

SUPERVISOR'S ENDORSEMENT

—I hereby declare that I have read through this report entitle —walking motion trajectory generation of hip powered orthotic device”” and found that it has comply the partial fulfillment for awarding the degree of Bachelor of Electrical Engineering (Mechatronics)”

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Date : 23 JUNE 2016

**WALKING MOTION TRAJECTORY GENERATION OF HIP POWERED
ORTHOTIC DEVICE**

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**A report submitted in partial fulfilment of the requirements for the degree
of Mechatronics Engineering**

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STUDENT'S DECLARATION

I declare that this report entitle "walking motion trajectory generation of hip powered orthotic device" is the result of my own research except as cited in the references. The report has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

Signature:

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Date : 23 JUNE 2016

To my beloved mother”WAHBAH” and father “QAID”

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ABSTRACT

Nowadays, increasing number of people who could utilize from the powered lower limb orthotic devices because of some of diseases such as spinal cord injured and stroke, especially in elderly age, which means that effects of these diseases lead to disability either Hemiplegia or total paralysis. According to the National Spinal Cord Injury Statistical center (NSCISC), 40 cases per million of American population is sustained every year (12,000 cases annually at average 40 years old). Thus, there are many methods have been used to move Exoskeletons device that accurately matches real human motion in different terrains and environments. Although most of methods are aimed to generate smooth walking like human based on flexion angle, velocity and acceleration data of three joint movements (ankle, knee and hip joints) but these methods do not accurately match real human motion. In this project, there are two objectives that should be achieved, first objective is to generate a walking motion profile in the leg's hip joint that accurately matches real human movement whereas the second objective is to analyze and evaluate the accuracy of the generated trajectory profiles and make a comparison between a generated trajectory motion profiles with real human trajectory profiles. In order to achieve the objectives aforementioned, trajectory motion profile is generated by using both quintic polynomial equation (high-order-polynomial) and cubic polynomial equation (third-order-polynomial) that are coded into MATLAB to get the motion profile for position, velocity and acceleration. Then, generated trajectory profiles were compared with the reference hip trajectory profile to obtain root mean square error (RMS error) and dynamic error. The result shows that quintic polynomial equation generates trajectory profile that accurately matches reference trajectory profile of human's hip during terminal stance and mid-swing with RMS error 0.00575rad and 0.006025rad respectively. Where, it becomes less match during initial swing and terminal swing with RMS error 0.03015rad and 0.01923rad respectively. Besides that, cubic polynomial equation generates trajectory motion profile that accurately matches reference trajectory profile of human's hip during terminal stance and initial swing with RMS error 0.00486rad and 0.00883rad respectively. However, it becomes less match during mid-stance and pre-swing with RMS error 0.017001 rad and 0.01566 rad respectively.

ABSTRAK

Pada masa kini, penggunaan peranti anggota badan Ortotik di kalangan warga tua semakin meningkat. Ianya adalah disebabkan oleh beberapa penyakit seperti kecedera pada saraf tunjang dan strok dimana, ianya akan mengakibatkan ketidakupayaan seperti 'Hemiplegia' atau lumpuh keseluruhan. Berdasarkan pada 'National Spinal Cord Injury Static Center' (NSCISC) sebanyak 40 juta per kes populasi di Amerika diterima sepanjang tahun (12000 kes tahunan dalam lingkungan 40 tahun). Terdapat banyak kaedah yang telah digunakan bagi memindahkan peranti 'Exoskeleton' dimana lebih menepati pergerakan manusia sebenar dalam berlainan rupa bentuk dan persekitaran. Walaupun kebanyakan kaedah adalah bertujuan untuk menjana pergerakan manusia berjalan dengan lebih lancar berdasarkan sudut 'flexion', halaju dan pemecutan ke atas 3 sendi iaitu buku lali, lutut dan sendi pinggul. Akan tetapi, kaedah ini tidak sepadan dan tidak tepat dengan pergerakan manusia yang sebenar. Dalam projek ini, terdapat 2 objektif. Objektif yang pertama adalah bagi menjana profil pergerakan berjalan di sendi pinggul sepadan dan tepat dengan pergerakan sebenar manusia. Manakala objektif kedua pula, ialah untuk menganalisis dan menilai ketepatan penjanaan profil 'trajectory' dan membuat perbandingan antara penjanaan profile 'trajectory' bergerak dengan manusia sebenar. Dalam mencapai objektif yang dinyatakan diatas, profil pergerakan 'trajectory' ini dijana oleh penggunaan kedua-dua 'quintic polynomial equation' (high order polynomial) dan 'cubic polynomial equation' (third order polynomial) yang telah dikodkan ke dalam MATLAB untuk mendapatkan profil pergerakan bagi kedudukan, halaju dan pemecutan. Kemudian, profil trajectory yang telah dijana akan dibandingkan untuk rujukan profile hip trajectory untuk mendapatkan 'root mean square error (RMS error) dan 'dynamic error'. Keputusan menunjukkan persamaan 'quintic' polynomial menjana profil trajectory sepadan dan tepat dengan rujukan manusia biasa 'terminal stance' dan 'mid swing' dengan RMS error 0.00575rad and 0.006025rad. Dimana, ianya menjadi kurang sesuai semasa hayunan permukaan dan hayunan terminal dengan RMS error 0.03015rad and 0.01923rad. Selain itu, persamaan kubik polynomial menjana profil pergerakan trajectory yang sepadan dan tepat dengan rujukan profil trajectory pinggul manusia. Semasa terminal stance dan hayunan permulaan dengan RMS error 0.00486rad and 0.00883rad. Walaubagaimanapun, ianya menjadi kurang sepadan semasa 'mid stance' dan 'pre swings' dengan RMS 0.017001 rad and 0.01566 rad.

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CHAPTER 1

INTRODUCTION

1.1 Motivation

Nowadays, many human diseases will cause gait disorder or movement disorder, especially happen in the elderly age. This will cause those falls, walking with ugly walking pattern, always feeling of fearing to fall, and so on. About three fourths of deaths that are due to falls in the United States occur in the 13% of the population that is aged 65 and older[1]. The most significant impact among them is falls, because for an elderly person, falls will give them a very big impact and sequel. From the studies, the 31% out of 12 large studies of falling is due to gait disorder [1]. According to the National Spinal Cord Injury Statistical center (NSCISC), 40 cases per million of American population is sustained every year (12,000 cases annually at average 40 years old) [2].

In Malaysia, according to the statistical report that is done by World Health Organization (WHO), 92 percentages of Malaysian people who are above 60 years old, they do sever from disability motion. Moreover, based on the annual report of the Malaysian Ministry of Health, about 7.2 percentages to 11 percentages of children between 0 to 18 years old are sustained with physical and cerebral palsy disabilities [2].

Nowadays, increasing the people who could utilize from the powered lower limb orthotic devices because of some of diseases such as spinal cord injured and stroke, especially in elderly age, which means that effects of these diseases lead to disability either Hemiplegia or total paralysis.

All disable people aforementioned need a device to help them during their movement. Many exoskeleton devices have been developed but it is not all exoskeleton devices are satisfied to be a good helper because of friction that occurs during motion due to less accuracy of exoskeleton. In order to avoid the injuries that can be occurred for users during walking, reducing the friction between the exoskeleton and the leg is one solution, which means that smooth movement has to be generated by the exoskeleton to reduce this friction. Thus, generating trajectory motion that accurately matches real human motion, the smooth generated movement can be the solution to protect the user from injury.

1.2 Problem Statement

Trajectory generation is a hard task either for lower or upper part of body, which means many equations have to be involved in order to generate an accurate motion compared to real human motion. Accuracy of trajectory motion is the most concerned problem in this project. To get an accurate trajectory motion for hip joint, many methods of trajectory generation have been conducted but it is not all generate an accurate gait motion that matches reference motion trajectory of human's hip.

Moreover, generating an accurate motion gait that matches real human motion trajectory is not guaranteed to make a stable walking because falls commonly appear during either increasing the speed of walking or changing the terrain condition.

Quintic and cubic polynomial are the equations that proposed to overcome problems aforementioned. Quintic and cubic polynomial are chosen to be applicable equations in this project. Furthermore, both quintic and cubic polynomial equation allow a smooth transitions between walking phases, reduce possibility of falling under different terrain condition and generate gait motion that accurately matches real human motion .

However, the suitability of using cubic and quintic equations have not studied yet. Thus, this project is aimed to study the polynomials equations especially quintic and cubic polynomial and how quintic and cubic polynomial equations generate a motion that accurately matches the real human motion without injuring the human's leg during walking.

Based on the comparison between cubic and quintic polynomial trajectory profile related to real human hip motion profile, the best polynomial equation will be chosen to be the solution for problems aforementioned.

1.3 Objectives

The objectives of this project are:

1. To generate a motion trajectory profiles relative to hip joint that accurately matches real human motion profile.
2. To analyze and evaluate the accuracy of the quintic and cubic polynomial trajectory profiles related to real human hip motion.
3. To make a comparison between the both cubic and quintic trajectory motion profiles with real hip trajectory profile.

1.4 Scope

The scope of this project will be focused on several items that are related to the movement of human's hip. Studying the movement of healthy human, especially for hip part is one way that leads me to understand the trajectory generation planning. Besides that, trajectory generation methods are highlighted in order to choose the best method that can generate motion accurately and stably compared to real human hip motion. By doing all studies aforementioned, learning MATLAB is the way that leads to generate trajectory planning by using cubic and quintic polynomial equations in this project.

Furthermore, studying the gait will be focused on the hip instead of whole leg and using one leg's hip instead of both legs' hip. Moreover, the experiment is not conducted to the real human because it is only a simulation experiment.

CHAPTER 2

LITERATURE REVIEW

2.1 Theoretical Background

2.1.1 Background of Walking Model

There are different methods which are used in order to overcome the difficulties imposed by the extraction of human gait features. Model-based and non-model-based methods are two approaches methods which are being used for human gait analysis [3].

The non-model-based is considered the first method analysis which is applied in image-based gait analysis. The prediction, velocity, shape, texture and color is feature correspondence between successive frames that is depend on it .Additionally, small motion is the main assumption between consecutive frames but using different geometric constraints, the feature correspondence can be conducted. According to the non-model-based, a priori shape model has been founded in order to match actual data to this predefined model. So, extracting the corresponding features during the best match is obtained. Thus, the most common method that have been used for human gait movement analysis is model based approach and that because of its advantages. Furthermore , detailed and exacted motion data can be extracted by the model based method such as stick models and volumetric models [3].

The extraction gait movement methods of human has been established and founded along previous ten decades and recently decades. Thus, as I noticed that the history of the

extraction method of motion for human has already started for a long time that what we will mention below.

Nash (Nash et al., 1998) suggested a parametric gait model consisting of a pair of articulated lines which are jointed at the hip in order to extract movement of articulated objects from a temporary sequence of images. Whereas, Cunado et al has used Pendulum model that help him to extract and describes human gait easily. So, Cunado et al have presented the human leg as two pendulums which are joined in series [3].

Additionally, a model-based method was proposed by Zhang in term of gait identification. In this experiment, a 5-link biped locomotion human model is used. Whereas, a model that is consisting of six segments is divided as (two legs, two arms, the torso and the head Akita) is proposed by Akita. Also, a 7-ellipse model was suggested by Lee, Lee has used this model in order to describe a representation of gait appearance in term of person identification and classification [3].

Furthermore, Yoo et al was used A 2D stick figure model that is consisted of 7 segments. 2D stick figure model was used to represent the human body relative to joint angles and angular velocities because joint angles and angular velocities have to be calculated in order to describe the gait motion. Hence, also a stick figure model which has 10 sticks articulated with six joints is used by Guo in order to represent the human body structure in the silhouette. represented the human body as a stick figure is represented by Cheng which was considered to be consist of 12 rigid parts [3].

The use of a hierarchical and structural model of the human body was suggested by Dockstader. To add to that, volumetric model for the analysis of human trajectory motion was proposed by Rohr (Rohr, 1994). Rohr has been used volumetric model which is 14 elliptical cylinders; this type of model helped him to present the human body in term of motion. Whereas, Karaulova has used the stick figure model in order to build a novel hierarchical model of human dynamics which is represented by using hidden Markov models [3].

Whereby, the oldest branch of physics that focused on the study of motion is Newtonian, mechanics branches, in this branches, the forces and internal forces is applied which affect on the body in order to generate a motion and act within the body. Thus, one of the applications of Newtonian mechanics is Biomechanics that is used to study the neuromuscular skeletal system. So, there are many uses of Biomechanics that has been

conducted in different field such as physical medicine, orthopedics, rehabilitation characterizing function and dysfunction of the muscular skeletal system. From aforesaid , gait analysis human motion is one branch of biomechanics that has developed in early studies ,especially in the late 1900s. Thus, During the past two decades, gait analysis has been expanded in order to investigate various other activities[4].

Recently, the studies is focusing on lower limbs, upper limbs , stairs ascending and stairs descending based on the techniques of biodynamic and trajectory motion analysis. In contrast, the mathematical techniques of dynamics are required in order to present the trajectory motion analysis [4].

2.1.2 Basic of Trajectory Generation

Trajectory is essentially related to velocity, acceleration and position for every degree of freedom. This trajectory includes the human-interface in term of how trajectory or path is specified through space[5].

The easiest way to describe the manipulator motion for user of a robot system is that the human user should be required to write uncomplicated function in order to determine the task. In contrast; capability of determining trajectories with simple descriptions has to be in consideration in order to get the desired motion. For instance, sometimes the user has only a desire to determine the desired goal position and orientation of the end effector. at the same time , the user leaves it to the system in order to choose the exact shape of the trajectory to get the other details such as duration and the velocity profile[5].

Additionally, representation of the trajectories by using computer after the trajectories that have been planned is the big concerned, at the end, computing the trajectory from the generating trajectory or internal representation is a problem that is available.

In the most common case, three parameters have to be computed which are velocity, position and acceleration. Thus, these trajectories are calculated on the digital

computers, so the trajectory points are computed at a certain rate which is called the path-update rate. This rate is located between 60 and 2000 Hz [5].

2.1.3 Generation of Walking Motion

Repetition of one-step motion that is executed in a period of time is considered a walking motion.

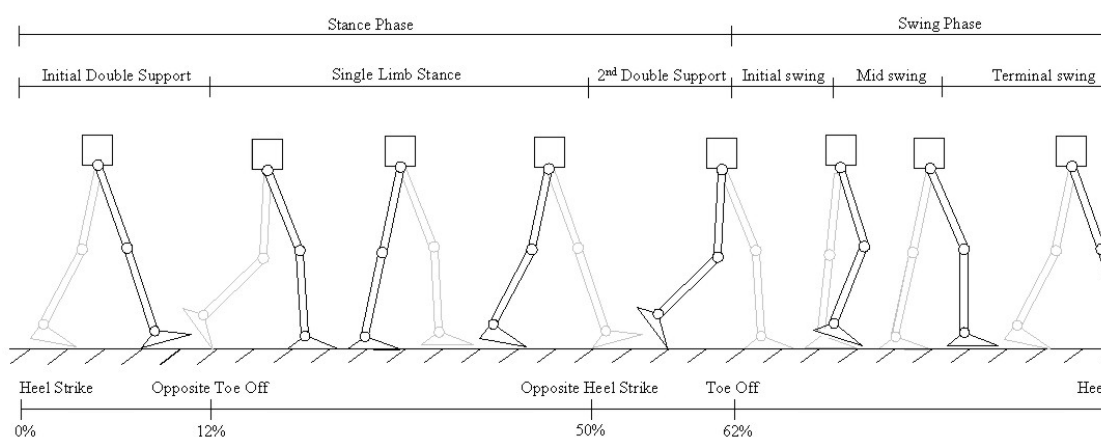


Figure 2. 1: Trajectory walking motion[6].

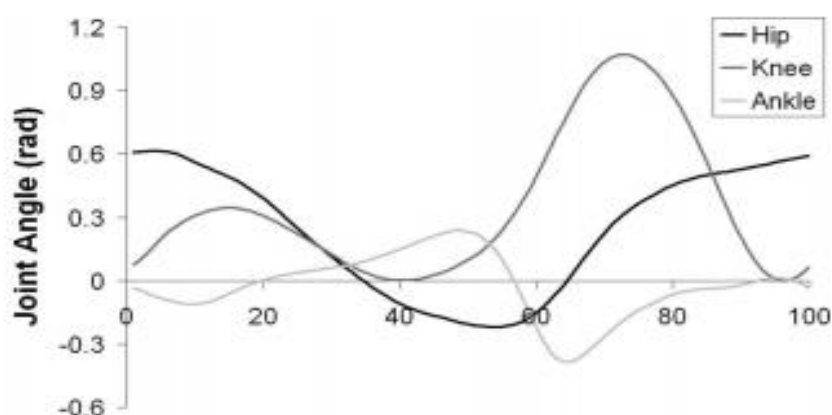


Figure2. 2: Average curves from 7 walking trials (Joint Angle)[7]

The Figure 2.1 shows the phases of the one step and how the step is made during the walking. At the beginning, the foot which is in a movement leaves the ground and raise up, and then extend the step length as desire in order to achieve the goal position, at the end, the foot returns back to the ground to make one step motion. Thus ,there are two phase that can be illustrated during the movement which are single support phase and double support phase .As we can notice from the Figure 2.1 , the single phase can be seen or occurred when one foot on the ground and the second foot is raised up , where the double support phase can be occurred when the foot that is raised during the single phase returns back to the ground[6].

In order to accomplish dynamic walking, the alteration between single supports phase and double supports phase has to be executed smoothly. Controlling the hip and swing foot trajectories in the sagittal and frontal plane can be used to determine the walk sequence of a biped robot. The easiest way to get a solution for the inverse kinematics problem for generating joint trajectories from the walking motion in Cartesian space is by Knowing the hip and swing foot movements [6].

On the other side, definition of gait cycle is the cycle the can be made from heel contact of one foot to the next heel contact of the same foot.

In the gait cycle, there are two parts or two phases which are stance phase and swing phase. So, the gait cycle is almost one second in duration with 60% in stance and 40% in swing. Stance phase is one part of gait cycle that have mentioned; which includes an initial double stance that followed by a period of single stance and final period of double stance. Thus, the single stance is used to indicate the one foot that is in contacted with ground whereas the double stance can be used to indicate both feet that is in contact with ground too. To add to that , During walking, a period of double stance have to be available Although during running, a period of double stance is replaced by a flight phase which the foot is neither in contact with the ground nor not in contact with ground[4] .

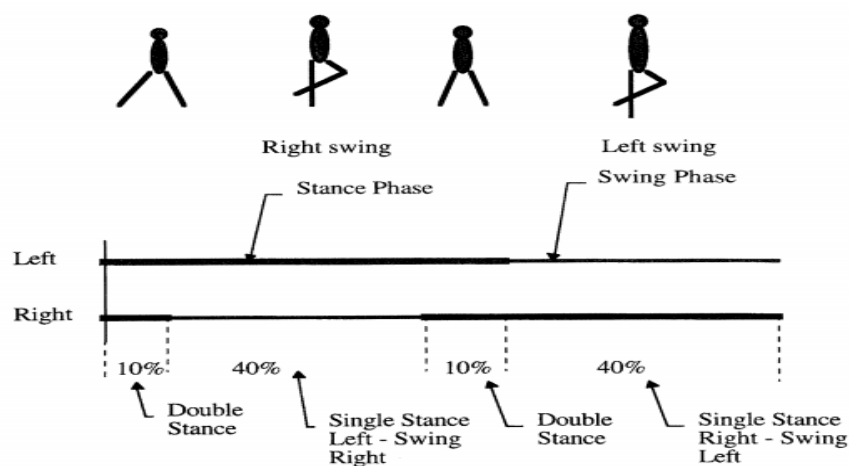


Figure2. 3: The Gait Cycle adaptive from[4]

Figure 2.3 shows walking gait .From that, we can notice that in the early part of stance phase, the heel is actually in contact with the ground, advancing to foot-flat during single stance and then going to the forefoot contact during the final double stance phase ending with toe-off. This gait cycle can be the normal contact areas of the plantar surface of the foot with the ground although it may differ extremely with pathological gait[4] .



Figure 2. 4: Divisions of the Gait Cycle[8]

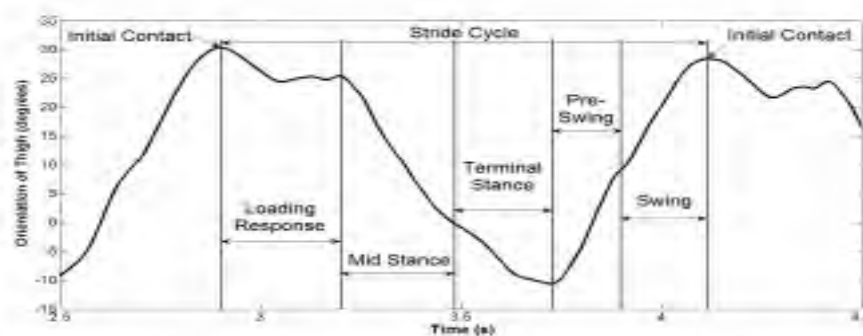


Figure2. 5: Gait phases identified based on thigh angle[8]

Thus, walking or gait movement already has been defined as a series of falls from one limb to the other and it is clearly that the greatest danger of an actual fall is during this period of transferring weight [4].

2.1.3.1 Joint-Space Schemes

Methods of path generation in which the path shapes (in space and in time) are described in terms of functions of joint angles. every path point is usually indicated in terms of a goal position and orientation of the tool frame, {T}, with regard to the station frame, {S} [5].

By using application relative to inverse kinematics, each one of these through points is "transferred" into a set of desired joint angles. Thus, each segment is required the same time for every joint so all joints can be able to reach through the via point at the same time, so the desirable Cartesian position of the tool frame will achieve at each via point .moreover, each joint has its own desired join angle which is independent of the other joints. Thus, join space schemes achieve the desired position and orientation through via points. In addition, the path shape is considered a simple in joint space compared to the Cartesian space due to the easy computing in the join space schemes and also because there is no correspondence between Cartesian space and joint space, there is basically no issue with singularities of the mechanism [5].thus, four items will be illustrated such as ; (a) cubic polynomials (b) Cubic polynomials for a path with via points (c) Higher-order polynomials[a quintic polynomial] (d) Linear function with parabolic blends.

a. Cubic Polynomial

Cubic polynomials are a method of moving the tool from its initial position to a desired position in a definite amount of time. Inverse set of joint angles which correspond to the desired position and orientation can be calculated by using Inverse kinematics. The initial position of the manipulator is also known in the form of a set of joint angles. In addition to that, in the cubic polynomials, acceleration linearly varies with time and is continuous; thus, the trajectory does not require infinite accelerations. Equation 2.1 shows the Four constraints that must be specified such as a_0 , a_1 , a_2 , and a_3 . [5][9]

$$\theta(t) = a_0 + a_1t + a_2t^2 + a_3t^3 \quad (2.1)$$

In addition, the velocity and the acceleration along the path is illustrated in the 2.2 and 2.3

$$\dot{\theta}(t) = a_1 + 2a_2t + 3a_3t^2 \quad (2.2)$$

$$\ddot{\theta}(t) = 2a_2 + 6a_3t \quad (2.3)$$

These four constraints can be categorized as two constraints are related to the initial and final positions and other two are related to the initial and final velocities. Moreover, these four constraints can be determined by using the quantities as shown in Equation 2.4

$$\begin{aligned} a_0 &= \theta_0 \\ a_1 &= 0 \\ a_2 &= \frac{3}{t_f^2}(\theta_f - \theta_0) \\ a_3 &= -\frac{2}{t_f^3}(\theta_f - \theta_0) \end{aligned} \quad (2.4)$$

Thus, the cubic polynomial form [at least third degree] is used to determine these four constraints. Hence, we can calculate the cubic polynomial that connects any initial joint angle position with any desired final position. This solution is used for the case that initial and final velocity of the joint are zero [5].