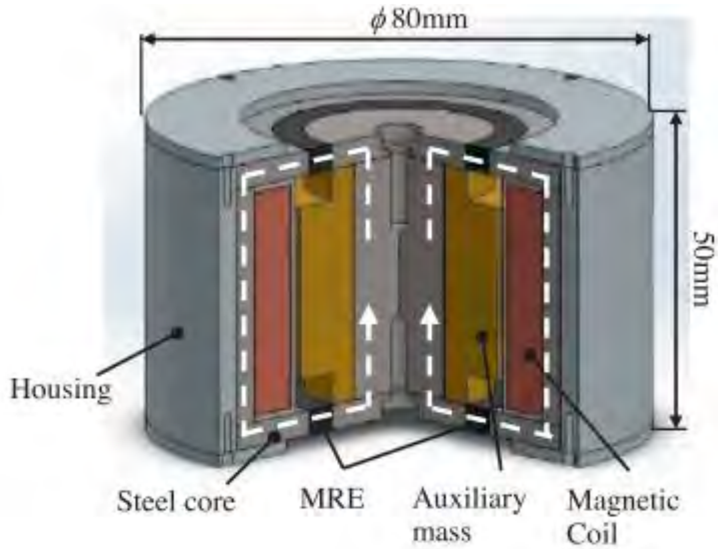


Figure 2.5: Proposed magnetorheological elastomer-based tuned absorber [Source: (Toshihiko.K, et al., 2016)].

To reduce the structural vibration of a machine platform subject to varying excitation frequency, Chen and Lee (2015) come up with a solution of using DVA with tunable mass driven by micropump. Micropump is more suitable to be used in tight spatial condition compared to mechanical transmission in variable inertia type semi-active DVA. At two natural modes of the beam, two chambers were placed. Liquid were filled and withdrawn from chambers by using micropump and solenoid valve. Stiffness and damping of the absorber kept constant. For varying excitation frequency, different mass will set different natural frequency to mitigate the vibration. Sloshing of liquid surface results in increased modal damping. Large damping will cause the performance of the DVA to drop. To overcome this problem, separation panel was installed inside the chamber.

2.3 Active Type Dynamic Vibration Absorber

Dwivedy and Mohanty (2016) in their research compared linear and nonlinear vibration absorber with acceleration feedback. Active vibration absorber is considered in this research. Active vibration absorber is a type of absorber which uses sensor or actuator to provide counteracting force to the primary system to reduce its vibration. A lead zirconate titanate (PZT) actuator is connected in series with absorber by a spring. Optimum tuning condition for both linear and nonlinear systems were calculated. The mass ratio is set to be 0.05 and damping of the primary system is set to be zero. When the actuating force α is 0.0001 the response of the primary system is 6.45. but as the α increases response of the system decreases. When α is 0.99 response of primary system is only 1.15. For the nonlinear setup, when the α is small response of the function is infinity. When α is bigger than the response is smaller. It should be noted that response of nonlinear system is much smaller than linear system for the same 0.99 α. For nonlinearity of the spring, as the nonlinearity increases bends of frequency response curve shows hardening effect because of stable and unstable region. In time response graph, it is shown that displacement of primary system decays with or without application of controlling force. But it should be noted that with controlling force it decays much faster compared to without controlling force.

Another solution for narrow band width of operation for DVA is by using active vibration control. Active DVA can be mounted directly to the structure at the vibrating point to counteract the vibration by using proper algorithm. But, that is not possible for all condition. Wu et al. (2007) recently proposed an absorber setup which use the main structure as part of the absorber as shown in Figure 2.6. The aim of the experiment was to eliminate the vibration at the midsection. The mechanism is realized as hard mechanical elements by using linear motor and tuned properly using an emulated feedback algorithm and tuned for various frequencies. Active element of the absorber uses a linear variable differential transformer (LVDT). When the structure is excited, the midsection will be static and rest of the structure will swing in harmonic way.

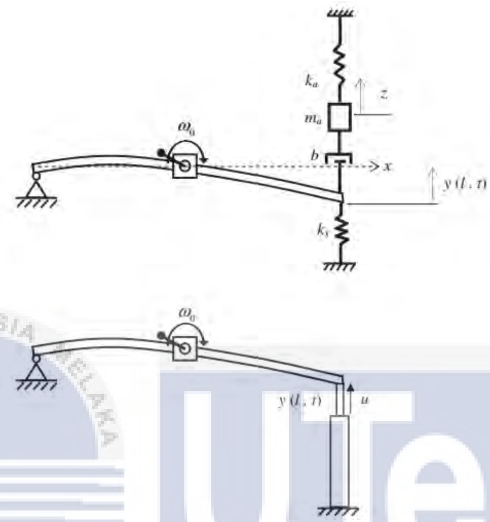


Figure 2.6: Realization of proposed model as hard mechanical element [Source: (Wu.S.T, et al., 2007)].

Chatterjee (2010) in his research paper investigates the active absorber and compared its performance with optimally tuned passive vibration absorber and system without absorber. Delayed Resonator (DR) type of active absorber was considered. This DR uses time-delayed position, velocity, or acceleration feedback of the absorber mass. This makes the absorber look like its own structure running on its internal feedback without getting any feedback from primary system as shown in Figure 2.7. In this kind of absorber an actuator is placed in parallel with the absorber spring. rest of the structure will swing in harmonic way. As for the comparison, both passive and active absorbers are optimally tuned to get maximum performance. Based on the result shown in Figure 2.8, active vibration absorber transmits less vibration to the main system compared to the optimally tuned passive absorber. Other than less transmissibility, with active absorber the frequency does not have to be natural frequency as with all other absorbers.

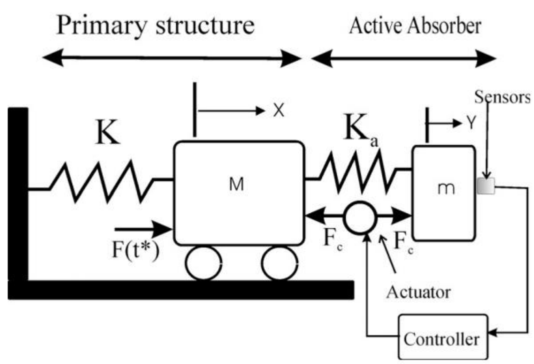


Figure 2.7: Setup of active absorber. [Source: (Chatterjee.S, 2010)].

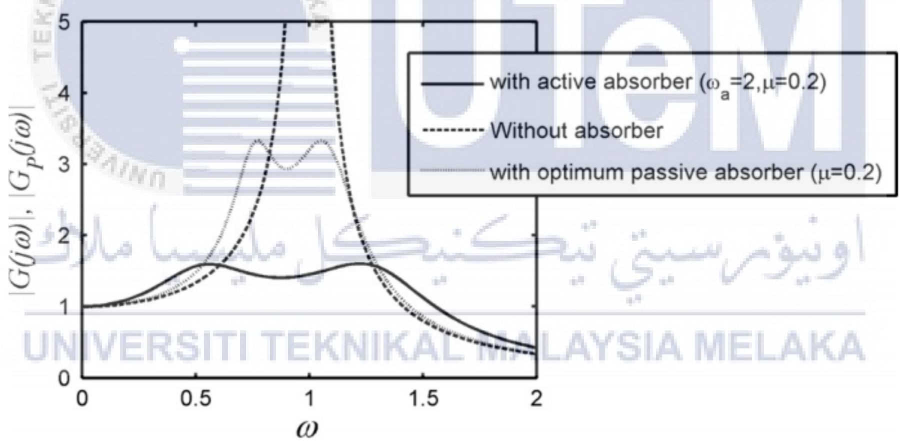


Figure 2.8: Comparison between active absorber, without absorber and optimum passive absorber. [Source: (Chatterjee.S, 2010)].

2.4 Nonlinear Dynamic Vibration Absorber

In this research paper Tang et al. (2016) try to find out the parameters of the nonlinear vibration absorber by experimental method using free vibration. Primary system was modelled as the shaker and absorber was modelled as small mass attached to thin circular brass plate sandwiched between two aluminum rings as shown in Figure 2.9. When the system excited at low level vibration the system acts linearly. But, as the vibration increases the in-plane stretching of the plates gives the stiffness and causes the hardening of nonlinear spring. The shaker was working in its mass controlled region and excited at its jump down frequency, then the source signal was switched off. This makes the system vibrate freely. Vibration of the absorber mass was sampled and use as reference in characterization of the nonlinear absorber.

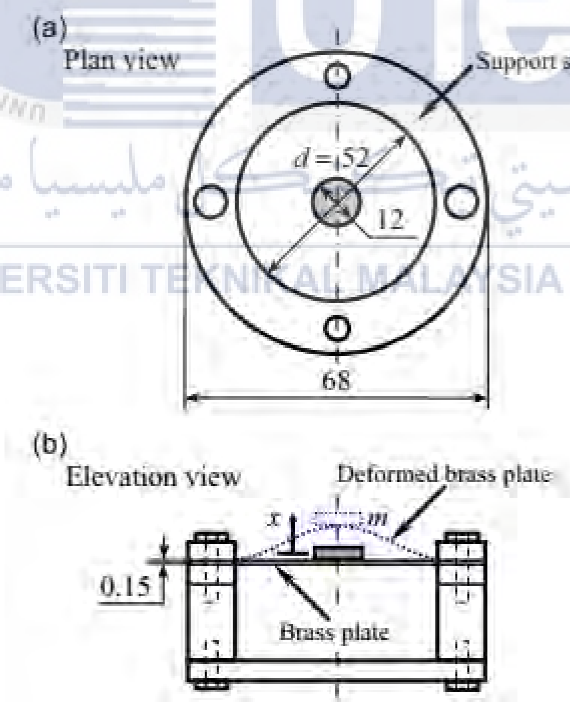


Figure 2.9: Setup of nonlinear absorber [Source: (Tang.B, et al., 2016)].

Linear DVA is used to suppress the vibration of linear primary system. For nonlinear primary system, linear DVA cannot suppress vibration for all forcing amplitudes. This is because of limitations of linear DVA. When the forcing amplitude is higher, hardening behavior of cubic spring with positive coefficients is also present in the second resonance peak. At this point, primary system becomes strongly nonlinear and linear DVA unable to suppress vibration of primary system. Habib et al. (2015) proposes that using nonlinear DVA would overcome the problem. This nonlinear DVA composes of mass, damper, linear spring for weak nonlinearity of primary system and nonlinear spring for strong nonlinearity of primary system. The nonlinear DVA used in this paper is tuned to the rule of choose mathematical form nonlinear DVA restoring force so that it is mirror of the primary system. By using this method Den Hartog and Ormondroyd's (1928) equal peak method can be generalized for nonlinear DVA. Nonlinear DVA performance is superior to linear DVA because, nonlinear DVA is 'mirror' of primary system. But, for very strong nonlinear amplitudes nonlinear DVA exhibits detached resonances curves.

In the previous literature, proposed nonlinear absorber encounter problems at higher nonlinearity of primary system. To counter this problem Febbo and Machado (2013) propose that nonlinear dynamic absorber with saturation is used. Finite extensibility elastic (FENE) potential was used to model the saturable nonlinearity of the absorber. Averaging method is used to solve the equation of motion of this nonlinear absorber. To show the effectiveness of this new absorber, its performance was compared to the cubic nonlinear absorber as shown in Figure 2.10. For linear system, both absorber have similar characteristics excepts for proposed FENE absorber have larger attenuation in the stable region. When the nonlinearity of primary system increased, response of the FENE absorber is smaller compared to cubic absorber for whole frequency range. Downside of this FENE absorber is the presence of quasi-periodic oscillation of high amplitudes coexisting with unstable periodic solutions.

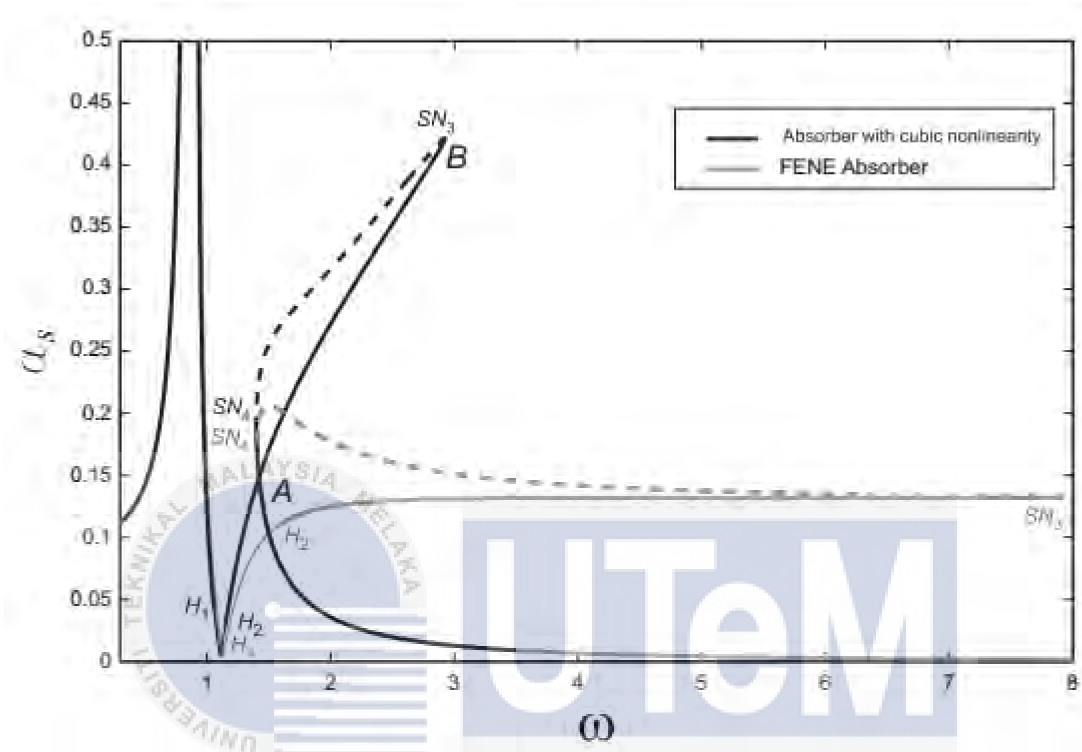


Figure 2.10: Comparison between FENE absorber and absorber with cubic nonlinearity.

[Source: (Febbo.M & Machado.S.P, 2013)].

Vibration of weakly nonlinear primary system can be suppressed by using nonlinear absorber. Ji (2014) suggests that by using three-to-one internal resonance based absorber can be best solution. Absorber contains light-weight mass, linear damper and spring of linear and weak nonlinear characteristic. Linearized natural frequency of the primary system and absorber are tuned to be under three-to-one internal resonance. This makes the absorber needs small linear stiffness to mitigate vibration of nonlinear primary system. This nonlinear absorber can increase critical forcing amplitudes, modify frequency response of primary resonance and prevent saddle-node bifurcations in FRF of primary system.

In transportation industry travelling loads on host structure is common problem. Critical speed for repetitive load is different from critical speed of static load. Linear DVA is best solution for suppress vibration at certain frequency, when the frequency varies vibration level will be even worse than it was without DVA. Pellicano and Samani (2012) suggest that, this type of problem can be overcome by using nonlinear DVA having linear stiffness and linear-quadratic damper. It is very effective compared to linear DVA in suppressing vibration around critical velocity where deflection and velocity amplitude increase.

2.5 Nonlinear vibration absorber using hardening mechanism

Ramlan et al. (2009) investigated the effects of using nonlinear vibration absorber in an energy harvesting device. Two types of nonlinear absorbers are considered. First type was by using non-linear bi-stable snap-through mechanism. This snap-through mechanism contains two linear oblique springs connected with mass and damper in Figure 2.11. In this specific setup springs produce nonlinear restoring force in x direction. When the frequency increase, the period become shorter and this system unable to produce square -wave-like-response. At high frequency, this setup behaves like linear system. Snap-through mechanism can be efficiently applied to systems that have higher natural frequency than the excitation frequency. Second type is nonlinearity by hardening mechanism. This mechanism can give wide bandwidth of operation for absorber. In this system, nonlinear spring is connected in parallel with damper and in series with mass. The nonlinear spring have cubic stiffness. Bandwidth of hardening system mainly dependent on damping ratio, nonlinearity, and input acceleration.

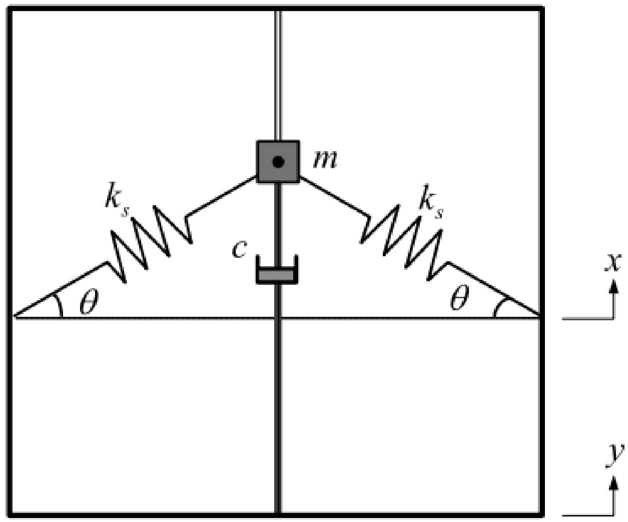


Figure 2.11: Snap-through mechanism setup [Source: (Ramlan, 2009)].

In this research, Zhang et al. (2016) use nonlinear absorber to suppress vibration and harvest energy. Hardening by magnetic stiffness method is used to produce nonlinearity of spring. Magnets of same polarity were placed opposite to each other to produce repulsiveness. Smaller the gap, higher the nonlinearity of spring. Based on the results, nonlinear vibration absorber is useful in vibration mitigation and energy harvesting in wider bandwidth.

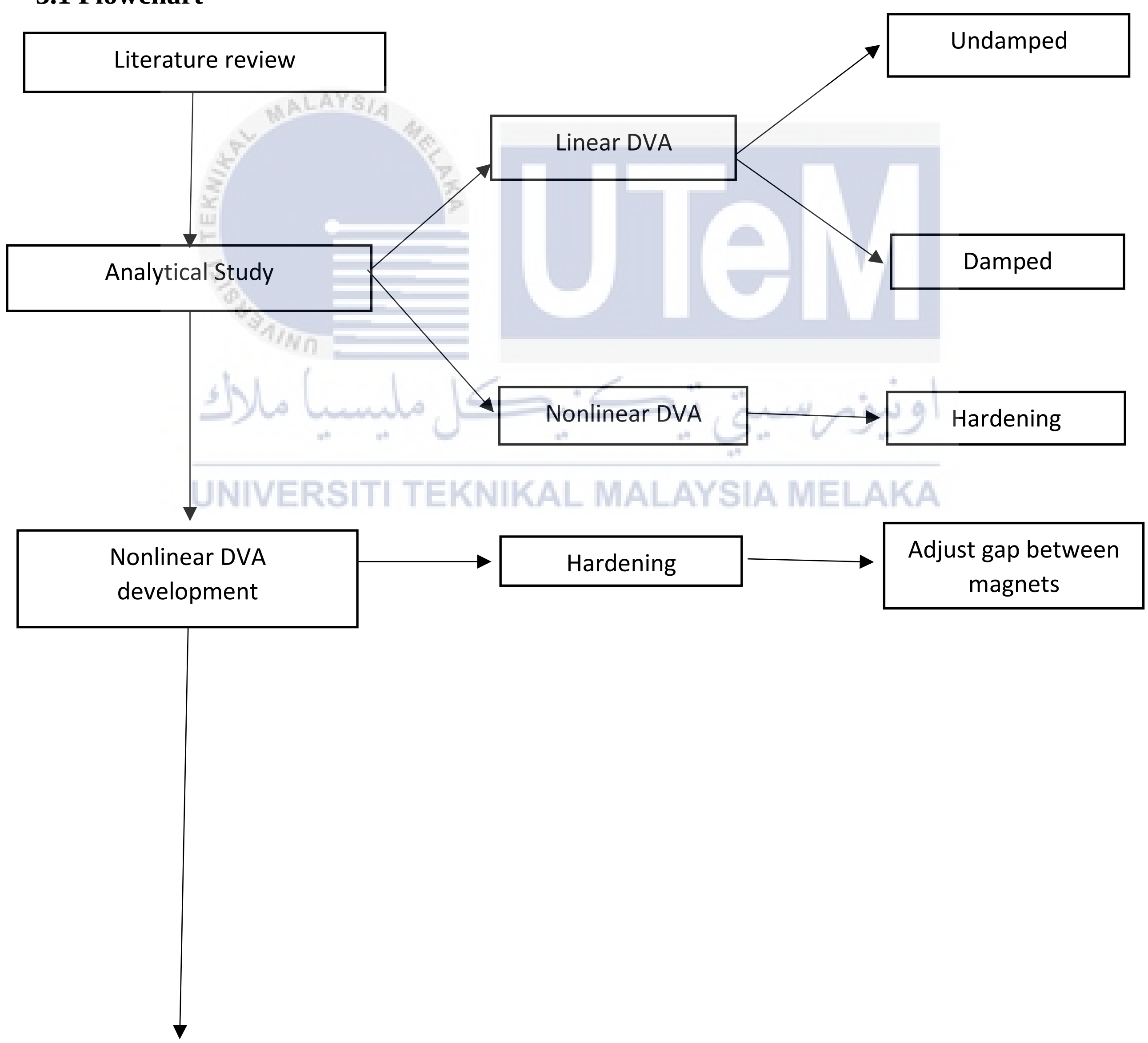
Low et al. (2015) investigated the hardening and softening mechanism to widen the operational bandwidth of absorber. Three types of magnetic configurations were investigated. By moving the axial gaps between the magnets, stiffness of beam was altered. Altering the distance cause the stiffness either soften or harden. Both softening and hardening produce broad bandwidth of operation for absorbers.

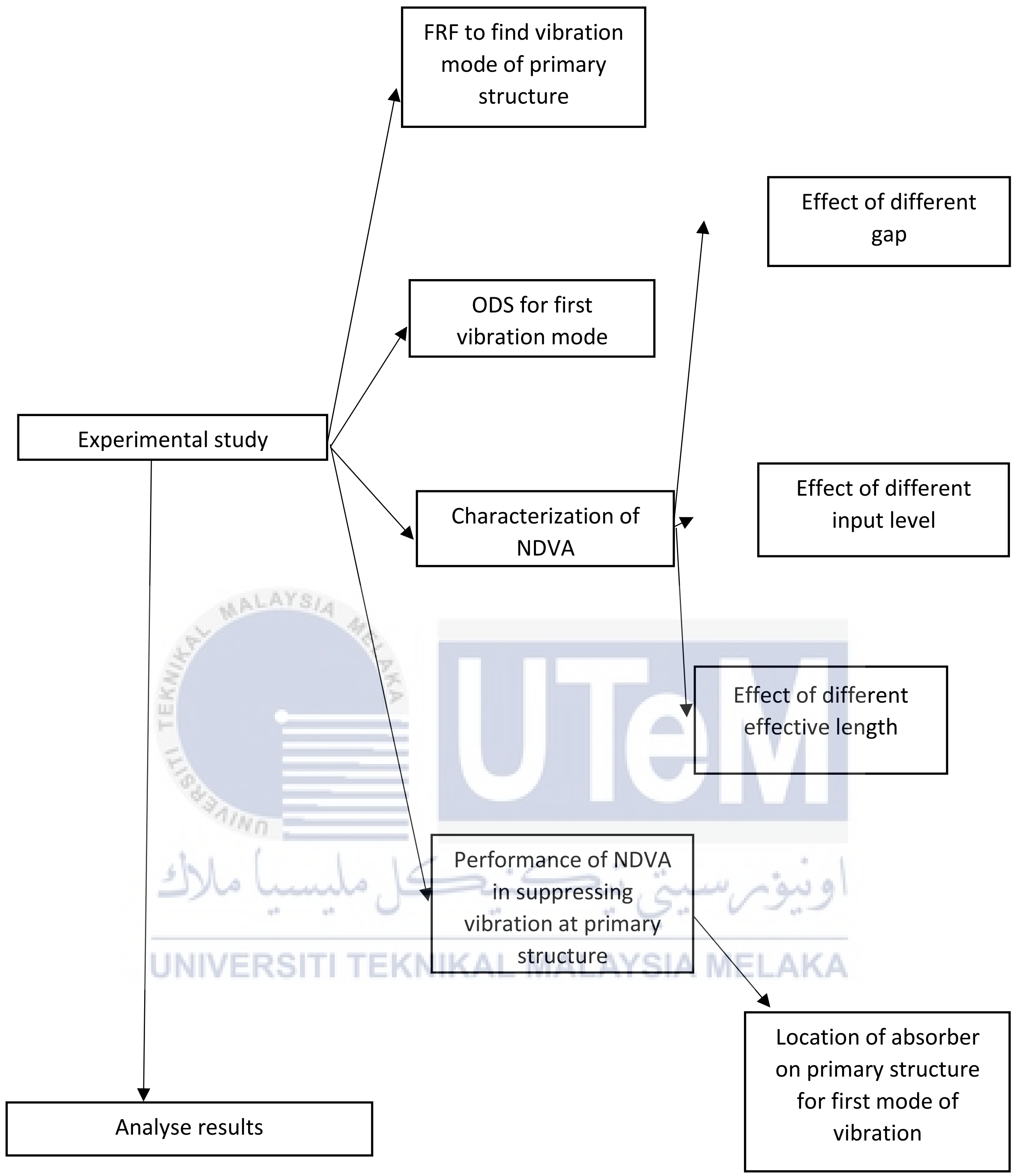
In this research, nonlinear stiffness of absorber is induced by using hardening method. Magnets will be used to hardened the absorber beam, thus making the absorber nonlinear. Characterisation of nonlinear absorber will be done in three separate experiments. They are effect of gap between magnets, effect of level of input displacement and effect of length of absorber beam. Then, effectiveness of nonlinear DVA in suppressing vibration of primary system was investigated by placing absorber at different location and different gap. Investigation is carried out for first vibration mode of primary structure.

CHAPTER 3

METHODOLOGY

3.1 Flowchart




FRF to find vibration mode of primary structure
Effect of different gap
ODS for first vibration mode
Experimental study
Characterization of NDVA
Effect of different input level
Effect of different effective length
Performance of NDVA in suppressing vibration at primary structure
Location of absorber on primary structure for first mode of vibration
Analyse results

3.2 Analytical Study on Linear and Nonlinear absorber

3.2.1 Linear absorber

Frahm (1911) patented the very first absorber. Absorbers are naturally single degree of freedom, SDOF type systems. They have only one mass and move in one direction, mostly vertical. There are two types of absorber under linear absorber category. They are undamped and damped linear absorber.

Taipei 101 building is a perfect example of functionality of dynamic vibration absorber. In this building, a massive ball shaped mass placed between 87th and 89th floor. When the building sways, the secondary mass sways in opposite direction (Poon.D, et al., 2004). This will keep the building stable during typhoon or earthquake.

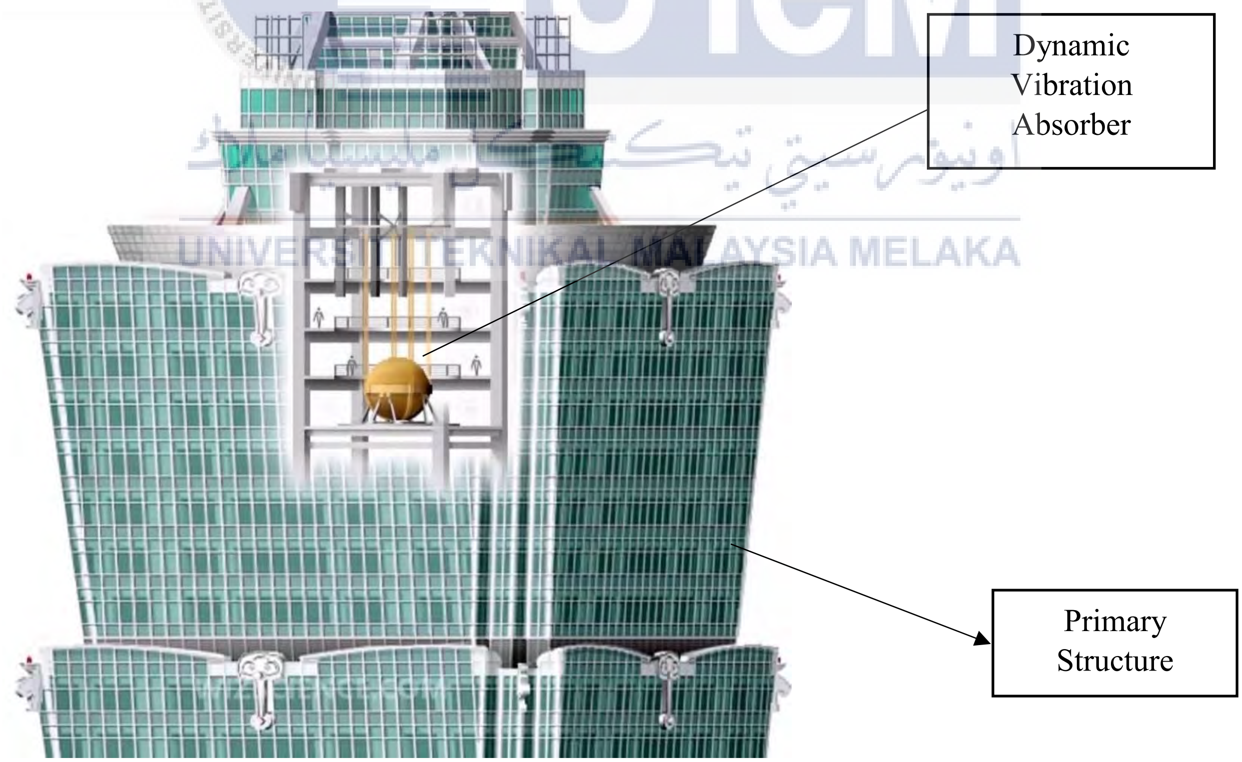


Figure 3.1: Image of absorber inside Taipei 101 building [Source: (Poon.D, et al., 2004)].

3.2.1.1 Undamped Linear Absorber

Undamped linear absorber contains small mass and linear spring. Figure 3.2 illustrates setup of undamped linear absorber. Taipei 101 tower shown in Figure 1 can be modelled as Figure 3.2 to simplify calculations. If the system analyzed using newton's method, equation of motion of the system will be obtained. Equation of motion is given below as m_1 is the primary structure's mass, m_2 is absorber mass, K_2 is linear absorber stiffness, x_1 and x_2 are deflection of primary and secondary mass respectively. Relative motion of absorber spring can be denoted as z, where $z = x_2 - x_1$.

$$m_1\ddot{x}_1 + (K_1 + K_2)\dot{x}_1 - K_2\dot{x}_2 = F(t) \qquad (1)$$

$$m_2\ddot{x}_2 + K_2 z = 0 \qquad (2)$$

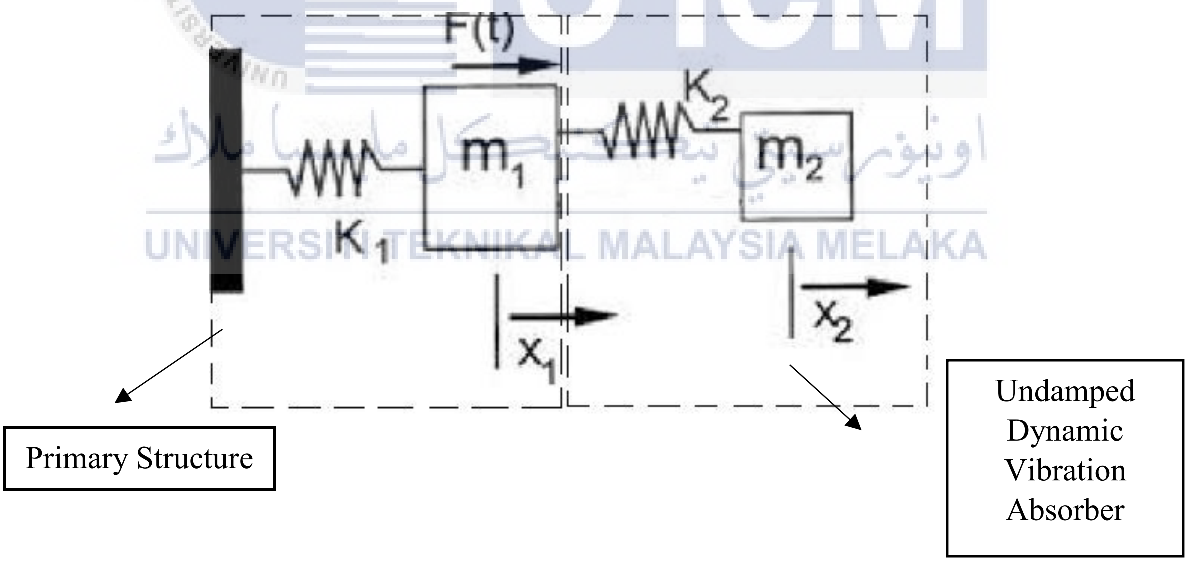


Figure 3.2: Setup of linear undamped vibration absorber.

In linear absorber spring of the absorber is a linear spring. It acts as in Hooke's Law. Hooke's Law suggest that when an elastic object is stretched, the increased length is called its extension. This extension of an elastic object is directly proportional to the force applied to it.

$$F_m = kx_2 \quad (3)$$

In Equation 2, F_s is spring restoring force, k is linear stiffness constant and x_2 is extension of spring. Figure 3.3 shows that, if the elastic limit not exceeded, graph of force against extension produces straight line passes through the origin. Greater the value of k, the stiffer the spring becomes.

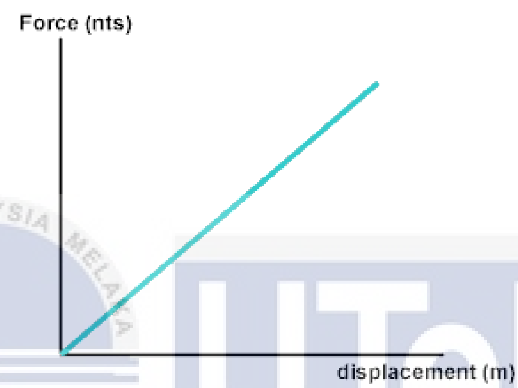


Figure 3.3: Graph of force against extension of spring. [Source: (Physicslab, 2016)].

In a forced response system, the area of interest is to see how the absorber respond to various excitation frequency. The response in which the magnitude of the response varies against the excitation frequency is called frequency response function (FRF).

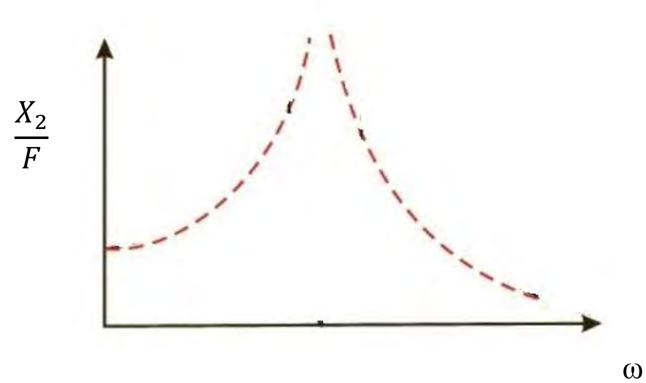


Figure 3.4: FRF of linear undamped vibration absorber. [Source: (Siemens Corporation, 2016)].

As can be seen in Figure 3.4 response of the system become infinity at its resonance frequency. Resonance is a phenomenon in which a vibrating system or external force drives another system to oscillate with greater amplitude at specific preferential frequency. This what exactly happens when primary system excited at it natural frequency without absorber.

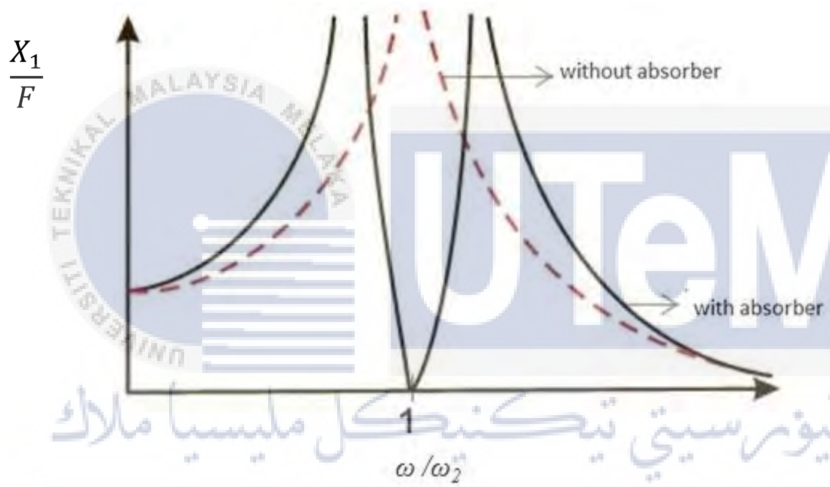


Figure 3.5: FRF of primary structure with linear absorber.

[Source: (Siemens Corporation, 2016)].

When absorber tuned to primary system's natural frequency the response cancels out each other as seen in Figure 3.5. But, this systems bandwidth of operation is very narrow. Den Hartog and Ormondroyd (1928) came up with the solution of damping a damper to the absorber.

3.2.2 Damped Linear Absorber

Dynamic Vibration absorber consist of mass-spring system is cheap and efficient method to suppress constant frequency harmonic excitation on a primary structure. The problem with this system is that it has a very narrow bandwidth of operation and its efficiency drops when excitation frequency varies. To counter this problem Den Hartog and Ormondroyd (1928) presented mathematical theory of damped linear absorber consist of mass-spring-damper as shown in Figure 3.6. Equation of motion for primary structure and damped linear absorber is given below and c is damping coefficient of absorber.

$$m_1\ddot{x}_1 + c\dot{x}_1 - c\dot{x}_2 + (K_1 + K_2)x_1 - K_2x_2 = F(t) \tag{4}$$

$$m_2\ddot{x}_2 + c\dot{z} + K_2 z = 0 \tag{5}$$

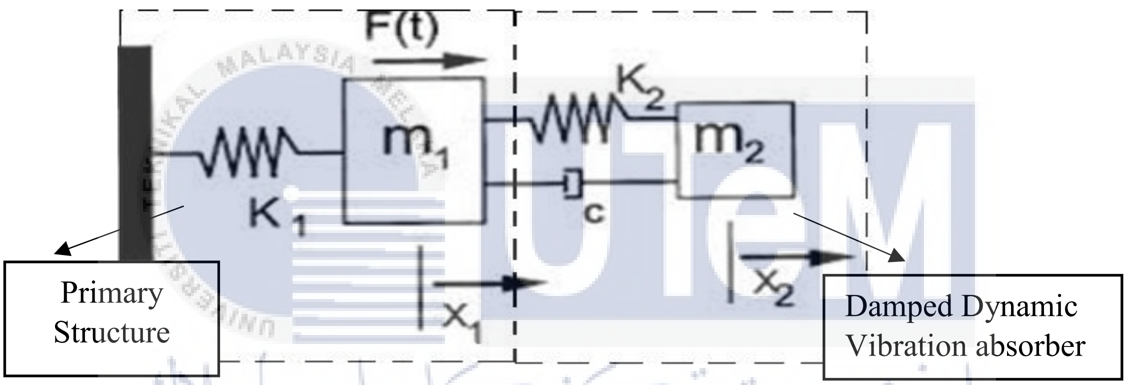


Figure 3.6: Setup of linear damped absorber.

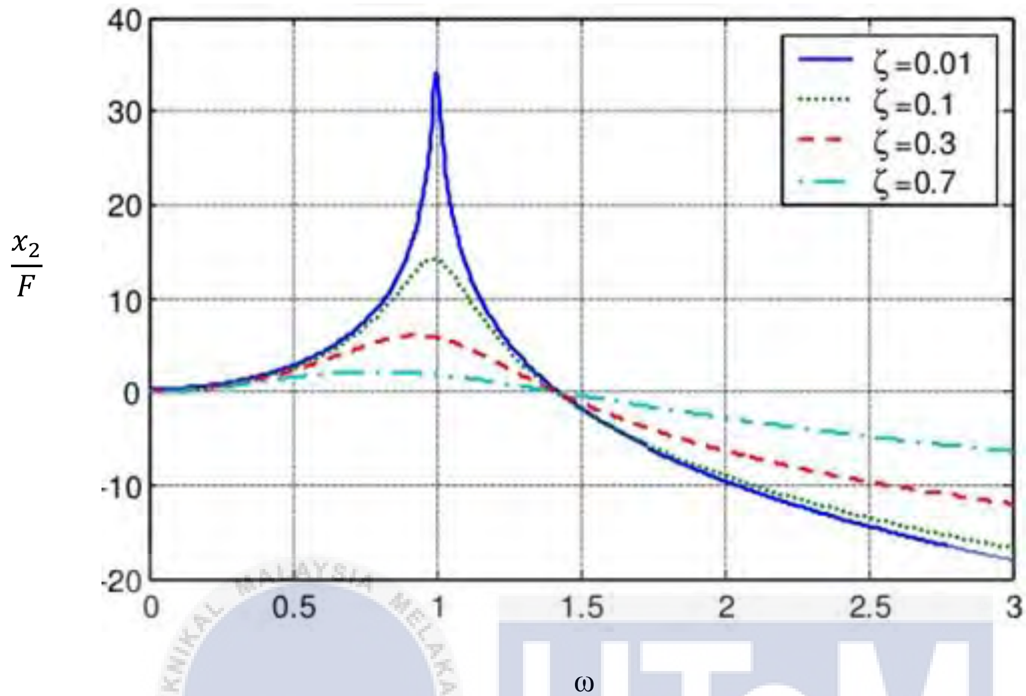


Figure 3.7: FRF of linear damped vibration absorber [Source: (Purdue University, 2002)].

From Figure 3.7 it can see that at resonance frequency the response of the absorber is not infinite. As the damping coefficient increase the peak gradually become smaller. Tuning parameter and damping ratio is the main design parameters of damped linear absorber. Den Hartog first tackled the optimum solution of the damped linear absorber attached to primary system consist of only mass and spring. He studied on ‘fixed-point’ features where, response amplitude of primary mass is independent of absorber damping.

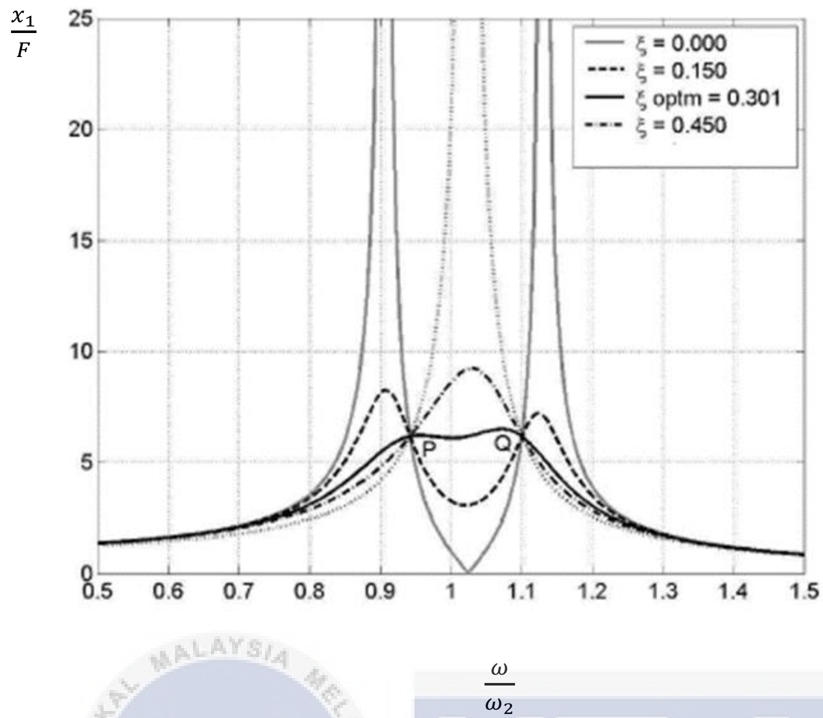


Figure 3.8: FRF of primary structure for different damping coefficient level [Source: (Purdue University, 2002)].

In Figure 3.8, fine dotted line represents the FRF when no absorber is attached to the primary structure. Grey line represents when undamped absorber is attached to the primary structure. These two conditions have been explained earlier in Figure 3.5. when damped absorber attached to the primary structure FRF of black dotted line will be obtained. As seen in Figure 3.8, when damped DVA attached to primary structure it will not completely cancel out the vibration at desired frequency. Peaks at either side of the selected frequency also not infinity and not on same when damped DVA is used. When optimally tuned damped DVA is used peaks at point P and Q are almost level and operable bandwidth of absorber is also wider compared to undamped and untuned dynamic vibration absorber. Further increase in damping ratio results in convergence of two peaks into a single peak. This coupling of two peaks means that two masses function as a single mass and making the damped DVA useless.

In the classic mass-spring primary system there is no damper, so ζ=0, optimum tuning parameter is $\beta^*$and μ is $m_2/m_1$and known as mass ratio. When the mass ratio increases, the FRF curve of primary structure with damped vibration absorber widens. This will increase the operating bandwidth of absorber. But, this option is not practical. Because, as rule of thumb mass ratio should be between $0.05 \leq \mu \leq 0.25$.

$$\beta^* = \frac{1}{1+\mu} \tag{6}$$

Optimum damping ratio ζ_a^* of absorber is given by

$$\zeta_a{}^* = \sqrt{\frac{3\mu}{8(1+\mu)}} \tag{7}$$

When the mass ratio is set at appropriate value, there will be curves join at fixed points called P and Q. As predicted by Den Hartog optimum tuning parameter ensures that amplitude at point P is same with amplitude at point Q as shown in Figure 3.8.

3.2.3 Nonlinear Absorber

In nonlinear absorber, absorber is made of mass, spring and damper. The difference is spring used in this absorber is nonlinear spring. Nonlinearity of springs can be induced by two methods, namely hardening and softening. Nonlinearity by hardening method will be investigated in this research. It will be setup as shown in Figure 3.9. In a nonlinear absorber, cubic stiffness term x^3 and nonlinear stiffness constant K_3 will be added to left side of Equation 5. The equation now will be given as

$$m_2\ddot{x}_2 + c\dot{z} + K_2 z + K_3 z^3 = 0 \tag{8}$$

Since $x_2 = z + x_1$, Equation 8 can be rewritten as

$$m_2\ddot{z} + c\dot{z} + K_2 z + K_3 z^3 = -m_2\ddot{x}_1 \tag{9}$$

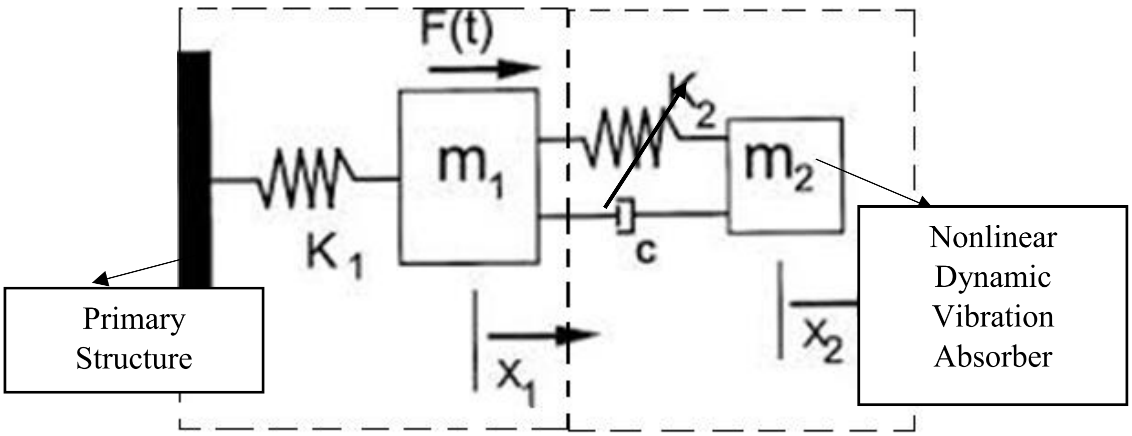


Figure 3.9: setup of nonlinear vibration absorber.

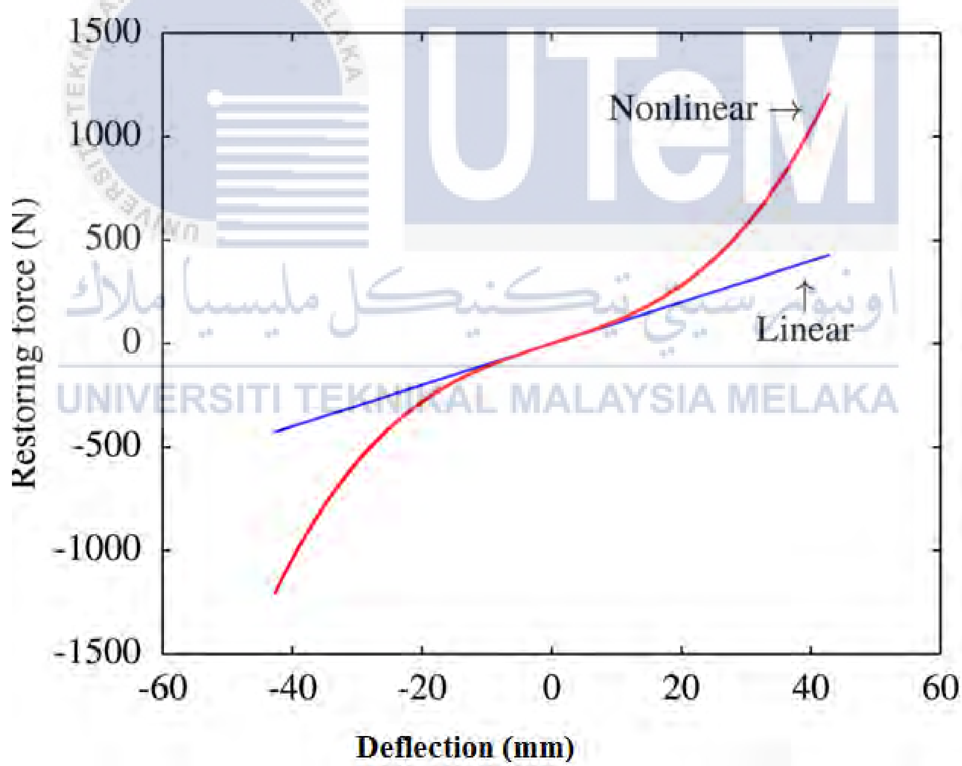


Figure 3.10: Difference between linear and nonlinear stiffness. [Source: (Russel, 2016)].

From Figure 3.10, it can see that deflection and restoring force have a linear relationship. That is when the deflection increases the restoring force also increases linearly. Slope of gradient for linear stiffness is same everywhere. For nonlinear stiffness, spring acts as a linear spring for small deflection. For large deflection, rate of restoring force increment over deflection is not uniform. For a small increase in deflection at this region will have large restoring force.

Figure 3.10 shows that when cubic stiffness constant increases nonlinearity of spring increases. This is called nonlinearity by hardening. In hardening case, the graph curves upwards compared to downward curve of softening spring. When the nonlinear absorber excited, theoretically it will have the effect of pushing over the resonance peak compared to linear absorber. Level of distortion will be higher for stronger nonlinearity. In linear absorber, response of the absorber is infinity at resonance frequency. But, in nonlinear absorber it will be finite for finite excitation frequency. Nonlinearity of absorber increases as cubic stiffness constant K_3 increases.

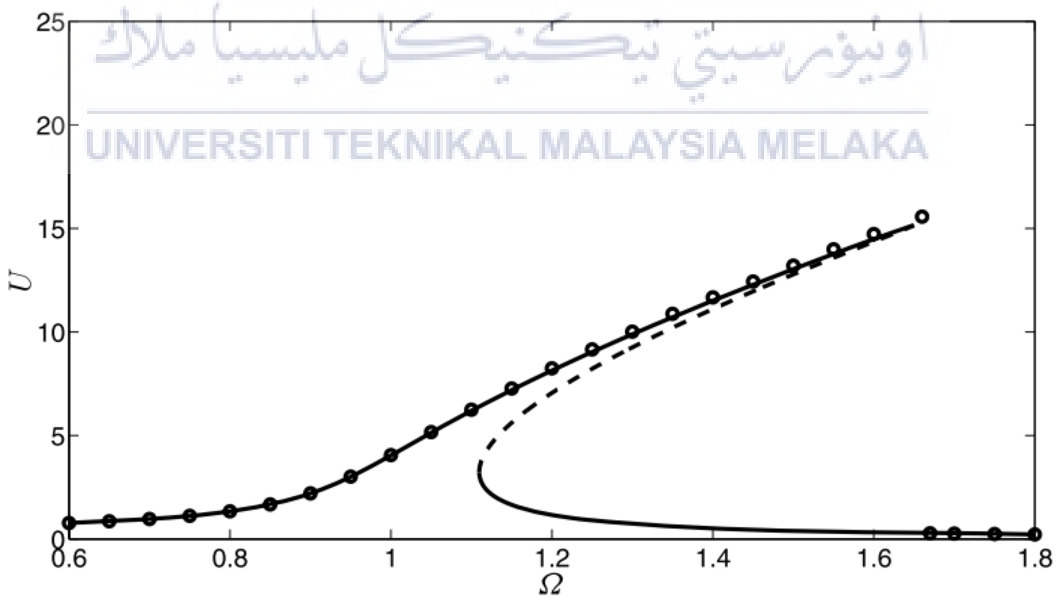


Figure 3.11: FRF for nonlinear vibration absorber [Source: (Ramlan, 2009)].

Figure 3.11 shows nonlinear absorber's FRF. Peak for nonlinear absorber is not straight as in llinear absorber. It's peak will be skewed to right if the nonlinearity induced using hardening method. It will skewe to left if softening method is used.. The degree of skewe depeends on nonlinearity of the absorber. Higher the nonlinearity, to the right side will be peak of nonlinear absorber.

3.3 Experimental Study

3.3.1 Mode of Vibration of Beam

a) BUMP TEST

One of the ways to identify natural frequency of the beam is by performing bump test on it. Bump test is a method of identifying natural frequency, modal masses, modal damping ratio and mode shapes of an object by hitting it with an impact hammer. All the equipment was setup as shown in Figure 3.12.

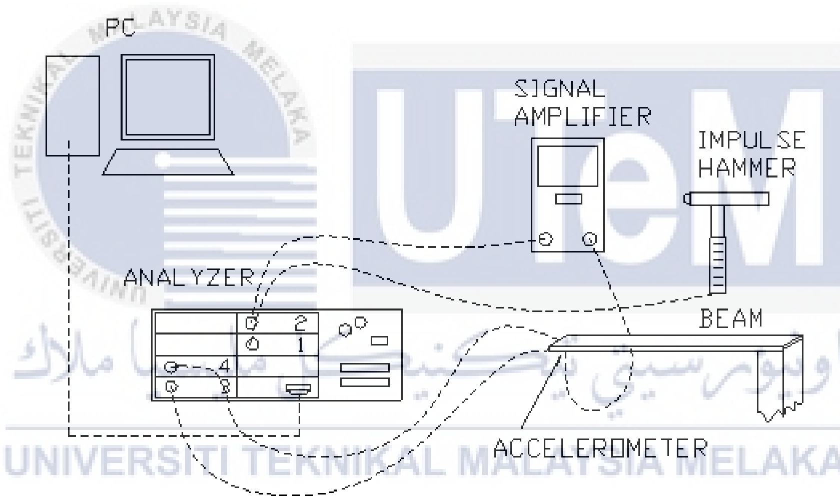


Figure 3.12: Bump test setup.

In this method, when the beam hit with impact hammer all the natural frequency of the beam will be excited. The excited natural frequencies will be represented as peaks in the frequency spectrum.

b) SWEPT SINE TEST

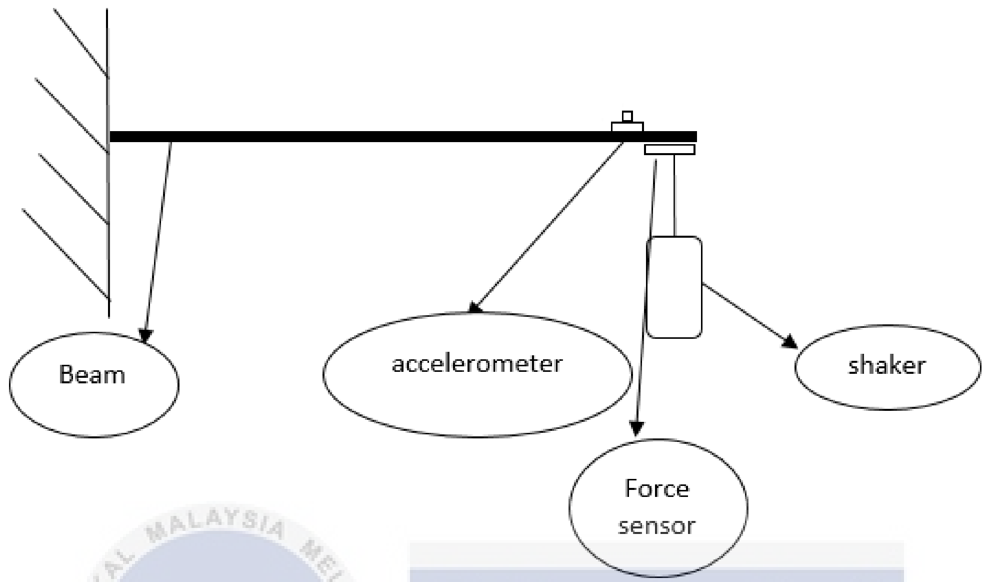


Figure 3.13: Setup of beam excited without absorber.

The beam modelled as primary structure was excited at random excitation frequency without attaching absorber as shown in Figure 3.13. From the resulting FRF, the first mode of vibration will be obtained for the primary structure. Mode of vibration is also known as natural frequency or resonant frequency of a structure. Resonant vibration amplifies vibration response of structure beyond design levels of static loading (Schwarz.B & Richardson.M, 1999). This is because, at resonant frequency energy is easily absorbed by the structure and retain the energy in the form of mode shape which is a deformation wave form. The type of FRF used in this research is the ratio of acceleration over force A_2/F called accelerance. This type of FRF is used because accelerometer is used to measure vibration from primary beam. Receptance response of the absorber, x_2/F can be calculated by double integration of accelerance.

First mode of vibration of primary structure was analytically calculated first. Vibration modes of beam depends on Young's modulus, E, moment of inertia, I, mass per unit length, m, and length of beam, L. Equation to calculate the first mode of vibration of beam is given as

$$\omega_1 = (1.875)^2 \sqrt{\frac{EI}{mL^4}} \tag{10}$$

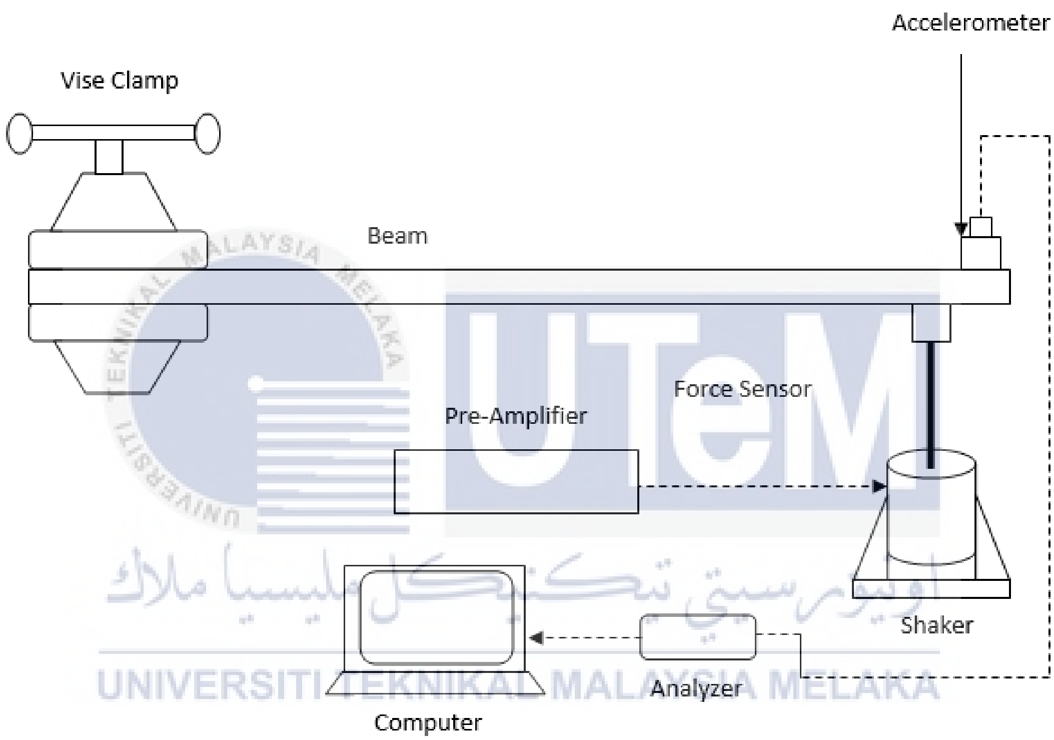


Figure 3.14: Setup to identify vibration mode of primary structure.

Figure 3.14 shows how the apparatus was setup before the beam is excited. The beam was excited at random frequency. The resulting FRF curve for magnitude have several peaks. First peak will be the first mode of vibration for the beam. at this point the phase diagram will have 90° slope. Second mode of vibration of the beam will be the peak next to the first mode of vibration.

3.3.2 Operating Deflection Shape

Operating deflection shape (ODS) can be defined as deflection of structure at any given frequency. In other words, ODS is movement of one point in the structure relative to all other points. Generally, ODS is measured to see how much structure moves at excitation frequency. Which part is moving the most and at which direction it moves also can be identified using ODS.

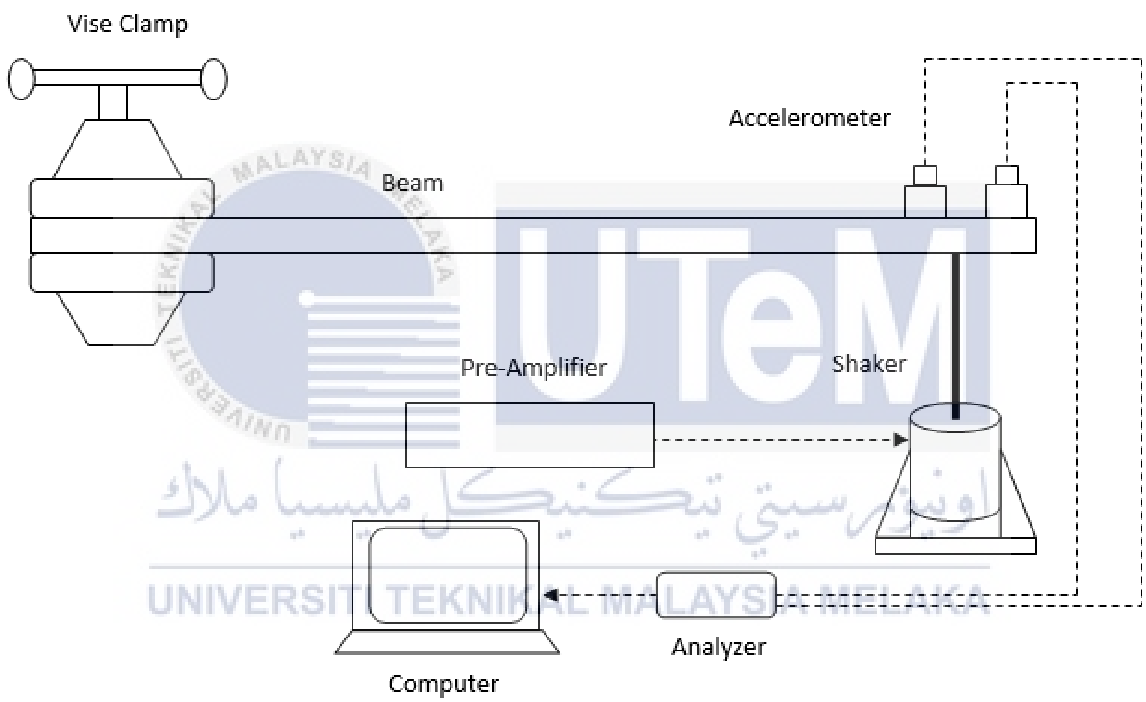


Figure 3.15: Schematic diagram of experimental setup to obtain ODS of primary structure.

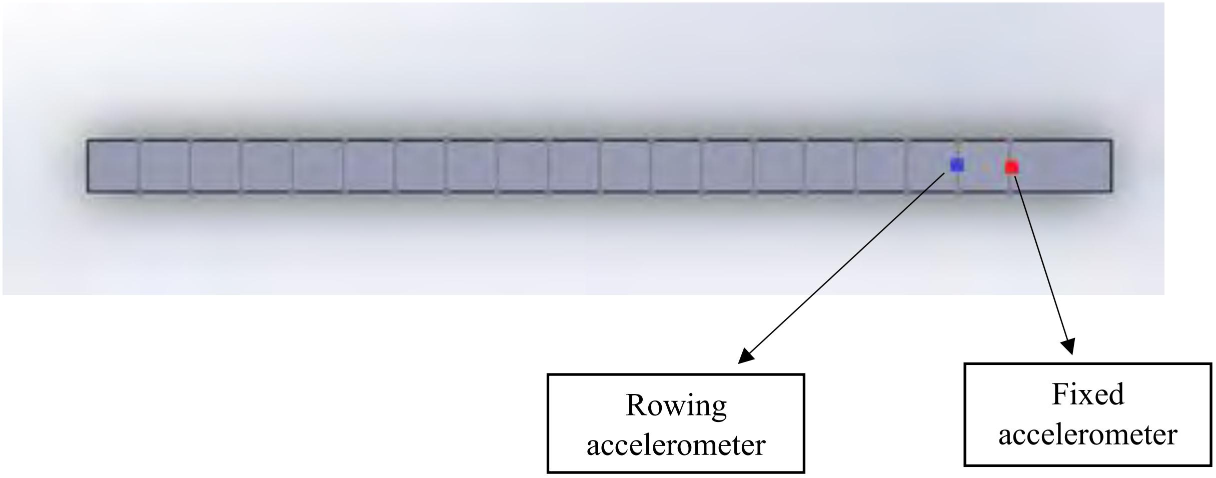


Figure 3.16: Primary structure marked with points to measure ODS.

All the apparatus was setup as shown in Figure 3.15, 18 lines were marked on beam as shown in Figure 3.16. Red point marked on the beam indicates reference point. Accelerometer attached to this point will not be altered. Another accelerometer was placed on the blue point. This point is called rowing point. The beam will be first excited at its first mode of vibration, ω_1. After taking magnitude and cross-phase at this point, accelerometer will be attached to the next rowing point. All the points were drawn in VibShape software. Points are drawn same as they are in the beam. For each point, their respective magnitude and cross-phase were inserted.

3.3.3 Experimental Setup Configuration

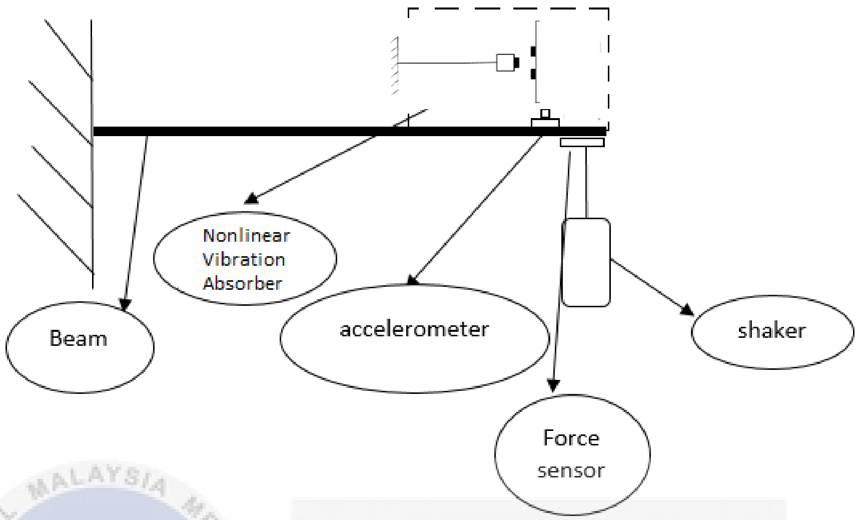


Figure 3.17: Schematic diagram of experimental setup of nonlinear absorber.

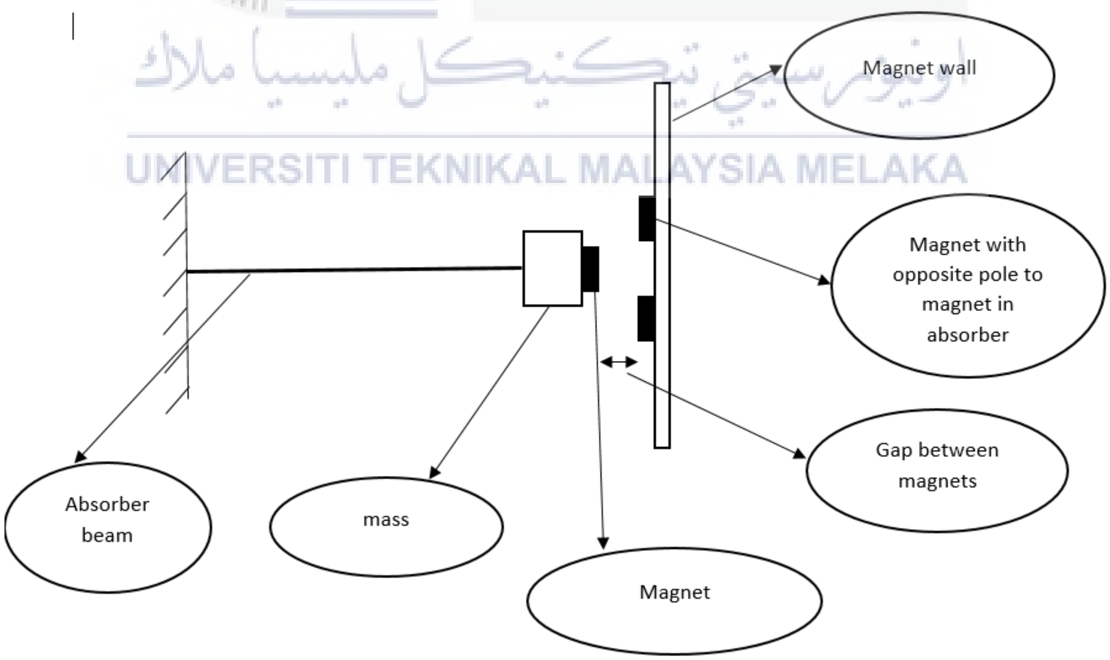


Figure 3.18: Design of nonlinear dynamic vibration absorber.

Experiment was setup as shown in Figure 3.17. Absorber was setup as shown in Figure 3.18. Gap between magnets changed by moving the magnet wall. Magnets used in this experiment made from Neodymium iron boron 'rare earth' material. It has high resistance to de-magnetization. Specifications of this magnets are shown in Table 3.1.

Table 3.1: Specifications of magnets

Parameter	Specifications
Type	Neodymium
Width, mm	16
Thickness, mm	4.5
Pull force, kg	0.95

Primary beam used in this experiment is made from Tool Steel. The beam is fully annealed and free from carburization. This type of beam is highly resistant to wear. Specification of the beam is shown in Table 3.2

Table 3.2: Specifications of beam

Characteristics	Specifications
Length, mm	1000
Width, mm	50
Thickness, mm	10
Density, kg/m^3	7850
Young's Modulus, N/m^2	2.1e11
Moment of inertia, m^4	1.0667e-9

3.3.4 Characterisation of Nonlinear Dynamic Vibration Absorber

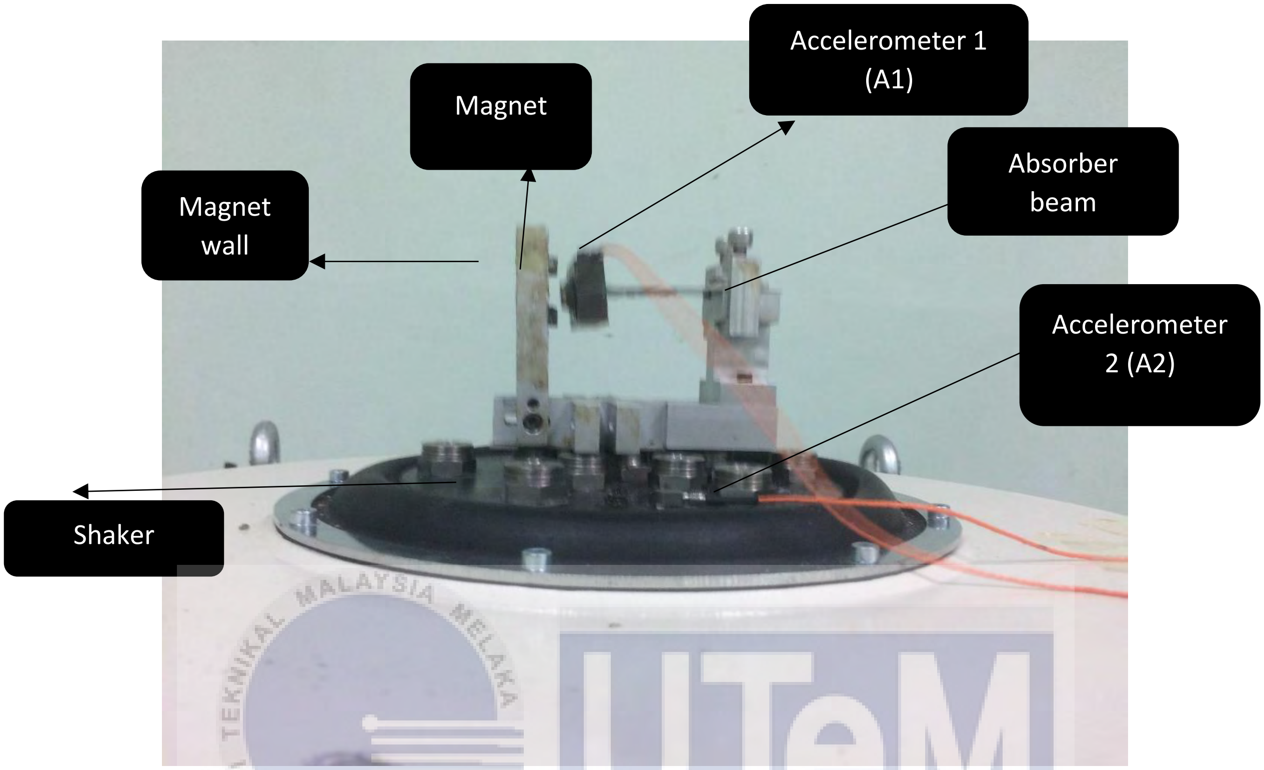


Figure 3.19: Characterisation of nonlinear dynamic vibration absorber on shaker.

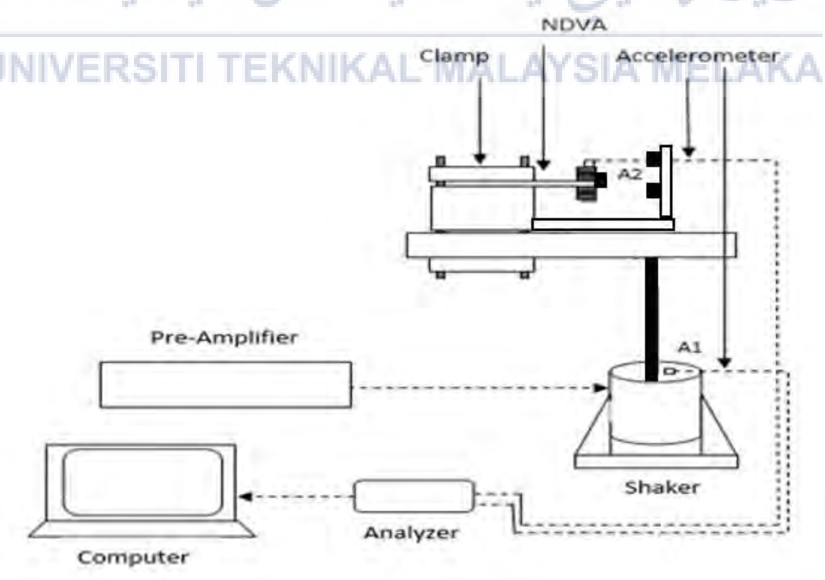


Figure 3.20: Schematic diagram of characterisation of NDVA.

Characterisation of absorber was investigated on large shaker as shown in Figure 3.19. ETS solution shaker model MPA101-L215M as shown in Figure 3.21 was used for the characterization experiment. This shaker can support specimen payload up to 70 kg. It has armature diameter of 150 mm and have frequency range up to 4500 Hz. VR9500 controller from Vibration Research (VR) was used as analyzer. It has 13000 lines of resolution and maximum control frequency up to 20000 Hz. Experiment was setup as shown in Figure 3.20. For each characterization process two accelerometers are needed. One accelerometer was placed on the top surface of shaker. This accelerometer take reading from shaker which is the input signal. Another accelerometer was placed on mass of the absorber beam. This accelerometer measures the output signal which is the vibration level of absorber. Ratio of output signal over input signal was plotted against frequency in the frequency spectrum to analyze effect of all three variables on the nonlinearity of the absorber. For each experiment, readings were taken in ascending order, from 10 Hz to 35 Hz. This ascending order called sweep up frequency. Then, sweep down frequency is continued at the similar frequency range. Three variables that were manipulated in characterization process are gap between magnets, level of input displacement, and length of absorber beam.

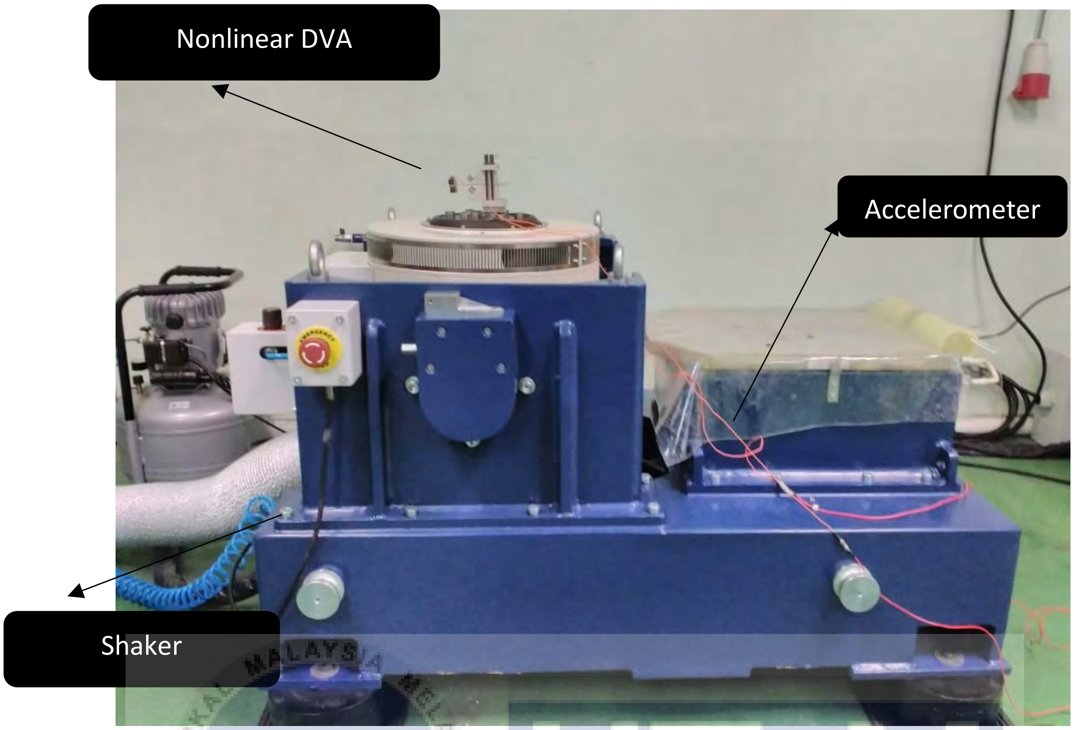


Figure 3.21: Shaker used for characterisation of absorber.

a) Effect of gap between magnets

For this characterisation process, horizontal gap between magnet attached to absorber beam and magnet wall was altered.

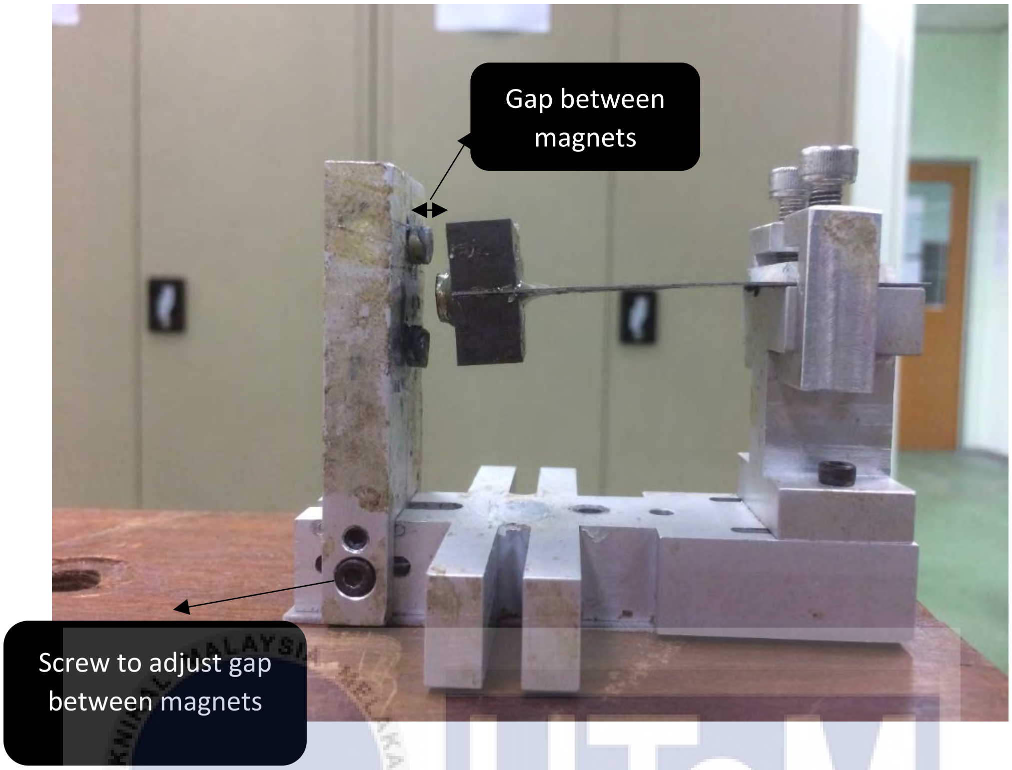


Figure 3.22: Method to increase or decrease gap between magnets

As shown in Figure 3.22, position of magnet wall can be adjusted using the screw. Effect of gap between magnets was investigated for six different gap levels. They are 0.5 mm, 1.0 mm, 1.3 mm, 2 mm, 2.5 mm, and 5 mm. During this characterisation process, other two variables, which are level of input displacement and length of absorber beam were kept constant. Level of input displacement was maintained at 2.5 mm. Length of absorber was kept constant at 5 cm.

b) Effect of input level of displacement

For this characterisation process, absorber's level of input displacement was manipulated. Level of input displacement can be increased or decreased by controlling the maximum peak-peak (pk-pk) amplitude of the shaker. If the pk-pk amplitude is smaller, input from shaker will be small. Effect of level of input displacement was investigated for four input levels. They are 0.05 mm pk-pk, 1 mm pk-pk, 1.5 mm pk-pk, and 2.0 mm pk-pk. During this characterisation process, gap between magnets was kept constant at 1mm. Length of absorber was maintained at 5 cm.

c) Effect of absorber beam length

For this characterisation process, effective length of absorber beam was manipulated. Effective length of beam is from the clamped end of beam to the free end of beam as shown in Figure 3.23. Effect of length of absorber was investigated for four different length. They are 4.5 cm, 5.0 cm, 5.2 cm, and 5.5 cm.

Figure 3.23: Starting point of effective length.

3.3.5 Performance of Nonlinear Dynamic Vibration Absorber at Different Location.

For vibration mode of vibration, performance of absorber partly depends on its position in the structure. Most suitable position of absorber will be determined by placing the absorber at three different location for each vibration mode.

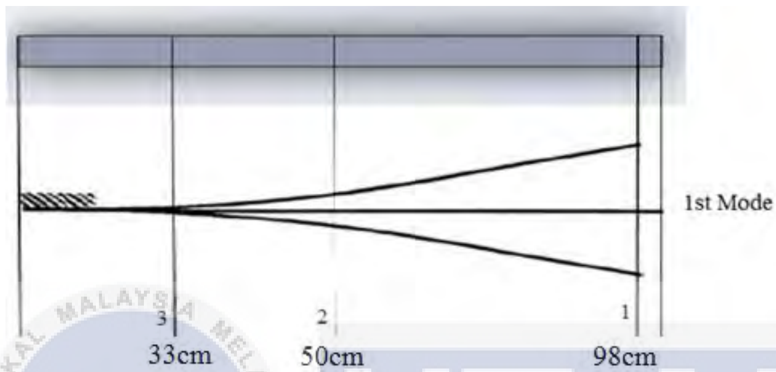


Figure 3.24: Points nonlinear vibration absorber will be placed on primary structure.

Three points will be marked on primary beam as shown in Figure 3.24. Absorber will be placed at each point and beam will be excited at its first and second mode of vibration. FRF of A_1/F for primary beam will be investigated for performance of NDVA at each marked point. For the position with greatest performance, absorber with different gap levels will be investigated.

CHAPTER 4

THEORETICAL RESULTS

4.1 Vibration Mode of Primary Structure

When the beam is subjected to harmonic force excitation close to the natural frequency of the system it will experience a phenomenon called resonance. Resonance is a situation where the response of the system will be infinite if damping is neglected. At these frequencies, small excitation force will produce very large vibrational amplitude oscillation. Natural frequencies are also called vibration modes. First vibration mode of cantilever beam can be calculated by using Equation 10. Equation 10 can be modified as

$$\omega_1 = 1.875^2 \sqrt{\frac{E\left(\frac{bh^3}{12}\right)}{\rho(bh)L^4}} \tag{11}$$

When the similar parameters are taken out, this equation will be obtained.

$$\omega_1 = 1.875^2 \sqrt{\frac{Eh^2}{12\rho L^4}} \tag{12}$$

First vibration mode of the beam will be

$$1.875^2 \sqrt{\frac{2.1\times10^{11}\times0.01^2}{12\times7850\times0.67^4}} = 116.9330 \text{ rad/s} \tag{13}$$

116.9330 rad/s = 18.6104 ≈ 19 Hz

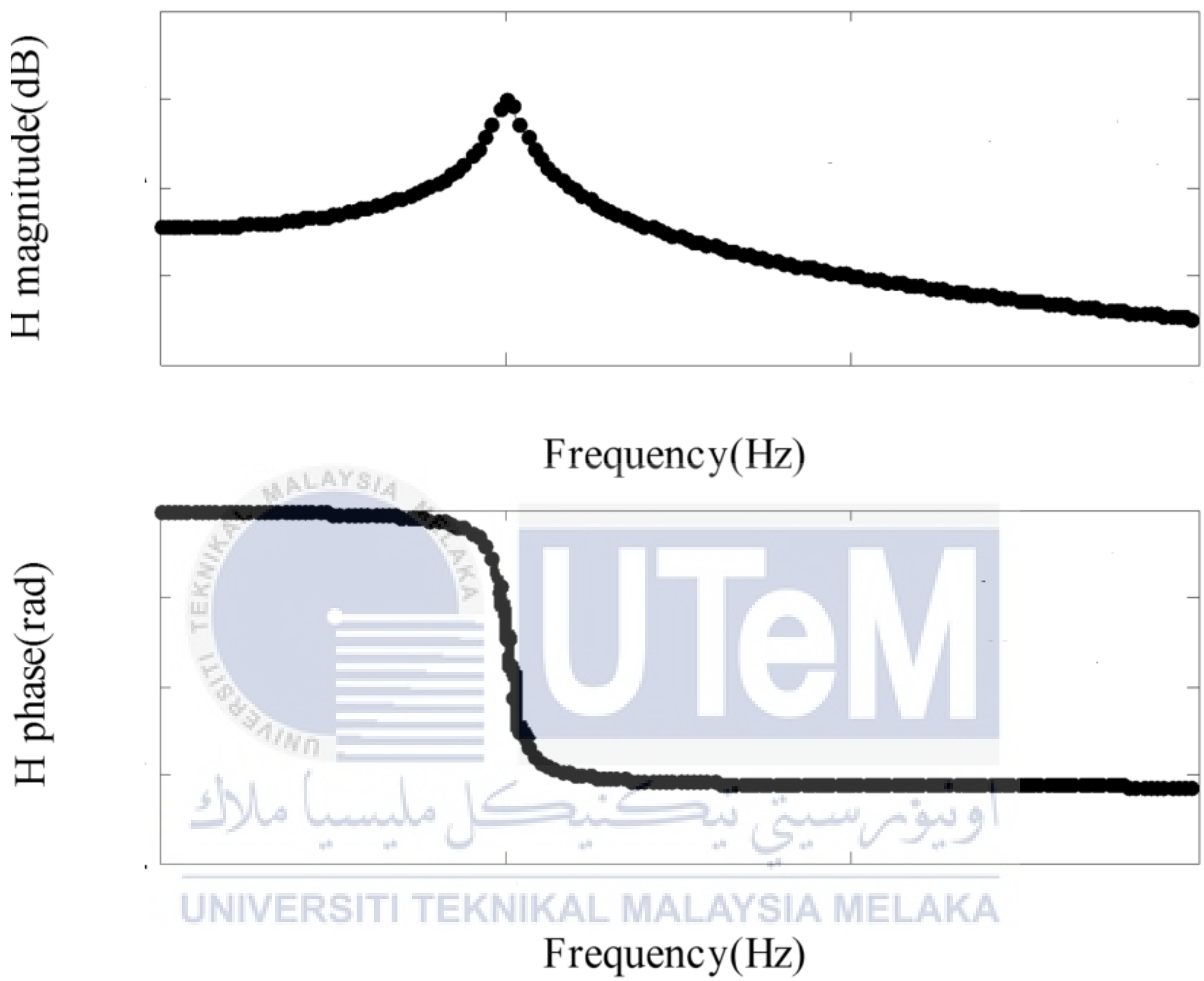


Figure 4.1: FRF for primary structure indicating its vibration mode.

Figure 4.1 shows the typical FRF of Single Degree of Freedom (SDOF). Peak in the magnitude spectrum and 90° slope in the phase spectrum are the indication of vibration mode of a SDOF system.

4.2 Operating Deflection Shape (ODS)

ODS will help visualize the deflection of beam at its natural frequencies. Visualization of beams vibrating pattern will help later in identifying perfect location to place absorber. For the first vibration mode, the beam will vibrate as shown in Figure 4.2. Deflection at the free end marked 1 will be maximum. In this resonant frequency, absorber should be placed at position 1 to suppress the vibration of the beam.

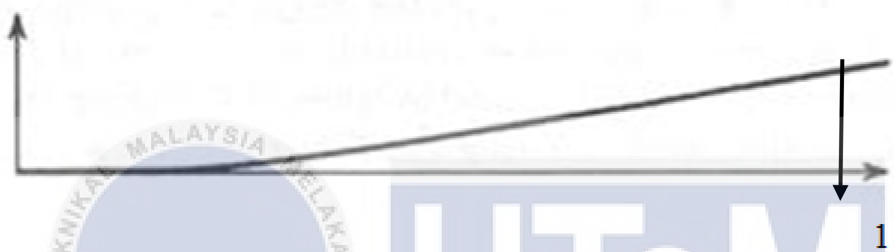


Figure 4.2: ODS for first mode of vibration.

4.3 Characterization of Nonlinear Dynamic Vibration Absorber

a) Effect of Gap Between Magnets

When the gap between magnets is smaller, the gap between jump down and jump up frequency will be bigger. According to Ramlan et al (2016) sudden changes of the amplitude occur at increasing frequency (jump-down) and decreasing frequency (jump-up).

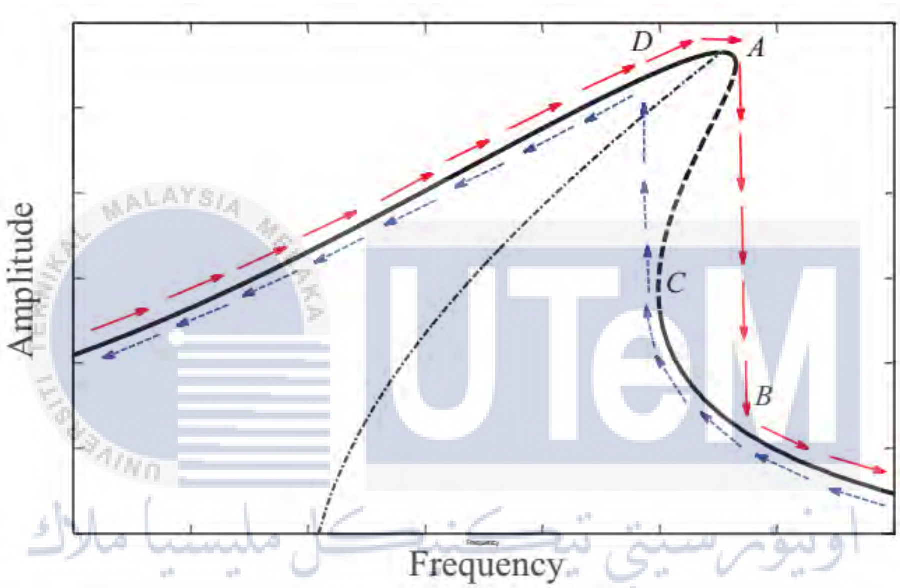


Figure 4.3: FRF for nonlinear absorber indicating jump-down and jump-up frequency [Source: (Ramlan, 2016)].

Figure 4.3 shows the typical FRF curve for nonlinear material by hardening method. Red arrows in the figure represents sweep up frequency, while blue arrows represent sweep down frequency. Point A to B represents jump-down frequency and point C to D represents jump-up frequency. These jump-down and jump-up frequencies are referred to as the unstable region in the FRF curve. This is because, between the jump-down and jump-up frequency region, there are multiple stable solutions on the branches of FRF. This region is the part which separates the linear and nonlinear absorbers. Traditionally this region is seemed undesirable by researchers. Expected results in this study suggests that, if the boundaries of these regions can be identified then this unstable region can be exploited. When this region becomes wider, effective bandwidth of absorber increases. Magnet restoring force F_m is inversely proportional to square of gap between magnet attached to beam and magnet attached to magnet wall. The gap should be minimal to have higher stiffness. Their relationship is given by

$$F_m \propto \frac{1}{(gap)^2} \tag{14}$$

When the input and length is kept constant, this effective bandwidth can be increased if the gap between magnets are reduced.

b) Effect of Level of Input Displacement

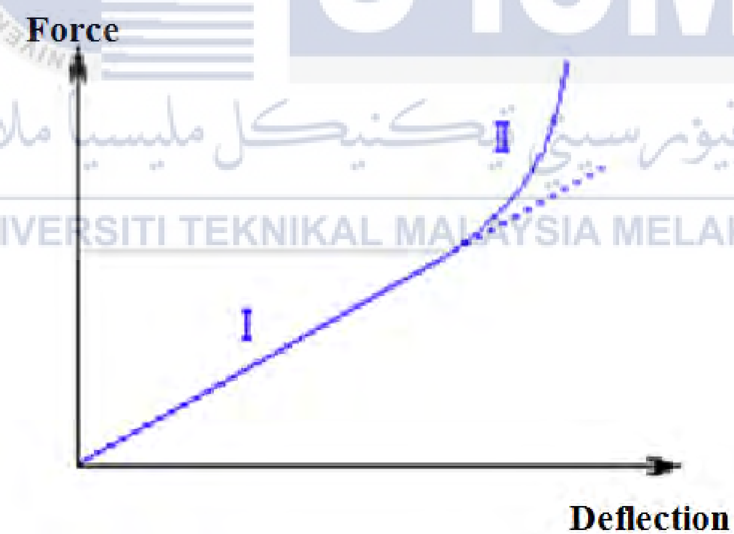


Figure 4.4: Graph showing difference between linear and nonlinear spring.

Figure 4.4 shows, linear and nonlinear spring graph plotted on the same axis. Continuous line represents nonlinear spring, while dotted line represents linear spring. In the section marked as

I, both linear and nonlinear spring behave like a linear spring. At section II, the linear spring continue its increase in force linearly against deflection. For nonlinear spring, even for small change in deflection increase in force will be exponential. When the absorber is in nonlinear phase, increase in pk-pk amplitude will result in increase of absorber's nonlinearity. Gap between jump-down and jump-up frequency will be widened when there is increase in input level of shaker.

c) Effect of Absorber Beam Length

Adjusting length of absorber will not affect the nonlinearity of absorber. According to linear stiffness equation,

$$K_L = \frac{3EI}{L^3} \tag{15}$$

Where, K_L is linear stiffness, E is Young's modulus, I is second moment of inertia and L is length of beam.

changing length of beam will result in change of absorber's linear stiffness. Change in linear stiffness will change the natural frequency of absorber. Shortening of absorber beam length will increase the linear stiffness of absorber. Increase in stiffness will increase the natural frequency of absorber. Hence, shortening of absorber beam's effective length will result in natural frequency peak in FRF to shift right side.

4.4 Performance of Nonlinear Vibration Absorber on Primary Structure

Nonlinear dynamic vibration absorber is an auxiliary, small mass-spring system attached to the primary system as shown in Figure 4.5. By adding this second mass to the primary system, a new multiple degree of freedom (MDOF) system will be formed. This system will have its own new natural frequencies.

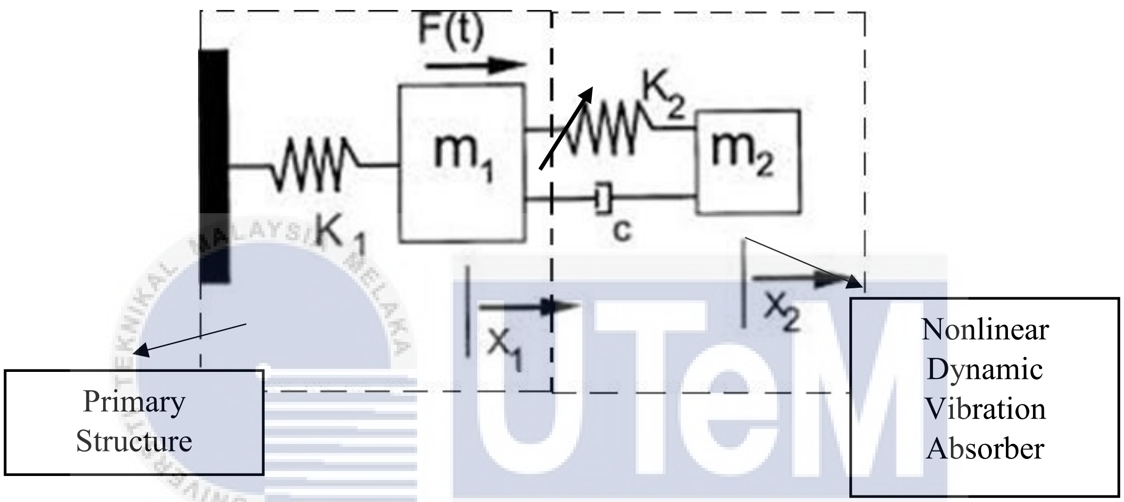


Figure 4.5: Nonlinear dynamic vibration absorber

Stiffness of nonlinear absorber is $K_2x_2 + K_3x_2^3$. To simplify the equation $K = K_2 + K_3$. Equation of motion of the combine system will be

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x_1} \\ \ddot{x_2} \end{Bmatrix} + \begin{bmatrix} c & -c \\ -c & c \end{bmatrix} \begin{Bmatrix} \dot{x_1} \\ \dot{x_2} \end{Bmatrix} + \begin{bmatrix} K_1 + K & -K \\ -K & K \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F(t) \\ 0 \end{Bmatrix} \quad (16)$$

Damping parameter will be ignored because of its insignificant small value. When the system is differentiated in complex form of

$$x = Xe^{j\omega t} \quad (17)$$

$$\dot{x} = j\omega Xe^{j\omega t} \quad (18)$$

$$\ddot{x} = -\omega^2 X e^{j\omega t} \tag{19}$$

Two new Equations 20 and 21 will be obtained.

$$(K_1 + K - \omega^2 m_1)X_1 - KX_2 = F(t) \tag{20}$$

$$(K - \omega^2 m_2)X_2 - KX_1 = 0 \tag{21}$$

Response of the system for output over input will be

$$\frac{X_1}{F} = \frac{(K-\omega^2 m_2)}{(K_1+K-\omega^2 m_1)-K^2} \tag{22}$$

When the nonlinear absorber attached to beam, deflection of the beam X_1= 0. Then,

$$K - \omega^2 m_2 = 0 \tag{23}$$

$$\omega = \sqrt{\frac{K}{m_2}} \tag{24}$$

When the nonlinear absorber is tuned to natural frequency of primary structure, it will suppress the vibrational amplitude of the primary structure. Figure 4.6 shows the theoretical FRF for absorber on primary structure.

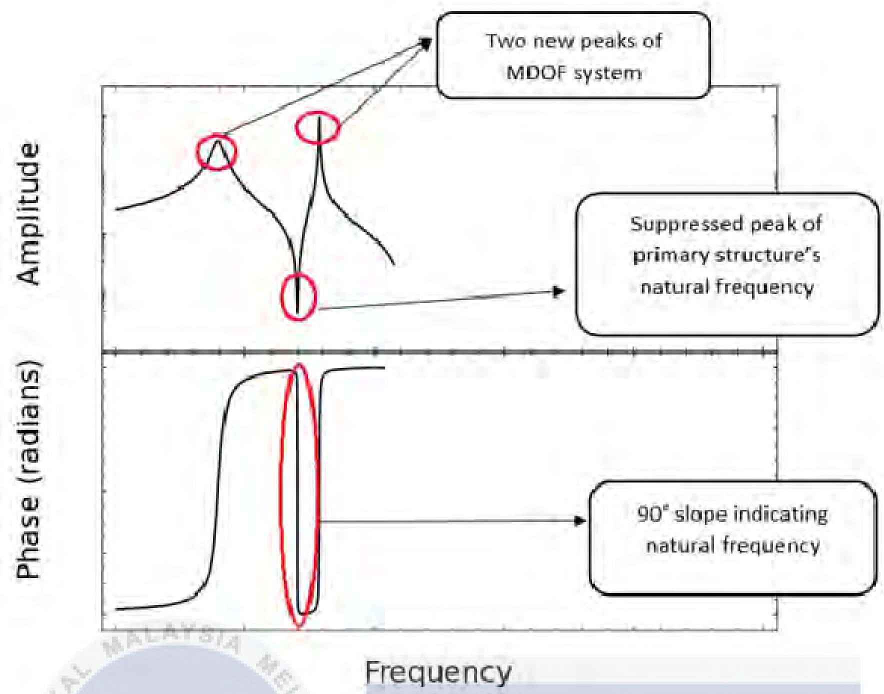


Figure 4.6: FRF for properly tuned absorber on primary structure. [Source: (Inman.D, 2001)].

When absorber attached to primary structure, it changes from being a (SDOF) system to Multiple Degree of Freedom (MDOF) system. In this case, the system is Two Degree of Freedom system because there are two masses in the system. One is primary mass of primary structure and the second is mass of absorber. Two new peaks on the side of old peak in the FRF indicates the new MDOF system. Amplitude of original peak of primary structure is at lowest because at that frequency, the vibration absorber will 'absorb' the vibration of primary structure. If tuned properly, both primary structure and vibration absorber will have same natural frequency. When the input frequency matches the natural frequency, both primary structure and vibration absorber will be in resonance. Thus, vibration amplitude of primary structure and vibration absorber will cancel out each other. That is why the amplitude of original peak of primary structure is small.

CHAPTER 5

EXPERIMENTAL RESULTS

5.1 Vibration Mode of Primary Structure

a) Bump Test

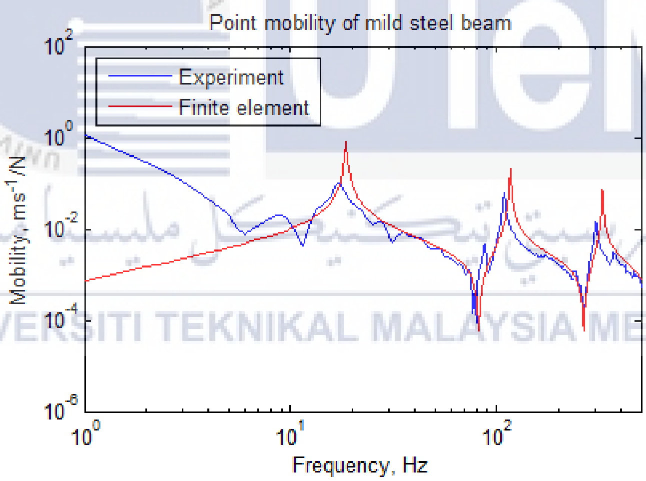


Figure 5.1: FRF of primary structure from impact test.

Figure 5.1 shows the result of impact test on primary structure. Blue line represents the result of experiment and red line obtained from Finite Element (FE). Three peaks from blue line almost matches the peak in red line. This shows that the experimental result matches the theoretical value obtained from FE. First peak of the blue line is at 17 Hz. Experimental result differs slightly from analytical result which is 19 Hz. Possible reason for this difference is the actual damping value of primary structure is different from the value used in calculation of FE. Stringer that holds force transducer must be in straight line during the impact test to prevent it from tamper the data.

b) Swept Sine Method

Figure 5.2 shows the accelerance graph for primary structure without nonlinear absorber. Peak in this graph shows the first vibration mode of the primary structure. It is at 16 Hz. It is slightly lower than theoretical result of 19 Hz. Reason for this difference is the difference in mass loading specified in simulation and experiment. During the theoretical calculation damping of the beam was ignored.

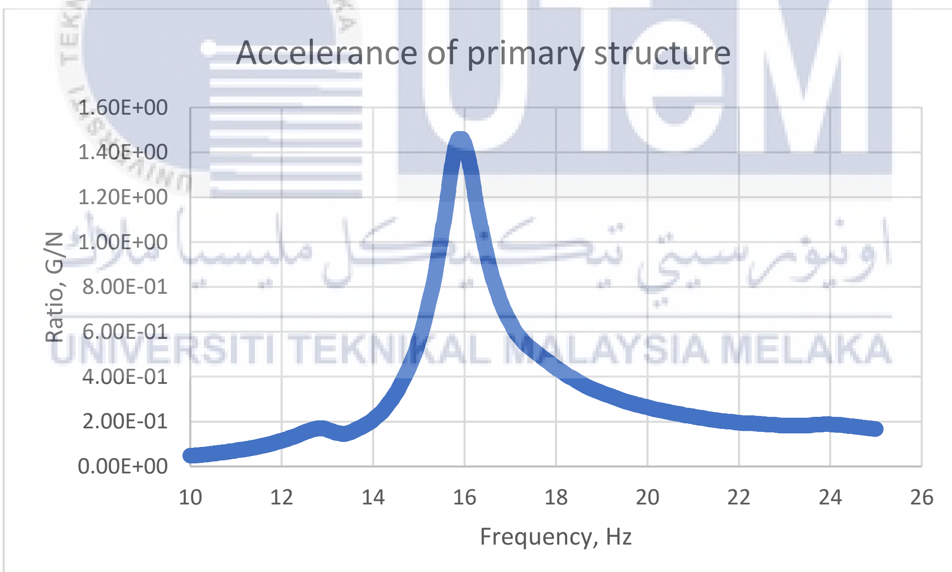


Figure 5.2: Accelerance of primary structure without dynamic vibration absorber.

5.2 Operating Deflection Shape (ODS)

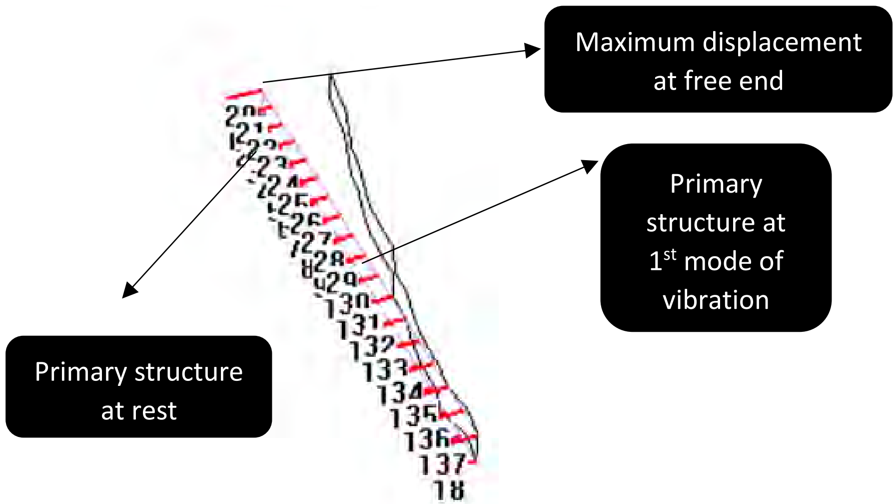


Figure 5.3: ODS for first vibration mode of primary structure.

After identifying the frequency for first vibration mod of primary structure, ODS for the first vibration mode was constructed. Figure 5.3 shows the ODS for 16 Hz. Purple line indicates the shape of primary structure when it is at rest. Black line indicates shape of primary structure when it is excited at its first mode of vibration. Shape of beam during this mode of vibration is consistent with the theoretical results. Vibration amplitude is maximum at the free end of beam while, at clamped end vibration amplitude is zero.

5.3 Characterisation of Nonlinear Dynamic Vibration Absorber

Characterisation of nonlinear vibration absorber can be divided into three parts. All three parts are shown in Figure 5.4. Nonlinear property of the absorber influenced by three characteristics. They are Gap between magnets, level of input displacement from the shaker and length of beam. During characterisation of each criteria, other two criteria must be kept constant to avoid disturbance to measured characteristic.

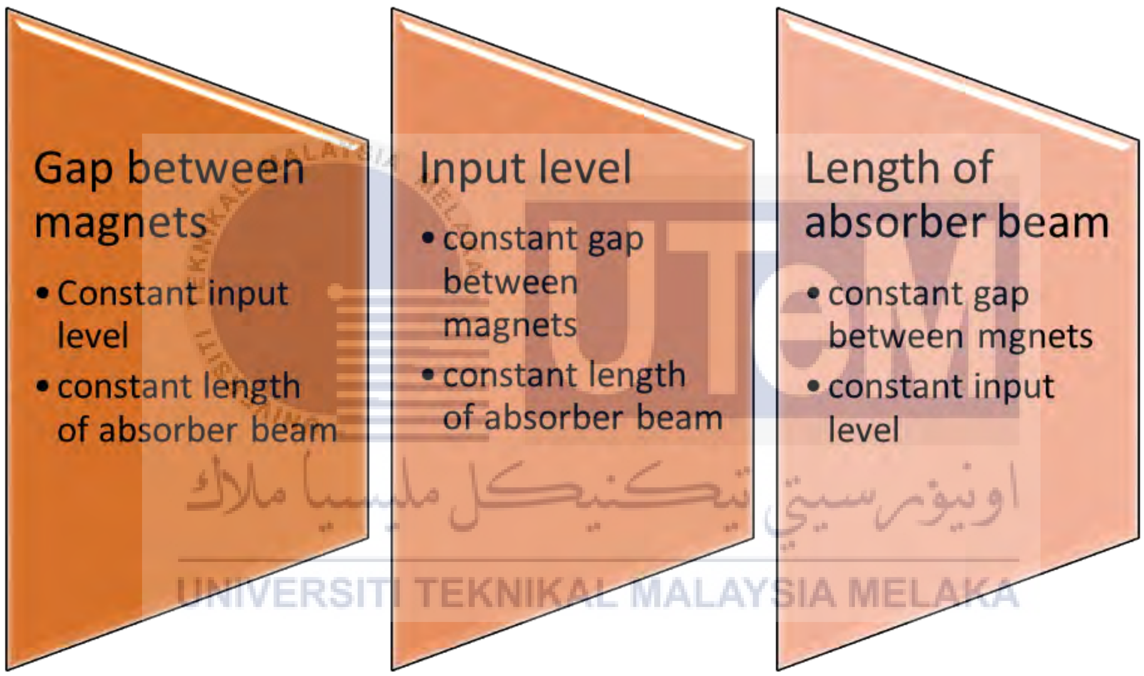


Figure 5.4: Three types of characterisation nonlinear dynamic vibration absorber.

5.3.1 Effect of Gap Between Magnets

Magnet at the absorber beam and magnet at magnet holder are placed facing opposite poles so that they attracted to each other. At rest, because of attraction of opposite poles the beam becomes softer and easy to vibrate. When the beam reaches its highest amplitude, it gets harder and hardening of absorber beam occurs. This will induce nonlinearity of the absorber. Effect of gap between magnets and nonlinearity of absorber was tested for six different gaps. They are 0.5 mm, 1 mm, 1.3 mm, 2 mm, 2.5 mm, and 5 mm. Results for these gaps are shown in Figures 5.5, 5.6, 5.7, 5.8, 5.9 and 5.10 respectively. For this experiment level of input displacement was kept constant by maintaining amplitude at 2.5 mm pk-pk. Length of absorber beam was also kept constant at 5 cm.

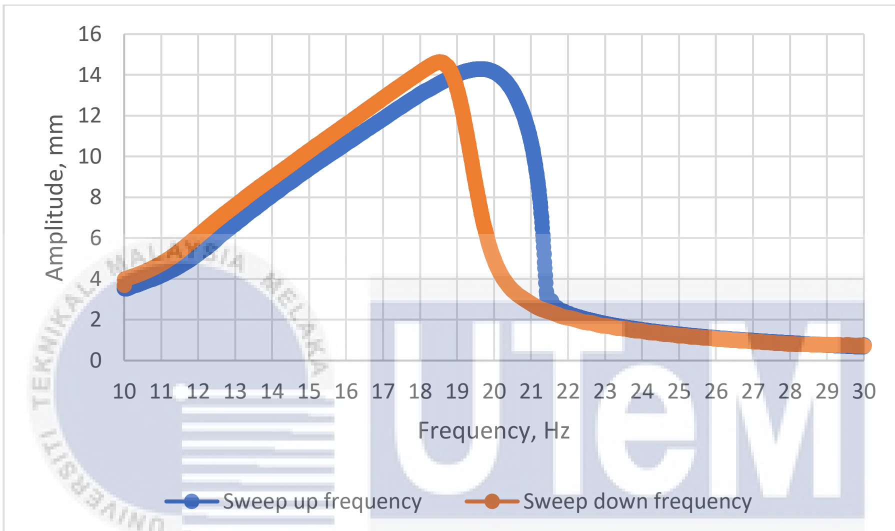


Figure 5.5: Displacement response for gap of 0.5 mm [Input: 2.5 mm pk-pk, Length: 5 cm].

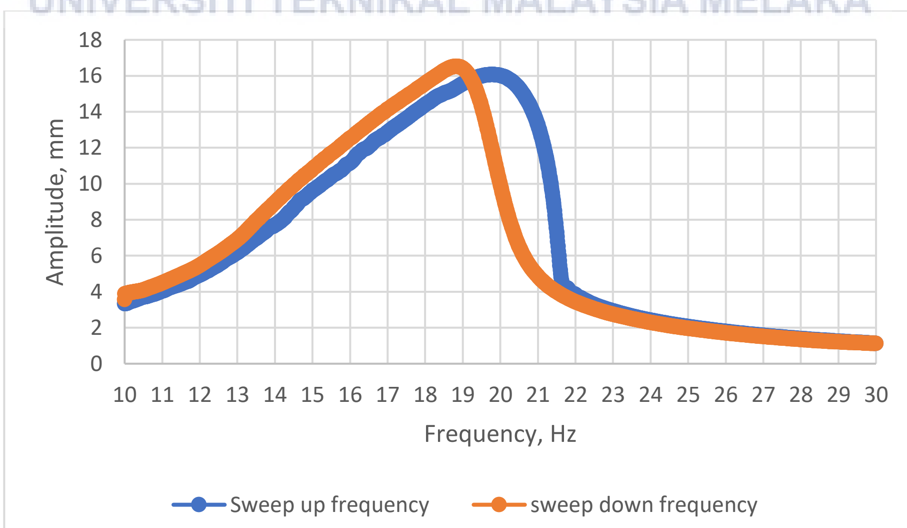


Figure 5.6: Displacement response for gap of 1.0 mm [Input: 2.5 mm pk-pk, Length: 5 cm].

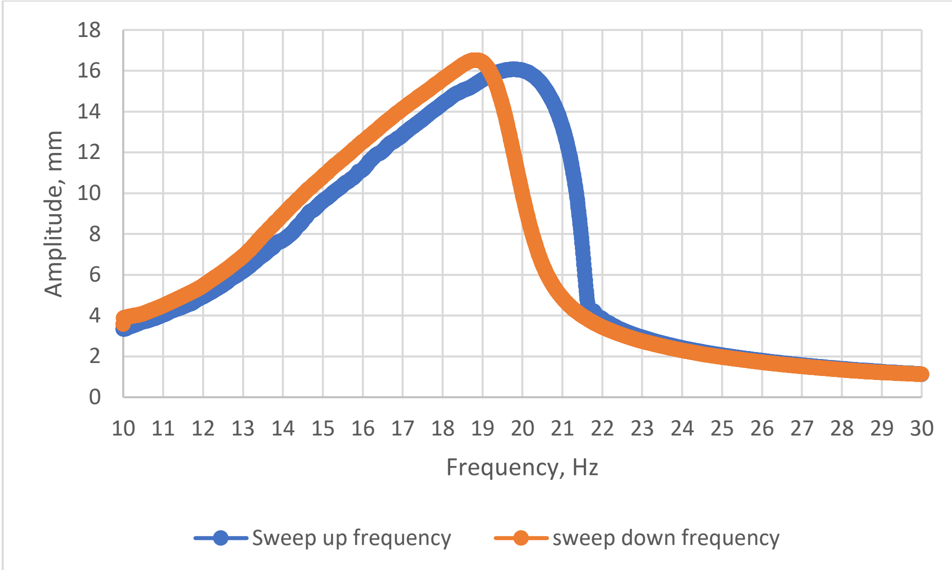


Figure 5.7: Displacement response for gap of 1.3 mm [Input: 2.5 mm pk-pk, Length: 5 cm].

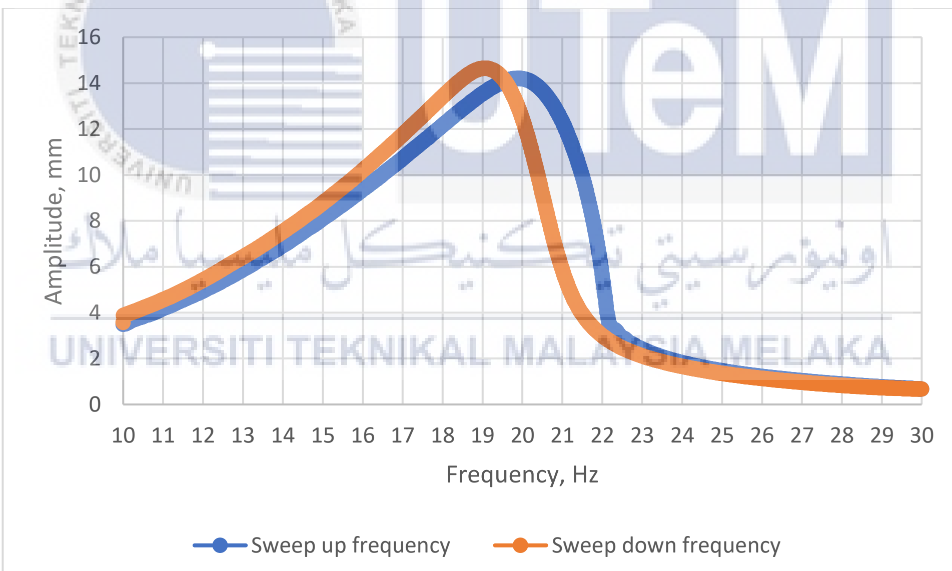


Figure 5.8: Displacement response for gap of 2.0 mm [Input: 2.5 mm pk-pk, Length: 5 cm].

Figures 5.5 to 5.8 show clear jump down in the sweep up frequency and jump up at sweep up frequency. Jump down frequency is the sudden drop in sweep up frequency. Meanwhile, jump up frequency is the sudden frequency surge in the sweep down frequency. As the gap between magnets increase, the gap between jump down and jump up frequency decreases.

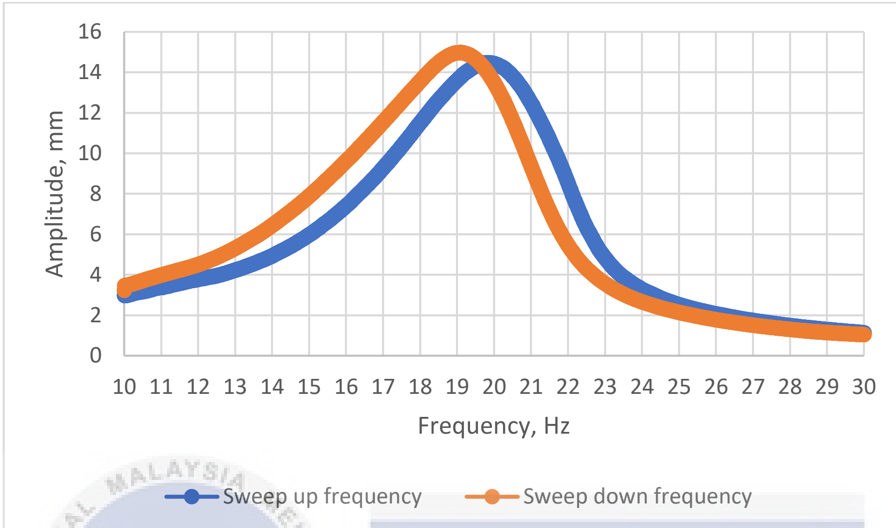


Figure 5.9: Displacement response for gap of 2.5 mm [Input: 2.5 mm pk-pk, Length: 5 cm].

For the gap of 2.5 mm, the absorber approaches linear as shown in Figure 5.9. This is because there is no visible jump down and jump up frequency. But, it is still in nonlinear region because sweep up frequency and sweep down frequency are slightly different.

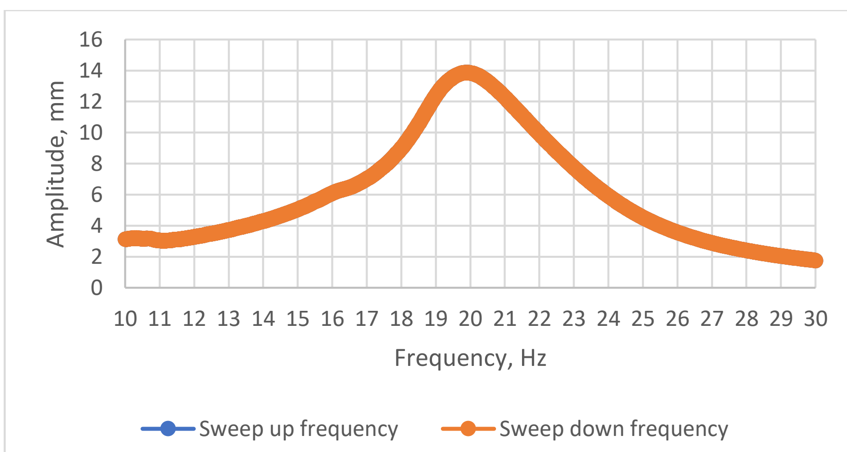


Figure 5.10: Displacement response for gap of 5.0 mm [Input: 2.5 mm pk-pk, Length: 5 cm].

When the gap is increased to 5 mm, the absorber becomes fully linear as in Figure 5.10. This is because, both sweep u and sweep down frequencies are similar.

Gap between magnets and nonlinearity of absorber shares inversely proportional relationship. When the gap between magnets decrease, nonlinearity of absorber increases. When nonlinearity of absorber increases gap between jump down and jump frequency increases. Gap between jump down frequency and jump up frequency for gap of 0.5 mm is much bigger compared to gap of 2.0 mm. when the gap between magnets increase, absorber converges into linearity. When the gap between magnets is 2.5 mm, FRF is almost like linear graph except for the slight difference in sweep up and sweep down frequency. Attractive force between magnets decrease when the distance between magnets increase. At 5 mm, the attractive forces are so minimum that it has no effect on vibration of the absorber.

5.2.2 Effect of Level of Input Displacement

For this characterisation part, level of input displacement is manipulated while gap between magnet and absorber beam kept constant at 1 mm. Length of absorber beam was kept constant at 5 cm. This characterisation experiment was carried out to investigate whether the input level will have effect on the increasing or decreasing of effective bandwidth of absorber. Level of input displacement controlled by controlling the pk-pk amplitude of the absorber beam. Four different input level were tested for this characterization experiment. They are 0.005 mm, 1.0 mm, 1.5 mm, and 2.0 mm. Results of these test are shown in Figures 5.11, 5.12, 5.13, and 5.14.

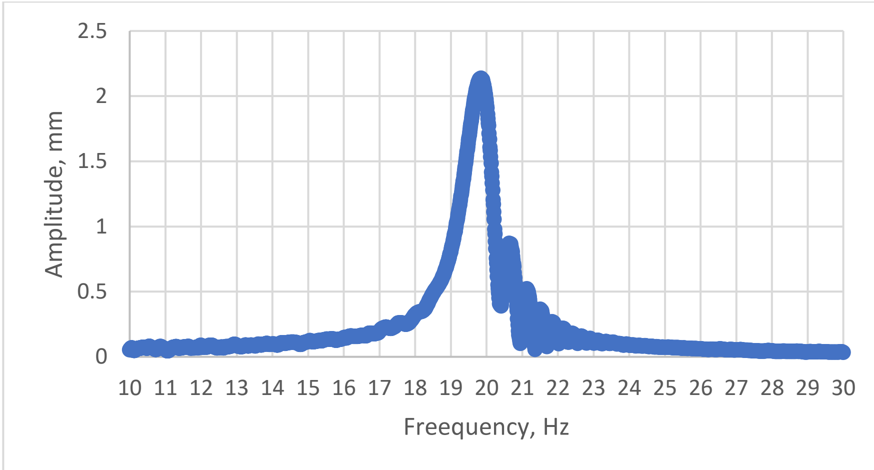


Figure 5.11: Displacement response from input excitation 0.05 mm pk-pk [Length 5 cm Gap of 1.0 mm].

When the input from shaker is 0.05 mm pk-pk, the absorber behaves like a linear absorber. There is no visible jump down or jump up frequency and it has only single peak at natural frequency like a linear absorber's FRF as shown in Figure 5.9.

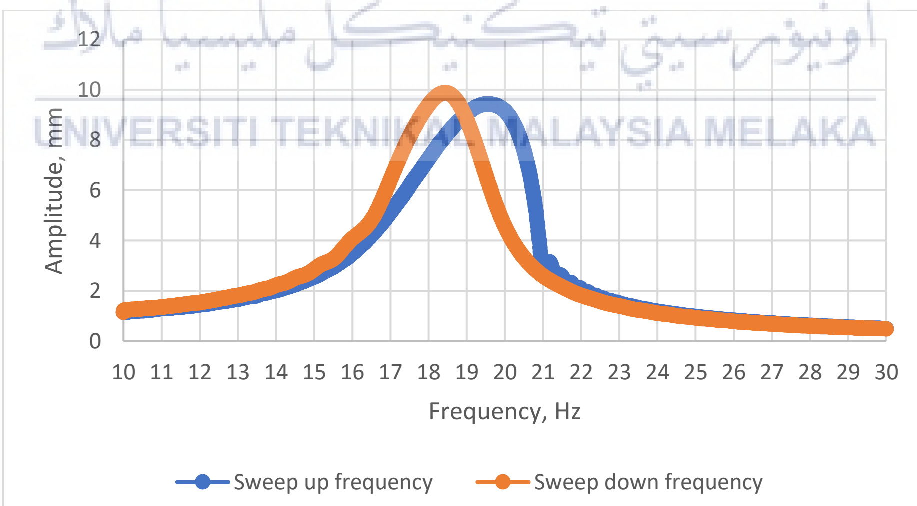


Figure 5.12: Displacement response from input excitation 1.0 mm pk-pk [Length 5 cm Gap of 1.0 mm].

Figure 5.12 shows FRF for input level of 1mm pk-pk. There is no obvious jump down and jump up frequency. But, sweep up frequency is different to sweep down frequency. The absorber is in weak nonlinear stage.

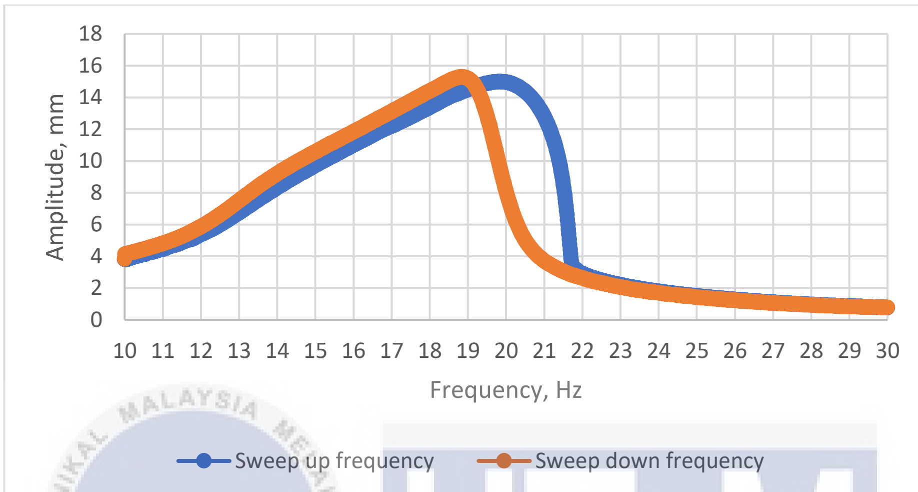


Figure 5.13: Displacement response from input excitation 1.5 mm pk-pk [Length 5 cm Gap of 1.0 mm].

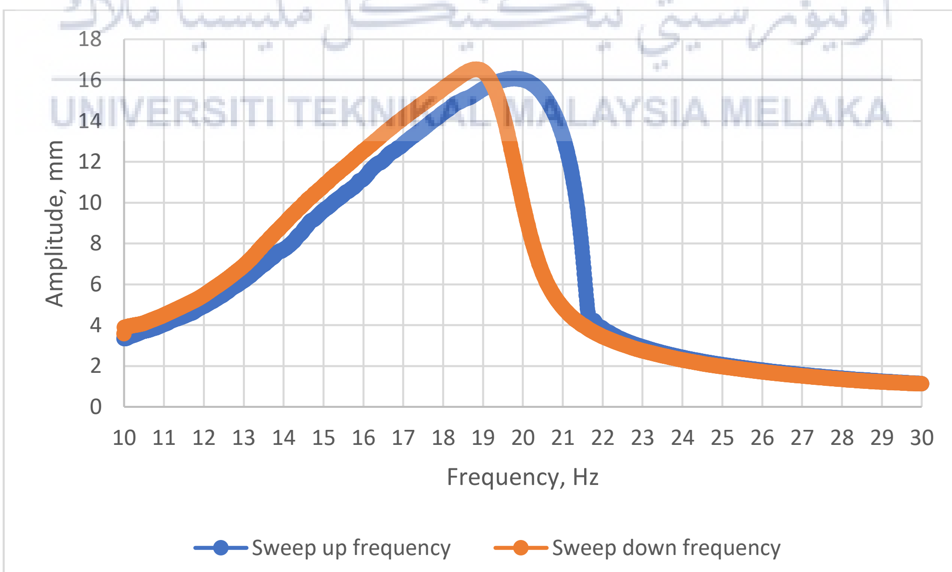


Figure 5.14: Displacement response from input excitation 2.0 mm pk-pk [Length 5 cm Gap of 1.0 mm].

For both input level of 1.5 mm and 2.0 mm, there are visible jump down and jump frequencies as shown in Figures 5.13 and 5.14. Gap between jump down and jump up frequency widens as input level increases.

When the input level is increased, the nonlinearity of absorber increases. Gap between jump-down and jump-up frequency widens when input level increases.

$$\alpha = \frac{k_3}{k_1} y^2 \qquad (24)$$

Where, α is nonlinearity, k_3 is nonlinear constant, k_1 is linear constant and y is input level. In Equation 14 input level is squared. Because it is squared, any change in its value will have bigger impact compared to ratio of spring constants. When input level is, increased nonlinearity increases. This means even at same gap between magnets if the input level is minimum the nonlinearity of absorber will be smaller. One of the reason for this situation is that when the amplitude is lowered, maximum level of vibration of absorber beam also decreased. As the input level increases, amplitude level of absorber also increases. Smaller amplitude means the absorber beam will not reach till the level where the attractive force of magnet is quite strong for hardening process to occur. Thus, the absorber will remain in the state of linear absorber. When the input level is increased, the beam will have enough energy to reach the magnet. From the attractive force between magnet and absorber beam hardening is induced in the beam.

5.2.3 Effect of Absorber Beam Length

For this characterisation part, length of beam of the absorber is manipulated while gap between magnet and absorber beam kept constant at 1 mm. Level of input displacement was kept constant by maintain the amplitude at 1 mm. This characterisation experiment was carried out to investigate whether the input level will have effect on the increasing or decreasing of effective bandwidth of absorber. Length of the beam was controlled by increasing and decreasing the effective length of the beam. Effective length of beam measured from tip of the beam holder as shown in Figure 5.15. Four different effective length were tested for this characterization experiment. They are 4.5 cm, 5.0 cm, 5.2 cm, and 5.5 cm. Results of these test are shown in Figures 5.16, 5.17, 5.18, and 5.19.

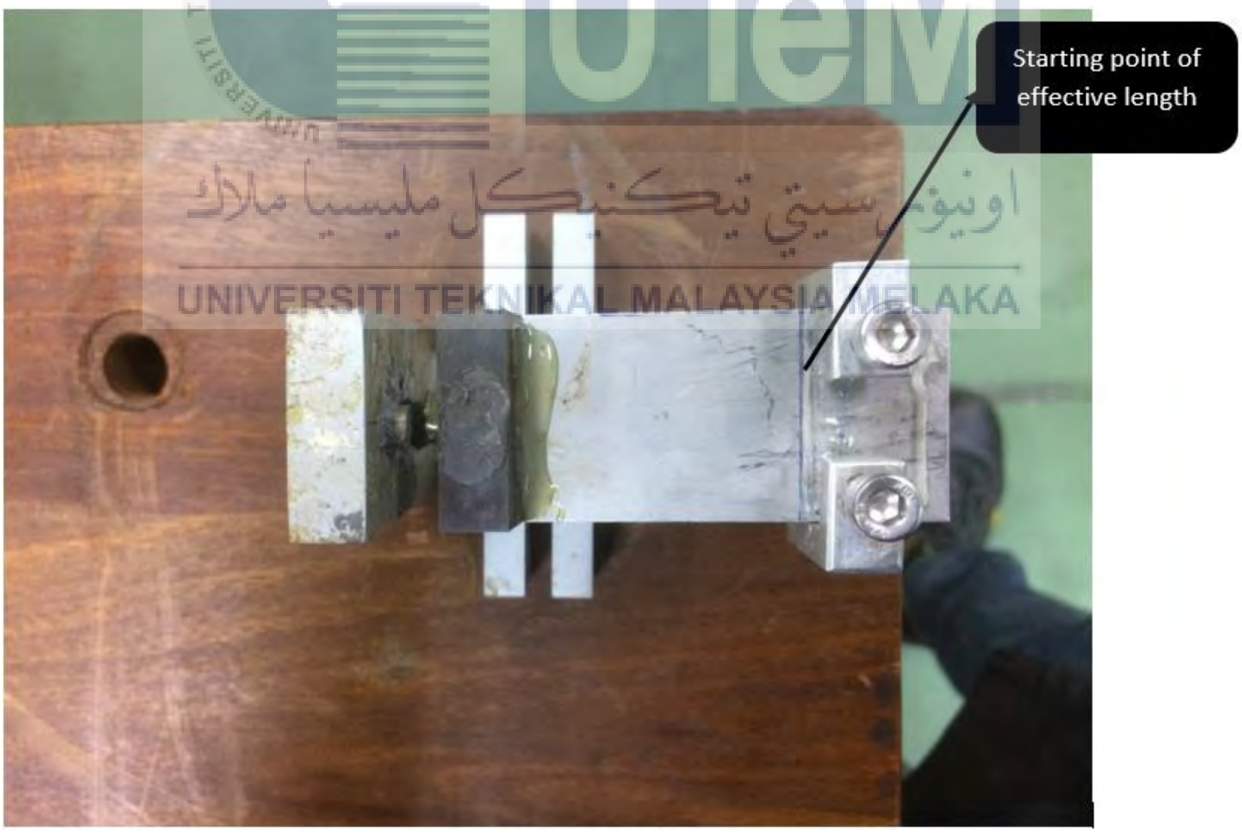


Figure 5.15: Starting point of effective length

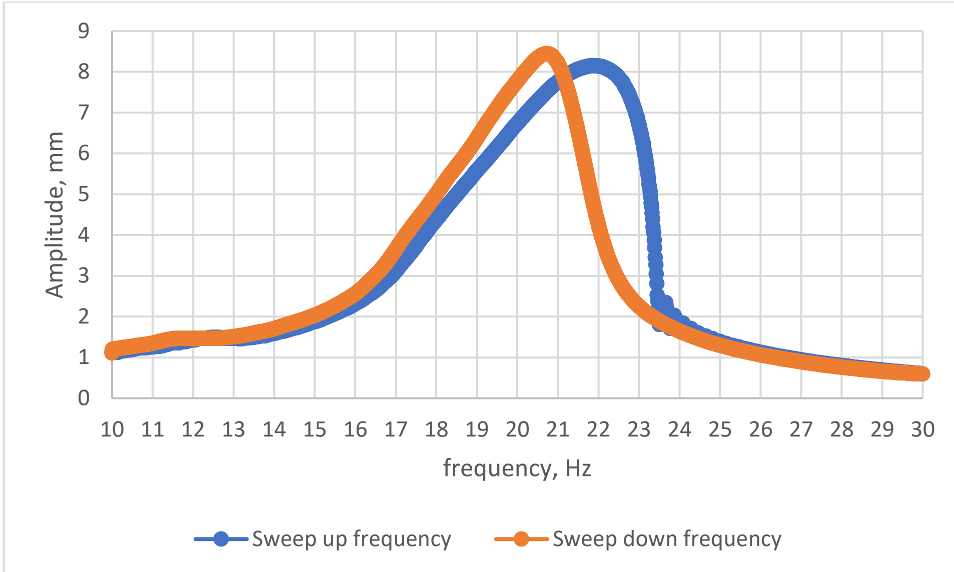


Figure 5.16: Displacement response for length 4.5 cm [Input: 1 mm pk-pk, Gap: 1 mm].

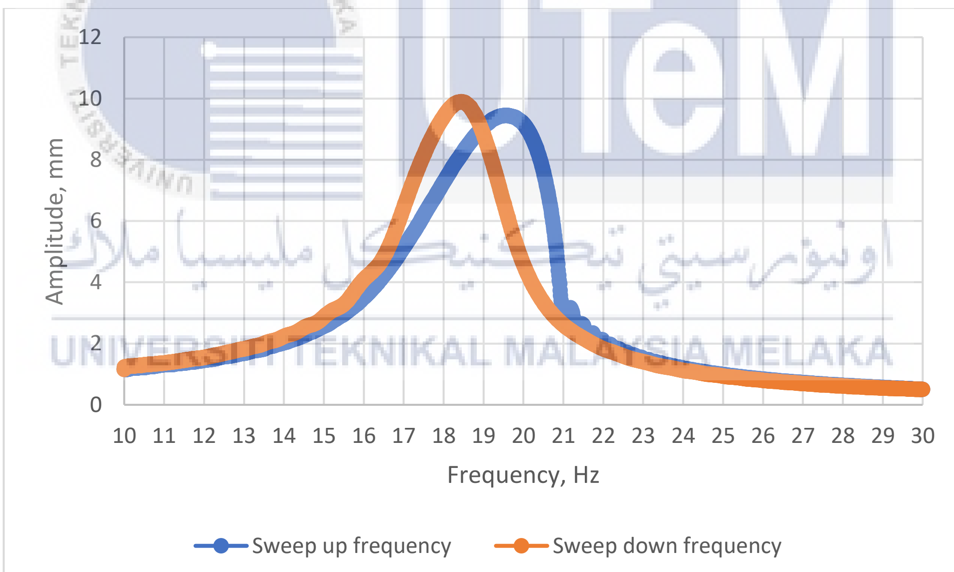


Figure 5.17: Displacement response for length 5 cm [Input: 1 mm pk-pk, Gap: 1 mm].

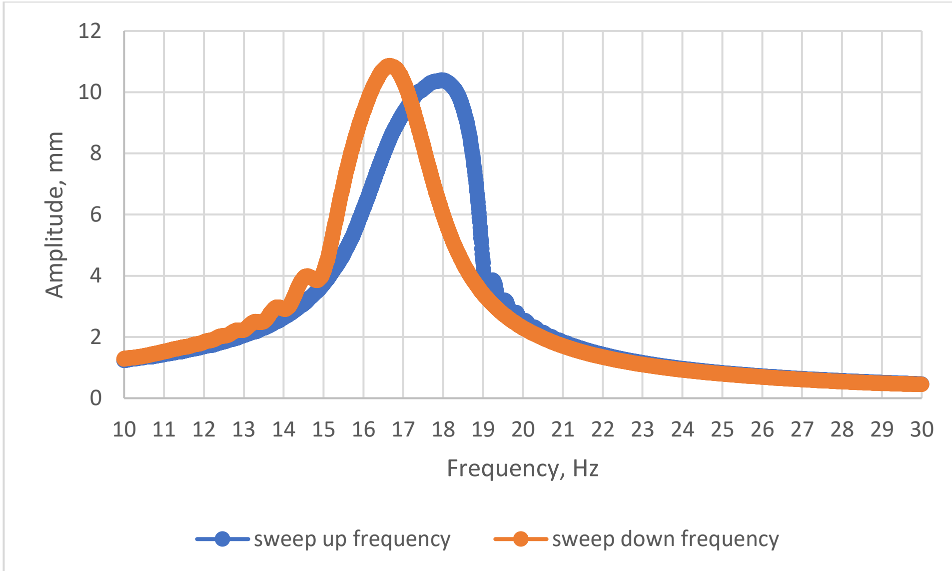


Figure 5.18: Displacement response for length 5.2 cm [Input: 1 mm pk-pk, Gap: 1 mm].

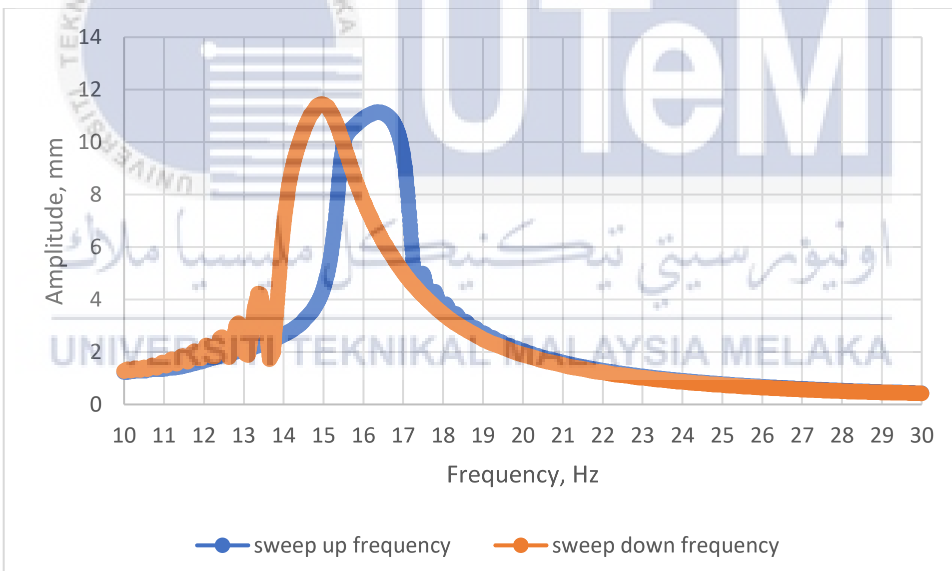


Figure 5.19: Displacement response for length 5.5 cm [Input: 1 mm pk-pk, Gap: 1 mm].

During the characterisation of varying input, 1 mm input does not exhibit strong nonlinear property. For this reason, 1mm input was selected for this characterisation process. If adjusting length have any effect on widening of effective bandwidth, it will be clearer to identify. From the results above, it is easy to say that adjusting the length does not widen the gap between the jump down frequency and jump up frequency. Changing the effective length change the natural frequency of the absorber beam. From Figures 5.16 to 5.19, the peak of FRF curve is shifting to the right and changing the natural frequency of the beam. For fixed mass, second moment of inertia and Young's modulus of elasticity linear stiffness of absorber depends on cube of absorber length as in Equation 14.

$$k = \frac{3EI}{L^3} \tag{15}$$

Where k is linear stiffness, E is Young's modulus, I is second moment of inertia and L is the length of absorber.

When the absorber length is increased linear stiffness of absorber decreases. As a result, natural frequency of the absorber decreases. This statement is based on Equation 15 that is used to calculate natural frequency of an object.

$$\omega_n = \sqrt{\frac{k}{m}} \tag{26}$$

Where ω_n is natural frequency, k is linear stiffness and m is mass of object.

When linear stiffness increase, natural frequency of absorber increase. To increase the linear stiffness, length of absorber beam must be decreased. Adjusting the length can be used to tune the absorber to match the natural frequency of the structure the absorber will be applied to.

5.4 Performance of Nonlinear Dynamic Vibration Absorber on Primary Structure

First part of performance test was to test which position is the most effective to place absorber for the first mode of vibration of the primary structure. Three positions were marked on primary structure as shown in Figure 3.21. For this experiment, gap between magnets was kept constant at 1.5 mm. Input gain of shaker was maintained at 1G and length of absorber beam is kept constant at 5 cm. Result of accelerance of primary structure for each position is as shown in Figures 5.20, 5.21 and 5.22.

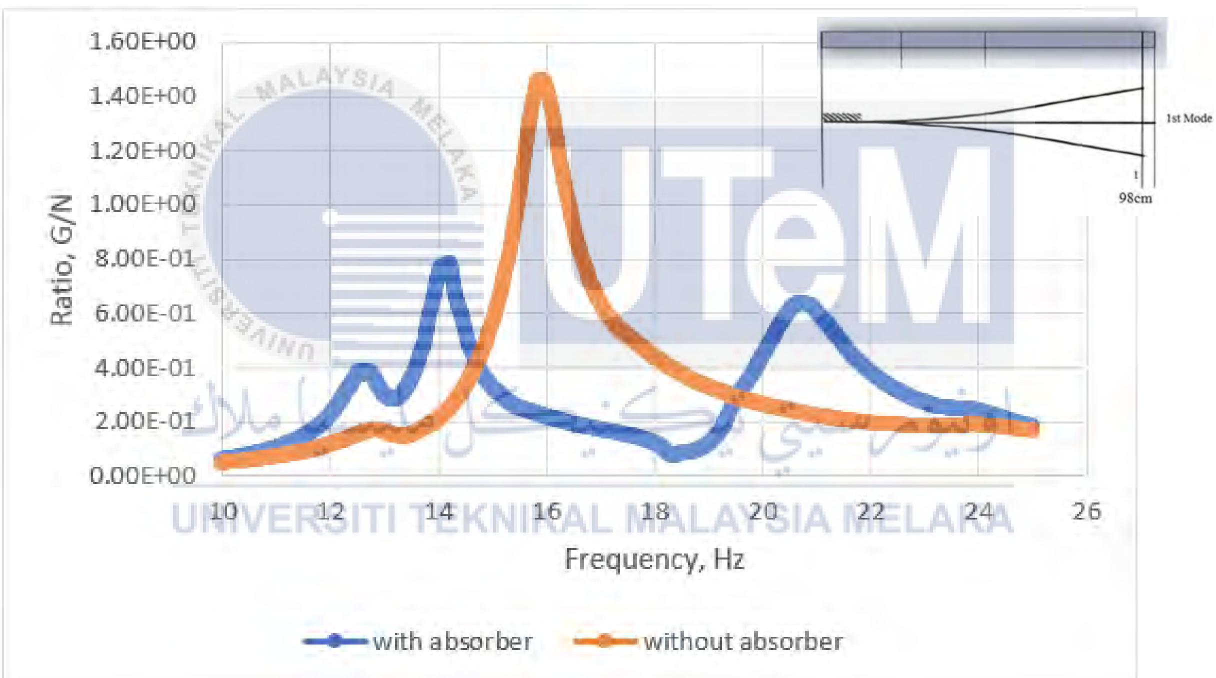


Figure 5.20: Accelerance graph of primary structure with absorber attached to position 1 with gap of 1.5 mm between magnets.

From Figure 5.20, amplitude ratio of Gain / Force (G/N) drops drastically when absorber is attached on the primary structure. Amplitude ratio without absorber for first vibration mode is between 1.4 and 1.6. when absorber is attached, it is almost 0.2. lowest amplitude ratio is between 18 and 20 Hz. There are two new peaks in the blue line indicating it is a MDOF system. There is also a small peak between 12 and 14 Hz.

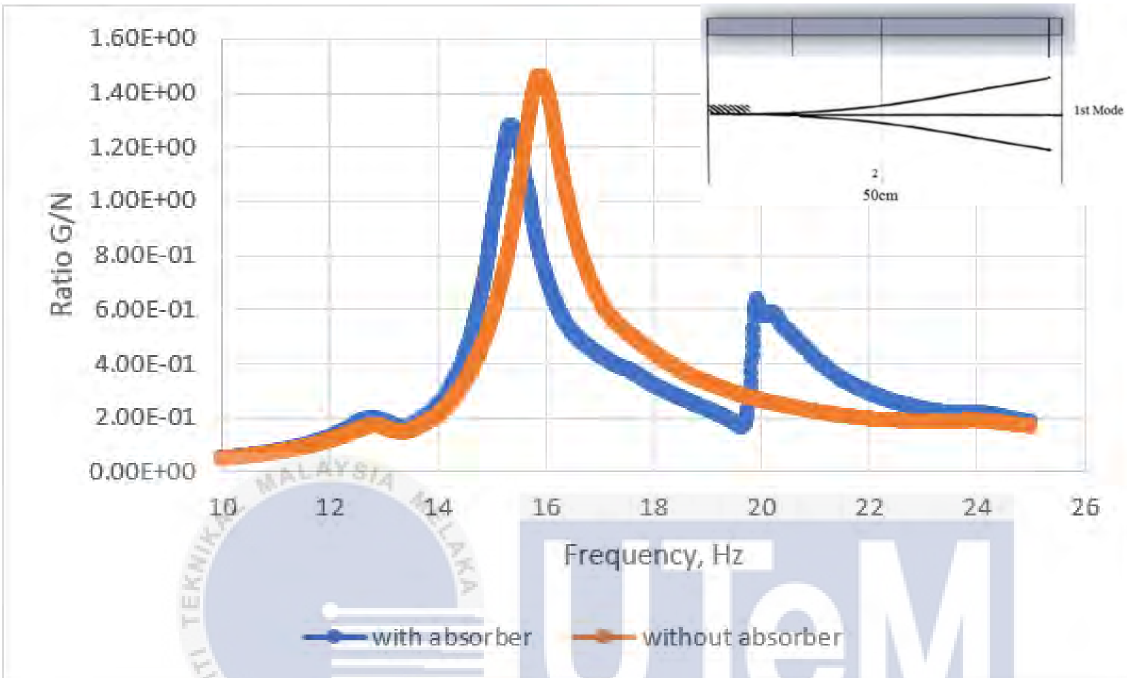


Figure 5.21: Accelerance graph of primary structure with absorber attached to position 2 with gap of 1.5 mm between magnets.

When absorber is attached at position 2, the peaks with and without absorber are at almost same position. First peak for the blue line (with absorber) is between 14 and 16 Hz, meanwhile peak for orange line (without absorber) is almost at 16 Hz. There is also second peak at 20 Hz for the blue line. Amplitude ratio of blue line at first vibration mode is between 0.6 and 0.8, which is lower than without absorber. But, theses figure is higher compared to amplitude ratio for position 1 at the same frequency.

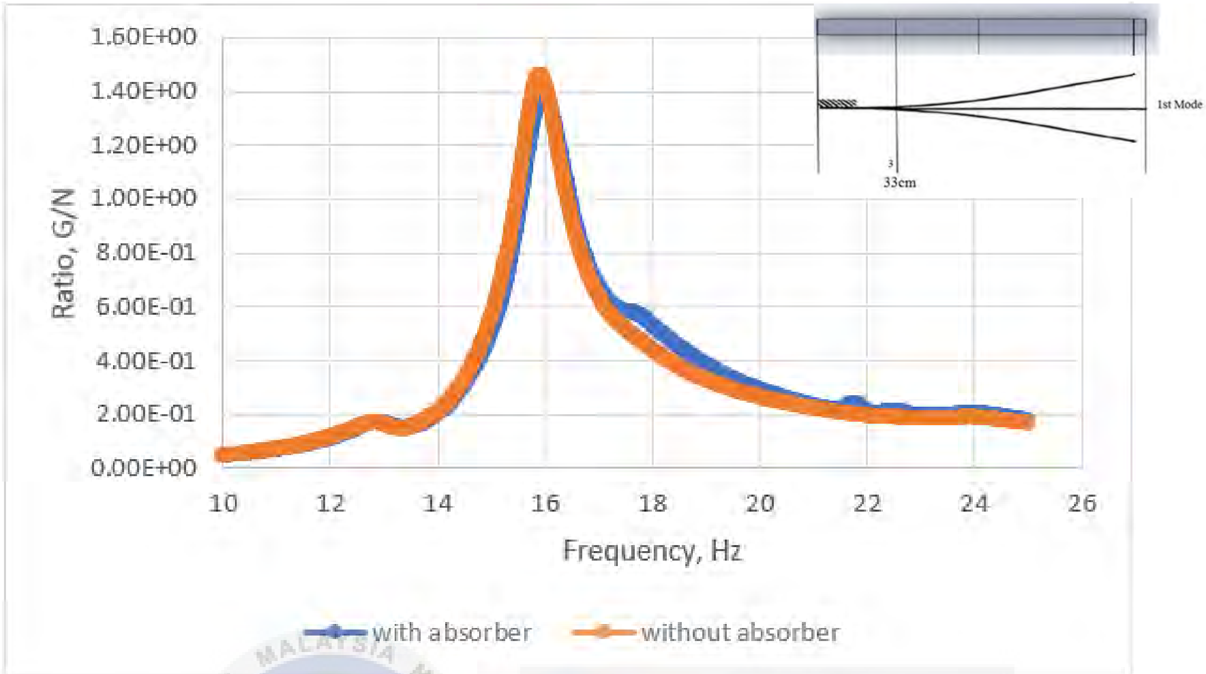


Figure 5.22: Accelerance graph of primary structure with absorber attached to position 3 with gap of 1.5 mm between magnets.

When the absorber is attached to position 3, accelerance graph for both with and without absorber are almost identical. There is only one peak at each graph and both at the first vibration mode of the primary structure. Amplitude ratio with and without absorber at first vibration mode are same which is between 1.4 and 1.6.

From ODS at first vibration mode, highest displacement of the beam is at the free end of beam. When absorber is attached at position 1, amplitude ratio is at its lowest at 16 Hz, which is the first vibration mode of beam. absorber can perform at its best when it is attached at the location of highest displacement, because it can cancel out vibration of primary structure by moving at opposite direction. As seen in Figure 5.18 lowest amplitude ratio is not at the first vibration mode of primary structure. This is because natural frequency of absorber is not 16 Hz. Natural frequency of absorber is at round-off value of 18 Hz. This situation proves that nonlinear absorber has wider bandwidth of operation than linear absorber. As for linear absorber, it can only suppress vibration at its natural frequency. Figure 5.18 shows that although natural frequency of absorber and primary structure are different, vibration amplitude of primary structure can be reduced if the excitation frequency is within the effective bandwidth of operation of nonlinear absorber. When the absorber is at position 2, it did not suppress vibration as well as it has in position 1. At position 2, displacement of beam is not as high as it was at position 1. When absorber was attached at position 3, it is as if there is no absorber attached to primary structure. There is very minimum displacement at position 3. Figures 5.19 and 5.20 indicates that absorber cannot suppress vibration properly if it is not attached to the position where displacement of primary structure is maximum.

Second part of the experiment is by changing the gap between magnets in the absorber. For this experiment, position of absorber is kept constant at position 1. Input from shaker and length of absorber beam was maintained at 1G and 5 cm respectively. Gaps tested for this experiment is 1 mm, 1.5 mm, and 2 mm. Results of these experiment are shown in Figures 5.23, 5.24 and 5.25.

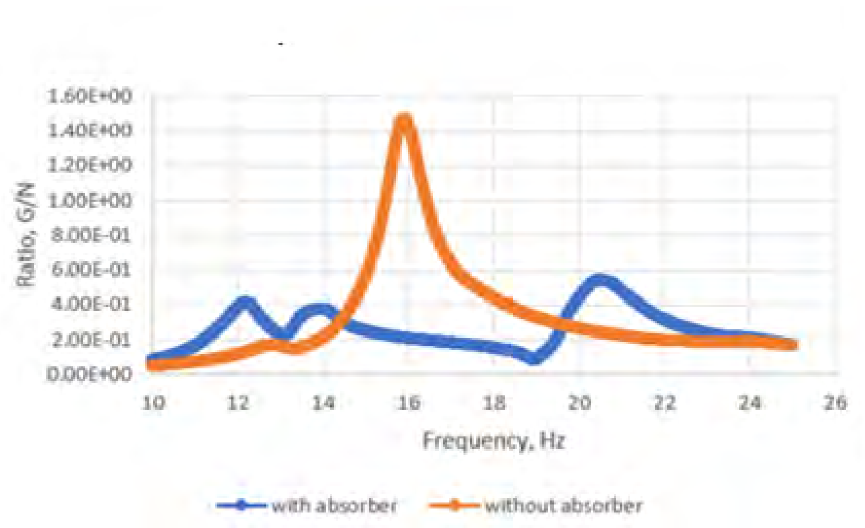


Figure 5.23: Accelerance graph of primary structure with absorber attached to position 1 with gap of 1 mm between magnets.

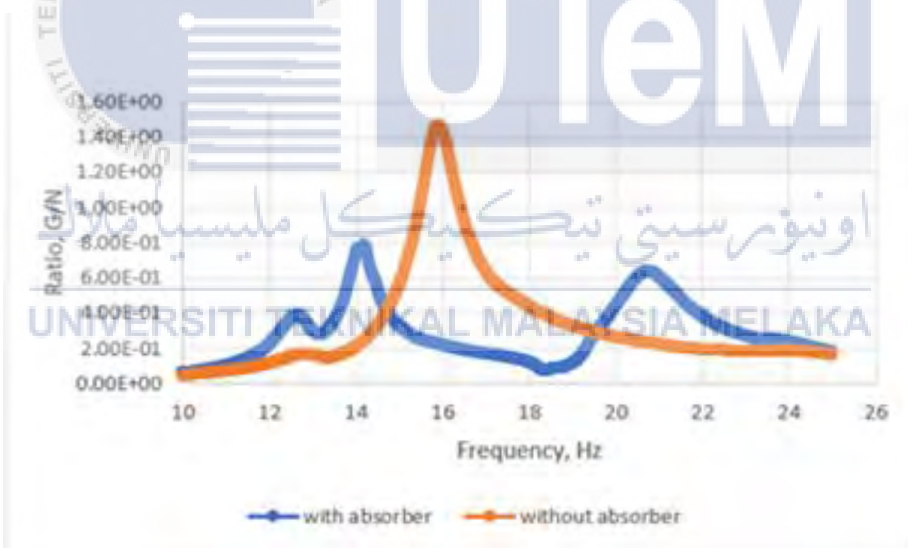


Figure 5.24: Accelerance graph of primary structure with absorber attached to position 1 with gap of 1.5 mm between magnets.

Figures 5.23 and 5.24 shows the accelerance of primary structure for absorber with different gap levels. Amplitude ratio for the gap 1 mm is lower compare to gap of 1.5 mm. Amplitude ratio for last peak of blue line is above 0.6 for gap of 1.5 mm while it was below 0.6 for gap of 1 mm. Lowest amplitude ratio for both gap is between 18 Hz and 20 Hz. The lowest drop point for gap of 1 mm is much sharper compared to drop point for gap of 1.5 mm.

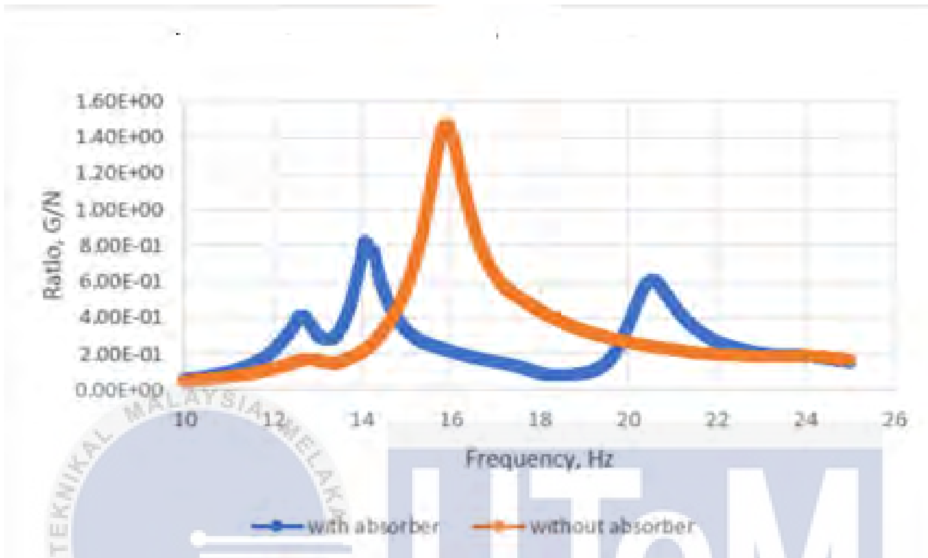


Figure 5.25: Accelerance graph of primary structure with absorber attached to position 1 with gap of 2 mm between magnets.

Figure 5.25 shows the accelerance graph for gap of 2 mm. Amplitude ratio is much higher compare to gap of 1.5 mm. The lowest drop point also not as sharp as in Figures 5.23 and 5.24. The line is much smoother than previous two graphs.

For all three gaps, amplitude ratio at first vibration mode is at around 0.2. This shows gap does not affect the performance of nonlinear absorber, if the natural frequency of absorber and primary structure are different. At the natural frequency of absorber, accelerance level is at its lowest when the gap is smaller. At smaller gap level nonlinearity of absorber is higher. Thus, it can suppress vibration better.

CHAPTER 6

CONCLUSION

Dynamic vibration absorber is an auxiliary, small mass-spring system attached to the primary system to lower vibration of primary system at its resonance frequency. Literature review was conducted to understand various type of dynamic vibration absorber. Passive vibration absorber completely suppress vibration at desired frequency but, it has a very narrow operational bandwidth. There are also several other types of absorbers such as semi-active and active type of absorbers. Nonlinear dynamic vibration absorber is characterised using adjustable magnetic stiffness in this project.

First part of experiment was to obtain vibration modes of beam and ODS for vibration modes of the beam modelled as primary structure. First vibration mode of primary structure was 19 Hz by analytical calculation, 17 Hz by impact test and 16 Hz by sweep sine test. ODS was constructed using VMI VibShape software. Maximum displacement of primary structure was at its free end. This result matches the theoretical prediction.

Characterisation of absorber was divided into three parts. For the first part, gap between magnets was changed to study its effect on nonlinearity of absorber. When the gap is decreased, nonlinearity of absorber increases. Second part is to investigate the effect of input level of shaker on nonlinearity of absorber. When the input level is higher, nonlinearity of absorber is higher.

Last characterisation part is to study the effect of changing effective length of absorber beam on nonlinearity of absorber. When effective length of absorber is increased, natural frequency of absorber will decrease. Changing effective length of absorber does not have any effect on nonlinearity of absorber.

Nonlinear vibration absorber was tested at 3 different places on primary structure. Performance of nonlinear absorber was at its most efficient when it was attached at the free end of the primary structure for its first mode of vibration. Then, effect of different gap level on performance of nonlinear absorber was investigated. When the gap is smaller, amplitude ratio of vibration of primary structure was at its lowest. Nonlinear absorber performs at its maximum limit when the gap between magnet is smaller.

RECOMMENDATION FOR FUTURE WORKS

For future improvement of this project, suppression of primary structure's second and third mode of vibration can be investigated. For first mode of vibration, absorber should be placed at position 1 to suppress vibration effectively. For vibration mode two and three that may not be the case as shown in Figure 6.1. Future researchers can conduct experiment to determine best possible position for second and third mode of vibration.

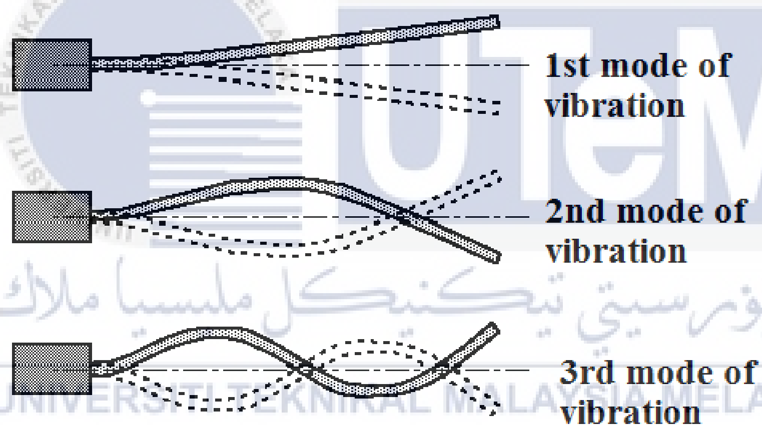


Figure 6.1: Modes of vibration

Other than that, future researchers can explore new characterization variables to improve the effective bandwidth of operation of nonlinear dynamic vibration absorber. In this project, effect of horizontal gap between magnets were investigated. In future, effect of adjusting vertical gap as shown in Figure 6.2, between magnets can be investigated to determine its possible effect on nonlinearity of absorber.

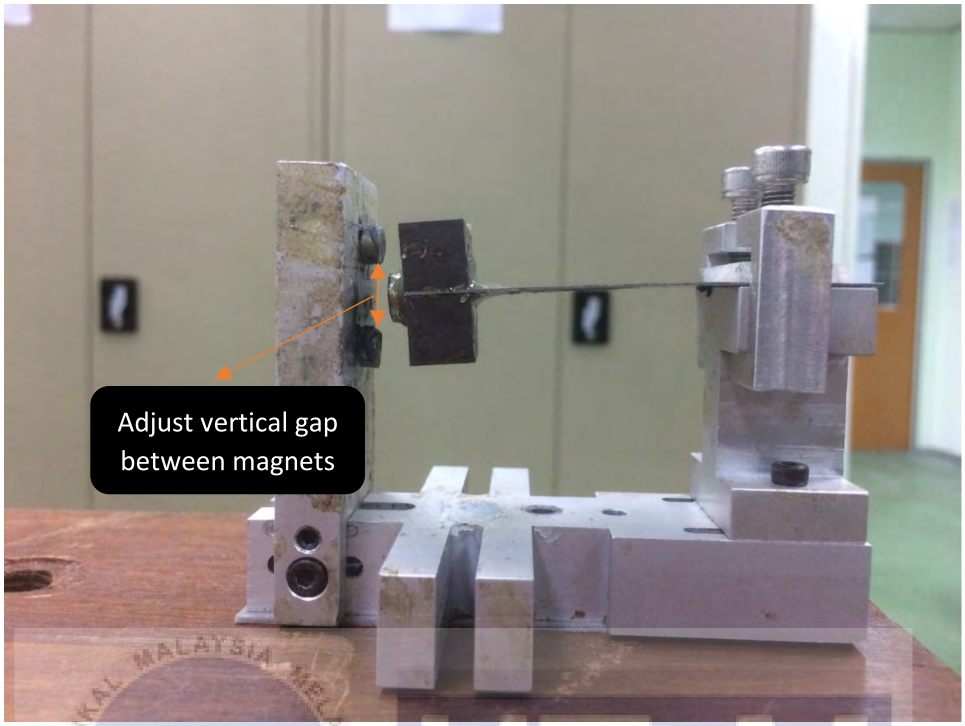


Figure 6.2: Changing vertical gap between magnets

REFERENCES

Bekdas.G & Nigdeli.S.M, 2011. Estimating optimum parameters of tuned mass dampers using harmony search. *Engineering Structures,* pp. 2716-2723.

Chatterjee.S, 2010. Optimal active absorber with internal state feedback for controlling resonant and transient vibration. *Journal of sound and vibration,* Volume 329, pp. 5397-5414.

Chen.C.Y & Lee.C.Y, 2015. Experimental application of a vibration absorber in structural vibration reduction using tunable fluid mass driven by micropump. *Journal of sound and vibration,* Volume 348, pp. 31-40.

Cheung.L.C, Wong.W.O & Cheng.L.S, 2015. A subsystem approach for analysis of dynamic vibration absorbers suppressing broadband vibration. *Journal of sound and vibration,* Volume 342, pp. 75-89.

Deng.H & Gong.X, 2008. Application of magnetorheological elastomer to vibration absorber. *Communications in nonlinear science and numerical simulation,* Volume 13, pp. 1938-1947.

Den Hartog.J.P & Ormondroyd.J, 1928. The theory of the dynamic vibration absorber. *Transactions of the American Society Mechanical Engineers, Applied Mechanics Division APM-50-7,* pp. 9-22.

Dwievedy & Mohanty, 2016. Linear and nonlinear analysis of piezoelectric based vibration absorber with acceleration feedback. *Procedia Engineering,* Volume 144, pp. 584-591.

Febbo.M & Machado.S.P, 2013. Nonlinear dynamic vibration absorbers with saturation. *Journal of sound and vibration,* Volume 332, pp. 1465-1483.

Frahm, H., 1911. *Device for damping vibrations of bodies.* Germany, Patent No. US989958 A.

Habib.G, Detroux.T, Viguie.R & Kerschen.G, 2015. Nonlinear generalization of Den Hartog's equal-peak method. *Mechanical system and signal processing,* Volume 52-53, pp. 17-28.

Inman.D, 2001. *Engineering Vibration, Second Edition.* New Jersey: Prentice Hall.

Ji.J.C, 2014. Design of nonlinear vibration absorber using tree-to-one internal resonances. *Mechanical system and signal processing,* Volume 42, pp. 236-246.

Ji.J.C & Zhang.N, 2011. Suppression of super-resonance response using a linear vibration absorber. *Mechanics research communications,* Volume 38, pp. 411-416.

Kerschen.G & Viguie.R, 2009. Nonlinear vibration absorber coupled to a nonlinear primary system:A tuning methodology. *Journal of sound and vibration,* pp. 780-793.

Kerschen.G & Viguie.R, 2010. On the functional form of a nonlinear vibration absorber. *Journal of sound and vibration,* pp. 5225-5232.

Ko.Y.H, Lee.X.M & Ripin.Z.M, 2011. Tuned vibration absorber for suppression of hand-arm viibration in electric grass trimmer. *International Journal of Industrial Ergonomics,* Volume 41, pp. 494-508.

Koo.J.H, Shukla.A & Ahmadian.M, 2008. Dynamic performance analysis of non-linear tuned vibration absorbers. *Communications in Nonlinear Science and Numerical Simulation,* Volume 13, pp. 1929-1937.

Low.P.S, Ramlan.R, Muhamad.N.S & Ghani.H.A, 2015. Analysis on degree of nonlinearity in hardening nonlinear system of a vibration based energy harvesting device.. *Procedings of Mechanical Engineering Research day 2015,* pp. 109-110.

Ormondroyd, 1928. The theory of the dynamic vibration absorber. *Transactions of ASME.*

Physicslab, 2016. *Springs: Hooke's Law.* [Online]
Available at:
http://dev.physicslab.org/Document.aspx?doctype=3&filename=Dynamics_HookesLawSprings.xml

Poon.D, Tomasetti.T, Shieh.S.S & Joseph.L, 2004. *Structural Design of Taipei 101, the World's Tallest Building,* Taipei: Taipei 101.

Purdue University, 2002. *Introduction to structural motion control,* s.l.: s.n.

Ramlan.R, et al., 2016. Exploiting Knowledge of Jump-up and Jump-down Frequencies to Determine the Parameters of a Duffing Oscillators. *Commun Nonlinear Sci Numer Simulat,* pp. 282-291.

Ramlan.R, Brennan.M.J, Mace.B.R & Kovacic.I, 2009. Potential benefits of a non-linear stiffness in an energy harvesting device. *Nonlinear Dynamics,* pp. 545-558.

Russel, 2016. *The piano hammer as nonlinear spring.* [Online]
Available at: http://www.acs.psu.edu/drussell/Piano/NonlinearHammer.html

Samani.F & Pellicano.F, 2012. Vibration reduction of beams under successive travelling loads by means of linear and nonlinear dynamic absorbers. *Journal of sound and vibration,* Volume 331, pp. 2272-2290.

Shen.Y, Peng.H, Li.X & Yang.S, 2016. Analytically optimal parameters of dynamic vibration absorber with negative stiffness. *Mechanical systems and Signal processing,* Volume 85, pp. 193-203.

Siemens Corporation, 2016. *LMS Test.Lab Modal Analysis: Modification Prediction.* [Online] Available at: https://community.plm.automation.siemens.com/t5/tkb/articleprintpage/tkb-id/Simcenter_Test_tkb/article-id/71

Tang.B, Brennan.M.J, Gatti.G & Ferguson.N.S, 2016. Experimental characterization of a nonlinear vibration absorber using free vibration. *Journal of sound and vibration,* Volume 367, pp. 159-169.

Toshihiko.K, Toshio.I & Osamu.T, 2016. Broadband vibration control of a structure by using a magnetorheological elastomer based tuned vibration absorber. *Mechatronics,* Volume 000, pp. 1-9.

Weber.F, 2014. Semi-active vibration absorber based on real-time controlled MR damper. *Mechanical system and signal processing,* Volume 46, pp. 272-288.

Wong.W.O & Cheung.Y.L, 2008. Optimal design of a damper dynamic vibration absorber for vibration control of structure excited by ground motion. *Engineering Structures,* Volume 30, pp. 282-286.

Wu.S.T, Chen.J.Y, Yeh.Y.C & Chiu.Y.Y, 2007. An active vibration absorber for a flexible plate boundary-controlled by a linear motor. *Journal of sound and vibration,* Volume 300, pp. 250-264.

Zhang.Y, et al., 2016. Energy havesting using nonlinear vibration absorber. *Transactions of the Canadian Society of Mechanical Engineers, Vol. 40,* pp. 221-229.