DEVELOPMENT OF ACTIVE STEERING CONTROL BASED ON SLIDING MODE CONTROL WITH THE INCLUSION OF AN OBSERVER

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This report is submitted in partial fulfilment of the requirements for the degree of Bachelor of Electronic Engineering (Industrial Electronics) with Honours

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DECLARATION

"I declare that this report is the result of my own work except for quotes as cited in the references."

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APPROVAL

"I hereby declare that I have read this thesis and in my opinion this thesis is sufficient in terms of scope and quality for the award of Bachelor of Electronic Engineering (Industrial Electronic) with Honours."

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Date : 01 JUNE 2017

DEDICATION

Special dedication to my beloved parents

Ng Kek Seng and Ong Guat Cheng

Their encouragement and guidance has always be an inspiration to me along this journey of education.

ABSTRACT

The purpose of this project is to enhance the performance of steering control of a vehicle. A nonlinear active steering control system which will overcome the disturbances such as road conditions and crosswind is designed in order to achieve that purpose. Based on a single track car model, the model of the active steering control system is derived and simulated. A nonlinear active steering controller is designed by using the Sliding Mode Control (SMC) strategy which the side slip angle is used as the control parameters and an observer for Sliding Mode Control (SMC) had been developed. The proposed controller is applied to the nonlinear system, simulated and tuned using Matlab/Simulink platform. The various types of disturbance such as crosswind disturbance and braking torque disturbance and the road friction coefficient are been applied to the proposed controller in order to analyze the stability and controllability of the proposed controller. The performance of the developed controller is compared with the performance of other controllers such as Linear Quadratic Regulator (LQR) and pole placement controller to determine its robustness and stability.

ABSTRAK

Objektif utama projek ini adalah untuk menambahbaik prestasi kawalan stereng untuk kenderaan. Sistem kawalan stereng aktif yang tidak linear akan mengatasi gangguan seperti keadaan jalan dan angin lintang akan direka untuk mencapai tujuan itu. Berdasarkan model kereta trek tunggal, model sistem kawalan stereng aktif akan diwujudkan dan disimulasikan. Pengawalan stereng aktif yang tidak linear akan direka dengan menggunakan strategi "Sliding Mode Control" (SMC), sudut gelincir sisi akan digunakan sebagai parameter kawalan dan pemerhati untuk "Sliding Mode Control" (SMC) akan dibangunkan. Pengawal yang dicadangkan akan digunakan untuk sistem yang tidak linear akan disimulasi dan diubah menggunakan platform Matlab / Simulink. Pelbagai jenis gangguan seperti gangguan angina lintang dan gangguan brek tork dan geseran jalan raya telah digunakan untuk pengawal yang dicadangkan untuk menganalisis kestabilan dan prestasi pengawalan yang dicadangkan. Prestasi pengawalan yang diwujudkan untuk membandingkannya dengan prestasi pengawalan yang lain seperti "Linear Quadratic Regulator" (LQR) dan pengawalan "pole placement" untuk menentukan keteguhan dan kestabilannya.

ACKNOWLEDGEMENTS

I have just completed my Final Year Project (FYP) and report. In the beginning, I would like to take this opportunity to express my gratefulness to some organization and individuals who kindly contributed for my final year project in UTeM. With the cooperation and contributions from all parties, the objectives of the project which are soft-skills, knowledge and experiences were gained accordingly even this is just part of the whole project. Furthermore, I would like thank to my supervisor, Dr. Ahmad Sadhiqin Bin Mohd Isira for the proper guidance, cooperation and involvement throughout my Final Year Project. His effort to ensure the successful and comfort ability of students under his responsibility was simply not doubtful. Moreover, I would like to extend my sincere acknowledgement to my parent and family members who have been very supportive for the past one year. Their understanding and support in term of moral and financial were entirely significance towards the project completion. Last but not list, my appreciation goes to my fellow student in UTeM, especially for who are from FKEKK. Their willingness to help, opinions and suggestions on some matters, advices and technical knowledge are simply precious while doing upon completion of my final year project.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Nowadays, there is a common problem that will cause the road accident is losing control of a car at high speeds. Due to the driver's failure to understand the many road conditions and situations that could cause the road accidents are being happened in term of loss of control. A small city car is not designed to be driven at highway speeds at the same time that the car's tires with poor grip will increase the chances of losing control. A new driver or in young age and inexperience driver will not able to fully control the car during there are the hidden dangers of disturbances.

Besides that, a well maintained vehicle will also have chances to lose control due to external factors. There are many external disturbance that may cause a car to lose control such as crosswind, braking torque rainy day, conditions of the road and others. Although there are many controllers such as antilock brake system, traction control, dynamic stability control and active steering are being installed on a vehicle, these system is still unable to defy extreme forces. Thus, the active steering control is developed in order to improve the drivability of a vehicle when there are any disturbances such as poor road adhesion and crosswinds.

Recently, there are a lot of the researches from all over the world are doing research about the area of vehicle stability in sequence to the increasing of vehicle capabilities. This includes the steering control by using different controller strategies to improve safety and handling for the car steering in order to make sure that the driver and the passenger are safe. There are a lot of analysis about the steering and the controller strategy of the dynamic system. However, it is still difficult to value the improvements.

A complementary system for a front-steered vehicle that adds or subtracts a component to the steering signal which is performed by the driver is known as a steering control system. An angular movement on the steering wheel is the steering signal from the driver. The resulting steering angle is thus composed by the component performed by the driver and the component contributed by the steering system. Thus, the input of the system came from the front wheel angle of the driver. There are many types of control strategy can be used to stabilize the vehicle during disturbances happened whether using conventional controller such as PID, fuzzy logic control and sliding mode control. In this project, a single track car mathematical model and Sliding Mode Control are implemented and analyzed. The main aimed of this project is to implement a steering aid system to help the driver.

1.2 Project Overview

There are already exist the driver assistant systems which use a braking method that been applied on each wheel [4]. Due to the hardware that is consisting the exist of ABS braking system with an additional yaw rate sensor and does not require a new actuator, thus these systems are cheap.

There are several reasons that the causes of an active steering system is considered as a good alternative. The first reason is a torque which is tire force times lever arm is generated to compensate yaw disturbance torques. The second reason is the cause of the disturbance torque is the different friction coefficient, μ on the left and right sides (μ-split braking). On the other side, the braking torque can be compensated and a straight short braking path can be achieved by a steering torque. While the third reason of an active steering should be a good alternative is energy conservation, reduced wear of tires and brakes and smooth operation around zero correction.

Practically, braking systems are not capable of immediately react to an emergency situation and cannot immediately compensate small errors. However, it will late react in detected emergency situations when the car is close to skidding or rolling. The active steering system is the only feasible way for continuous operation and also for better comfort under continuous disturbances.

Due to modeling inaccuracies, the vehicle dynamics are subjected to various uncertainties [5]. Thus, this situation is not capable to be handled by the conventional linear control. Hence, by applying a robust controller to the vehicle system design, the robust performance capabilities against uncertainties can be overcome.

The main aim of this project is to revise and define the automatic steering control of passenger cars for general lane-following maneuver. A 2-DOF controller based on H loop-shaping methodology is used by lateral vehicle control system is successfully designed [5]. The 2-DOF controller supplies good lane-keeping and lane-change abilities on both curved and straight road segments. Moreover, it provides a computationally efficient algorithm and does not require explicit knowledge of the vehicle uncertainty. But, the test results show that the higher the vehicle's speed, the less stable the vehicle system.

1.3 Objectives

In order to measure the outcome of the project the goals are stated below:

1. To analyze a single-track car model.

2. To develop a controller and observer for a single track car that base on the robust control strategy which is Sliding Mode Control (SMC) to overcome uncertainties and disturbances of a road handling.

3. To evaluate and analyze the performance of the single track car model with a proposed controller and compared with others controller such as Linear Quadratic Regulator (LQR).

The main objective for this project is to emphasize more on the performance of the active steering control when there is a disturbance applied on the car. In order to verify the performance of the proposed controller, various parameters such as slip angle and yaw rate will be observed. Thus, the performance of the proposed controller will be compared to others controller such as Linear Quadratic Regulator (LQR) and pole placement.

Theoretically, the stability of the proposed controller will be accomplished by using the Lyapunov's second method. The performance of the proposed controller will be evaluated and analyzed by using MATLAB software and SIMULINK toolbox with respect to various types of parameters.

1.4 Scope of Project

This project is focused on the car steering system and using the single track model as the mathematical model which is described that all to represent the dynamic system of the steering which lumping the front and rear tire into one side only [4]. The other side of the car acts as the passive side due to both side which are left side and right side of the car are symmetrical. The Sliding Mode Control technique is used to design a new controller method in active steering. An active car steering system is evaluated and analyzed on various disturbance profiles and road friction coefficient. The input parameter of the car denoted by the front steering angle δ_f, the rear steering angle δ_r, side slip angle β and the yaw rate ψ will be used for this active steering controller. As for the result of the project it will be shown clearly in MATLAB/SIMULINK by showing the comparison between controllable system and uncontrollable system. Besides that, the performance of the proposed SMC will be compared with pole placement and Linear Quadratic Regulator (LQR) techniques. In order to control the stability and to reject unwanted steady state error of the sliding mode control technique used in this project.

1.5 Methodology

There are certain procedures and methods are used in order to complete the research and make sure the project is running smoothly. Figure 1.1 shows the flow chart of the project methodology. The study of the literature review is firstly done, mostly using the IEEE database as the main source to find the papers and journals regarding to the related field of this project. The book about the controller including modern strategy controller book also is the main source of the literature review to find the appropriate mathematical model that will be used. The study on the MATLAB/SIMULINK software has to be done in order to simulate the final results of the controller system. The single track mathematical model is used in this research. This mathematical model is very useful as to represent the dynamic system. After implementing the mathematical model to the MATLAB/SIMULINK, the simulation of controllable system can be observed and analyzed. Then the controller is designed to enhance the stability and to reject the undesired steady state error.

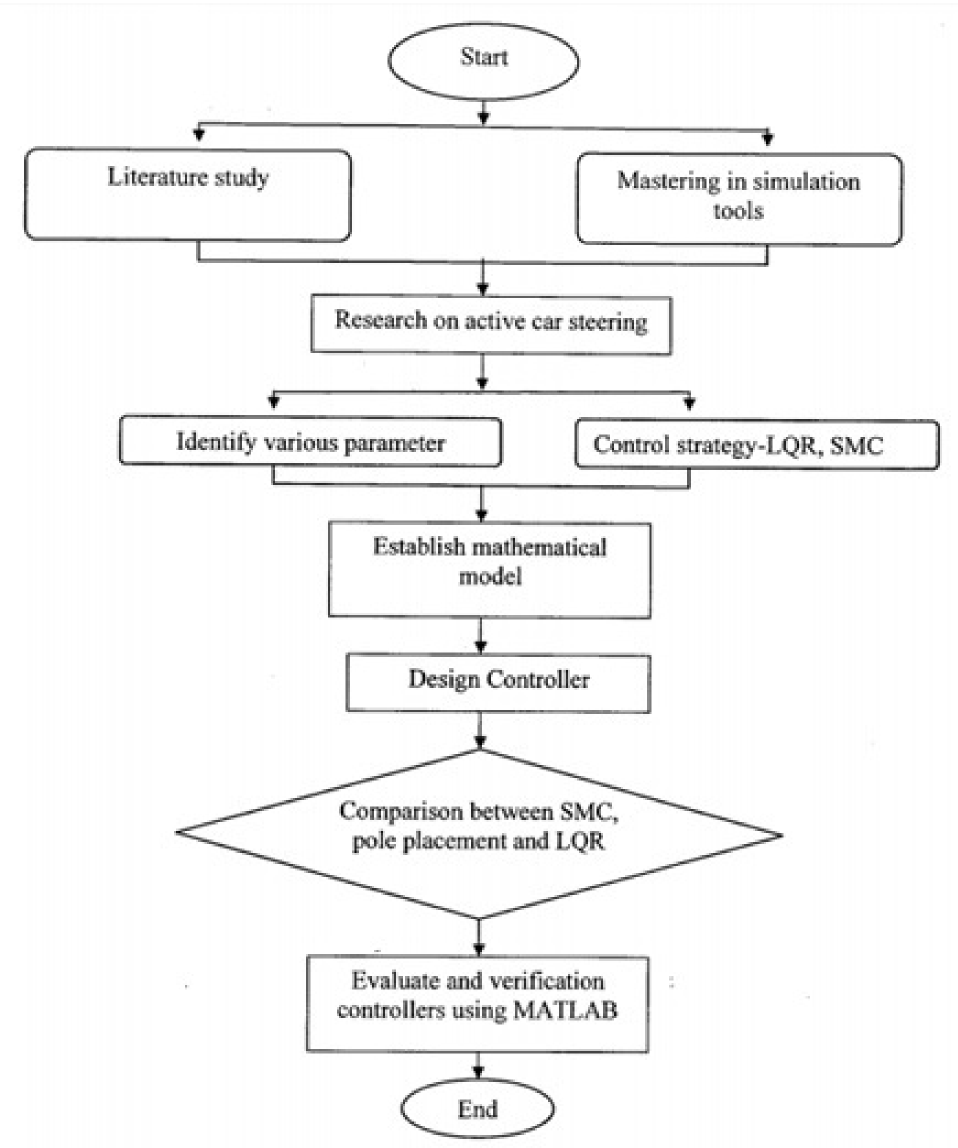

Figure 1.1: Methodology FLow Chart.

1.6 Thesis Structure

The contents of this thesis are about the flow of the project report. This thesis consists of five chapters. In the chapter I, the project overview which the objective, scope of work, problem statement and project methodology are briefly deliberated which purposely to provide the reader an understanding of the project introduction.

Chapter II, embracing the literature review of the project which includes the conception, principle, perspective, and the method of the project that is used in order to solve the problem occurs and any assumption that related with the research of methodology.

Chapter III is about the investigation methodology of the project. This chapter will discuss the method or approach that used in project development such as mathematical modeling and also includes the software aspect.

Chapter IV discusses briefly on the observation, result and the analysis of the project that the achievement during the development of project. This chapter also consists of the final result of the project.

Chapter V covers the discussion of whole contents of the thesis and project and the recommendation for improvement process in the future research and overall conclusion of the project.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The derivations on the modeling of the active car steering systems with linearization model plant are presented in detail in this chapter. A mathematical derivation is established in state space equation of the active car steering. Furthermore, the assumptions that have been added to the single track car model will be described. In order to differentiate dry and wet road performance, several values of road coefficient, μ are used. The disturbance profiles which are considered as the disturbance input torque are also presented in the end of this chapter.

2.2 Background

There are many researches on active safety systems for ground vehicles are having been up to recent years due to motivated by the ever increasing demand for safety against car accidents. The active safety systems are included active steering and independent brake intervention. Active safety systems are developed based on the "by-wire" technologies to drive the devices independently of the driver's operation [6]. The by-wire driven steering and braking systems which are dealt with this article are generated in large quantities lately [7]. Hence, the by-wire driven active four wheel steering and active braking system is assumed to be equipped in the vehicles. In order to improve vehicle lateral stability, the yaw moment is directly controlled by direct yaw moment control systems by generating differential braking forces between left and right wheels [6]. The lateral tire forces are controlled by the active four wheel steering systems by generating independent steer angles of front and rear wheels to improve vehicle handling and stability. Usually, the performance of each control system depends on the status of the vehicle and may be interfered with each other.

By modulating the lateral tire forces, the active steering can regulate the tire slip angle and affect the vehicle handling behavior. There are three types of active steering which are Active Front Steering (AFS) [8] [9] [10], Active Rear Steering (ARS) [11] and Active Four Wheel Steering (W4S) [12]. By using disturbance observer control method [8] [13], sliding mode control [14], predictive control [9], or other control techniques, this latter may be established. By applying an additional steering angle to the driver's steer command, normally the active handling control will serve a steering support system. The AFS potential is usable once Steer-by-wire technology is established because of the extra steering action.

The lateral acceleration of the front axle may be robustly triangularly decoupled from the yaw rate dynamics [15]. This is because it is using only the front wheel steering angle as a control input which is feeding back the yaw rate error through an integrator. For braked and unbraked driving condition, a PI active steering control on the yaw rate tracking error with different gains is used [16]. In order to ensure safety also during a system failures,the active front steering is designed [17]. The wheel steering angle, δ_f is the sum of the designed feedback control, δ_c and the driver input, δ_p [18].

The development of electro actuated differentials allows for new control strategies in vehicle systems dynamics control [19] [20]. The differential control system is semi-active which the electronic control system can decide the locking torque transferred but not its direction [19]. While the transfered torque is generated from the fastest wheel to the slowest one and the control operates when the rear wheels speed difference exceeds a given threshold. A proportional-integral control law computes the value on the measured and the desired rear wheel speed angular velocity. The proposed controller is designed following the Internal Model Control approach and since it can generate yaw moments of every amount and direction thus it is activated [20]. According to a Lyapunov analysis, it is electronically controlling the locking action of the rear differential [18] [21].

An integrated control of active front steering and direct yaw moment which generated by a distribution of braking forces is designed [22]. The electronic stability program (ESP) is integrated with the active front wheel steering, active suspension and active anti roll bar [23]. Four wheel steering are coordinated with wheel torque distribution by using an optimization approach is shown [24]. A non linear optimization is approached to determine the optimal force to be exerted by each tire controlled by active steering and brake pressures distribution [18] [25].

Active front steering is a newly developed mechatronic steering system for passenger cars that realizes an electronically controlled superposition of an angle to the hand steering wheel angle that is prescribed by the driver, cf [26].

A steering aid system integrated in cars is known as Active Steering. There are many different systems with different control strategies on the market. The main purpose is to improve safety and comfort of the vehicles by improving the stability and handling of the steering. The actuators are still used to influence the mechanical system even though the regulations demand a mechanical connection between the steering wheel and the steering rack.

Active steering is an effective way that can improve drivers comfort and handling. The vehicle handling and lateral stability can be controlled at the same time, if both the external yaw moment and active steering angle are adopted [27].

Active steering can be said as an integrated steering support system for cars. The system is like the steering on conventional cars but with additional functionality which can withstand with disturbances. For example, μ-split which is a split adhesion coefficient between wheels, wind gusts or decreased road adhesion conditions. There are various types of systems are conceptual and not intended for the market. However, BMW has a semi-mechanical system installed on the five hundred and thirty cars.

The main purpose for changing the steering characteristics of a car is to improve safety and comfort of the vehicle. The following sections will describe a specific theoretical solution for a steering system.

2.3 Research of Active Front Steering System

The steering system can be said is an important role of making car convenient to handle and enhance the vehicle stability. The steering system development has experienced many stages,and the newest technology of steering system for passenger cars is the Steer-by-Wire system (SBW). However, the Steer-by-Wire system is permitted by state regulations because of it has not yet accepted by public consumers in the consideration of the reliability and safety of the system. A newly technology for passenger cars which is developed by BMW is Active Front Steering (AFS). This technology is implementing an electronically controlled superposition of an angle to the hand steering wheel angle which is prescribed by the driver. However, the connection of permanent mechanical between steering wheel and road wheels are remained [1]. The vehicle performance can be adjusted by AFS through intervening the road wheel angle in condition of the driver have top priority.

For the active steering system, the variable steering gear ratio function will be experienced by the driver first. Then perceive the improvement of steering portability [1]. According to the vehicle's motion state, AFS enables continuous and situation-dependent variation of the steering ratio. Therefore, the maneuverability of the vehicle at low speed and the stability at high speed are improved by the AFS. The performances of the improvement of the stability with active steering system depend on the variation quality of the steering ratio to a certain extent. Thus, the variable steering gear ratio function is important to be investigated.

Active steering system is comprised of a double planetary gear and an electric actuator motor additionally which is compared with traditional mechanical steering system [1]. The AFS is reliable and safe due to all the links from the steering wheel to road wheel are mechanical. AFS can ensure that the vehicle is under driver's control and make driver have a clear road feel. From the Figure 2.1 below shows that the AFS's planetary gear have two degrees of freedom (DOF), the planetary gear output connects with the steering gear's pinion and one input connects with the steering wheel while the other input connects with an electric actuator motor.

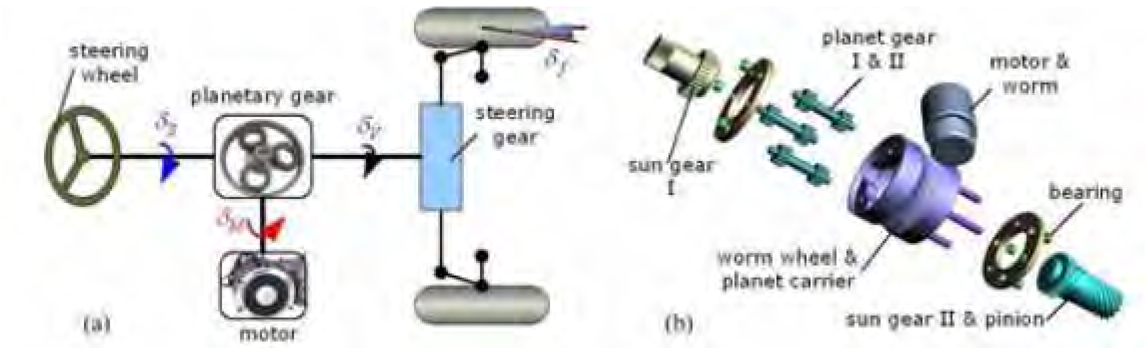

Figure 2.1: (a) Schematic view of AFS system; (b) 3D-model of planetary gear set and electric motor [1].

2.4 Vehicle Dynamics Modeling

According to the SAE standard which is described in SAE J670e, the vehicle axis system used throughout the simulation [28]. First, the derivation of that model which includes the tire model is discussed. The motion equations are converted into a state space form for easily to do integration and a Third Order Runge-Kutta integration is used as the integration algorithm. Lastly, in order to show its validity, the vehicle model is verified against results from Smith et al. [2].

2.4.1 Vehicle Axis System

According to SAE J670e [28] as shown in Figure 2.2 below, the coordinate system is used in vehicle dynamics modeling. The x-axis which is referred to the forward direction or the longitudinal direction. while the y-axis is referred to the lateral direction which is positive when it points to the right of the driver. And lastly, the z-axis is represented to the ground satisfying the right hand rule.

Only the X-Y plane of the vehicle is considered which among most studies related to handling and directional control. The Z-axis which is vertical axis is often

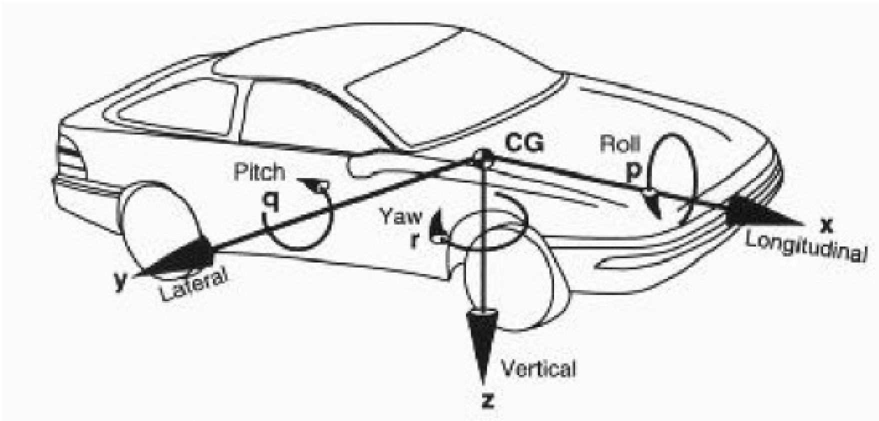

Figure 2.2: Vehicle Axis System after SAE [2].

used in the study of ride, pitch, and roll stability type problems [2]. The relevant definitions for the variables associated with this research are defined in the following list.

Longitudinal direction which is a forward moving direction of the vehicle. There are two different types of methods to look at the forward direction. One of them is with respect to the vehicle body itself and the another is with respect to a fixed reference point. When dealing with acceleration and velocity of the vehicle, the former is often used. The latter is used during the location information of the vehicle with respect to a starting point or an ending point is desired [2].

While the lateral direction is a sideways moving direction of the vehicle. Same as the longitudinal direction, the lateral direction also has two ways of looking which are with respect to the vehicle and with respect to a fixed reference point. The extreme values of lateral acceleration or lateral velocity can decrease the stability and controllability of vehicle.

The tire slip angle is equivalent to heading in a given direction but walking at an angle to that direction by displacing each foot laterally. Because of the presence of lateral forces the foot is displaced laterally.

Lastly, the body-slip angle is the angle between the X-axis and the velocity vector that represents the instantaneous vehicle velocity at that point along the path which is shown in Figure 2.3. This is different from the slip angle related with tires.

Although the concept is same, but each tire may have different slip angle at the same time. Sometimes the body slip angle is calculated as the ratio of lateral velocity to longitudinal velocity [2].

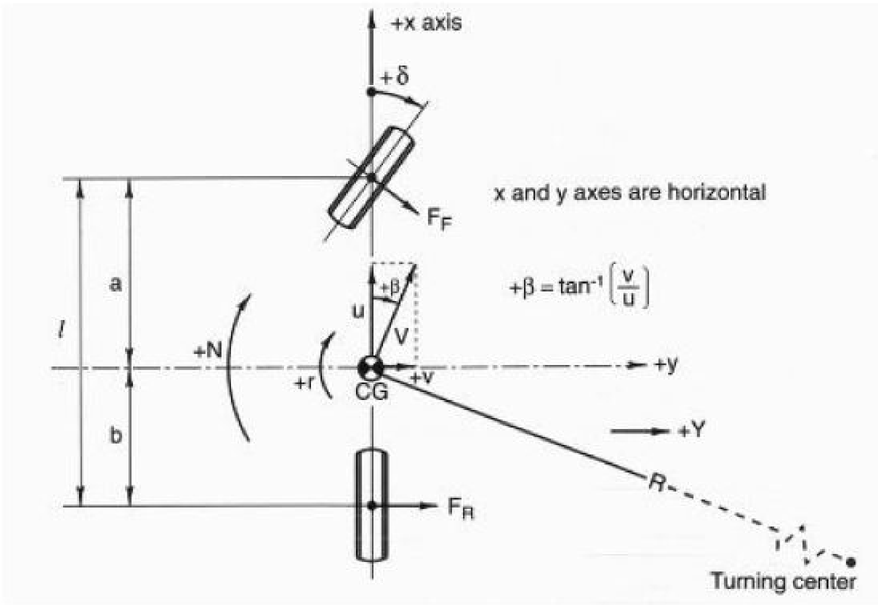

Figure 2.3: Vehicle Axis System after SAE [2].

2.4.2 Vehicle Models

There are many types of degrees of freedom (DOF) which are associated with vehicle dynamics. A two-degree-of-freedom bicycle model is the most simplified vehicle dynamic model which is representing the lateral and yaw motions. The idea for this model is that sometimes the longitudinal direction is not necessary or desirable to be included due to the lateral or yaw stability of the vehicle does not be affected by the longitudinal direction. Usually this model is used in teaching purposes. Figure 2.4 shows the two-degree-of-freedom model.

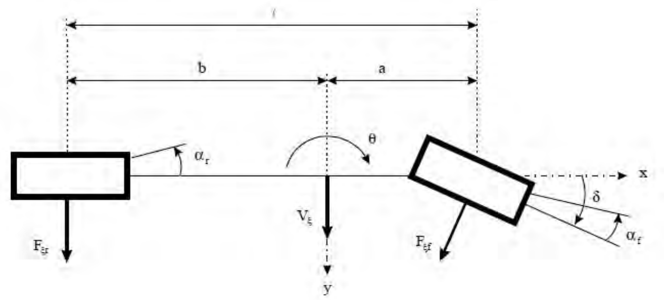

Figure 2.4: Two Degree of Freedom Model [2].

While for a three-degree-of-freedom model, it adds longitudinal acceleration to the model. Thus, it enables one to describe the full vehicle motion in the X-Y plane. The longitudinal velocity, U, and the longitudinal force, F_{tf} and F_{tr}, are included into the model which is shown in the Figure 2.5. This is the model that is used in this project [2].

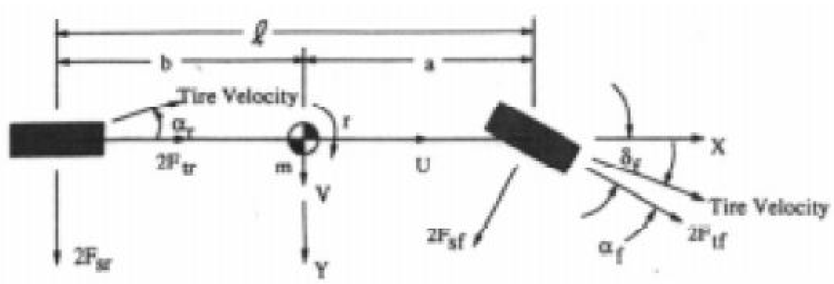

Figure 2.5: Three Degree of Freedom Model after Smith [2].

Lastly, the symmetry in dynamic behavior between right and left sides is no longer assumed for an eight-degree-of-freedom model. Instead of two tires, the rotational degree of freedom for each of the four tires is considered in this vehicle model. A rolling motion, ϕ_s is added between left and right sides of the vehicle. This model is frequently used in the suspension design or ride comfort analysis, especially for the effects of these issues with respect to roll and side-to-side load transfer [2].

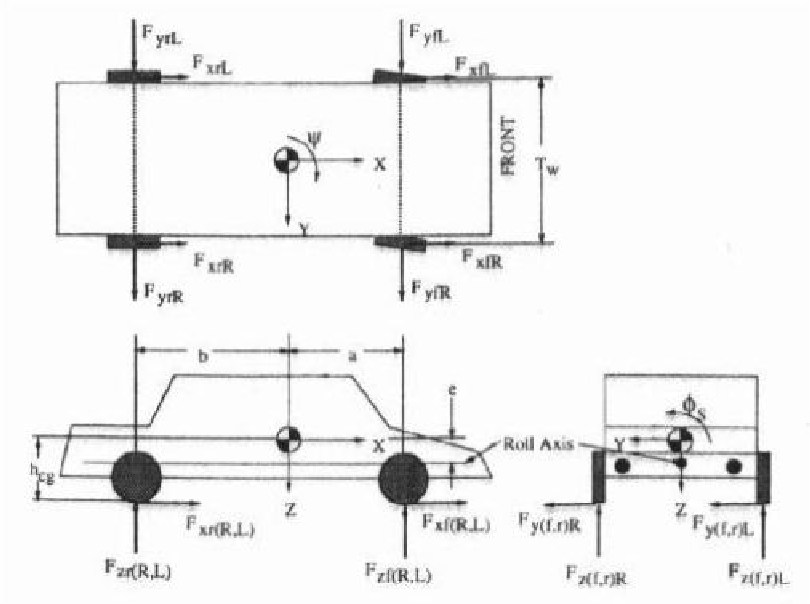

Figure 2.6: Eight Degree of Freedom Model after Smith [2].

2.5 Car Modeling

From the Figure 2.7 below shows that the car is modeled as a rigid body with mass m and moment of inertia J with respect to a vertical axis through the center of gravity (CG). The yaw angle ψ rotates the chassis coordinate system x, y with respect to an inertially fixed coordinate system x_0, y_0. The yaw rate is $r = \psi$ which is measured by a yaw rate sensor. The yaw rate will be used as one of the state variables in the state vector x [29].

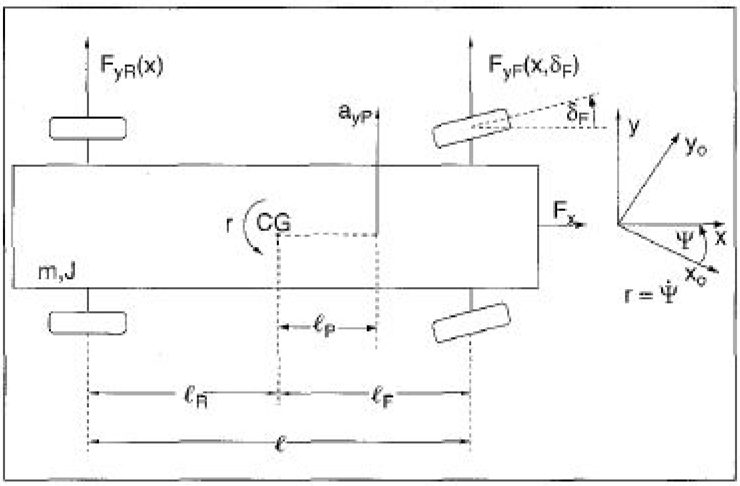

Figure 2.7: Unknown rear and front axle lateral forces act on the car body with mass m and moment of inertia J [3].

The most significant uncertainties for modeling the motion of this vehicle are the lateral forces F_{yR} which is at the rear axle and F_{yF} which is at the front axle. The lateral forces and rear axle are depended on the state x of the car. The front lateral force depends on the front wheel steering angle δ_F. In the automotive literature there are tire models that give the lateral forces. Yet, the forces are described in terms of other quantities, such as the road friction coefficient μ. Therefore, the forces F_{yR} and F_{yF} directly as the uncertain quantities.

A position at a distance l_p in front of the CG is needed to be chosen as the lateral acceleration a_{yP} at this point does not depend on F_{yR}. It is a calculation of a few lines to find the position and the calculation is shown in (2.1).

$$l_p = \frac{J}{ml_R} \tag{2.1}$$

Thus, the lateral acceleration is

$$a_{yP} = \frac{l}{ml_R} F_{yF}(x, \delta_F) \tag{2.2}$$

where $l = l_R + l_F$ is the wheelbase which can be seen at Figure 2.7

2.5.1 Mathematical Modeling For a Single Track Model

In the Figure 2.8, it shows the implementation of a steering control system. The driver is handling the main steering angle, δ_S from the steering wheel. An actuator sets a small corrective steering angle with input from a feedback controller. The yaw rate and the superposition $\delta_F = \delta_S + \delta_C$ is the main feedback signal which can be arranged mechanically. The tire forces are depended on the state variables of the chassis and steering angle δ_F.

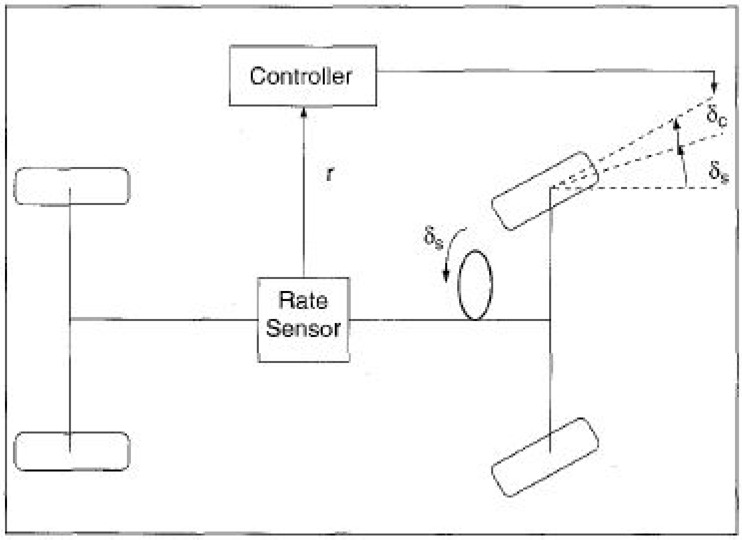

Figure 2.8: The steering angle $\delta_F = \delta_S + \delta_C$ is composed of the command δ_S, from the driver and the feedback controlled additional angle δ_C.

A single track car model is used to describe the dynamics of vehicle steering. By combining the two front wheels into one wheel which refers the center line of the car, same as the two rear wheels, the single track model is obtained. Thus, in Figure 2.9 below shows the car model is reduced which describes the yaw rate and lateral motions.

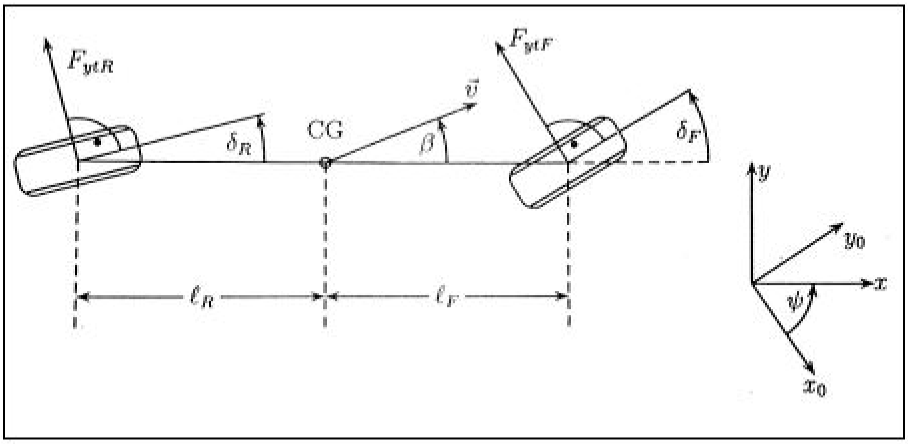

Figure 2.9: Single Track Model for Car Steering.

The angles δ_F and δ_R are referred to the front and rear steering angles respectively. The distance between the center of gravity (CG) and the front axle is represented as l_F and rear axle is represented as l_R. Thus, the wheelbase is referred to the sum of the front axle and rear axle, $l = l_R + l_F$. The vehicle sideslip angle which is the angle β is the angle between the vehicle center line and the velocity vector v at the CG. In the horizontal plane which is shown in Figure 2.9 shows that an initially fixed coordinates system (x_0, y_0) is together with a vehicle fixed coordinates system (x, y) that is rotated by a yaw angle ψ. In the dynamic equations, the yaw rate $r := \psi$ will appear as a state variable.

The model of automatic car steering will include the yaw angle ψ where the position of the vehicle relative to the lane is considered. The forces which are the side forces F_{ytF} and F_{ytR} that are transmitted between the road surface and the car chassis through the wheels are shown in Figure 2.9. The forces in the longitudinal direction of the tires are assumed to be zero for example the wheels are freely spinning.

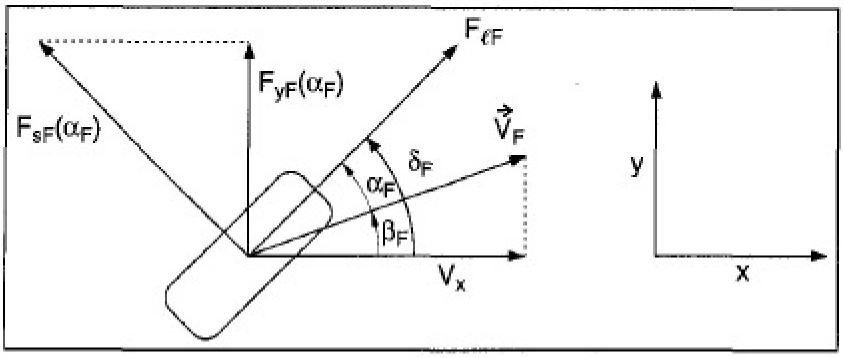

Figure 2.10: Lateral forces F_{ytF} at the front wheel in tire coordinates and F_{yF} in chassis coordinates.

The velocity vector v_F which under a sideslip angle β_F at the front axle with respect to the longitudinal axis (x-axis) of the chassis is represented in Figure 2.10. A function of the tire slip angle $\alpha_F = \delta_F - \beta_F$ is known as the lateral force F_{ytF} in tire coordinates. The dominants component in chassis coordinates is shown in equation (2.3).

$$F_{yF} = F_{ytF} cos\delta_F \tag{2.3}$$

If it occurs symmetrically at the left and right wheel, the small retarding component $F_{xF} = -F_{ytF} sin\delta_F$ does not generate a yaw torque. The longitudinal effect is compensated by speed control. The speed control is automatic or controlled by the driver. In a static tire description, the tire side forces F_{ytF} is a function of the tire slip angle α_F.

$$F_{ytF} = f(\alpha_F) = f(\delta_F - \beta_F), \tag{2.4}$$

$$F_{yF} = f(\delta_F - \beta_F) cos\delta_F \tag{2.5}$$

The front wheels are indicated by the index F which it is replaced by R for the rear wheels. The lateral force is zero, $f(0) = 0$ if the velocity vector v_F is aligned with the tire. The lateral force is close to saturate for $\alpha_A > 10$. A control system cannot overcome on the physical limits. Thus, design steering controllers is very important for that only small tire sideslip angles occur.

The lateral forces at the front and rear axles are the inputs to the vehicle dynamics which shown in equation below:

$$\begin{aligned} F_{yF} &= F_{ytF} cos\delta_F \\ F_{yR} &= F_{ytR} cos\delta_R \end{aligned} \tag{2.6}$$

While for longitudinal force component:

$$F_x = -F_{ytF} sin\delta_F - F_{ytR} sin\delta_R \tag{2.7}$$

These forces represents the sum of the forces at the left and right tire. Through the dynamics model, the forces control state variables are β, v and r. The motions equations for 3-Degree-of-Freedom in horizontal plane are:-

1. Lateral motion

$$mv(\beta+\psi)cos\beta+mvsin\beta=F_{yF}+F_{yR} \tag{2.8}$$

2. Longitudinal motion

$$-mv(\beta+\psi)sin\beta+mvcos\beta=F_x \tag{2.9}$$

3. Yaw motion

$$J\psi=F_{yF}l_F-F_{yR}l_R+M_{zD} \tag{2.10}$$

It is obtained from equations (2.8) to (2.10) with $r:=\psi$.

$$\begin{bmatrix} mv(\beta+r) \\ mv \\ Jr \end{bmatrix} = \begin{bmatrix} \text{-}\sin\beta\cos\beta 0 \\ \cos\beta\sin\beta 0 \\ 001 \end{bmatrix} \begin{bmatrix} F_x \\ F_{yF}+F_{yR} \\ F_{yF}l_F-F_{yR}l_R+M_{zD} \end{bmatrix} \tag{2.11}$$

From the kinematic model which denotes the steering angles δ_F,δ_R and the state variables β,r and v, the sideslip angles α_F and α_R at the front and rear tires are obtained. The β_F and β_R are the front and rear chassis sideslip angles. The velocity components in the longitudinal direction of center line of the vehicle is equal to:-

$$v_R\cos\beta_R=v_F\,\beta_F=v\cos\beta \tag{2.12}$$

The yaw rate, r will affect the relationship between the velocity components and the center line, as it is shown as below:

$$\begin{aligned} v_F \sin\beta_F &= v\sin\beta + l_F r \\ v_R \sin\beta_R &= v\sin\beta + l_R r \end{aligned} \tag{2.13}$$

By the corresponding terms from equation (2.12), the velocity terms v_F and v_R are eliminated by division. So that, the kinematic model is

$$\begin{aligned} \tan\beta_F &= \frac{v\sin\beta + l_F r}{v\cos\beta} = \tan\beta + \frac{l_F r}{v\cos\beta} \\ \tan\beta_R &= \frac{v\sin\beta + l_R r}{v\cos\beta} = \tan\beta + \frac{l_R r}{v\cos\beta} \end{aligned} \tag{2.14}$$

In Figure 2.10, it shows that the tire sideslip angles are

$$\begin{aligned} \alpha_F &= \delta_F - \beta_F \\ \alpha_R &= \delta_R - \beta_R \end{aligned} \tag{2.15}$$

By the non-linear tire model, the feedback-structured model is completed:

$$\begin{aligned} F_{ytF} &= f_F(\alpha_F) \\ F_{ytR} &= f_R(\alpha_R) \end{aligned} \tag{2.16}$$

2.5.2 Linearization for Constant Velocity and Small Angles

The vehicle dynamics which is shown in equation (2.11) are nonlinear. By taking the assumption $v = 0$, these equations can be linearized. It can be justified because of the velocity, v is changing more slowly than the state variables r and β. The velocity, v is now treated as an uncertain constant parameter. In addition, the force component $F_x \sin\beta$ is neglected. Next, the linearized version of equation (2.11) is:-

$$\begin{bmatrix} mv(\dot{\beta}+r) \\ \dot{J}r \end{bmatrix} = \begin{bmatrix} (F_{yF}+F_{yR})\cos\beta \\ F_{yF}l_F - F_{yR}l_R + M_{zD} \end{bmatrix} \tag{2.17}$$

Due to the chassis sideslip angles such as β, β_F and β_R are small, thus $\cos\beta = 1$ and becomes:-

$$\begin{aligned} \beta_F &= \beta + l_F r/v \\ \beta_R &= \beta + l_R r/v \end{aligned} \tag{2.18}$$

Then, the steering angles such as δ_F, δ_R are small as well, for the rear wheels the $\cos\delta_F = 1$ and $\cos\delta_R = 1$. Thus,

$$\begin{aligned} F_{yF} &= F_{yF}(\alpha_F) \\ F_{yR} &= F_{yR}(\alpha_R) \end{aligned} \tag{2.19}$$

The equations above are the unknown characteristics. Hence, the equation (2.17) will become:-

$$\begin{bmatrix} mv(\dot{\beta}+r) \\ \dot{J}r \end{bmatrix} = \begin{bmatrix} F_{yF}(\alpha_F)+F_{yR}(\alpha_R) \\ F_{yF}l_F - F_{yR}l_R + M_{zD} \end{bmatrix} \tag{2.20}$$

Note: The $\alpha_F = \delta_F - \beta_F$ and $\alpha_R = \delta_R - \beta_R$.

From the Figure 2.8 for the feedback control , the steering angles δ_F is the sum of the driver command δ_s with the corrective angle δ_c which both of them are generated by the feedback system. The relationship between δ_s, δ_c and α_F, β_F are demonstrated by Figure 2.10.

The lateral tire forces are now linearized about a zero tire sideslip angle as in equation (2.16) in order to allow for a linear analysis of the car.

$$\begin{aligned} F_{yF}(\alpha_F) &= \mu c_F(\alpha_F) \\ F_{yR}(\alpha_R) &= \mu c_R(\alpha_R) \end{aligned} \tag{2.21}$$

The cornering stiffness is the slope c_F and c_R of the tire characteristic. The friction coefficient $\mu <= 1$ is assumed that both the front and rear wheels are same. The representative values of the friction coefficient μ are:

$$\begin{aligned} \mu &= 1(dryroad) \\ \mu &= 0.5(wetroad) \\ \mu &= 0.15(icyroad) \end{aligned}$$

Thus, equation (2.20) is interpreted for β and r by substituting the linearized tire characteristics F_{yF} and F_{yR}:

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{\mu}{mv}(c_F\alpha_F + c_R\alpha_R) - r \\ \frac{\mu}{J}(c_F l_F \alpha_F + c_R l_R \alpha_R) + \frac{1}{J}M_{zD} \end{bmatrix} \tag{2.22}$$

The both equations (2.15) and (2.18) are substituted and becomes: