STABILIZATION CONTROL OF ROTARY INVERTER PENDULUM SYSTEM WITH DOUBLE-PID AND LQR CONTROLLER

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A report submitted in partial fulfillment of the requirements for the degree of Mechatronics Engineering

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2014/2015

" I hereby declare that I have read through this report entitle "Stabilization Control of Rotary Inverted Pendulum with Double-PID and LQR Contrroller" and found that it has comply the partial fulfillment for awarding the degree of Bachelor of Mechatronics Engineering."

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Date : ....

I declare that this report entitle "Stabilization Control of Rotary Inverted Pendulum System with Double-PID and LQR Controller" is the result of my own research except as cited in the references. The report has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

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ACKNOWLEDGEMENT

In preparing this report, I have met with many researchers, academicians and friends. They have contributed towards my understanding and thought. In particular, I wish to express my sincere appreciation to my project supervisor, Dr. Chong Shin Horng, for encouragement, guidance, advice and friendship.

I am also very thankful to researchers in Motion Control Lab, Fakulti Kejuruteraan Elektrik (FKE), Universiti Teknikal Malaysia Melaka (UTeM), Hee Wai Keat and Vasanthan A/L Sakthivelu for their advices and motivation. Without their continued support and interest, this project would not have been same as presented here. Lab assistant of Motion Control Lab and lab assistant of CIA Lab, FKE, UTeM, Khairulddin Hashim and Mohamed Khairy bin Ali should also be recognized for their support.

My sincere appreciation also extends to all my friends who have provided assistance at various occasions. Their views and tips are useful indeed. I am grateful to all my family members for moral support throughout this project.

ABSTRACT

Rotary inverted pendulum (RIP) system is a well-known control system with its non-linearity and underactuated system. There are many applications of RIP system in industry especially the application of balancing of a robot. The RIP system in this project is driven by DC Servo motor. The DC Servo motor is used because it is easy to setup and control, has precise rotation and most importantly is low cost. As for RIP system itself, has smooth motion, not easy to wear out and high mechanical efficiency. However problems will occur when inverted pendulum of the RIP system is required to stable at an upright position. The non-linearity and underactuated characteristics cause the system to be highly unstable when maintaining at upright position. Therefore, the objective of this project is to propose double Proportional-Integral-Derivative (PID) and Linear Quadratic Regulator (LQR) controller to stable the inverted pendulum at an upright position by a rotary arm which is actuated by a DC Servo motor. The performance between the double-PID and LQR controller is compared in order to validate the performance of each controller. Mathematical modelling of the RIP system is carried out to obtain a state space model of the RIP system for ease of designing controller. Designing of double-PID and LQR controller is carried out in two phases. In Phase 1, LQR controller is designed for a basic stabilization control. In phase 2, double-PID controller is designed based on the RIP system with LQR controller for improving the stabilization performance. Throughout all the designing procedure, the stabilization performance of inverted pendulum and settling time are examined and compared. At the end of this project, double-PID with LQR controller was designed successfully to stable the RIP system.

ABSTRAK

Sistem bandul balik yang berputar adalah satu sistem kawalan yang terkenal dengan ketidaklinearan dan sebagai sistem underactuated. Applikasi sistem bandul balik yang berputar dalam industry termasuk menstabilkan robot. Sistem bandul balik yang berputar dalam projek ini adalah digerakkan dengan DC servo motor. DC servo motor digunakan adalah disebabkan penggunaan yang mudah, tepat penggerakan dan kos yang rendah. Masalah dalam sistem kawalan ini berlaku semasa mendirikan bandul balik pada kedudukan yang tegak. Sistem tersebut akan mengalami masalah ketidakstabilan. Justeru, objektif projek ini adalah mencadangkan pengawal Double-PID dan LQR untuk menstabilkan bandul balik tersebut. Sistem bandul balik berputar ini dimodelkan dalam matematika model dengan menggunakan state space model. Terdapat 2 fasa untuk mereka bentuk pengawal Double-PID dan LQR untuk sistem tersebut. Fasa 1 adalah fasa untuk mereka bentul pengawal LQR. Fasa 2 adalah fasa untuk mereka bentuk pengawal Double-PID pada sistem bandul balik berputar yang berdasarkan pengawal LQR yang sedia ada. Melalui prosedur-prosedur rekaan bentuk pengawal, kestabilan sistem bandul balik berputar telah dianalisis untuk mengkaji pengawal Double-PID dan LQR. Dengan itu, pengawal Double-PID dan LQR telah berjaya menstabilkan sistem bandul balik berputar.

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CHAPTER 1

INTRODUCTION

1.1 Research background

The problem of balancing a broomstick in a vertical upright position on a person's hand is well known to the control engineering community. For any person, a physical demonstration of the broomstick-balancing act constitutes a challenging task requiring intelligent, coordinated hand movement based on visual feedback. The instability corresponding to the broomstick vertical upright position leads to the challenge inherent in the problem [1]. Since Furuta's Pendulum established in year 1992, RIP is one of the imperative systems for testing various control technique. It is highly nonlinear, severely unstable, multivariable and an under-actuated system in the field of control theory. It is most helpful for testing self-tuning regulator kind of control technique. Craig *et. al.* designed and built a course "Mechatronic System Design" at Rensselaer. They summarized a mechatronic system design case study for the RIP system [1]. Inverted pendulum system is a typical experimental platform for the research of control theory. The process of inverted pendulum can reflect many key programs, such as stabilization problem, nonlinear problem, robustness, follow-up and tracking program etc [2]. The RIP system consists of an actuator and two degrees of freedom, which makes it under-actuated, only robot arm is being actuated while pendulum is indirectly controlled, so as to result in the balance problem. Segway is one of the most significant succeed research based on control theory of inverted pendulum system. Its ability to self-balance brings development of modern vehicle to a new milestone. The inverted pendulum concept can be applied in control of a space booster rocket and a satellite, an automatic aircraft landing system, aircraft stabilization in the turbulent air flow, stabilization of a cabin in a ship and others [3-4].

1.2 Motivation and significance of research

In this project, the stabilization controllers were proposed and compared by using the RIP system driven by servo motor module as a medium of experiment. The stabilization controllers that proposed in this project are double PID controller and LQR controller. Both designs of controller are compared with some specifications of performances.

1.3 Problem Statement

In order to control the stabilization of the RIP system at upright position driven by DC motor, many different controller approaches have been introduced in global academic researches. All of the approaches aim to achieve a better transient performance, low steady state error and low overshoot condition. The problem arises when there are two outputs giving feedback to control system, tuned signal is giving one input only back to the plant. This Single Input Multiple Output (SIMO) system cannot be controlled by a single conventional PID controller. Furthermore, certain parameter of RIP system is changed when disturbance is given to high non-linear characteristic of the RIP system. The single conventional PID controller cannot adapt to the changes of system parameter that occur on the system. Therefore, tuning multiple outputs of the RIP system at the same time become a challenge throughout the whole project. Double PID with LQR controllers are proposed to solve this SIMO RIP system.

1.4 Objectives

The basic objectives of this project are as follows:

a) To propose a double-PID with LQR controller for stabilization control of a rotary inverted pendulum system.

b) To evaluate the stabilization performance of the rotary inverted pendulum system experimentally.

1.5 Scope

The scope of this project are:

a) The design of the stabilization controller for the RIP system is required to maintain the pendulum at upright position, within ±20º from vertical upright position.

b) After the pendulum is within ±5°, rotary arm of the RIP system is required to maintain moving range within ±22.5°.

c) The RIP system is modelled mathematically using Lagrange's equation.

d) This system with two Degree of Freedom (DOF) is actuated by a DC motor, which categorize this system as underactuated system.

CHAPTER 2

LITERATURE REVIEW

2.1 Theory and Basic Principle of RIP System

Figure 2.1 shows the RIP system manufactured by TERASOFT. The RIP system is an under-actuated system which consists of one actuator and two Degree Of Freedom (DOF). The only actuator in the system is the DC motor. The rotary arm is driven by the DC motor where electrical energy is converted into mechanical energy, the torque to move it. The angular motion of the rotary arm gives energy to the pendulum to swing up and maintain stable at vertical upright position. The pendulum is set to be always perpendicular to the rotary arm. When the pendulum is at vertical upright position, the system is highly unstable, where a controller is needed to achieve stabilization and swing up mechanism of the RIP system. The amplitude of the supply voltage to the DC motor is proportional to the magnitude of the angular displacement of the rotary arm. Thus, the greater the supply voltage to the actuator, the greater the angular displacement of the rotary arm.

The angular displacement of RIP is indirectly moved by the DC motor torque. There are two types of movement mechanism in RIP system, which are swing-up mechanism and stabilize mechanism. In this project, swing-up mechanism is not discussed, position of RIP is assumed to be at upright position as initial condition. Second type of movement mechanism is stabilization mechanism of rotary inverted system which is the motion maintaining the RIP at vertical upright position and avoiding the pendulum falling down in its free fall of nature way.

Figure 2.1: TERASOFT Rotary Inverted Pendulum

2.2 Review of Previous Related Works

There were many researchers studied RIP system from different aspects especially in modeling and designing different controller of the system. For stabilization and swing-up of RIP system, numerous designs of controller approach have been suggested to achieve better stabilization performance. Therefore, the study of research that had been done by other researchers is important to get a rough idea of designing controller in this RIP system.

There are basically divided to classical controller and advanced controller in proposed controllers to stabilize the RIP system. Proportional-Integral-Derivative (PID) controller is one of the most widely used controller in field of control engineering. As the RIP system is an underactuated and non-linear system, PID controller is common to be designed in the RIP system, as it improves overshoot percentage and steady state error of the system with an easy approach. PID controller can be used although mathematical model of the system is not known. When the mathematical model is not known, Ziegler-Nichols rules can be applied. Ziegler Nichols tuning rules give an educated guess for the parameter values and provide a starting point for fine tuning. Thus, from year 2009 to 2012, 2DOF PID or Double-PID was designed as a controller in the RIP system [6,8,11,13,15]. As a stabilization controller in the RIP system, the controller stabilized the inverted pendulum and as well as the rotary arm.

For advance controller, hybrid strategy was applied on the controllers. Combining two types of controller into the RIP system was considered in designing controller procedure. The advanced controllers that were used in hybrid strategy in previous works are including Full State Feedback controller, fuzzy logic controller and Double-PID with LQR controller.

Full State Feedback (FSF) Controller

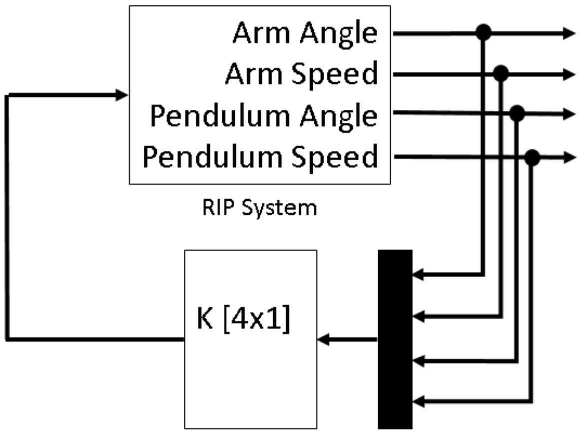

Figure 2.2: Full State Feedback/LQR Controller Block Diagram

Figure 2.2 shows a block diagram of a Full State Feedback (FSF) controller in the RIP system. The RIP system is was designed in state space model. Poles of the closed loop system may be placed at any desired locations by means of state feedback through an appropriate state feedback gain matrix *K*. There are a few approach to tune FSF controller, one of them is by pole placement method. According to M. Akhtaruzzaman [5], he designed FSF controller by placing stable poles of the RIP system, then used Ackermann's formula and Integral of Time-weighted Absolute Error (ITAE) table, state feedback control gain matrix, K which is a 4×1 matrix was obtained. FSF controller was considered relatively ease of design and effective procedure to obtain the gain matrix K. There are some drawbacks of designing FSF controller. It requires successful measurement of all state variables or a state observer in the system, where it needs control system design in state space model. It also requires experienced researcher to determine the desired closed loop poles of the system, especially when the system has a higher order system than second or third order.

Proportional-Integral-Derivative (PID) Controller

Figure 2.3 shows a double-PID controller instead of single conventional PID controller, it was because single PID controller can control only one variable of the system. In fact, rotary inverted pendulum system has one input and two outputs which single PID is incapable to control the system.

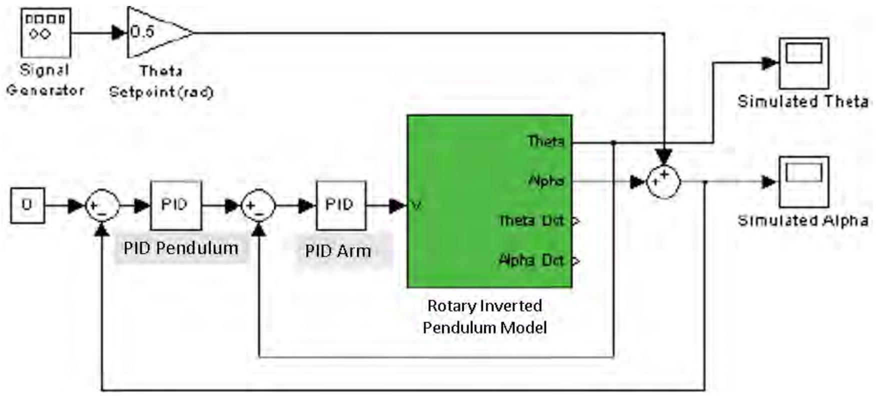

Figure 2.4: 2DOF PID Controller [5]

According to Figure 2.3, 2DOF PID was arranged cascaded. PID Arm was to maintain the rotary arm as zero, while PID pendulum maintained the speed and position of pendulum to remains stable [5]. PID Arm was tuned first then following by the PID Pendulum. Root locus analysis was used to tune the both PID.

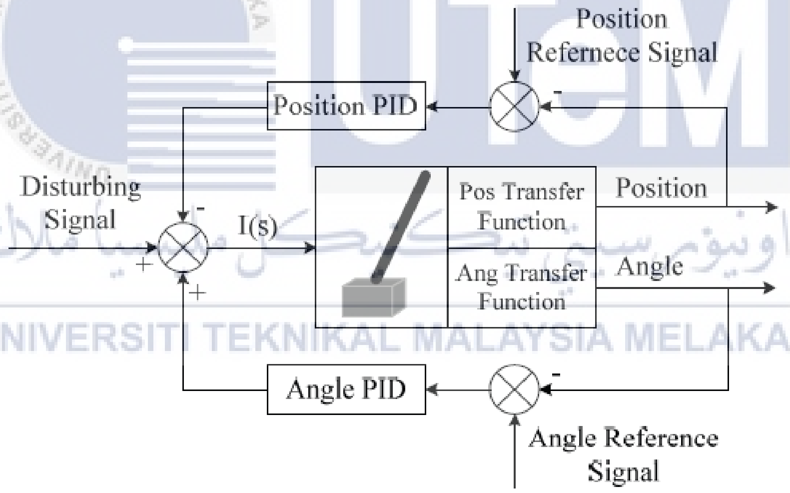

Figure 2.4: Model of cart inverted pendulum system with PID controller

The performance of double-PID in stabilization of RIP system was not satisfactory as its dynamic responses is not robust. Thus, researchers suggested to implement hybrid strategy, combining LQR controller with double-PID controller as stabilization controller.

Before suggesting double-PID controller in year 2011, another controller was also proposed for the RIP which is adaptive PID with sliding mode control [6]. In this paper, the researchers use sliding mode control to handle the nonlinear time varying part, and designed an adaptive law to tune up the system parameters online. The parameters of the PID controller were adjusted online by adaptive law. Unfortunately, the RIP control system possessed control difficulty with its non-linear and instable system. The design of the sliding mode controller took two steps, firstly, the sliding function was determined and then the control law was derived. The results showed that the proposed adaptive PID sliding mode controller has succeeded to make the system stable and robust effectively. The main advantage of the proposed control method is that the PID controller gains can be obtained online and converge efficiently.

In year 2011, from the paper [7-8], the double-PID was suggested as a stabilization controller in RIP system. The fundamental of the design of double-PID controller is taking position of the cart and angle of the pendulum rod as signals for feeding back to the system as shown in

In paper [9], Gan designed a composite controller according to the characteristics of LQR and PID controller to get a faster control speed and better effect of dynamic balance. The results show that the control algorithm combined by LQR and PID can obtain a good balance effect and has a good anti-disturbance effects, which can restore dynamic fast. For the PID controller, it can control robot balance but the robot vibrate larger in the vicinity of balance point, the static performance is poor [9]. For the LQR controller, it has a good control effect in small-angle scope, but, for larger disturbance, then the angle beyond the linearization constraint conditions, the LQR controller do not have good control effect, even cannot keep robot balance [9].

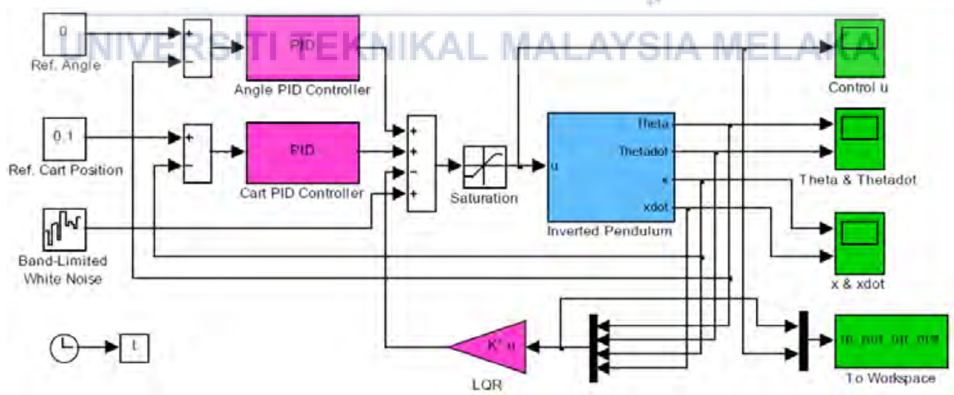

Figure 2.5: Double PID & LQR Control of Nonlinear Inverted Pendulum System [7]

Figure 2.5 shows the control structure of a double PID and LQR controller in [7]. LQR used state variables as the feedback variables in order to increase the stability of the system and obtain the desired system response. The optimal control value of LQR was added negatively with PID control value to have a resultant optimal control.

Linear Quadratic Regulator (LQR) Controller

By using modern control theory, the design of LQR controller is started by considering the first-order mathematical model of inverted pendulum system. By using linear quadratic control theory to design the control current signal, the pendulum rod can achieve a stable equilibrium point at upright position. According to the LQR optimal control law, its optimality is totally depended on the selection of Q and r, whereby Q and r are weighting matrixes that penalize certain states and control inputs of the system. The widespread method used to choose Q and r is by means of simulation and trial [10].

In paper [9], LQR controller is designed into a two-wheeled self-balancing robot. The results showed that an inverted pendulum system is unable to achieve the dynamic balance of the robot under the control of LQR. The limitations of LQR in a system are system modelling must be accurate and physical system parameters must be strict. In order to get faster control speed and better effect of dynamic balance, a composite controller is design according to characteristics of LQR and PID controller. Experiment summary in [9], the control algorithm combined by LQR and PID can obtain a good balance effect and has a good anti-disturbance effects and can restore dynamic fast.

In paper [11], the value of gain K is obtained by using the function *lqr* in MatLab. Furthermore, Taguchi Methods can be used to tune the gain K. The Taguchi methods are popular design of experiment (DOE) methods used in industry [11]. Due to the four gains in K, an orthogonal array that can include at least four factors needed to be used and L9 orthogonal is chosen for tuning the gain K because this array allows four three-level factors at most [11]. In the conclusion [11], the L9 orthogonal array in Taguchi methods can tune the feedback gain of the controller efficiently, which means that the RIP system able to stable at upright position. Yet, from A. Khashayar [12], the LQR controller could not set the pendulum position to zero degree, the reference angle at upright position, when it is at non zero initial condition. Thus, there were researchers suggesting to implement a double-PID controller into the system.

Fuzzy Logic Controller

Fuzzy logic control theory is very useful for systems with complicated structures such as non-linear and unstable of RIP system. The elements of fuzzy logic controller includes fuzzification, rule-based, inference mechanism and defuzzification. In paper [12], the fuzzy controller should use the real state variables, which are angular displacement of rotary arm, angular velocity of rotary arm, angular displacement of pendulum and angular velocity of pendulum, that are read by two encoders in the system. The fuzzy logic controller requires experts' previous experience about the operation of the real RIP system to create suitable experimental rules. The deviation on the angular displacement of pendulum from the reference angle is one of the input to the fuzzy logic controller, while the second input to the controller is angular velocity of the pendulum. These two inputs help the controller to diagnose how the motor decrease the angular position error to zero with rotating the rotary arm clockwise and anticlockwise repeatedly [13].

In paper [14], stabilization of RIP system is achieved by mapping linear optimal control law to the fuzzy inference system (FIS). A Mamdani FIS is designed which stabilizes the pendulum in the linear zone, emulating LQR control around the equilibrium point. The linear state feedback law is mapped to the rules of the fuzzy inference engine. The system consists of two fuzzy inference subsystems, one taking as inputs the angular position and speed of the pendulum arm, and other one taking as inputs the angular position and speed of the pendulum. The two output signals from both subsystems are then added to give a single control signal [14]. In the experiment, the observed performance of the system is smooth, and it is experimentally shown that the closed loop balancing system based on the fuzzy controller exhibits greater robustness to unmodeled dynamics and uncertain parameters than the LQR controller that it emulates [14]. Fuzzy logic controller requires complex linguistic expression. The linguistic expression which are the basic of the rule-base of the fuzzy logic controller must generated based on experts' whom have done many relevant experiments.

2.3 Summary and Discussion of the Review

In summary, the double-PID and LQR controller for stabilization control of RIP system are chosen in this project. The reason of proposing this design of controller is because, firstly, the PID controller is a common controller that used as in many applications, it has the effect of reducing overshoot and improving of transient response of a system. Secondly, the LQR controller has a good control effect with an easy and effective way of obtaining optimal feedback gain for RIP system stabilization.

CHAPTER 3

MODELLING AND CONTROLLER DESIGN

3.1 Experimental Setup

The RIP system setup requires two categories, which are host computer and Terasoft Electro-Mechanical Engineering Control System (EMECS). The EMECS is including three main components which are MicroBox 2000/2000C, servo-motor module, driver circuit and power supply. MicroBox 2000/2000C is a data acquisition unit to receive and send the signal via Ethernet cable connection with host computer. At the other end, the data acquisition unit interfaces with signal of rotary encoder of the system which has amplified by the driver circuit. According to Figure 3.1, the host computer is connected with each components in the system.

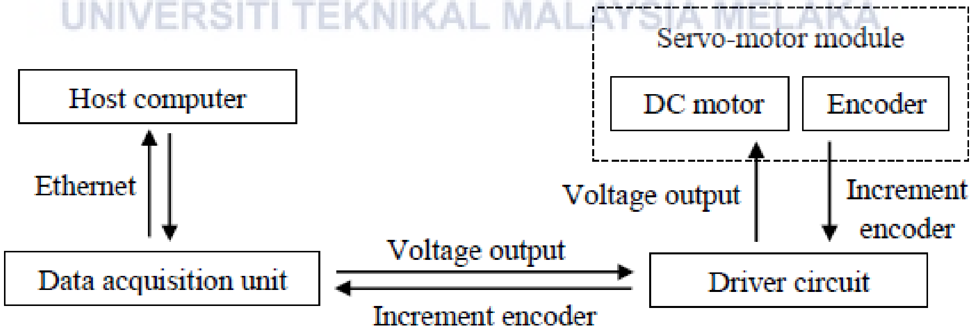

Figure 3.1: Connections between EMECS and host PC

The pendulum movement is actuated by the motor rotor indirectly where rotary arm is the connection link in between. The rotation of motor rotor result in angular displacement and its derivative, angular velocity of rotary arm. Then the change of the angular displacement and velocity each time the rotary arm moves, the kinetic energy and potential energy of the pendulum change instantly. The angular displacements of both rotary arm and pendulum are read by optical encoders which converts the analog displacement into digitized pulse signal. The digitize pulse signal is a voltage signal feeding into the MicroBox and the host PC.

The Electro-Mechanical Engineering Control System (EMECS) plant shown in Figure 3.2 was set up.

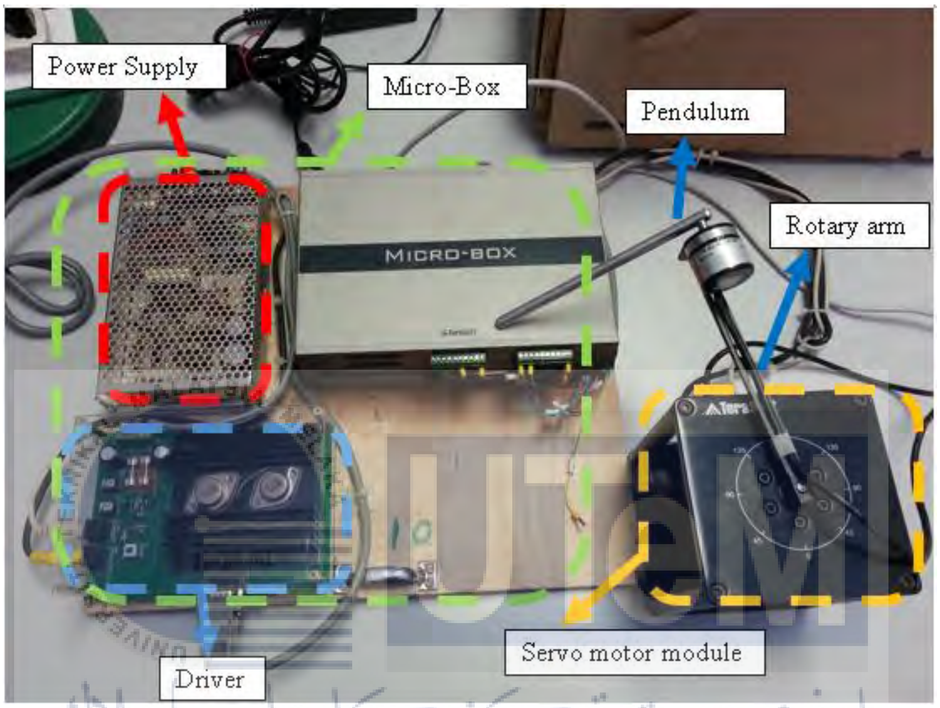

Figure 3.2: EMECS Plant

As to control the pendulum movement, polarity of the connection of servo motor module is considered. The different polarity of the connection of DC motor will result in different direction of rotation, either clockwise or anticlockwise. By interfacing between Simulink and the MicroBox, host PC can monitor all the signals sending from the MicroBox. All the signals are in form of digitized pulses which generated by the two rotary incremental encoders. Pin connections at MicroBox enable reading and sending digital or analog signals with the system. All the connections pin are shown in Appendix.

3.2 Modeling of RIP system

The study of system dynamics resides in modelling its behaviour. Systems models are simplified, abstracted structures used to predict the behaviour of the studied systems. Our interest is pointing towards the mathematical model used to predict certain aspects of the system response to the inputs. In mathematical notations a system model is described by a set of ordinary differential equations in terms of state variables and a set of algebraic equations that relate the state variable to other system variables [15].

Parameter of RIP System

The RIP system is an electro-mechanical engineering control system. The involved parameters consists of electrical and mechanical parameters. Table 3.1 shows the parameters of the whole system.

Table 3.1: System parameters for RIP system

Parameter	SI unit	Symbol	Numerical Value
Mass of arm	kg	m_1	0.056
Mass of pendulum	kg	m_2	0.022
Length of arm	m	l_1	0.16
Length of pendulum	m	l_2	0.16
Distance to centre of arm mass	m	c_1	0.08
Distance to centre of pendulum mass	m	c_2	0.08
Inertia of arm	kgm^2	J_1	0.00215058
Inertia of pendulum	kgm^2	J_2	0.00018773
Gravitational acceleration	m/s^2	g	9.81
Angular position of arm	°	α	-
Angular velocity of arm	rad/s	$\dot{\alpha}$	-
Angular position of pendulum	°	β	-
Angular velocity of pendulum	rad/s	$\dot{\beta}$	-
Viscous friction co-efficient of arm	kgm^2/s	C_1	0
Viscous friction co-efficient of pendulum	kgm^2/s	C_2	0
Motor torque constant	Nm/A	K_t	0.01826
Motor back-emf constant	Vs/rad	K_b	0.01826
Motor driver amplifier gain	V/count	K_u	850
Armature resistance	Ω	R_m	2.56204
Armature inductance	L	L_m	0.0046909

DC Motor Characteristics

According to Figure 3.3, a voltage signal is generated and it is supplied to a PWM amplifier which drives servo-motor to control the rotary arm and indirectly to pendulum. After Kirchhoff's voltage law is applied, the equation are as follows:

$$V_m = I_a R_a + L_a \frac{dI_a}{dt} + E_b \tag{3.1}$$

The motor back EMF, E_b, is proportional to the rate of change of magnetic flux and hence proportional to the angular velocity of the motor.

$$E_b = K_b \frac{d\alpha}{dt}$$
$$E_b = K_b \dot{\alpha} \tag{3.2}$$

For a constant field current, the torque exerted by the motor, τ_m, is proportional to the armature current. Assuming that the effects of the coil inductance, L_a, are negligible.

The torque can be written as:

$$\tau_m = K_t I_a$$
$$\tau_m = \frac{K_t(V_m - E_b)}{R_a}$$
$$\tau_m = \frac{K_t K_b}{R_a} u - \frac{K_t K_b}{R_a} \dot{\alpha} \tag{3.3}$$

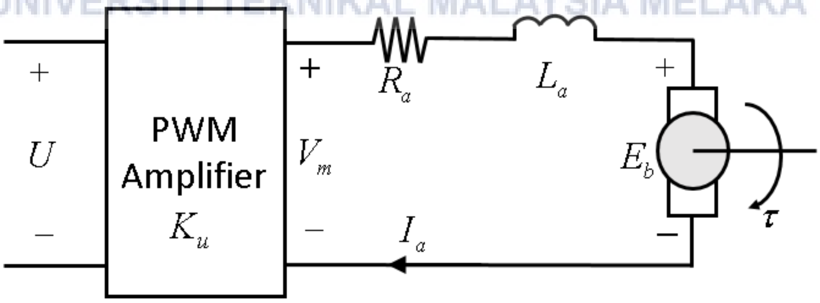

Figure 3.3: Schematic diagram of DC motor of RIP system

Coordinate System

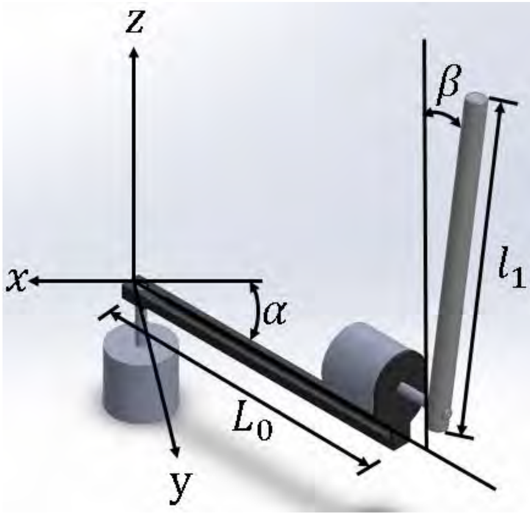

Figure 3.4: Mechanical Model of RIP

Figure 3.4 shows the coordinate system used for the derivation of the dynamic model. The RIP system consists of two parts, namely rotary arm and pendulum rod. The pendulum is rotating freely in a vertical plane with the objectives of swinging up and balancing the pendulum in the inverted position and it is attached to the rotary arm that is mounted on the shaft of the servo-motor. Thus, the rotary arm can be rotated horizontally in its plane by the servo motor. Optical encoders are installed on the rotary arm and pendulum to detect the angular displacement. The standard right-handed Cartesian co-ordinate system is used. The angular position of the arm α is assigned to be increasing when the rotary arm is rotating about the z-axis in the right handed sense (Right-Hand-Grip Rule). The angular position of the pendulum, β and α, are assigned to be increasing when the pendulum is rotating about an axis passing through the arm section from the origin to the pivot point of the pendulum, in the right handed case. The reference of β is taken from the upward vertical.

Lagrange's Equation of Motion (Rotary Systems)

The following is Lagrange's equation of motion. It will be used in the derivation of the dynamic model [16].

General Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\Theta}_i}\right) - \frac{\partial L}{\partial \Theta_i} = Q_i \qquad (3.4)$$

$$L = L_0 + L_1 \tag{3.5}$$

$$L_0 = T_0 - V_0 \tag{3.6}$$

$$L_1 = T_1 - V_1 \tag{3.7}$$

where,

T = the total kinetic energy of rotating body

V = the total potential energy of rotating body

Θ_i = angular position of body about axis i.

Q = the total torque applied at axis i

Non-linear Dynamic Model

The non-linear dynamic model describes the system by giving the exact relationships among all the variables involved. As shown in Appendix A, the parameters of corresponding symbols represent, the dynamic model with the pendulum, with motor torque characteristic, in the upright position is:

$$[A]\begin{bmatrix}\ddot{\alpha}\\ \ddot{\beta}\end{bmatrix} = [D]u - [B]\begin{bmatrix}\dot{\alpha}\\ \dot{\beta}\end{bmatrix} - [C] \tag{3.8}$$

Matrix A is inertia matrix of the system, matrix B represents Coriolis and gyroscopic of the system. While Matrix C represents gravity terms in Cartesian space of the system, and Matrix D is the torque on the end-effector of the pendulum. Thus, from Equation 3.9, the RIP system nonlinear state space model is:

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}\ddot{\alpha}\\ \ddot{\beta}\end{bmatrix} = \begin{bmatrix}j\\ 0\end{bmatrix}u - \begin{bmatrix}e & f\\ g & h\end{bmatrix}\begin{bmatrix}\dot{\alpha}\\ \dot{\beta}\end{bmatrix} - \begin{bmatrix}0\\ i\end{bmatrix} \tag{3.9}$$

where,

$$a = J_0 + m_1 L_0^2 + m_1 l_1^2 \sin^2\beta$$

$$b = -m_1 L_o l_1 \cos\beta$$

$$c = -m_1 L_o l_1 \cos\beta$$

$$d = J_1 + m_1 l_1^2$$

$$e = C_0 + \frac{K_t K_b}{R_a} + \frac{1}{2} m_1 l_1^2 \dot{\beta} \sin 2\beta$$

$$f = -m_1 L_o l_1 \dot{\beta} \sin\beta + \frac{1}{2} m_1 l_1^2 \dot{\alpha} \sin 2\beta$$

$$g = -\frac{1}{2} m_1 l_1^2 \dot{\alpha} \sin 2\beta$$

$$h = C_1$$

$$i = m_1 g l_1 \sin\beta$$

$$j = \frac{K_t K_u}{R_a}$$

Linearized RIP Model

The model is then linearized by expanding the nonlinear model into a Taylor Series about the operating point and the retention of only the linear terms. The model is linearized about the upright position of the pendulum, whereby, the pendulum is at static, angle and velocity of pendulum, β and $\dot{\beta}$ are zero, and the rotary arm is not moving as well, velocity of arm, $\dot{\alpha}$ is zero. Equation 3.10 can be linearized and summarized in a dynamic equations below,

$$\begin{bmatrix} \dot{\alpha} \\ \ddot{\alpha} \\ \dot{\beta} \\ \ddot{\beta} \end{bmatrix} = \frac{1}{af - c^2} \begin{bmatrix} 0 & af - c^2 & 0 & 0 \\ 0 & -df & ch & -cC_1 \\ 0 & 0 & 0 & af - c^2 \\ 0 & -cd & ah & -aC_1 \end{bmatrix} \begin{bmatrix} \alpha \\ \dot{\alpha} \\ \beta \\ \dot{\beta} \end{bmatrix} + \frac{1}{af - c^2} \begin{bmatrix} 0 \\ ef \\ 0 \\ ce \end{bmatrix} u \qquad (3.10)$$

where,

$$a = J_0 + m_1 L_0^2 \qquad d = C_0 + \frac{K_t K_b}{R_a} \qquad h = m_1 g l_1$$

$$b = m_1 l_1^2 \qquad e = \frac{K_t K_u}{R_a}$$

$$c = m_1 L_o l_1 \qquad f = J_1 + m_1 l_1^2$$

By having values of all the parameters, the equation is rearranged:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \ddot{\alpha} \\ \ddot{\beta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 5.98 & -0.05267 & 0 \\ 0 & 57.68 & -0.04514 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 28.84 \\ 24.72 \end{bmatrix} u \qquad (3.11)$$

$$y = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + [0] u$$

Equation 3.11 is the mathematical state space model of the RIP system. Design of LQR controller requires state space model of the system for states vector feedback to LQR controller itself.

3.3 Controller Design Procedure

As mentioned in the literature review, Double-PID with LQR controller is chosen to achieve stabilization of the system. With the state space model of the system, LQR controller is designed then followed by Double-PID controller.

<u>Phase 1: LQR Controller Design</u>

This phase of the project aims to design a LQR controller to achieve stabilization of the RIP system.

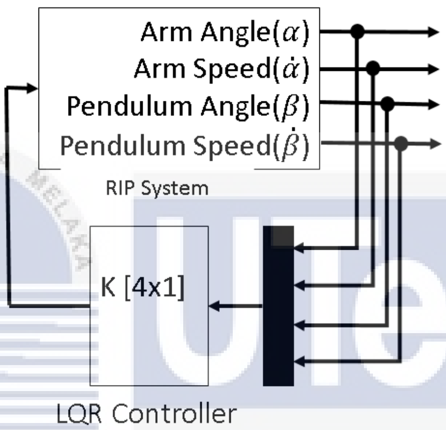

Figure 3.5: Block diagram of LQR controller in RIP system.

Figure 3.5 shows block diagram of the system with LQR controller. To design LQR controller, vector of optimal state-feedback control gains K is required to be determined. Weighting matrices of Q and R are assumed as the error matrix (deviation from reference angle) and control efforts of the system respectively.

$$A^{*}P + PA - PBR^{-1}B^{*}P + Q = 0 \tag{3.12}$$

Equation (3.12) is the reduced-matrix Riccati equation. Matrices A and B are the matrix constant of the state vector and the matrix constant of the control signal respectively. The equation is solved to obtain a positive definite matrix P. Matrix P is used to solve the following equation:

$$K_{ref} = R^{-1}B^{*}P \tag{3.13}$$

The optimal gain, K_{ref} is obtained in Equation (3.13). The optimal gain, K_{ref} is a [4×1] matrix which takes output states of the RIP system as input, including the angle and velocity of the pendulum $(\beta, \dot{\beta})$, and angle and velocity of the rotary arm $(\alpha, \dot{\alpha})$. Then, voltage control signal will minimize the error and increase the control effort of the system.

In this phase, the desired specification for the LQR controller to achieve is:

1. The inverted pendulum stabilizes within $\pm 3°$.
2. Settling time of the RIP system is within 15 seconds. The settling time, is the time taken for the inverted pendulum to stabilizes within $\pm 3°$.

A ground-tuned optimal gain value of K_{ref} is obtained by solving Equation (3.13). To achieve stabilization performance, K_{ref} is tuned by using Taguchi Method of Design of Experiments.

Tuning via Taguchi Method of Design of Experiments

Design of Experiments (DOE) is a statistical technique used to study the effects of multiple variables simultaneously. Taguchi Method is a quality engineering method. Taguchi Method is an experimental strategy in a form of DOE with special application principles. L9 orthogonal array of Taguchi Method is used to tune K_{ref}. L9 orthogonal array is used to replace a full factorial experiment with four-three level factors which requires 81 different combinations of the gains. Optimal gain, K is determined based on the stabilization performance.

Table 3.2: L9 Orthogonal Array of Taguchi Method

Trial \ Data Set	*K1*	*K2*	*K3*	*K4*
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

Data set 1: 110% of optimal gain, K_o
Data set 2: Original values of optimal gain, K_o
Data set 3: 90% of optimal gain, K_o

The LQR controller is designed when the optimal gain, K is determined. However, LQR controller alone is not sufficient to stabilize the RIP system. LQR controller cannot solves overshoot and settling time problem. With this remark, a Double-PID controller is designed in the LQR-RIP system.

Phase 2 Double-PID with LQR Controller Design

Double-PID with LQR controller aims to improve the stabilization performance of the settling time of the angle of pendulum (beta) and the moving range of rotary arm (α). Double-PID controller is start with designing PID_alpha. Figure 3.6 shows the block diagram of the RIP system with stabilization controller.

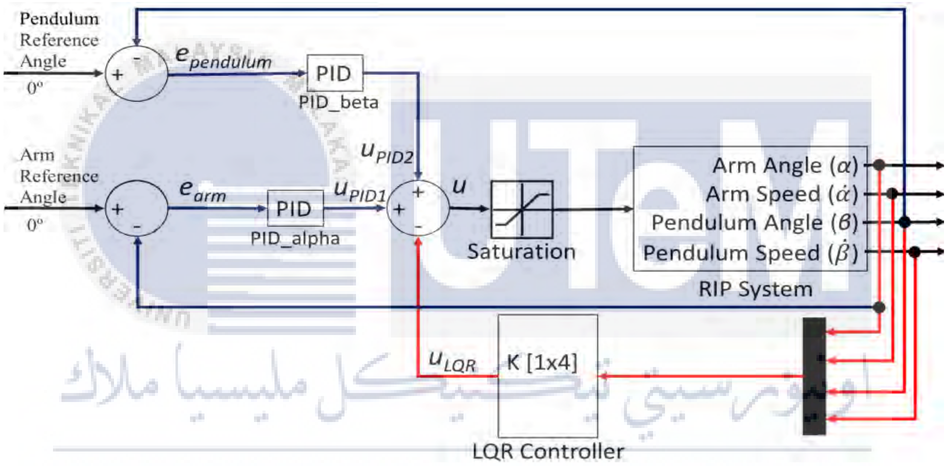

Figure 3.6: Block diagram of Double PID and LQR controller

PID_alpha Controller Design

Based on Figure 3.6, the PID_alpha takes the error signal of arm angle (α) as input and gives the voltage control signal together with voltage control signal from LQR controller. PID_alpha aims to stabilize the rotary arm in a narrower range of movement of arm and control the rotary arm back to the initial position.

The Ziegler-Nichols Second Method is chosen to tune PID controllers. The LQR-RIP system is driven with a proportional gain to obtain an ultimate gain, K_u, where sustained oscillations occur. The period of the sustain oscillations, T_u, is measured. With the ultimate gain, K_u and period of the sustained oscillations, T_u, the values of the PID_alpha controller, $K_{p\alpha}$, $K_{I\alpha}$, and $K_{D\alpha}$ are calculated.

Table 3.3: Ziegler-Nichols Tuning Rule of PID Controller (Second Method)

Type of Controller	K_p	K_I	K_D
P	$0.5K_u$	∞	0
PI	$0.45K_u$	$\frac{1}{1.2}T_u$	0
PID	$0.6K_u$	$0.5T_u$	$0.125T_u$

The calculated PID gains ($K_{p\alpha}$, $K_{I\alpha}$, and $K_{D\alpha}$) are fined tuned to achieve stabilization performance of rotary arm (α). Next, the second PID of Double-PID controller is designed.

PID_beta Controller Design

The second PID controller, PID_beta, aims to stabilize the inverted pendulum (β) within $\pm 3°$ in shorter time. PID_beta is fined-tuned as referred to the gains of PID_alpha controller.

With both PID_alpha and PID_beta, the angle movement of α and β of the RIP system are examined to evaluate the stabilization performance. The stabilization performance is observed and analyzed.

CHAPTER 4

RESULT OF STABILIZATION PERFORMANCE

4.1 Stabilization Performance of RIP System with LQR Controller

Matrix Q is obtained by using trial and error method. Limitation of the RIP system setup is maximum voltage is $\pm 10v$. The sum of parameters of optimal gain K_{ref} is obtained around 10, in order to reduce wastage of energy. Thus, the parameter values of optimal gain K_{ref} is tuned by varying elements inside matrix Q. Matrix R is obtained by assuming it as [1].

$$Q = \begin{bmatrix} 50 & 0 & 0 & 0 \\ 0 & 26.9 & 0 & 0 \\ 0 & 0 & 55 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R = [1] \tag{4.1}$$

With matrix Q and R, an optimal gain values, K_{ref} is obtained.

$$K_{ref} = [K1\;\; K2\;\; K3\;\; K4] = [-0.48295 \quad 9.7741 \quad -0.6405 \quad 1.3495] \tag{4.2}$$

To further improve the quality of the stabilization performance of final selected optimal gain, nominal values for the design parameter variables, which are α, $\dot{\alpha}$, β, and $\dot{\beta}$, that yield the lowest impact on the RIP system's stabilization performance characteristic is determined.

A$\pm 10\%$ is added to the 4 arrays of optimal gain, K_{ref}. L9 orthogonal array is chosen to select the combinational number of the 4 arrays. Table 4.1 shows the arrangement of selected data set in L9 orthogonal array.

Table 4.1: Trials of values K using Taguchi Method

Trial \ Data	K_1	K_2	K_3	K_4
1	-0.5312	10.7515	-0.7046	1.4844
2	-0.5312	9.7741	-0.6405	1.3495
3	-0.5312	8.7967	-0.5765	1.2145
4	-0.4830	10.7515	-0.6405	1.2145
5	-0.4830	9.7741	-0.5765	1.4844
6	-0.4830	8.7967	-0.7046	1.3495
7	-0.4347	10.7515	-0.5765	1.3495
8	-0.4347	9.7741	-0.7046	1.2145
9	-0.4347	8.7967	-0.6405	1.4844

Table 4.1 shows the values of gain matrix K of each trial. There are total 9 trials were carried on to test the stabilization performance.

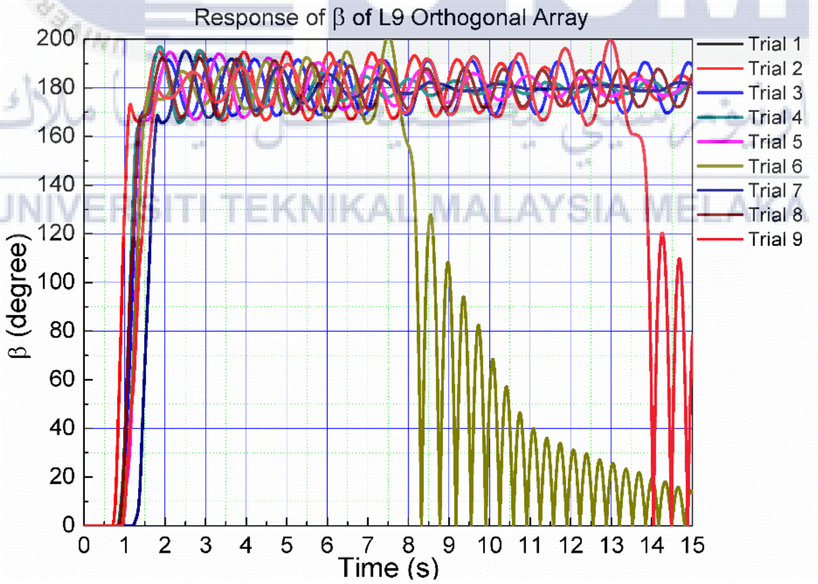

Figure 4.1: Response of β of all 9 trials of Taguchi Method

Figure 4.1 shows the quality of all the 9 trials of Taguchi Method. Out of 9 trials, Trial 6 and Trial 9 were not performing stabilization performance in the RIP system. Trial 7 has the best stabilization performance. The responses of each trial are shown in Appendix A. Thus, L9 orthogonal array is said to be satisfactory to inspect the quality of the gain matrix, K.

After testing all the trials, the stabilization performance of the inverted pendulum (β) and rotary arm (α) are observed. From all the 9 trials of Taguchi Method, the 7th trial shows a better result in stabilization performance of the RIP system. The optimal gain, K of 7th trial is:

$$K = [-0.4347 \quad 10.7515 \quad -0.5765 \quad 1.3495] \tag{4.3}$$

Figure 4.2 shows that the inverted pendulum (β) stabilizes within $\pm 3°$ in 8 seconds. The inverted pendulum oscillates in high overshoot percentage before it becomes stable. The rotary arm (α), it is not stable within $\pm 22.5°$. In short, the LQR controller alone is not sufficient to stable the inverted pendulum (β) and the rotary arm (α). It can be concluded that the Double-PID controller is needed to work together with the LQR controller in order to achieve a faster and more stable of stabilization performance.

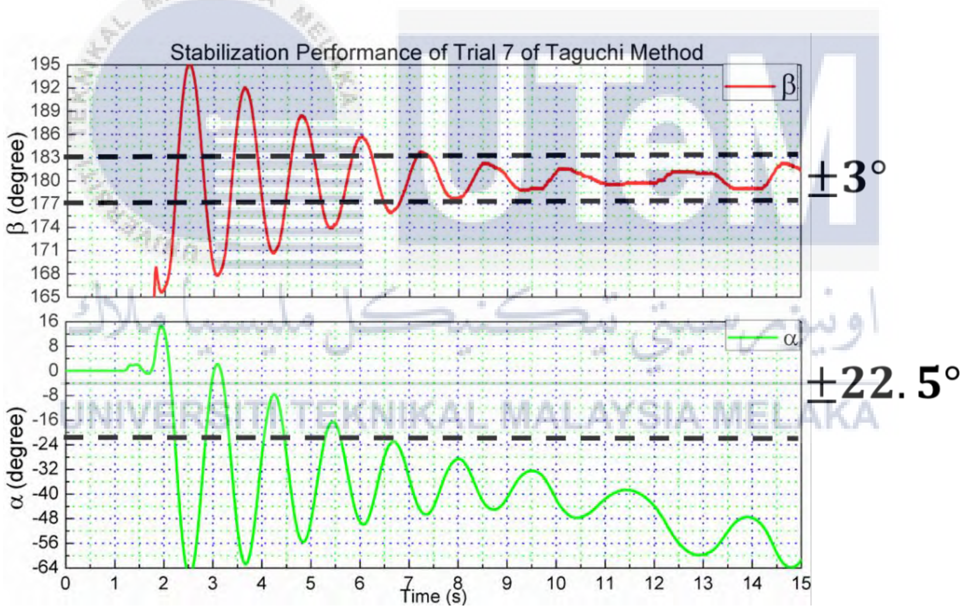

Figure 4.2: Response of beta and alpha of the RIP system with LQR controller

4.2 Stabilization Performance of RIP System with Double-PID with LQR Controller

As the unstable problem of rotary arm cannot be solved by LQR controller alone, Double-PID is designed in the LQR-RIP system. Double-PID is designed started with PID_alpha.

Design of PID_alpha Controller

The LQR-RIP system is driven with a proportional gain. After a few times of trial and error, the ultimate gain, K_u and the period of the sustained oscillations, T_u are obtained. Figure 4.3 shows the sustained oscillations when $K_u = 0.00007$ and $T_u = 1.25s$.

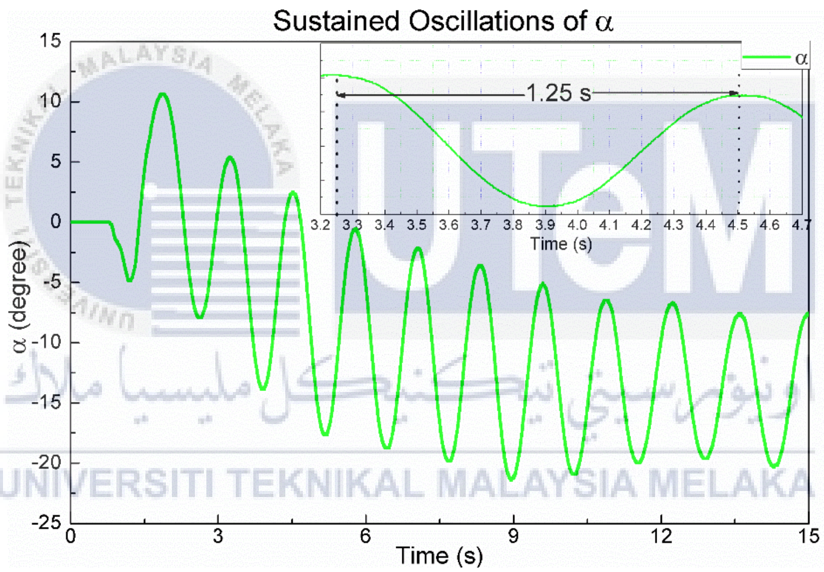

Figure 4.3: Response of rotary arm (α) when sustained oscillations occur with K_u=0.00007

Table 4.2 shows the value of $K_{p\alpha}$, $K_{I\alpha}$, and $K_{D\alpha}$ which are calculated by using Ziegler-Nichols Second Method.

Table 4.2: Ground tuned of PID_alpha Controller

Ziegler-Nichols Tuning Rule of PID_alpha Controller (Second Method)	
$K_{p\alpha}$	$0.6K_u = 0.000042$
$K_{I\alpha}$	$0.5T_u = 0.625$
$K_{D\alpha}$	$0.125T_u = 0.15625$

PID_alpha is fined-tuned to obtain stabilization of rotary arm (α). $K_{p\alpha}$, $K_{I\alpha}$, and $K_{D\alpha}$ are obtained as shown in Table 4.3.

Table 4.3: Fine Tuned of PID_alpha Controller

Final selection values of PID_alpha Controller	
$K_{p\alpha}$	0.000042
$K_{I\alpha}$	0.0000015
$K_{D\alpha}$	0.00001

As compared with the stabilization performance of LQR controller in Figure 4.2, the rotary arm (α) successfully stabilizes within$\pm 22.5°$. Figure 4.4 shows the stabilization performance of rotary arm (α). However, the inverted pendulum (β) is still not stable and it causes the rotary arm (α) becoming unstable also. Thus, PID_beta controller is designed.

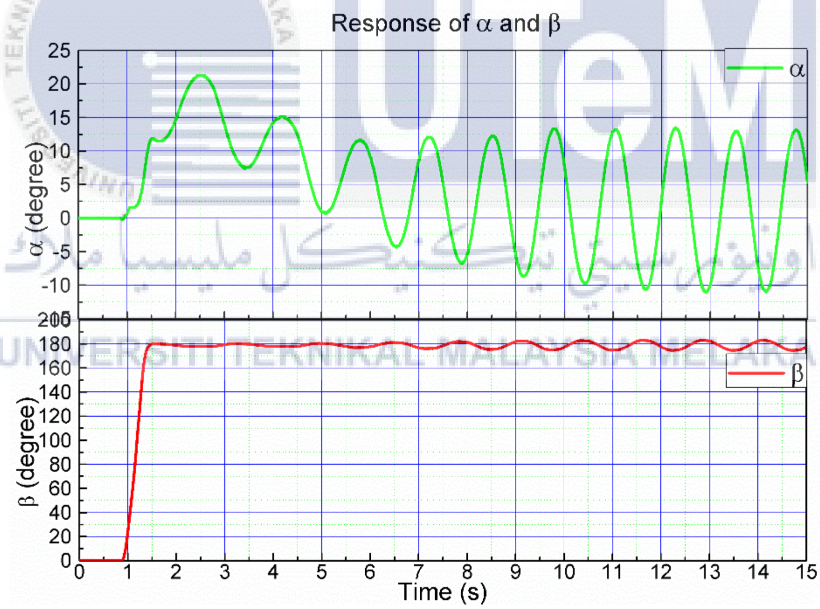

Figure 4.4: Response of rotary arm (α) and pendulum (β) after implementing PID_alpha into the RIP system

<u>Design of PID_beta Controller</u>

PID_beta controller is designed by referring to the value of PID_alpha. Initial values of $K_{p\beta}$, $K_{I\beta}$, and $K_{D\beta}$ are shown in Table 4.4.

Table 4.4: PID_beta Controller

Initial values of PID_beta Controller	
$K_{p\beta}$	0.000042
$K_{I\beta}$	0.625
$K_{D\beta}$	0.15625

PID_beta is fined-tuned to achieve stabilization performance. After a few times of trial and error, $K_{p\beta}$, $K_{I\beta}$, and $K_{D\beta}$ are obtained.

Table 4.5: Fined Tuned PID_beta Controller

Fined-Tuned PID_beta Controller	
$K_{p\beta}$	10
$K_{I\beta}$	0.004
$K_{D\beta}$	0.007

With the $K_{p\beta}$, $K_{I\beta}$, and $K_{D\beta}$ of PID_beta, a Double PID with LQR controller is considered designed successfully. The stabilization performance of the RIP system is evaluated and analyzed.

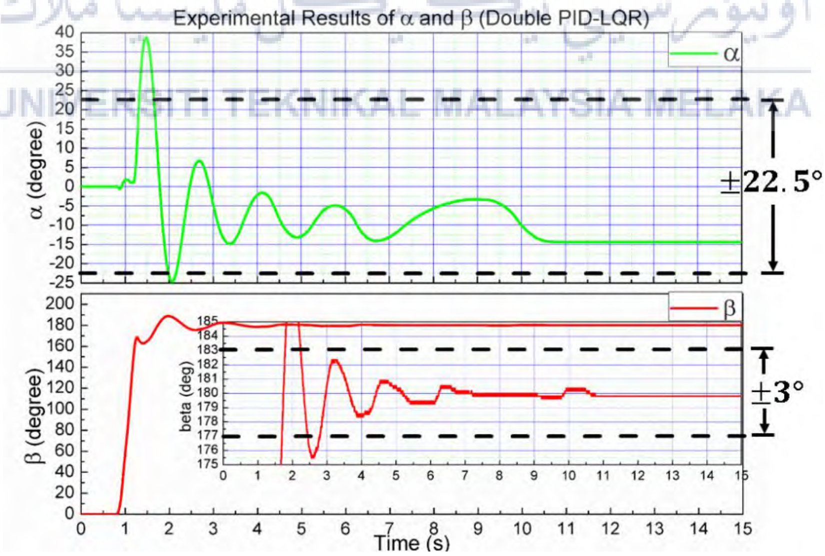

Figure 4.5: Response of rotary arm (α) and pendulum (β) of the RIP system

Figure 4.5 shows the desired stabilization performance has been achieved. The inverted pendulum (β) is stable within $\pm 3°$ in 3 seconds. The rotary arm (α) is stable within a range of $\pm 22.5°$.

Comparison of Stabilization Performance before and after implementing Double-PID Controller

A comparison on the stabilization performance before and after implementing Double-PID controller is made.

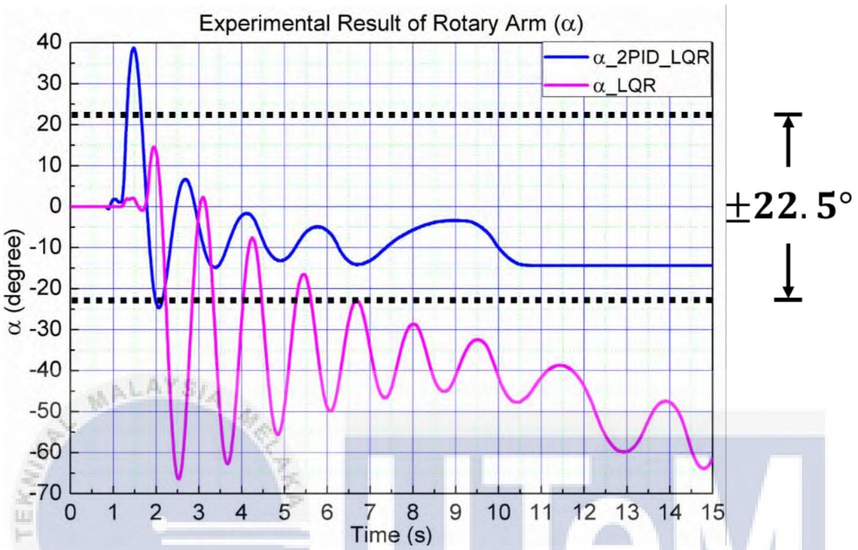

Figure 4.6: Comparison of Response of rotary arm (α) of Double-PID-LQR and LQR controller

Figure 4.6 shows the RIP system with Double-PID and LQR controller, the rotary arm (α) is oscillated less. It achieves the desired specification which is stable within $\pm 22.5°$. In the RIP system with Double-PID and LQR controller, the rotary arm (α) tends to move back to initial position and stable within the control time. Compared to the LQR-RIP system, Double-PID with LQR controller has solved the stabilization problem of rotary arm (α).

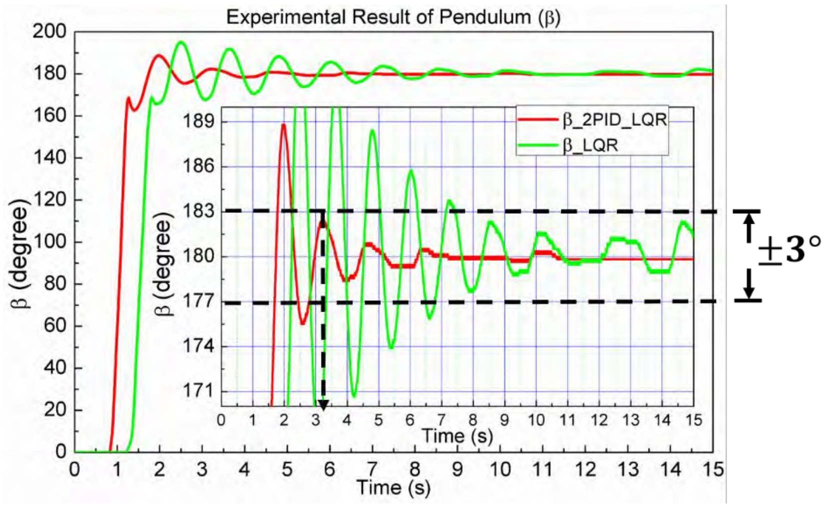

Figure 4.7: Comparison of response of pendulum (β) of Double-PID-LQR and LQR controller

Figure 4.7 shows the desired stabilization performance which is achieved in the RIP system with Double-PID and LQR controller. The overshoot and oscillations are reduced significantly. The inverted pendulum (β , red line) is stable within $\pm 3°$ in 3 seconds. Double-PID with LQR controller has solved the stabilization problem of the inverted pendulum (β) in the LQR-RIP system. The desired stabilization performance is achieved.

Table 4.6: Summary of results of LQR and Double-PID with LQR controller

Controller	**Stabilization angle range (°)**	**Settling time (s)**
LQR	±3°	8
Double-PID with LQR	±0.5° (↑ 83.33%)	3 (↑ 62.5%)

Table 4.7 shows that the significant stabilization results of Double-PID with LQR controller. The inverted pendulum (β) is stable within 3 seconds, in the range of $\pm 0.5°$. After implementing Double-PID controller in the LQR-RIP system, the stabilization performance of pendulum (β) improves 83.33% while the settling time improves 62.5%. Then, adaptability of the controller is considered to be highlighted after achieving the desired stabilization performance. One controller must be adaptable in order to maintain desired performance when the system parameters change continuously. The adaptability of Double-PID with LQR controller in the RIP system is examined through repeatability test.

Repeatability Test

The adaptability of Double-PID with LQR controller is examined in repeatability test of 10 times. The average settling time is the time for rotary arm (α) and pendulum (β) to stable within $\pm 22.5°$ and $\pm 3°$ respectively.

Table 4.7: Repeatability test results of α

Data Set	**Time to reach $\pm 22.5°$ (s)**
1	2.786
2	3.005
3	2.684
4	2.594
5	2.801
6	2.030
7	2.5730
8	2.744
9	3.156
10	2.733
Average	**2.7106**

Table 4.8 shows the total 10 times of repeatability test, the average of the time for α to stable within $\pm 22.5°$ is 2.7106 seconds.

Table 4.8: Repeatability test results of β

Data Set	**Time to reach $\pm 3°$ (s)**
1	3.146
2	2.803
3	2.754
4	3.141
5	2.801
6	3.162
7	3.245
8	3.174
9	3.047
10	3.454
Average	**3.0727**

Table 4.9 shows the same total 10 times of the repeatability test, the time for β to stable within $\pm 3°$ is 3.0727 seconds.

CHAPTER 5

CONCLUSION AND FUTURE WORK

5.1 Conclusion

The application of concept of RIP system has become wider in the industry, robotics and field of research due to its simplicity of system setup with highly unstable and underactuated characteristics. To achieve the first objective previous researches are studied, the stabilization controller of LQR and Double-PID is proposed to be the stabilization controller to maintain upright position of RIP. There were many studies and researches have been conducted to model a rotary inverted pendulum system. Lagrange's equation is one of the suggested approaches to model the RIP system. With the consideration of kinetic energy and potential energy of the RIP system, a mathematical model of the RIP system is obtained successfully by using Lagrange's Equation. The system mathematical model is then linearized by Taylor Series. It is linearized when it is static at an upright position. Without a controller, a RIP cannot be stable at upright position. Thus, second objective of my project is to design a Double-PID with LQR stabilization controller in the RIP system. Designing of LQR controller and Double-PID controller is carried out respectively. LQR controller is designed. L9 orthogonal array in Taguchi methods of design of experiments (DOE) is used to tune the LQR controller. The trial 7 of optimal gain value of K with the best quality in stabilization performance is obtained. In designing double-PID controller, Ziegler-Nichols Second method is chosen to tune PID_alpha. Fined tuned Double-PID controller is obtained to achieve stabilization performance of the RIP system. The stabilization performance is evaluated and analyzed. This project has given a brief explanation of the RIP system from modeling to designing stabilization controller.

5.2 Future Work

Tuning of LQR Controller

LQR controller is an easy and effective way to obtain an optimal feedback gain for a RIP system. A ground tuned value of the optimal feedback gain is not suitable to be selected as final decision. L9 orthogonal array of Taguchi Methods offers an inspection on the quality of the values of optimal feedback gain on stabilization performance of the RIP system. In future, instead of adding a $\pm 10\%$ to the optimal gain values, a smaller variation of percentage from the original optimal feedback gain value K_{ref} should be considered. It may achieved a more detailed inspection on the quality of the stabilization performance of the system.

Adaptability of Controller in RIP System

From the repeatability test on the stabilization performance of the RIP system, it showed that long control time affected the result of stabilization performance. Shorter control time and longer time interval between each attempt in repeatability test can result a better stabilization performance. In LQR, it seeks to minimize energy of the controlled output and energy of the control signal. LQR controller consumes energy of the signal continuously. In future, a predictor or corrector type observer for example, Kalman's Filter should implemented into the RIP system to reduce the consumption of energies by the state feedback controller.

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APPENDIX A

Trials of L9 Orthogonal Array of Taguchi Methods

There are 9 trials obtained from L9 orthogonal array. Each trial has shown its own stabilization performance in beta and alpha of the RIP system.

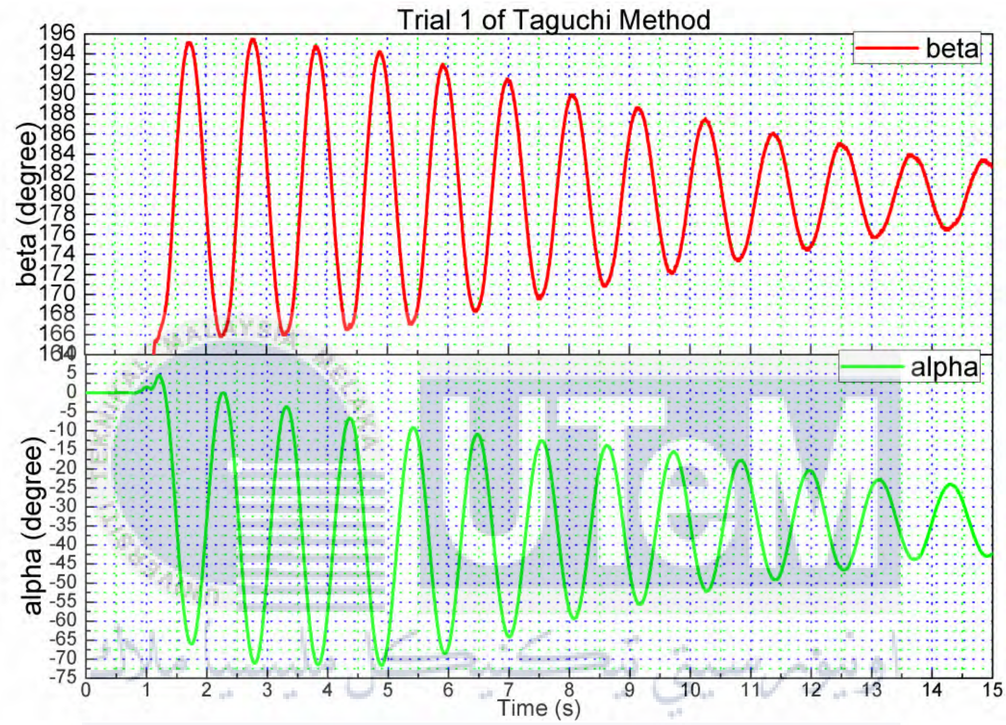

Figure A1: Graph of beta and alpha of Trial 1 of Taguchi Method

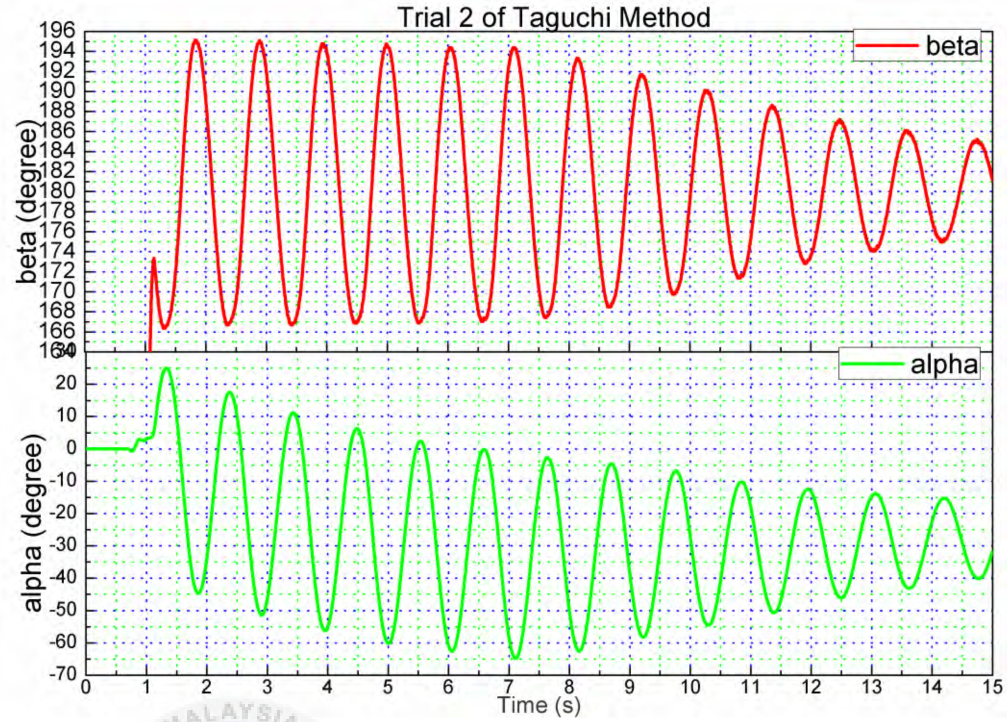

Figure A2: Graph of beta and alpha of Trial 2 of Taguchi Method

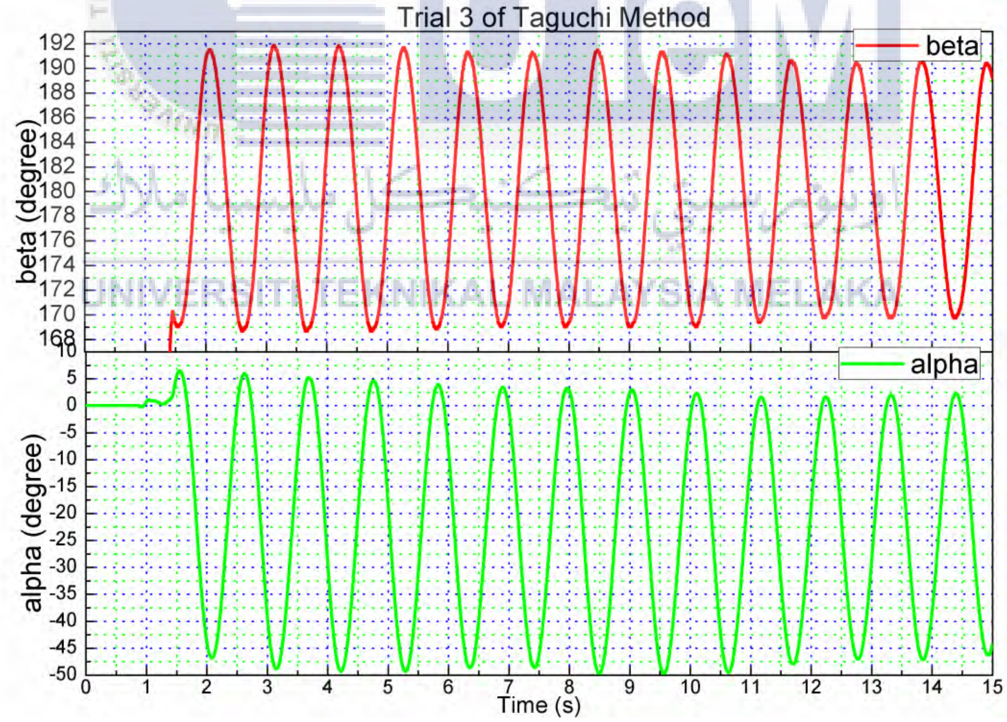

Figure A3: Graph of beta and alpha of Trial 3 of Taguchi Method

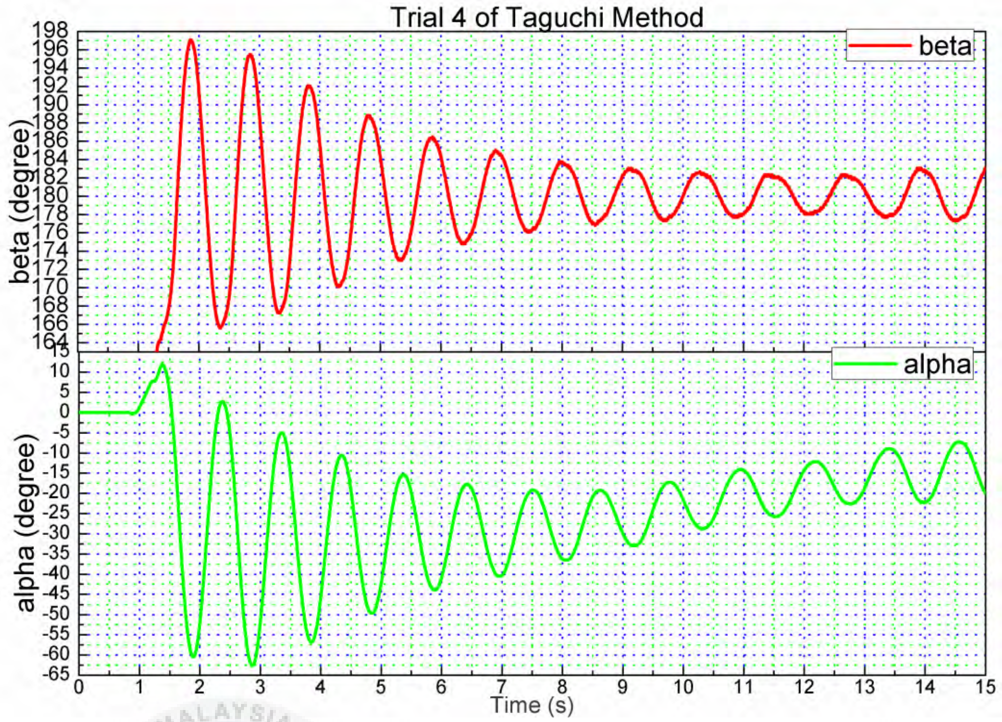

Figure A4: Graph of beta and alpha of Trial 4 of Taguchi Method

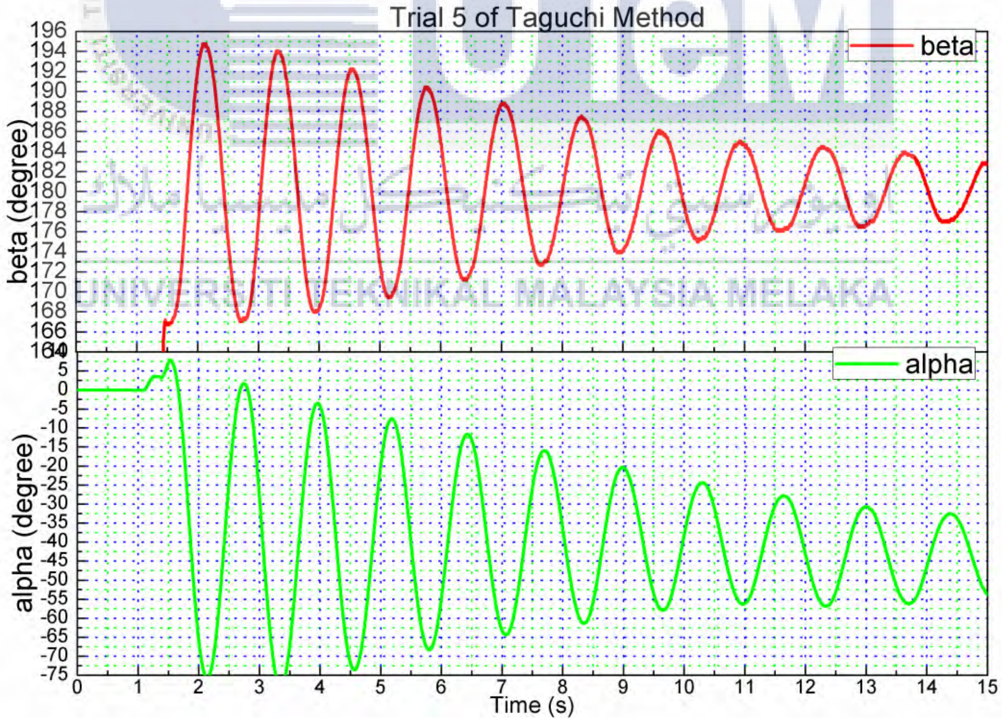

Figure A5: Graph of beta and alpha of Trial 5 of Taguchi Method

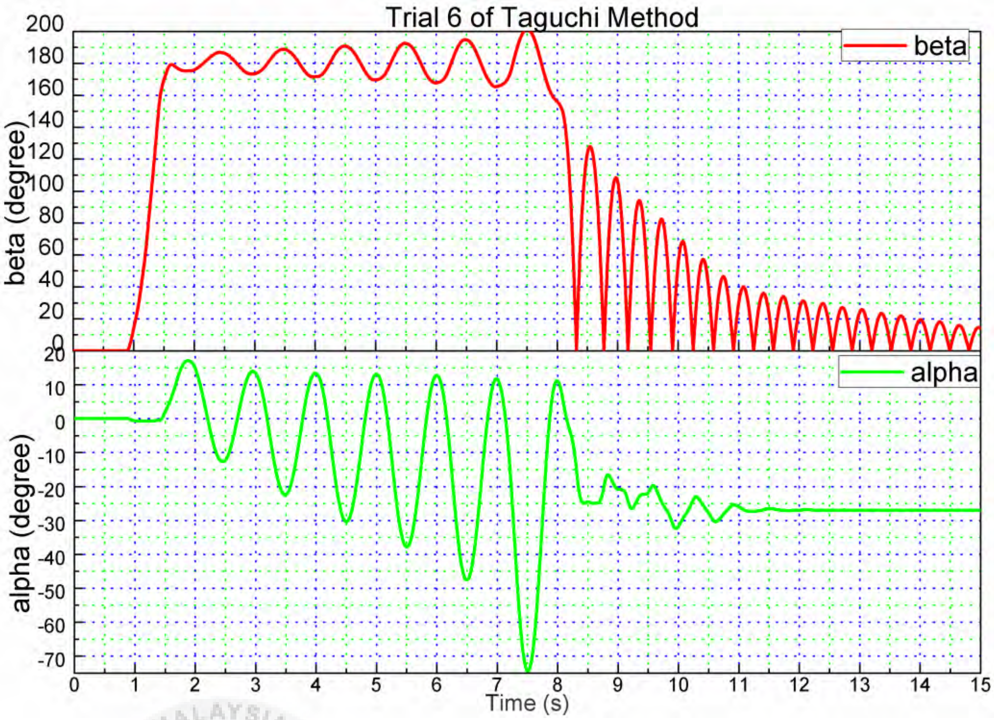

Figure A6: Graph of beta and alpha of Trial 6 of Taguchi Method

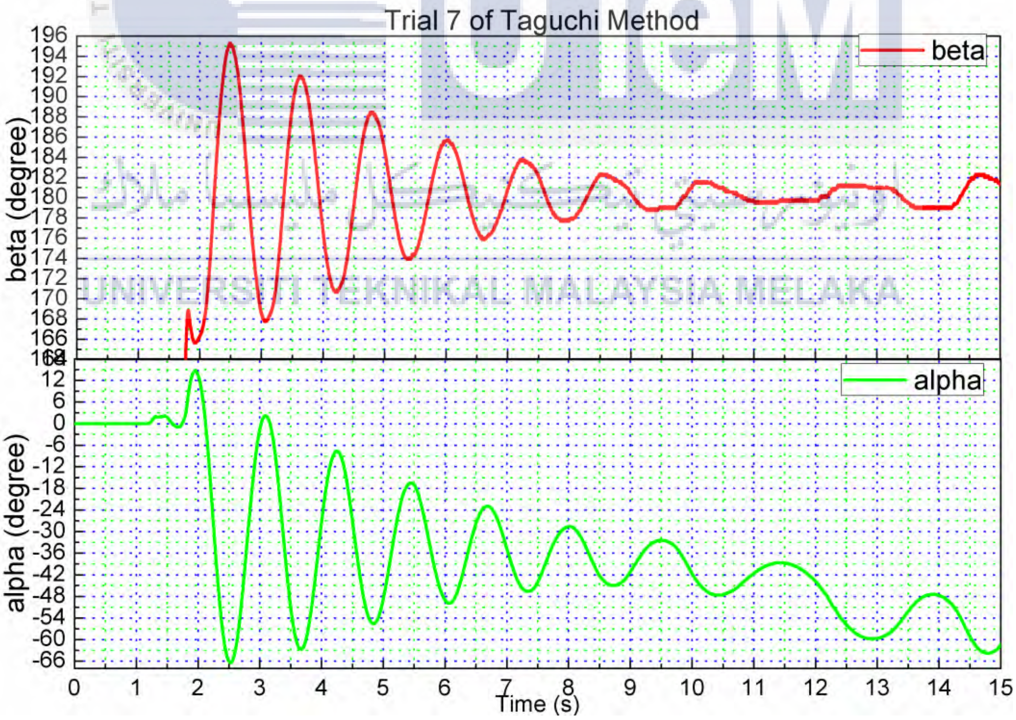

Figure A7: Graph of beta and alpha of Trial 7 of Taguchi Method

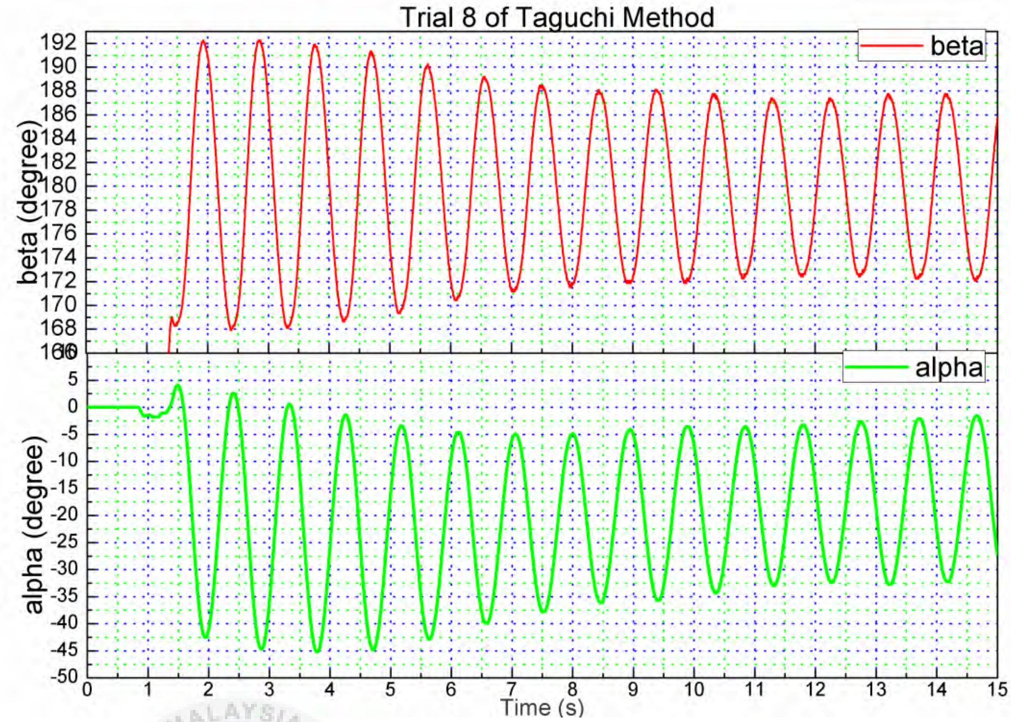

Figure A8: Graph of beta and alpha of Trial 8 of Taguchi Method

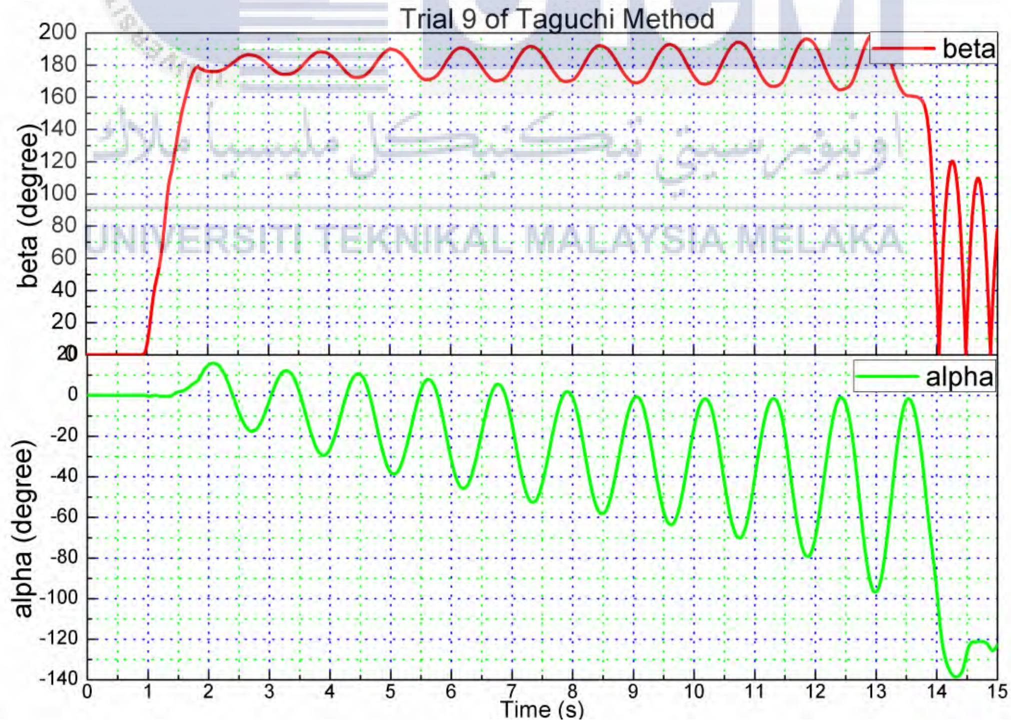

Figure A9: Graph of beta and alpha of Trial 9 of Taguchi Method

APPENDIX B

Repeatability Test on Stabilization Performance of Double-PID and LQR Controller

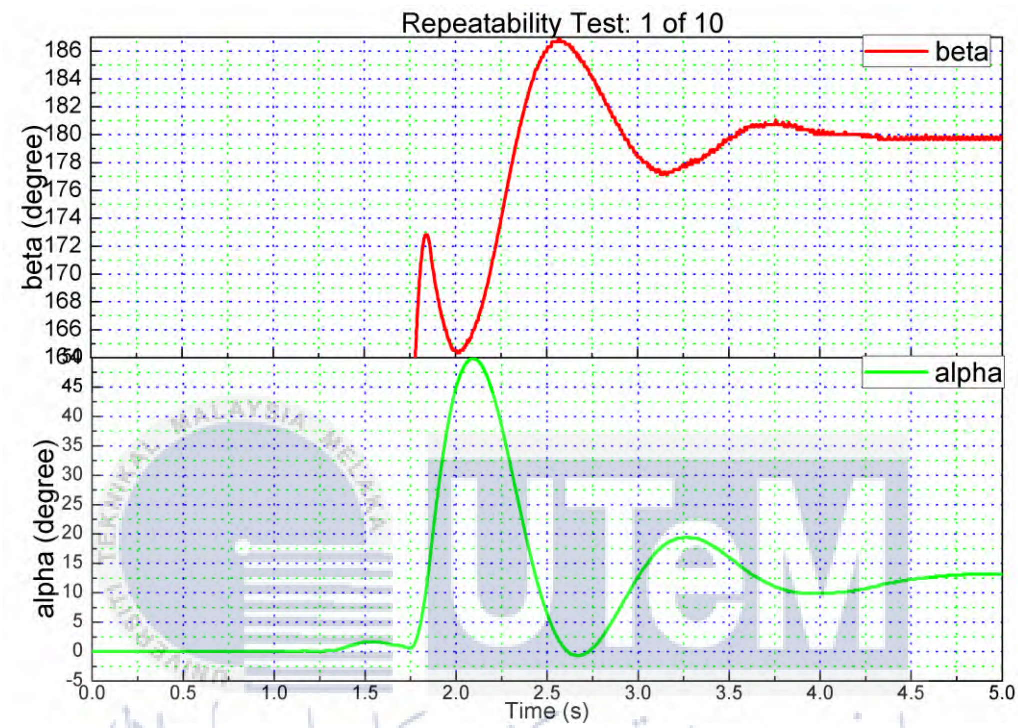

Figure B1: Graph of beta and alpha of repeatability test of 1 of 10

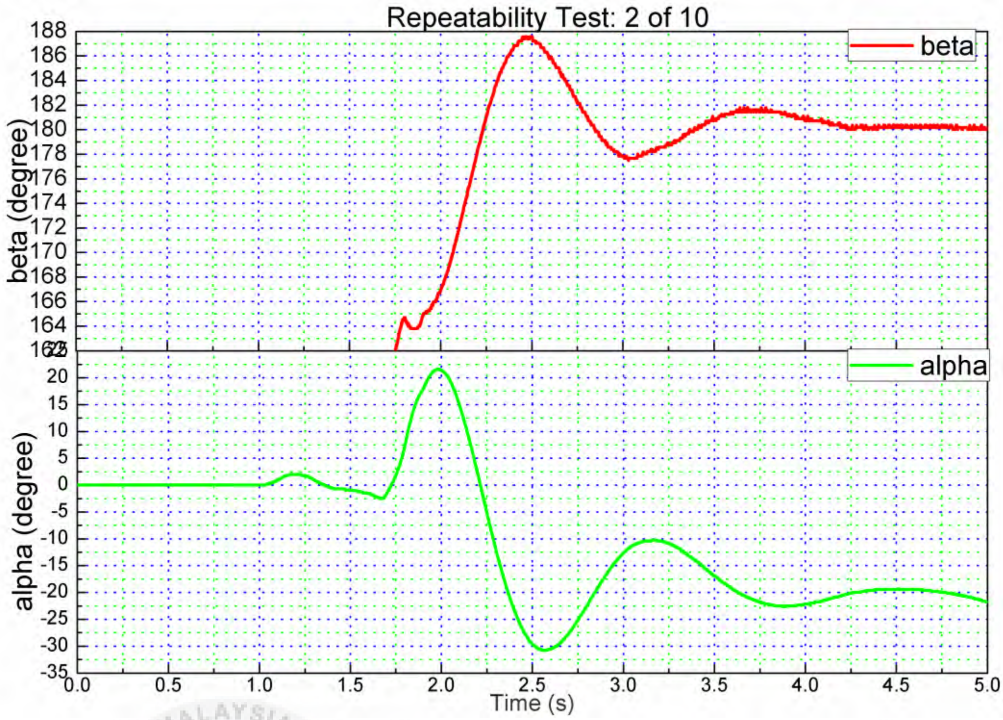

Figure B2: Graph of beta and alpha of repeatability test of 2 of 10

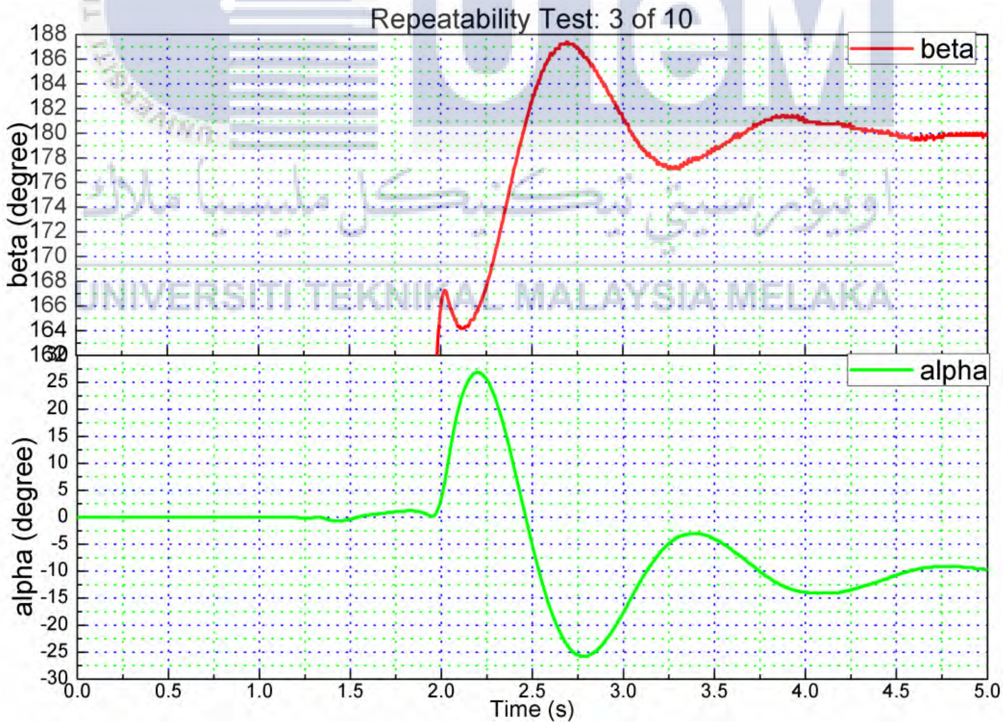

Figure B3: Graph of beta and alpha of repeatability test of 3 of 10

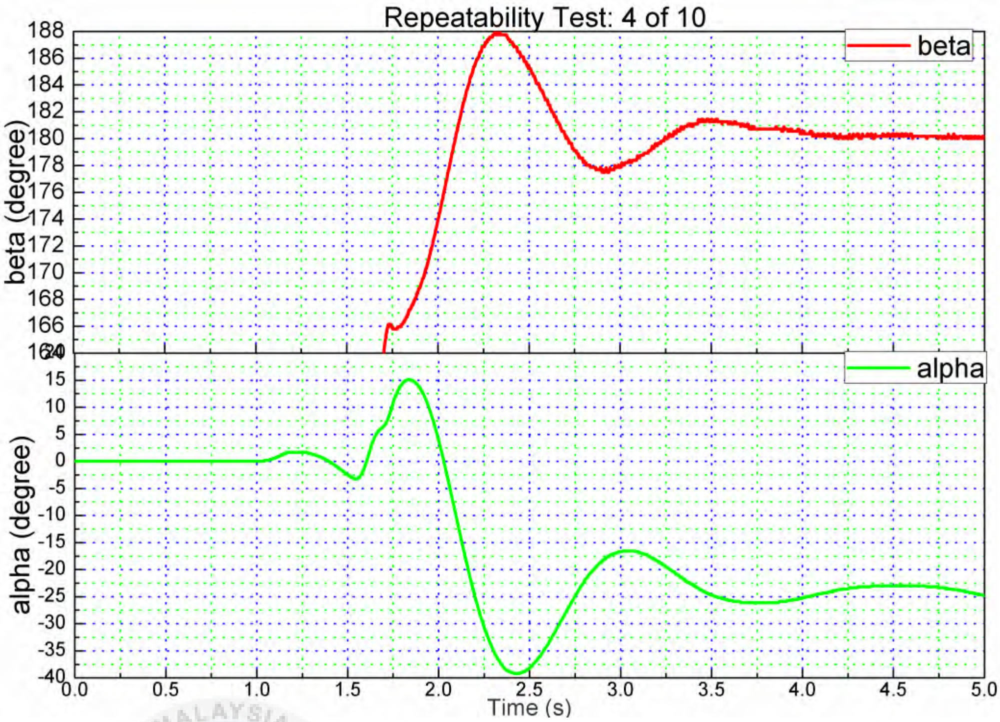

Figure B4: Graph of beta and alpha of repeatability test of 4 of 10

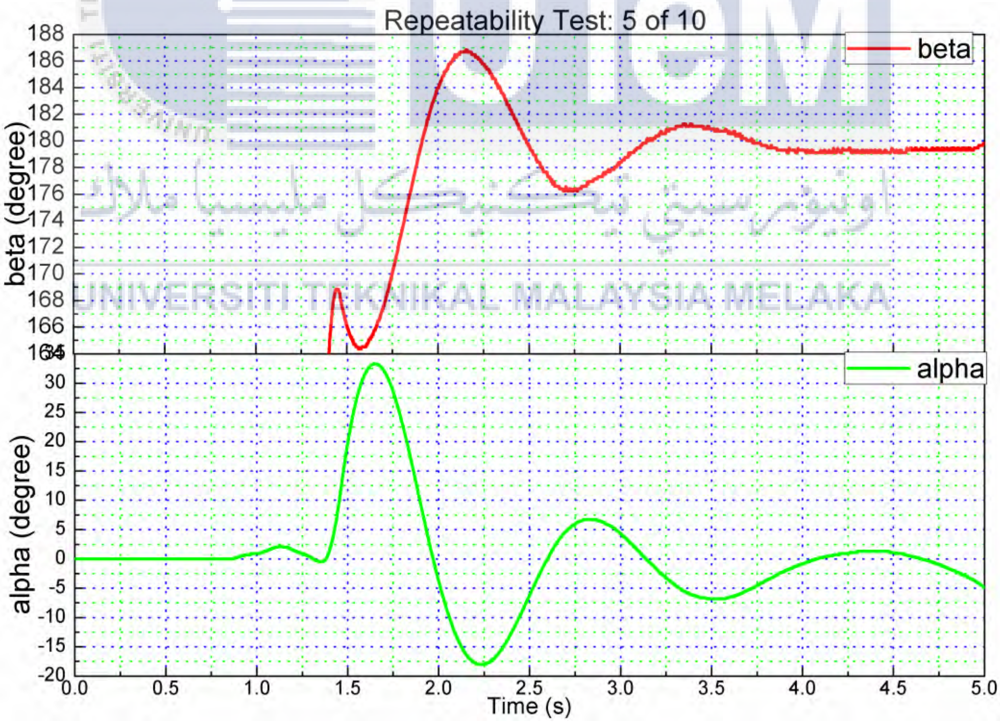

Figure B5: Graph of beta and alpha of repeatability test of 5 of 10

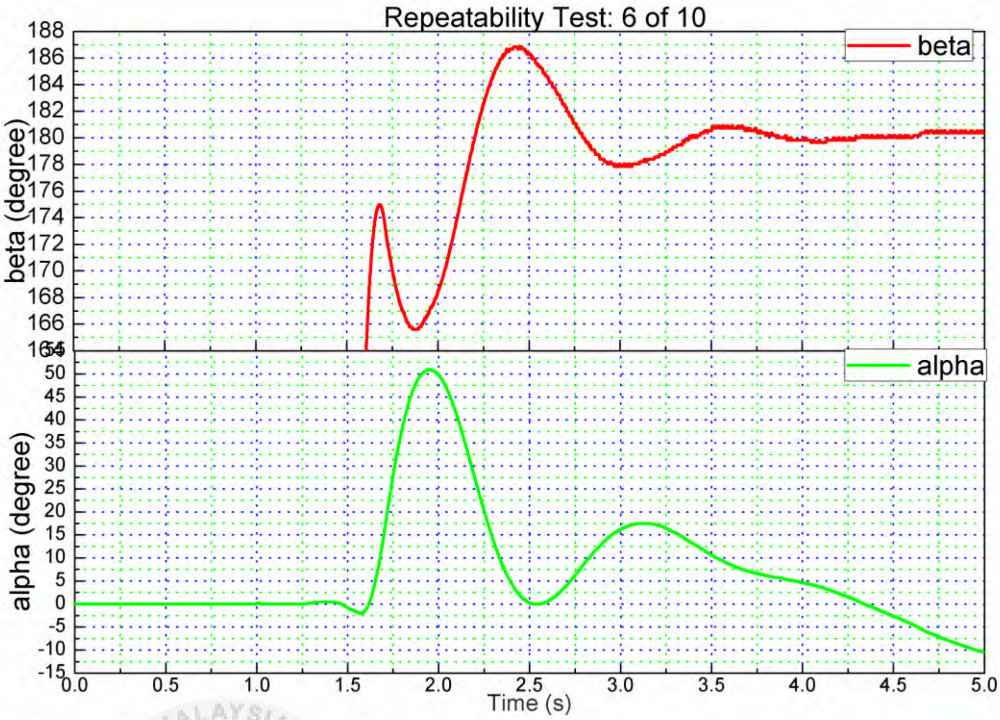

Figure B6: Graph of beta and alpha of repeatability test of 6 of 10

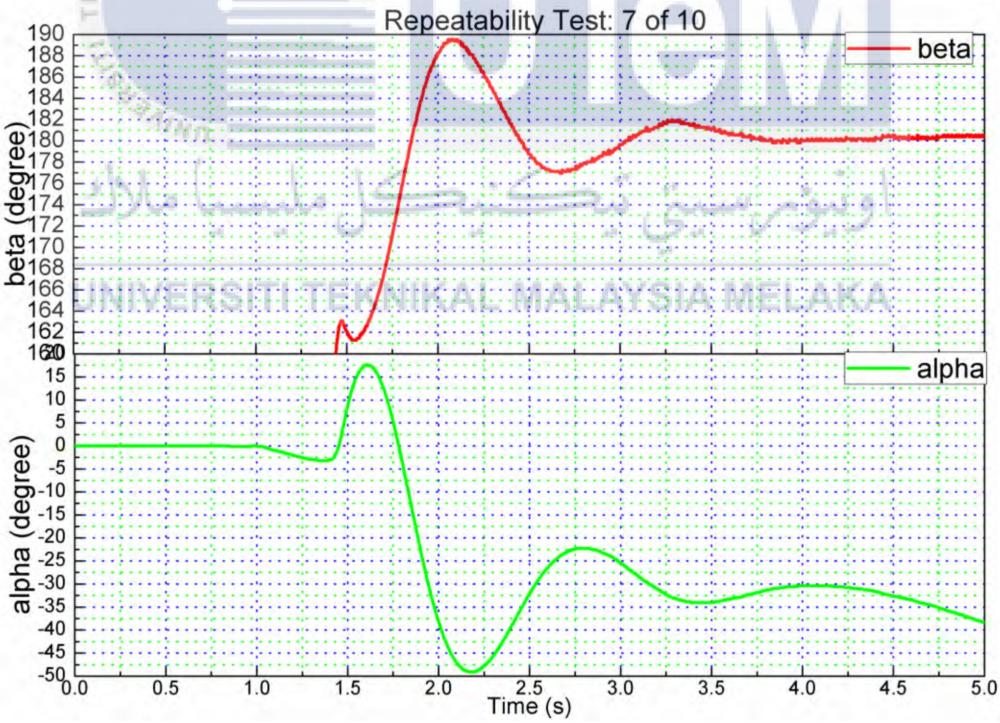

Figure B7: Graph of beta and alpha of repeatability test of 7 of 10

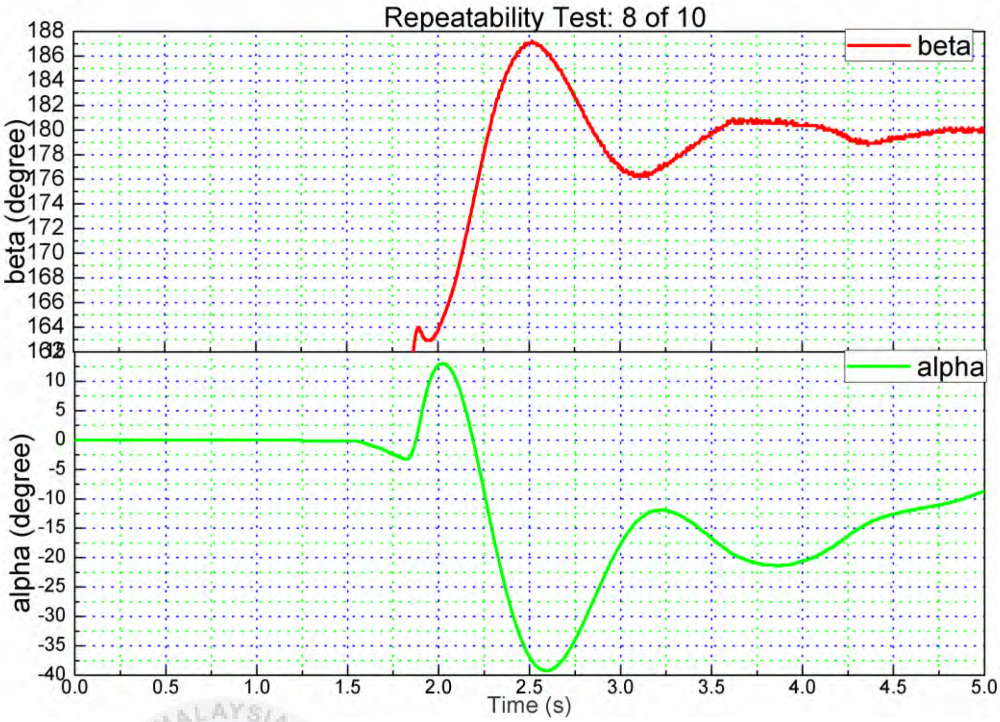

Figure B8: Graph of beta and alpha of repeatability test of 8 of 10

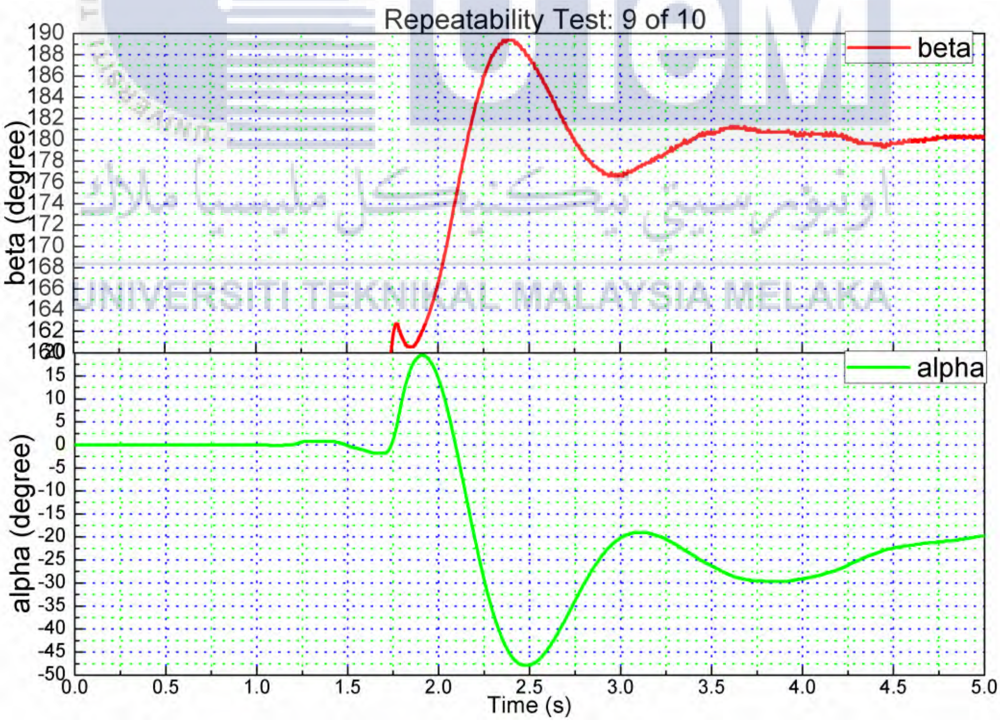

Figure B9: Graph of beta and alpha of repeatability test of 9 of 10

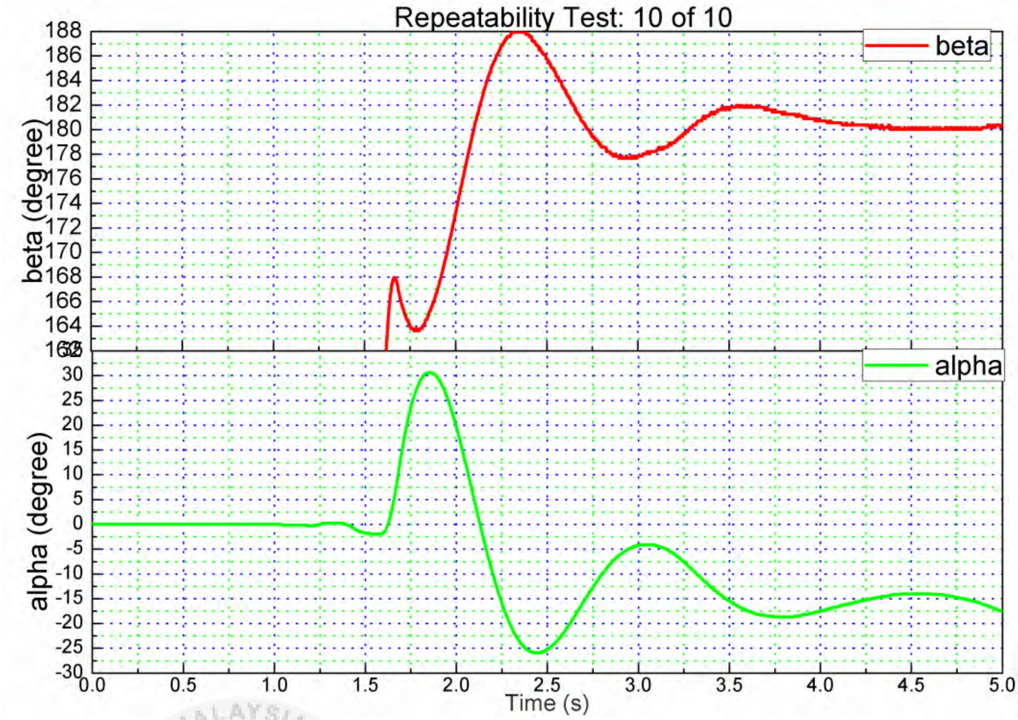

Figure B10: Graph of beta and alpha of repeatability test of 10 of 10