



## **UNIVERSITI TEKNIKAL MALAYSIA MELAKA**

### **Numerical Solution of Nanofluids Flow and Heat Transfer over a Moving Surface with Thermal Radiation**

This report submitted in accordance with requirement of the Universiti Teknikal Malaysia Melaka (UTeM) for the Bachelor's Degree in Manufacturing Engineering Technology (Process and Technology) with Honours

by

**MUHAMMAD ASYRAF BIN AZLI**

**B071410505**

**950828-04-5129**

FACULTY OF ENGINEERING TECHNOLOGY

2017

## DECLARATION

I hereby, declared this report entitled “Numerical Solution of Nanofluids Flow and Heat Transfer over a Moving Surface with Thermal Radiation” is the results of my own research except as cited in references.

Signature : .....  
Author’s Name : .....  
Date : .....

## **APPROVAL**

This report is submitted to the Faculty of Engineering Technology of UTeM as a partial fulfillment of the requirements for the degree of Bachelor's Degree in Manufacturing Engineering Technology (Process and Technology) with Honours. The member of the supervisory is as follow:

.....

(En Iskandar Bin Waini)

## ABSTRAK

Bendalir nano merupakan sejenis bendalir yang mengandungi sejumlah kecil zarah nano yang terampai secara seragam dan stabil dalam bendalir asasnya. Ia mempunyai konduktiviti terma yang tinggi dan dapat meningkatkan pekali pemindahan haba berbanding bendalir asasnya. Dalam kajian ini masalah lapisan sempadan bagi bendalir nano atas permukaan yang bergerak telah dijalankan melalui penyelesaian berangka dengan kesan sinaran haba. Model matematik yang digunakan untuk aliran bendalir nano ini menggabungkan kesan gerakan Brownian dan juga kesan termoforesis. Persamaan terbitan separa tak linear dijemakan kepada sistem persamaan terbitan biasa tak linear dengan menggunakan penjelmaan keserupaan dan seterusnya diselesaikan secara berangka dengan menggunakan skema beza terhingga yang dikenali sebagai kaedah kotak Keller. Keputusan berangka untuk bilangan pengurangan Nusselt dan bilangan pengurangan Sherwood menunjukkan bahawa apabila termoforesis meningkat, bilangan Sherwood dan bilangan Nusselt akan berkurangan. Sebaliknya, penurunan bilangan Nusselt cenderung meningkatkan bilangan Sherwood kerana kesan gerakan Brownian meningkat. Kesan radiasi haba membuktikan bahawa ketebalan lapisan sempadan terma meningkat disebabkan peningkatan nilai parameter radiasi. Oleh itu, sinaran terma meningkatkan penyebaran haba. Keputusan menunjukkan bahawa gerakan Brownian dan termoforesis mempengaruhi ciri-ciri aliran bagi bendalir nano dengan memanaskan lapisan sempadan atau meningkatkan pelanggaran di antara zarah-zarah nano dan bendalir asasnya.

## ABSTRACT

Nanofluid is an advanced kind of fluid containing small quantities of nanoparticles that are uniformly and stably suspended in a base liquid. It has a higher thermal conductivity and enhanced heat transfer coefficient than its base fluid. In this study, the boundary layer of nanofluids problems has been carried out through the numerical solutions over a moving surface in the appearance of the thermal radiation. This mathematical model describes the flow of nanofluid that incorporates the effects of thermal radiation, Brownian motion and thermophoresis. The governing nonlinear partial differential equations are transformed into a system of nonlinear ordinary differential equations using similarity transformation which is then solved numerically using a finite difference scheme known as the Keller-box method. The numerical results for reduced Nusselt and reduced Sherwood numbers obtained shows that when the thermophoresis increase, the Sherwood number and the Nusselt number will decrease. On the other hand, the decreasing of the Nusselt number tends to increase the Sherwood number as the Brownian motion effects are increased. The effect of thermal radiation proves that the thickness of the thermal boundary layer increases due to the increases of the values of the radiation parameter. Thus, thermal radiation enhances thermal diffusion. The results show that the Brownian motion and thermophoresis influence the flow characteristics of nanofluids by either warming the boundary layer or aggravating the collisions between the nanoparticles and molecules of the base fluid.

## **DEDICATION**

To my beloved mother, father, family, lecturers and friends.

Thank you for everything.

## **ACKNOWLEDGMENT**

Firstly, thanks to Almighty Allah for graciously blessed me with the ability to undertake and finally complete this thesis.

I wish to express my most profound thanks to my supervisor, Mr Iskandar Bin Waini for his countless hours of reflecting, reading, encouraging, and most of all patience throughout the entire process. Also, I would like to recognize with much gratefulness to my co-supervisor, Mr Mohamad Ridzuan Bin Mohamad Kamal for his valuable guidance.

Thanks to my family particularly to my dearest mother and father, Atika Binti Berahim and Azli Bin Kasim for the unrestricted love and encouragement throughout the entire period of my study as well as for praying me to be successful in life. Finally, thanks to all my colleagues and friends who have assisted me directly or indirectly towards the consummation of this thesis. Their excitement and willingness to provide feedback made the completion of this research an enjoyable experience.

# TABLE OF CONTENTS

<b>Declaration</b>	i
<b>Approval</b>	ii
<b>Abstrak</b>	iii
<b>Abstract</b>	iv
<b>Dedication</b>	v
<b>Acknowledgement</b>	vi
<b>Table of Content</b>	vii
<b>List of Tables</b>	x
<b>List of Figures</b>	xi
<b>List Abbreviations, Symbols and Nomenclatures</b>	xii
<b>CHAPTER 1: INTRODUCTION</b>	
1.0 Introduction	1
1.1 Background of Study	1
1.2 Problem Statement	2
1.3 Objective of Project	2
1.4 Scope of Project	3
1.5 Gantt Chart and Flow Chart	4
<b>CHAPTER 2: LITERATURE REVIEW</b>	
2.0 Introduction	6
	vii



2.1	The Concept of Boundary Layer	6
2.2	Flow and Heat Transfer of Nanofluid over Moving Surface	7
2.3	Flow and Heat Transfer with Thermal Radiation Effects	12
<b>CHAPTER 3: METHODOLOGY</b>		
3.0	Introduction	18
3.1	The Governing Equations	18
3.2	Similarity Transformation	20
<b>CHAPTER 4: NUMERICAL FORMULATION</b>		
4.0	Introduction	32
4.1	Finite Difference Method	32
4.2	Newton's Method	36
4.3	The Block Tridiagonal Matrix	38
4.4	Starting Conditions	42
<b>CHAPTER 5: RESULT AND DISCUSSION</b>		
5.0	Introduction	45
5.1	Result Validation with Previous Results	45
5.2	Research Findings and Discussions	47
<b>CHAPTER 6: CONCLUSION AND RECOMMENDATION</b>		
6.0	Introduction	58
6.1	Conclusion of Research	58
6.2	Suggestions for Future Research	59

<b>REFERENCES</b>	61
<b>APPENDIX</b>	66

## LIST OF TABLES

- 5.1 Comparison of present results for reduced Nusselt number  $-\theta'(0)$ , 49  
and reduced Sherwood number  $-\phi'(0)$ , against Olanrewaju et al.  
(2012) and Shateyi and Prakash (2014) for different values of  $Pr$   
when  $Ha = 0, Le = 2, Nb = Nt = 0.1, R = 1, \lambda = 0.1$
- 5.2 Comparison of present results for reduced Nusselt number  $-\theta'(0)$ , 50  
and reduced Sherwood number  $-\phi'(0)$ , against Olanrewaju et al.  
(2012) and Shateyi and Prakash (2014) for different values of  $Le$   
when  $Pr = 0.71, Ha = 0, Nb = 0.1, Nt = 0.1, R = 1, \lambda = 0.1$
- 5.3 Comparison of present results for reduced Nusselt number  $-\theta'(0)$ , 51  
and reduced Sherwood number  $-\phi'(0)$ , against Olanrewaju et al.  
(2012) and Shateyi and Prakash (2014) for different values of  $\lambda$   
when  $Pr = 0.71, Ha = 0, Le = 2, Nb = Nt = 0.1, R = 1$
- 5.4 Comparison of present results for reduced Nusselt number  $-\theta'(0)$ , 52  
and reduced Sherwood number  $-\phi'(0)$ , against Olanrewaju et al.  
(2012) for different values of  $R$  when  $Pr = 0.71, Ha = 0, Le = 2, Nb$   
 $= Nt = 0.1, \lambda = 0.1$
- 5.5 Values of  $-\theta'(0)$  and  $-\phi'(0)$  at different values of  $Nb$  with 53  
parameters  $Pr = 0.71, Ha = 0, Le = 2, Nt = 0.1, R = 1, \lambda = 0.1$
- 5.6 Values of  $-\theta'(0)$  and  $-\phi'(0)$  at different values of  $Nt$  with 53  
parameters  $Pr = 0.71, Ha = 0, Le = 2, Nb = 0.1, R = 1, \lambda = 0.1$

## LIST OF FIGURES

1.5	Flow Chart of the Project	4
2.1	The development of the boundary layer for flow over a flat plate, and the different flow regimes	7
4.1	The grid for difference approximations	33
4.2	Plots of the velocity, temperature, and nanoparticle fraction profiles for the case $Pr = 0.71$ , $Ha = 0$ , $Le = 2$ , $Nb = Nt = 0.1$ , $R = 1$ , $\lambda = 0.1$	43
5.1	The effect of velocity parameter $\lambda$ , on the velocity profiles	54
5.2	The effect of magnetic field parameter $Ha$ , on the temperature profiles	54
5.3	The effect of radiation parameter $R$ , on the temperature profiles	55
5.4	The effect of thermophoresis parameter $Nt$ , on the temperature profiles	55
5.5	The effect of Brownian motion parameter $Nb$ , on the temperature profiles	56
5.6	The effect of magnetic field parameter $Ha$ , on the nanoparticle concentration profiles	56
5.7	The effect of Lewis number parameter $Le$ , on the nanoparticle concentration profiles	57
5.8	The effect of thermophoresis parameter $Nt$ , on the nanoparticle concentration profiles	57

# LIST OF ABBREVIATIONS, SYMBOLS AND NOMENCLATURE

## Abbreviations

UTeM	-	University of Technical Malaysia Melaka
FTK	-	Fakulti Teknologi Kejuruteraan
MHD	-	Magnetohydrodynamic
SRM	-	Spectral Relaxation Method
OHAM	-	Optimal Homotopy Asymptotic Method
SQLM	-	Spectral Quasi-Linearisation Method
HAM	-	Homotopy Analysis Method
DRA	-	Duan–Rach Approach
ChPDM	-	Chebyshev Pseudospectral Differentiation Matrix
HPM	-	Homotopy Perturbation Method

## Symbols

$\alpha$	-	Thermal diffusivity of the fluid
$\rho$	-	Fluid density
$\sigma$	-	Electrical conductivity
$\sigma^*$	-	Stefan-Boltzman constant
$\eta$	-	Similarity variable
$\lambda$	-	Velocity parameter
$\tau$	-	Nanoparticle heat capacity and base fluid capacity ratio
$\mu$	-	Coefficient of viscosity
$\nu$	-	Kinematic viscosity
$\theta$	-	Dimensionless temperature

## Nomenclature

$B_0$	-	Magnetic field of constant strength
$C$	-	Concentration of nanoparticle
$C_w$	-	Surface concentration of nanoparticle
$C_f$	-	Skin-friction coefficient
$D_B$	-	Brownian diffusion coefficient
$D_T$	-	Thermophoresis diffusion coefficient
$Ha$	-	Hartman number
$k$	-	Thermal conductivity coefficient
$K_s$	-	Rosseland mean absorption coefficient
$Le$	-	Lewis number
$Nb$	-	Brownian motion
$Nt$	-	Thermophoresis parameter
$Nu_x$	-	Local Nusselt number
$Pr$	-	Prandtl number
$q_w$	-	Heat flux from the plate
$q_m$	-	Mass flux from the plate
$Re_x$	-	Stagnation flow Reynold's number
$R$	-	Radiation number
$Sh_x$	-	Sherwood number
$(x, y)$	-	Cartesian coordinate
$(u, v)$	-	Velocity components
$U$	-	Free stream velocity
$T$	-	Temperature
$T_w$	-	Moving surface temperature
$T_\infty$	-	Free stream temperature

# CHAPTER 1

## INTRODUCTION

### 1.0 Introduction

This chapter will explain the overview of the study and the purpose of this study. The chapter includes the background of the study, problem statement, objectives that is expected to be achieved and the scope of the study that is going to be conducted.

### 1.1 Background of Study

Many engineering and industrial processes involve heat transfer by means of a flowing fluid in either laminar or turbulent regimes. If the heat transfer in the fluids were decreasing in the term of thermal resistance, it would benefit many of these processes. Nanofluids itself have the potential to reduce its thermal resistance and industrial groups, food and manufacturing, all these would be benefits from this heat transfer. The example of poor heat transfer of fluids is such as oil, water and ethylene glycol mixture. This is due to the thermal conductivity of the such fluids that have a job in the heat transfer coefficient between the medium heat transfer and the surface itself. Besides, Choi (1995) had introduced a technique which used a mixture of a nanoparticles and the base fluid. This is to see the presence of the fluids with the substantially in the higher conductivities. The mixture result of fluids and the nanoparticles was referred as a nanofluids.

The magnetic field effects on the convection flow of nanofluid which is free from any substance past a vertical semi-infinite flat plate, were examined by Hamad *et al.* (2011). Khan *et al.* (2012), studied the unsteady free convection boundary layer flow of nanofluid through the sheet with the thermal radiation and the viscous dissipation effects in the presence of magnetic fields. Yohannes and Shanker (2014)

investigated on the melting heat transfer in the MHD flow nanofluids on exponentially permeable stretching sheet.

This project is mainly focus on solving fluid flow, heat transfer and mass transfer problems by using numerical methods. The governing boundary layer equations are reduced to a system of the highly nonlinear ordinary differential equations by using the suitable similarity transformations. Then the resulting equations are solved numerically using the Keller-box method. The MATLAB software was used in order to solve the related problems.

## **1.2 Problem Statement**

Boundary layer behaviour over a moving continuous surface is an important type of flow occurring in several engineering processes. For example, many extrusion processes involve the cooling of persistent strips or filaments by drawing them through a quiescent fluid. These phenomena are same for the boundary layer along moving flat plate. The boundary layer equations must be used in order to solve boundary layer problem. There are many ways in order to solve boundary layer equation such as Taylor's series that the result sometimes is not really accurate. The problem may occur if the calculation that had been made got many error and not accurate. So, in order to solve the boundary layer equations, others numerical methods will be used.

## **1.3 Objective of Project**

The main objective of this project is to investigate the effect of thermal radiation on flow and heat transfer of nanofluid over a moving surface. The specific purposes for this project are summarized as follows:

- a) To formulate the governing partial differential equations of the nanofluid into the ordinary differential equations using similarity transformation.
- b) To develop the numerical algorithm of the related problem using MATLAB software.



- c) To investigate the effect of thermal radiation on the laminar boundary layer flow of nanofluid over a flat plat.

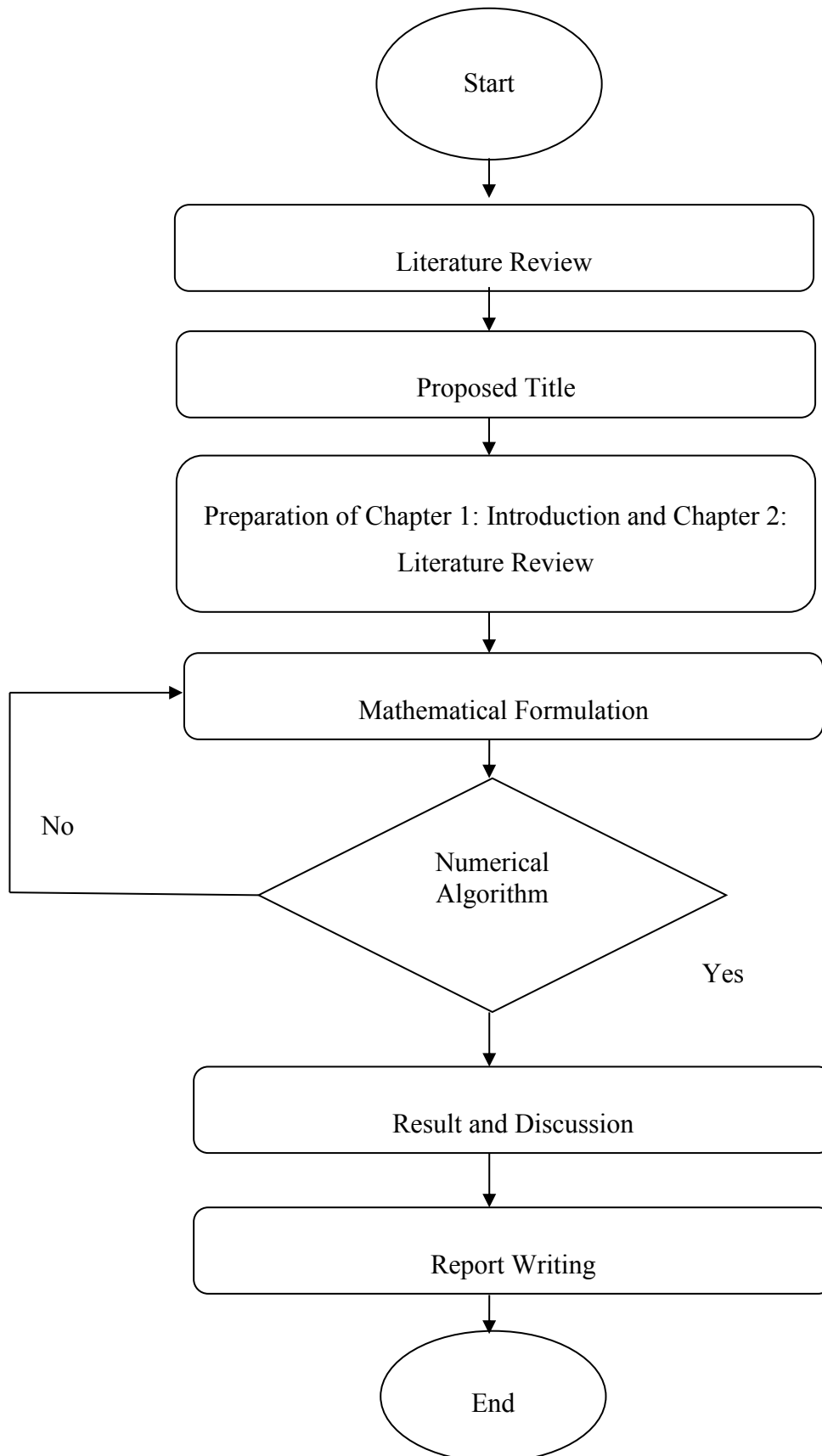
#### **1.4 Scope of Project**

Scopes for this project is based on objectives that have stated and there are the several scopes that will be carrying out:

- a) The laminar flow for a steady, two-dimensional and incompressible viscous flow are considered.
- b) The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis.
- c) This problem is formulated using similarity transformation and solves numerically using Keller-box method.
- d) The numerical algorithm is programmed using MATLAB software

### 1.5 Gantt Chart and Flow Chart

Project Activity	2017																		2017/2018																						
	FEB		MARCH				APRIL			MEI				JUN			SEPTEMBER		OCTOBER			NOVEMBER			DECEMBER			JANUARY													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18			
Submission of project title																																									
Drafting of proposal project																																									
Submission of proposal																																									
Literature Review																																									
Preparation of Chapter 1 Introduction																																									
Preparation of Chapter 2 Literature Review																																									
Preparation of Chapter 3 Methodology																																									
Development of Mathematical Formulation and Numerical Algorithm																																									
Preparation of Chapter 4 Numerical Formulation																																									
Preparation of Chapter 5 Result & Discussion																																									
Preparation of Chapter 6 Conclusion & Recommendation																																									
Slide presentation preparation																																									
Presentation																																									
Ending of presentation report																																									
Submission of project report																																									



**Figure 1.1:** Flow chart of the project

## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **2.0 Introduction**

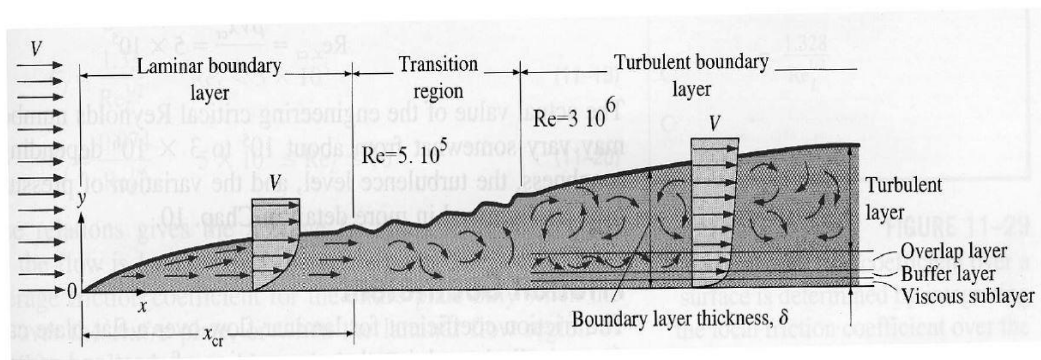
This section discusses the previous researches obtained from the internet, journals, articles and books about the topic related to this study. Section 2.1 explained the concept of boundary layer. The overview of flow and heat transfer of nanofluid over moving surface is discussed in Section 2.2. Lastly, flow and heat transfer with thermal radiation effects is presented in Section 2.3.

#### **2.1 The Concept of Boundary Layer**

Boundary layer is a very thin layer of fluid adjoining to the surface of an object around which the fluid is flowing. External flows past an object encompass an extremely wide variety of fluids mechanics phenomenon. Clearly, the character of the flow field is a function of the shape of the body. The characteristic of the flow depends very strongly on the various parameters for a given shaped object such as size, orientation, speed and fluid properties. The Reynolds number,  $Re = UL/\nu$ , where  $L$  is the characteristic dimension of the body is the most important of these parameters for the typical external flows. The Reynolds number were divided into two region which is a viscous boundary layer adjacent to the surface of the vehicle and the essentially inviscid flow outside the boundary layer.

The boundary layer concept was developed by Prandtl in 1904. It provides an important link between the ideal fluid flow and the real-fluid flow. The fluids have relatively small viscosity, where the effect of the internal friction in a fluid is appreciable only in a narrow region surrounding by the fluid boundaries. Since the fluid was zero velocity at the boundaries, there is a steep velocity gradient from the

boundary into the flow. The velocity gradient in real fluids sets up shear forces near the boundary that will reduce the flow speed to the boundary. The fluid layer which has had its velocity affected by the boundary shear is called the boundary layer. For smooth upstream boundaries the boundary layer starts out as a laminar boundary layer in which the fluid particles move in smooth layers. As the laminar boundary layer increases in thickness, it becomes unstable and finally transforms into a turbulent boundary layer in which the fluid particles move in haphazard paths. When the boundary layer has become turbulent, there is still a very thin layer next to the boundary layer that has laminar motion which is called the laminar sublayer which shown in Figure 2.1.



**Figure 2.1:** The development of the boundary layer for flow over a flat plate, and the different flow regimes

## 2.2 Flow and Heat Transfer of Nanofluid over Moving Surface

Buongiorno (2006) discussed the convective transport in nanofluid. The objective of this work is to develop an explanation for the abnormal convective heat transfer enhancement observed in nanofluid. He considered seven slip mechanisms that can produce a relative velocity between the nanoparticles and the base fluids. In his observations, only Brownian diffusion and thermophoresis are important slip mechanisms in nanofluid. From the findings, he developed a two-component nonhomogeneous equilibrium model for transport phenomena in nanofluids. Also, he proposed an alternative explanation for the abnormal heat transfer coefficient increases

which is the nanofluid properties may vary significantly within the boundary layer because of the effect of the temperature gradient and thermophoresis. He also developed a new correlation structure that can reproduce published nanofluid heat transfer data reasonably well.

Shateyi and Prakash (2014) studied a new numerical approach for magnetohydrodynamic (MHD) boundary layer flow and heat transfer of nanofluid over a moving surface in the presence of thermal radiation. The aim of this study is to develop numerical technique which is spectral relaxation method (SRM) to solve the problem. Results shows that an excellent agreement was observed between SRM method and those obtained using other method. On the other hand, it can be found that the local temperature rises as the Brownian motion, thermophoresis and radiation effects intensify.

Unsteady MHD boundary-layer flow and heat transfer of nanofluid over a permeable shrinking sheet in the presence of thermal radiation is investigated by Nandy *et al.* (2014). The main idea is to study the simultaneous effect of the magnetic field and thermal radiation on the flow and heat transfer due to the unsteady, two-dimensional laminar flow of a viscous nanofluid caused by a permeable shrinking sheet. The methodology used in this studied are fourth order Runge–Kutta method with shooting technique. The results obtained from the study is the dimensionless velocity increases with increasing magnetic and suction parameters for the first solution and decreases for the second solution. In addition, the dimensionless temperature and the rescaled nanoparticle volume fraction both decrease with increasing magnetic parameter for the first solution and increase for the second solution.

Nandy and Pop (2014) analyzed the effects of magnetic field and thermal radiation on stagnation flow and heat transfer of nanofluid over a shrinking surface. The objective is to extend the work of previous researcher by taking MHD flow over a shrinking sheet. The problem is solved numerically using a shooting technique. The authors observed that the velocity, temperature, the wall shear stress, the Nusselt number and the Sherwood number are strongly influenced by the magnetic parameter. Besides that, the authors also observed for the stretching sheet, similarity

solution exists for all values of the velocity ratio parameter  $\alpha$  ( $>0$ ) meanwhile for the shrinking sheet similarity solution may or may not exist.

Nanofluid flow over an unsteady stretching surface in presence of thermal radiation is studied by Das *et al.* (2014). The main purpose is to study the effect of thermal radiation on boundary layer flow of a nanofluid over a heated stretching sheet with an unsteady free stream condition. The methodology used are shooting method and Runge–Kutta–Fehlberg scheme. As a result, it is noticed that the fluid temperature and the thermal boundary layer thickness increase for increasing thermal radiation, Brownian motion and thermophoresis meanwhile the effect is opposite for unsteadiness parameter.

Flow and heat transfer over a moving surface with non-linear velocity and variable thickness in a nanofluids in the presence of Brownian motion is introduced by Abdel-wahed *et al.* (2015). This study investigated the effect of hydro-magnetic flow and heat transfer characteristic of a nanofluid over a steady moving surface with variable thickness on the mechanical properties of the surface in the presence of Brownian motion and heat source during the heat-treating process. The equations are solved using optimal homotopy asymptotic method (OHAM) for general conditions. It is noticed that Brownian motion and thermophoresis have a negative effect on the surface hardness and strength. On the other hand, it can be found that nanoparticle concentration near the non-flat surface is bigger and thinner than that on the flat surface.

Haile and Shankar (2015) researched a boundary-layer flow of nanofluids over a moving surface in the presence of thermal radiation, viscous dissipation and chemical reaction. The goal of this research is to present a new models and new improvements to include uniform magnetic field, viscous dissipation and chemical reaction in the momentum, energy and concentration equations respectively for more physical implications. The solutions of these problems are numerically solved using shooting technique followed by the classical fourth order Runge-Kutta method. The results that can be observed is the presence of Lewis number, Reynolds number, thermal radiation, thermophoresis, velocity, Brownian motion and viscous dissipation parameters in the flow field reduces the rate of thermal boundary layer

thickness whereas Prandtl number maximizes the rate of thermal boundary layer thickness. Other than that, viscous dissipation, thermal radiation, Brownian motion and thermophoresis parameters enhance the temperature profile whereas Prandtl number and the velocity parameter reduce it significantly.

Effect of chemical reaction on MHD boundary layer flow and melting heat transfer of Williamson nanofluid in porous medium is explored by Krishnamurthy *et al.* (2015). The objective of the study is to discuss the nanoparticles analysis for the Williamson fluid model. The resultant nondimensionalized boundary value problem are solved numerically by Runge–Kutta–Fehlberg fourth–fifth order method followed by shooting technique. It is concluded that velocity and the boundary layer thickness increase and temperature distribution decreases due to increase in melting parameter. Besides that, chemical reaction parameter decreases the concentration profile, whereas the velocity and temperature of the fluid are not significant with increase of chemical reaction parameter.

Abdel-wahed and Emam (2016) worked the MHD boundary layer behavior over a moving surface in a nanofluid under the influence of convective boundary conditions. The major objective in this study is to study the MHD boundary layer with convective boundary conditions over a flatness-moving surface taking the nanoparticles Brownian motion and thermophoresis force into account. The method used to solve the equation is fourth order classical Runge–Kutta method. The results gained from the work is the values of the Nusselt and Sherwood numbers for non-linear thermal radiation model are high comparing with linear model.

Unsteady MHD flow and radiation heat transfer of nanofluid in a finite thin film with heat generation and thermophoresis is explored by Li *et al.* (2016). The primary objective is to study the flow and heat transfer of MHD pseudo-plastic nanofluid in a finite film over unsteady stretching surface considering the internal heating effects. The problem is solved numerically by using `bvp4c` in Matlab. The results show that the thermophoresis parameter tends to slow down the temperature and to increase the nanoparticle volume fraction of the fluids. If the strength of the thermophoresis parameter increases, the thickness and temperature of the thin film decrease while the nanoparticle volume fraction increases.