“ I hereby declare that I have read through this report entitle “Transient Analysis of Electrical Circuits using Runge-Kutta Method” and found that it has comply the partial fulfillment for awarding the degree of Bachelor of Mechatronics Engineering”

Signature	:	....
Supervisor’s Name	:	....
Date	:	....

TRANSIENT ANALYSIS OF ELECTRICAL CIRCUITS USING RUNGE-KUTTA METHOD

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A report submitted in partial fulfillment of the requirements for the degree of Bachelor of Mechatronic Engineering(Hons.)

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DECLARATION

I declare that this report entitle "Transient Analysis of Electrical Circuits using Runge-Kutta Method" is the result of my own research except as cited in the references. This report has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

Signature : ....

Name : ....

Date : ....

To my beloved father and mother

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ABSTRACT

Transient analysis is an analysis of transient response of an electrical circuit. The purpose of transient analysis is to analyse the performance of an electrical circuit. Transient is a sudden application of source to a circuit or a brief increase in current or voltage in a circuit which cause damage to some sensitive components and instruments. Electrical transient phenomena can be generated due to some natural events and switching operations. In order to solve the transient analysis of electric circuit, numerical techniques for utilization in the companion analytical method of transient circuit analysis are used to solve second-order differential equations which generated from circuit equations of RLC circuit. The objective of this project is to solve the transient analysis of electric circuit using analytical method and Runge-Kutta method. Fourth-order Runge-Kutta method is chosen as the numerical method to solve transient analysis of electric circuit due to its high accuracy of approximations. In this project, transient analysis of RLC circuit is simulated using MATLAB and visualised in a graphical form. Next, the results of transient analysis obtained from analytical method and Runge-Kutta method are tabulated and analysed by using Microsoft Excel. Besides that, the aim of this project is to compare the transient analysis obtained form Runge-Kutta method with the analytical solution. There are two numerical simulations conducted in this project. First numerical simulation is to solve transient analysis of electric circuit on three different types of response which are underdamped response, critically damped response and overdamped response. Another numerical simulation is to solve transient analysis of a RLC circuit using different step sizes. The results obtained are analysed and compared in term of error in order to determine the accuracy of Fourth-order Runge-Kutta method. In the first simulation, Fourth-order Runge-Kutta method has a high degree of accuracy which is up to 99.99%. Besides, the accuracy of Fourth-order Runge-Kutta method can be improved as the time step size decimates. Fourth-order Runge-Kutta method provides an alternative solution for transient analysis of electrical circuits.

ABSTRAK

Transient analysis merupakan analisis *transient response* untuk litar elektrik. Tujuan *transient analysis* adalah untuk menganalisis prestasi sesebuah litar elektrik. Transient merupakan aplikasi sumber kepada litar elektrik secara tiba-tiba atau peningkatan yang singkat pada arus atau voltan dalam litar yang menyebabkan kerosakan kepada beberapa komponen dan instrument yang sensitive. Fenomena elektrik transient dapat dihasilkan daripada kejadian alam dan operasi bertukaran. Dalam usaha untuk menyelesaikan *transient analysis* bagi litar elektrik, teknik berangka digunakan bersama dengan kaedah analisis dalam analisis litar elektrik untuk menyelesaikan persamaan pembezaan yang dihasilkan daripada persamaan litar. Tujuan projek ini adalah untuk menyelesaikan *transient analysis* sesebuah litar elektrik dengan menggunakan kaedah analisis dan kaedah Runge-Kutta. Kaedah *Fourth-order Runge-Kutta* dipilih sebagai kaedah berangka yang digunakan untuk menyelesaikan *transient analysis* sesebuah litar elektrik kerana kaedah ini mempunyai tahap ketepatan yang tinggi. Dalam projek ini, MATLAB digunakan untuk mensimulasikan dan menggambarkan *transient analysis* bagi *RLC circuit* dalam bentuk grafik. Seterusnya, Microsoft Excel digunakan untuk merekodkan dan menganalisis keputusan *transient analysis* yang diperolehi daripada kaedah analisis dan kaedah Runge-Kutta. Di samping itu, projek ini bertujuan untuk membandingkan *transient analysis* diperolehi daripada kaedah Runge-Kutta dengan penyelesaian analitikal. Terdapat dua simulasi berangka yang dijalankan dalam projek ini. Simulasi berangka pertama adalah untuk menyelesaikan *transient analysis* litar elektrik ke atas tiga jenis tindak balas yang merupakan *underdamped response, critically damped response* dan *overdamped response.* Simulasi berangka kedua adalah untuk menyelesaikan *transient analysis of RLC circuit* menggunakan saiz langkah yang berbeza. Keputusan yang diperolehi daripada kaedah-kaedah tersebut akan dianalisis dan dibandingkan dari segi ralat untuk menentukan ketepatan kaedah *Fourth-order Runge-Kutta.* Dalam simulasi pertama, kaedah *Fourth-order Runge-Kutta* mempunyai tahap ketepatan yang tinggi iaitu sehingga 99.99%. Selain itu, tahap ketepatan kaedah *Fourth-order Runge-Kutta* dapat diperbaiki dengan menggunakan saiz lengkah masa yang lebih kecil. Kaedah *Fourth-order Runge-Kutta* menyediakan penyelesaian alternative untuk *transient analysis* litar elektrik.

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CHAPTER 1

INTRODUCTION

1.1 General Overview

Electric circuit is known as the path for electric current transmission. Electric circuit is also defined as the interconnection of electric elements or electrical devices [1]. Electrical elements are the electrical components of the circuit. The electrical elements in electric circuit are basically divided into two types which are active elements and passive elements. Generators, batteries and operational amplifiers are the active elements meanwhile resistors, capacitors and inductors are the passive elements. The main difference between the active element and passive element is active element is capable to generate energy while passive element is not.

First-order circuit is characterised by a first-order differential equation. The electric circuit consists of only one single storage element which is a capacitor or an inductor. There are two types of simple circuits which are a circuit comprising a resistor and a capacitor and a circuit comprising a resistor and an inductor. These two types of circuits are well-known as RC circuit and RL circuit. Second-order circuit is characterised by a second-order differential equation. It consists of resistors and the equivalent of two storage elements which are capacitor and inductor. RLC circuit is a typical example of second-order circuits which the three types of passive elements are present in the circuit. All the first-order and second-order circuit are connected in two ways which are in series or in parallel.

In this research, transient analysis of RLC circuit is conducted using Runge-Kutta method. There are multiple applications for RLC circuit and some of the most important applications are oscillator and turners of radio or audio receiver. Besides, RLC circuit is also used to create low-pass filter, high-pass filter, band-stop filter and band-pass filter. Transient analysis is important for an electrical circuit because it is commonly used to analyse the performance of the circuit.

1.2 Motivation

Transient analysis is an analysis of transient response of an electrical circuit. Transient analysis is one of the significant analyses in fundamental electric circuit. The purpose of transient analysis is to analyse the performance of an electrical circuit. Transient analysis of electrical circuit is commonly solved by using analytical method. Analytical solution is defined as the exact solution and usually derived using analysis. Analytical solutions in transient analysis are derived by analysing the circuit using circuit equations and a transient response complete solution is formed.

In this research, numerical methods are proposed to solve the transient analysis of electrical circuit in an alternative way. Numerical solution is an approximate solution obtained by using methods of numerical analysis. In the process of analysing RLC circuit which is a second-order circuit, second-order differential equations is utilized to obtain the transient analysis of the circuit. Runge-Kutta method is chosen as the numerical solution for transient analysis in order to solve the second-order differential equations. Runge-Kutta method is the analogue of Simpson’s rule for differential equation. This method is commonly used for numerical analysis due to its high accuracy of approximations. Runge-Kutta method is easy to implement, stable and accurate. However, Runge-Kutta method requires enormous computation time and the error estimation are hard to be done.

Equation (1.1) shows a simple differential equation with the initial condition $x(0) = 1$. The computed point is obtained by simulating equation (1.1) using Fourth-order Runge-Kutta formula with different step size. Table 1.1 shows the error obtained from the equation (1.1) by applying the Fourth-order Runge-Kutta method. Figure 1.1 indicates the graph plotted using fourth-order Runge-Kutta method in application of equation (1.1) with step size of 0.2.

$$\frac{dx}{dt} = x \tag{1.1}$$

Table 1.1: Error obtained with the Fourth-order Runge-Kutta method [2].

Time Step	**Exact Point**	**Computed Point**	**Error (%)**
$\Delta t = 0.5$	7.389056	7.383970	6.8828×10^{-4}
$\Delta t = 0.2$	7.389056	7.388889	2.2581×10^{-5}
$\Delta t = 0.1$	7.389056	7.389045	1.5335×10^{-6}
$\Delta t = 0.05$	7.389056	7.389055	9.9918×10^{-8}
$\Delta t = 0.02$	7.389056	7.389056	2.6226×10^{-9}
$\Delta t = 0.01$	7.389056	7.389056	1.6528×10^{-10}
$\Delta t = 0.001$	7.389056	7.389056	1.5385×10^{-14}
$\Delta t = 0.0001$	7.389056	7.389056	5.1686×10^{-15}

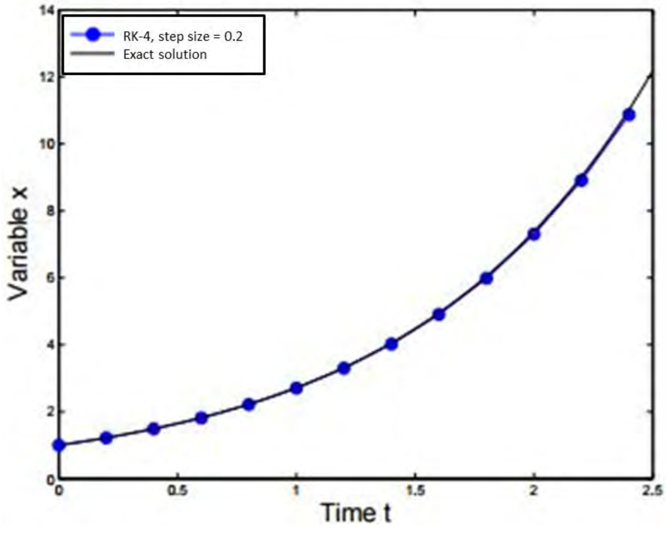

Figure 1.1: Fourth-order Runge-Kutta method in application with step size of 0.2 [2].

According to Table 1.1, as the step size (time) get smaller, the computed point get closer to the exact point and thus reducing the error. Small value of error obtained from the

Fourth-order Runge-Kutta method proves that this numerical method has a high degree of accuracy. In conclusion, Fourth-order Runge-Kutta method is suitable for solving differential equation because of its high degree of accuracy even with complex and high order differential equation.

1.3 Problem Statement

The complete response of an electric circuit can be decomposed into two components which are transient response and steady-state response [1]. Transient response is the temporary response of a circuit that will recede with time. Transient is a sudden application of source to a circuit or a brief increase in current or voltage in a circuit which cause damage to some sensitive components and instruments. Electrical transient phenomena can be generated due to some natural events and switching operations. One of the examples of the natural events is lightning strikes or surges and the switching operations mentioned are capacitor, load and transformer energizing [3]. The transient response can be categorized into three types of damping which are underdamped, critically damped and overdamped.

In previous research, transient analysis of electrical circuits is done in few methods which are the analytical method, Cauchy-Heaviside (C-H) operational method, Fourier Transformation method and Laplace Transformation method. C-H operational method is not as systematic as Laplace Transformation method. Hence, it has been abandoned in favour of the Laplace Transformation method. Laplace Transformation method is commonly used to solve differential equation especially in cases with discontinuous forcing terms or a periodic, non-sinusoidal forcing term [4]. Analytical method which worked with transient analysis of electric circuit requires higher knowledge of mathematics and less physical knowledge or matter related with transient behaviour of electric circuit compared to the others [5].

In order to solve the transient analysis of electric circuit, numerical techniques for utilization in the companion analytical method of transient circuit analysis are used to solve the differential equation. Numerical method is usually used to solve the differential equation of a second-order circuit. Numerical method is able to overcome the limitation of

Laplace Transformation method which numerical method is able to solve more complicated equation such as higher order differential equation. Besides, graph can be plotted easily based on the function given by using this method.

1.4 Objectives

The main objectives of conducting the project are:-

1. To solve the transient analysis of electric circuit using analytical solution and Runge-Kutta method.
2. To compare the transient analysis obtained from Runge-Kutta method with the analytical solution.
3. To validate the transient analysis of the electrical circuits obtained from Runge-Kutta method.

1.5 Scope of Project

The scopes of the research are listed as follow:-

1. Transient analysis conducted is emphasized only on second-order series RLC circuit.
2. Three types of transient analysis are conducted which are underdamped response, critically damped response and overdamped response.
3. Transient analysis is solved using analytical solution which derived from circuit analysis and equations.
4. The results of transient analysis are tabulated and analysed by using MATLAB software.
5. The results of transient analysis are performed in the graph of voltage against time.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Generally, the project named Transient Analysis of Electrical Circuit using Runge-Kutta method utilizes many resources from previous works of researchers. In this chapter, there are three major parts which are the transient analysis, analytical solution and types of numerical methods.

2.2 Background

2.2.1 Transient Analysis

Transients are the currents and voltages vary with time resulting from an unusual application of source. Transient is usually occurred due to switching [6]. There are many different analysis might be applied to the analogue circuit simulation and one of the examples is transient analysis. Transient analysis is the analysis most commonly used to analyse circuit performance. Besides, transient response analysis of doubly-fed induction generator (DFIG) is conducted during three-phase symmetrical grid voltage swell. Transient operation and steady-state operation are the two processes included in the transient process of DFIG. A series of transient processes is caused by the stator voltage swell and the transient processes are referring to overvoltage of DC-link, rotor impulse current and torque oscillation [7].

Transient response for non-uniform transmission line is usually analysed by using the theory of uniform transmission line. Transient analysis of transmission line becomes more significant as the connection line of integrated circuit. The main reason is transient analysis is applied to ensure the normal operation of high speed electronic information system [8].

A new Lightning Activity Monitoring System (LAMS) is installed on the mountain Lovćen. The new monitoring system is subjected to standard lightning impulse test to determine its reliability and energy capability when there is a lightning surge. Transient analysis due to the lightning event is applied on the wind turbine blade [9]. The transient caused by the lightning event can be very destructive for the system based on the result of simulations obtained.

In conclusion, many applications demand the result of transient response for systems. There are many applications of transient analysis such as double-fed induction generator (DFIG), non-uniform transmission line and Lightning Activity Monitoring System (LAMS). The importance of transient analysis is to determine the performance and stability of a system.

2.2.2 General Concepts of Electric Circuits

In transient analysis of electrical circuits, some fundamental laws that govern electric circuits are used to generate circuit equations for second-order circuits. There are some basic circuit equations frequently used for solving the electric circuit such as Ohm's law, Kirchhoff's Voltage Law (KVL) and Kirchhoff's Current Law (KCL) [1].

Ohm's law is defined as the voltage, v across a resistor is directly proportional to the current, i passes through the resistor. The mathematical form of Ohm's law is given by

$$v = iR \tag{2.1}$$

where v is voltage across resistor, i is current passes through resistor and R is resistance of resistor.

Since Ohm's law is not sufficient to analyse an electric circuit by itself, it is combined with Kirchhoff's law to analyse a large variety of electric circuits. Kirchhoff's first law is based on the law of conservation of charge while Kirchhoff's second law is based on the principle of conservation.

Kirchhoff's first law which is known as Kirchhoff's Current Law (KCL) states that the algebraic sum of currents entering and leaving a node is zero. In other words, the sum of current entering a node is equal to the sum of currents leaving the node. Kirchhoff's Current Law can be used to obtain the currents for each branch when the electric circuit elements are connected in parallel. Mathematically, Kirchhoff's first law is defined as

$$\sum_{n=1}^{N} i_n = 0 \qquad (2.2)$$

where N is the number of branches connected to the node and i_n is the nth current in or out of the node.

Kirchhoff's second law which is known as Kirchhoff's Voltage Law (KVL) is shown as equation (2.3) where M is the number of voltages in the loop or closed path and v_m is the mth voltage. This law states that the algebraic sum of voltages across various circuit elements in a closed path or loop is zero. In other words, the sum of voltage drops is equal to the sum of voltage rises. This law can be applied to obtain the total voltage of the circuit when voltage sources are connected in series.

$$\sum_{m=1}^{M} v_m = 0 \qquad (2.3)$$

2.2.2.1 Analysing of Second-Order Circuits

In second-order circuits, the initial conditions for circuit variables and their derivatives are important to analyse the circuits. The RLC circuits are considered for series circuits and parallel circuits in analysing transient response of circuits.

Equation (2.4) shows the capacitor voltage for the initial condition where $t = 0^-$ indicates the time just before a switching event and $t = 0^+$ indicates the time just after the switching event. The switching event is assumed taking place at $t = 0$. According to the equation below, the capacitor voltage is always continuous. This equation is used to find the initial condition of a RLC circuit especially for those variables that cannot change abruptly.

$$v(0^+) = v(0^-) \qquad (2.4)$$

Equation (2.5) indicates the inductor current for the initial condition where $t = 0^-$ is the time just before a switching event and $t = 0^+$ denotes the time just after the switching event. The switching event is assumed taking place at $t = 0$. Based on equation (2.5), the inductor current is always continuous. This equation is used to find the initial condition of a RLC circuit.

$$i(0^+) = i(0^-) \qquad (2.5)$$

A series RLC circuit with a voltage source is shown in Figure 2.1. Capacitor is a passive element that does not dissipate energy but store energy in its electric field meanwhile inductor is a passive element that does not dissipate energy but store energy in its magnetic field.