STABILIZATION CONTROL OF ROTARY INVERTER PENDULUM SYSTEM WITH DOUBLE-PID AND LQR CONTROLLER

PANG KEE KIAT

A report submitted in partial fulfillment of the requirements for the degree of Mechatronics Engineering

Faculty of Electrical Engineering
UINIVERSITI TEKNIKAL MALAYSIA MELAKA

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“ I hereby declare that I have read through this report entitle “Stabilization Control of Rotary Inverted Pendulum with Double-PID and LQR Contrroller” and found that it has comply the partial fulfillment for awarding the degree of Bachelor of Mechatronics Engineering.”

Signature : ....

Supervisor’s Name : Dr Chong Shin Horng

Date : ....

I declare that this report entitle “Stabilization Control of Rotary Inverted Pendulum System with Double-PID and LQR Controller” is the result of my own research except as cited in the references. The report has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

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ABSTRACT

Rotary inverted pendulum (RIP) system is a well-known control system with its non-linearity and underactuated system. There are many applications of RIP system in industry especially the application of balancing of a robot. The RIP system in this project is driven by DC Servo motor. The DC Servo motor is used because it is easy to setup and control, has precise rotation and most importantly is low cost. As for RIP system itself, has smooth motion, not easy to wear out and high mechanical efficiency. However problems will occur when inverted pendulum of the RIP system is required to stable at an upright position. The non-linearity and underactuated characteristics cause the system to be highly unstable when maintaining at upright position. Therefore, the objective of this project is to propose double Proportional-Integral-Derivative (PID) and Linear Quadratic Regulator (LQR) controller to stable the inverted pendulum at an upright position by a rotary arm which is actuated by a DC Servo motor. The performance between the double-PID and LQR controller is compared in order to validate the performance of each controller. Mathematical modelling of the RIP system is carried out to obtain a state space model of the RIP system for ease of designing controller. Designing of double-PID and LQR controller is carried out in two phases. In Phase 1, LQR controller is designed for a basic stabilization control. In phase 2, double-PID controller is designed based on the RIP system with LQR controller for improving the stabilization performance. Throughout all the designing procedure, the stabilization performance of inverted pendulum and settling time are examined and compared. At the end of this project, double-PID with LQR controller was designed successfully to stable the RIP system.

ABSTRAK

Sistem bandul balik yang berputar adalah satu sistem kawalan yang terkenal dengan ketidaklinearan dan sebagai sistem underactuated. Applikasi sistem bandul balik yang berputar dalam industry termasuk menstabilkan robot. Sistem bandul balik yang berputar dalam projek ini adalah digerakkan dengan DC servo motor. DC servo motor digunakan adalah disebabkan penggunaan yang mudah, tepat penggerakan dan kos yang rendah. Masalah dalam sistem kawalan ini berlaku semasa mendirikan bandul balik pada kedudukan yang tegak. Sistem tersebut akan mengalami masalah ketidakstabilan. Justeru, objektif projek ini adalah mencadangkan pengawal Double-PID dan LQR untuk menstabilkan bandul balik tersebut. Sistem bandul balik berputar ini dimodelkan dalam matematika model dengan menggunakan state space model. Terdapat 2 fasa untuk mereka bentuk pengawal Double-PID dan LQR untuk sistem tersebut. Fasa 1 adalah fasa untuk mereka bentul pengawal LQR. Fasa 2 adalah fasa untuk mereka bentuk pengawal Double-PID pada sistem bandul balik berputar yang berdasarkan pengawal LQR yang sedia ada. Melalui prosedur-prosedur rekaan bentuk pengawal, kestabilan sistem bandul balik berputar telah dianalisis untuk mengkaji pengawal Double-PID dan LQR. Dengan itu, pengawal Double-PID dan LQR telah berjaya menstabilkan sistem bandul balik berputar.

TABLE OF CONTENT

LIST OF FIGURES

LIST OF TABLES

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Research background

The problem of balancing a broomstick in a vertical upright position on a person's hand is well known to the control engineering community. For any person, a physical demonstration of the broomstick-balancing act constitutes a challenging task requiring intelligent, coordinated hand movement based on visual feedback. The instability corresponding to the broomstick vertical upright position leads to the challenge inherent in the problem [1]. Since Furuta's Pendulum established in year 1992, RIP is one of the imperative systems for testing various control technique. It is highly nonlinear, severely unstable, multivariable and an under-actuated system in the field of control theory. It is most helpful for testing self-tuning regulator kind of control technique. Craig *et. al.* designed and built a course "Mechatronic System Design" at Rensselaer. They summarized a mechatronic system design case study for the RIP system [1]. Inverted pendulum system is a typical experimental platform for the research of control theory. The process of inverted pendulum can reflect many key programs, such as stabilization problem, nonlinear problem, robustness, follow-up and tracking program etc [2]. The RIP system consists of an actuator and two degrees of freedom, which makes it under-actuated, only robot arm is being actuated while pendulum is indirectly controlled, so as to result in the balance problem. Segway is one of the most significant succeed research based on control theory of inverted pendulum system. Its ability to self-balance brings development of modern vehicle to a new milestone. The inverted pendulum concept can be applied in control of a space booster rocket and a satellite, an automatic aircraft landing system, aircraft stabilization in the turbulent air flow, stabilization of a cabin in a ship and others [3-4].

1.2 Motivation and significance of research

In this project, the stabilization controllers were proposed and compared by using the RIP system driven by servo motor module as a medium of experiment. The stabilization controllers that proposed in this project are double PID controller and LQR controller. Both designs of controller are compared with some specifications of performances.

1.3 Problem Statement

In order to control the stabilization of the RIP system at upright position driven by DC motor, many different controller approaches have been introduced in global academic researches. All of the approaches aim to achieve a better transient performance, low steady state error and low overshoot condition. The problem arises when there are two outputs giving feedback to control system, tuned signal is giving one input only back to the plant. This Single Input Multiple Output (SIMO) system cannot be controlled by a single conventional PID controller. Furthermore, certain parameter of RIP system is changed when disturbance is given to high non-linear characteristic of the RIP system. The single conventional PID controller cannot adapt to the changes of system parameter that occur on the system. Therefore, tuning multiple outputs of the RIP system at the same time become a challenge throughout the whole project. Double PID with LQR controllers are proposed to solve this SIMO RIP system.

1.4 Objectives

The basic objectives of this project are as follows:

a) To propose a double-PID with LQR controller for stabilization control of a rotary inverted pendulum system.

b) To evaluate the stabilization performance of the rotary inverted pendulum system experimentally.

1.5 Scope

The scope of this project are:

a) The design of the stabilization controller for the RIP system is required to maintain the pendulum at upright position, within ±20º from vertical upright position.

b) After the pendulum is within ±5°, rotary arm of the RIP system is required to maintain moving range within ±22.5°.

c) The RIP system is modelled mathematically using Lagrange's equation.

d) This system with two Degree of Freedom (DOF) is actuated by a DC motor, which categorize this system as underactuated system.

CHAPTER 2

LITERATURE REVIEW

2.1 Theory and Basic Principle of RIP System

Figure 2.1 shows the RIP system manufactured by TERASOFT. The RIP system is an under-actuated system which consists of one actuator and two Degree Of Freedom (DOF). The only actuator in the system is the DC motor. The rotary arm is driven by the DC motor where electrical energy is converted into mechanical energy, the torque to move it. The angular motion of the rotary arm gives energy to the pendulum to swing up and maintain stable at vertical upright position. The pendulum is set to be always perpendicular to the rotary arm. When the pendulum is at vertical upright position, the system is highly unstable, where a controller is needed to achieve stabilization and swing up mechanism of the RIP system. The amplitude of the supply voltage to the DC motor is proportional to the magnitude of the angular displacement of the rotary arm. Thus, the greater the supply voltage to the actuator, the greater the angular displacement of the rotary arm.

The angular displacement of RIP is indirectly moved by the DC motor torque. There are two types of movement mechanism in RIP system, which are swing-up mechanism and stabilize mechanism. In this project, swing-up mechanism is not discussed, position of RIP is assumed to be at upright position as initial condition. Second type of movement mechanism is stabilization mechanism of rotary inverted system which is the motion maintaining the RIP at vertical upright position and avoiding the pendulum falling down in its free fall of nature way.

Figure 2.1: TERASOFT Rotary Inverted Pendulum

2.2 Review of Previous Related Works

There were many researchers studied RIP system from different aspects especially in modeling and designing different controller of the system. For stabilization and swing-up of RIP system, numerous designs of controller approach have been suggested to achieve better stabilization performance. Therefore, the study of research that had been done by other researchers is important to get a rough idea of designing controller in this RIP system.

There are basically divided to classical controller and advanced controller in proposed controllers to stabilize the RIP system. Proportional-Integral-Derivative (PID) controller is one of the most widely used controller in field of control engineering. As the RIP system is an underactuated and non-linear system, PID controller is common to be designed in the RIP system, as it improves overshoot percentage and steady state error of the system with an easy approach. PID controller can be used although mathematical model of the system is not known. When the mathematical model is not known, Ziegler-Nichols rules can be applied. Ziegler Nichols tuning rules give an educated guess for the parameter values and provide a starting point for fine tuning. Thus, from year 2009 to 2012, 2DOF PID or Double-PID was designed as a controller in the RIP system [6,8,11,13,15]. As a stabilization controller in the RIP system, the controller stabilized the inverted pendulum and as well as the rotary arm.

For advance controller, hybrid strategy was applied on the controllers. Combining two types of controller into the RIP system was considered in designing controller procedure. The advanced controllers that were used in hybrid strategy in previous works are including Full State Feedback controller, fuzzy logic controller and Double-PID with LQR controller.

Full State Feedback (FSF) Controller

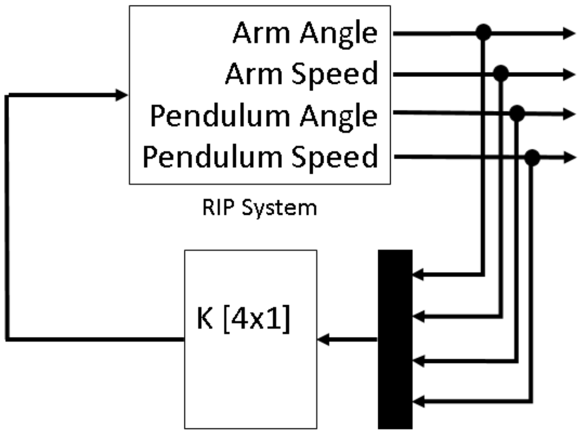

Figure 2.2: Full State Feedback/LQR Controller Block Diagram

Figure 2.2 shows a block diagram of a Full State Feedback (FSF) controller in the RIP system. The RIP system is was designed in state space model. Poles of the closed loop system may be placed at any desired locations by means of state feedback through an appropriate state feedback gain matrix *K*. There are a few approach to tune FSF controller, one of them is by pole placement method. According to M. Akhtaruzzaman [5], he designed FSF controller by placing stable poles of the RIP system, then used Ackermann's formula and Integral of Time-weighted Absolute Error (ITAE) table, state feedback control gain matrix, K which is a 4×1 matrix was obtained. FSF controller was considered relatively ease of design and effective procedure to obtain the gain matrix K. There are some drawbacks of designing FSF controller. It requires successful measurement of all state variables or a state observer in the system, where it needs control system design in state space model. It also requires experienced researcher to determine the desired closed loop poles of the system, especially when the system has a higher order system than second or third order.

Proportional-Integral-Derivative (PID) Controller

Figure 2.3 shows a double-PID controller instead of single conventional PID controller, it was because single PID controller can control only one variable of the system. In fact, rotary inverted pendulum system has one input and two outputs which single PID is incapable to control the system.

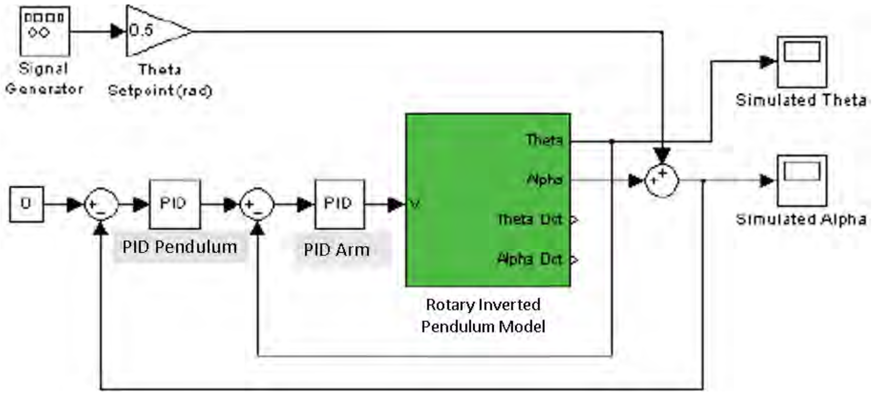

Figure 2.4: 2DOF PID Controller [5]

According to Figure 2.3, 2DOF PID was arranged cascaded. PID Arm was to maintain the rotary arm as zero, while PID pendulum maintained the speed and position of pendulum to remains stable [5]. PID Arm was tuned first then following by the PID Pendulum. Root locus analysis was used to tune the both PID.

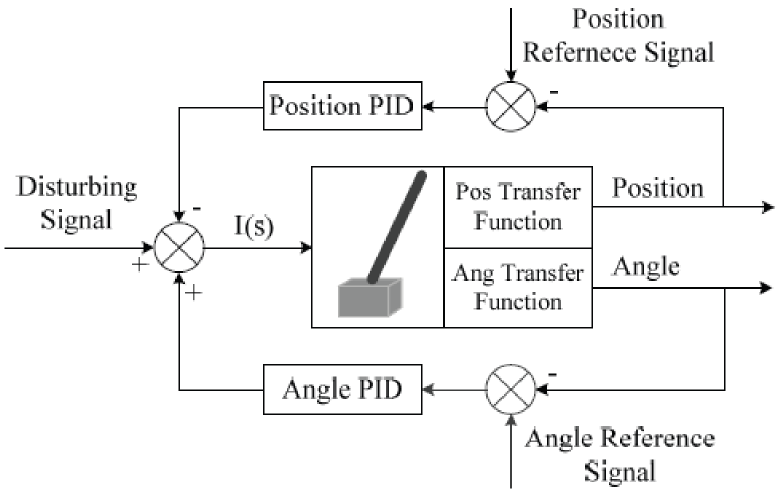

Figure 2.4: Model of cart inverted pendulum system with PID controller

The performance of double-PID in stabilization of RIP system was not satisfactory as its dynamic responses is not robust. Thus, researchers suggested to implement hybrid strategy, combining LQR controller with double-PID controller as stabilization controller.

Before suggesting double-PID controller in year 2011, another controller was also proposed for the RIP which is adaptive PID with sliding mode control [6]. In this paper, the researchers use sliding mode control to handle the nonlinear time varying part, and designed an adaptive law to tune up the system parameters online. The parameters of the PID controller were adjusted online by adaptive law. Unfortunately, the RIP control system possessed control difficulty with its non-linear and instable system. The design of the sliding mode controller took two steps, firstly, the sliding function was determined and then the control law was derived. The results showed that the proposed adaptive PID sliding mode controller has succeeded to make the system stable and robust effectively. The main advantage of the proposed control method is that the PID controller gains can be obtained online and converge efficiently.

In year 2011, from the paper [7-8], the double-PID was suggested as a stabilization controller in RIP system. The fundamental of the design of double-PID controller is taking position of the cart and angle of the pendulum rod as signals for feeding back to the system as shown in

In paper [9], Gan designed a composite controller according to the characteristics of LQR and PID controller to get a faster control speed and better effect of dynamic balance. The results show that the control algorithm combined by LQR and PID can obtain a good balance effect and has a good anti-disturbance effects, which can restore dynamic fast. For the PID controller, it can control robot balance but the robot vibrate larger in the vicinity of balance point, the static performance is poor [9]. For the LQR controller, it has a good control effect in small-angle scope, but, for larger disturbance, then the angle beyond the linearization constraint conditions, the LQR controller do not have good control effect, even cannot keep robot balance [9].

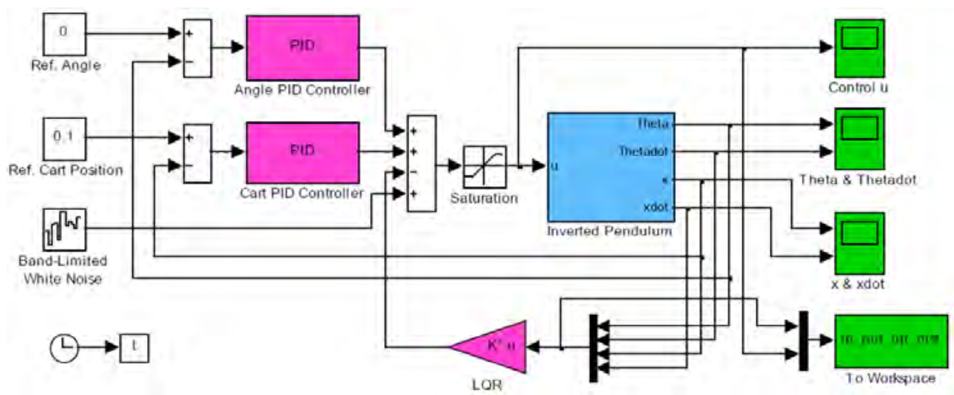

Figure 2.5: Double PID & LQR Control of Nonlinear Inverted Pendulum System [7]

Figure 2.5 shows the control structure of a double PID and LQR controller in [7]. LQR used state variables as the feedback variables in order to increase the stability of the system and obtain the desired system response. The optimal control value of LQR was added negatively with PID control value to have a resultant optimal control.

Linear Quadratic Regulator (LQR) Controller

By using modern control theory, the design of LQR controller is started by considering the first-order mathematical model of inverted pendulum system. By using linear quadratic control theory to design the control current signal, the pendulum rod can achieve a stable equilibrium point at upright position. According to the LQR optimal control law, its optimality is totally depended on the selection of Q and r, whereby Q and r are weighting matrixes that penalize certain states and control inputs of the system. The widespread method used to choose Q and r is by means of simulation and trial [10].

In paper [9], LQR controller is designed into a two-wheeled self-balancing robot. The results showed that an inverted pendulum system is unable to achieve the dynamic balance of the robot under the control of LQR. The limitations of LQR in a system are system modelling must be accurate and physical system parameters must be strict. In order to get faster control speed and better effect of dynamic balance, a composite controller is design according to characteristics of LQR and PID controller. Experiment summary in [9], the control algorithm combined by LQR and PID can obtain a good balance effect and has a good anti-disturbance effects and can restore dynamic fast.

In paper [11], the value of gain K is obtained by using the function *lqr* in MatLab. Furthermore, Taguchi Methods can be used to tune the gain K. The Taguchi methods are popular design of experiment (DOE) methods used in industry [11]. Due to the four gains in K, an orthogonal array that can include at least four factors needed to be used and L9 orthogonal is chosen for tuning the gain K because this array allows four three-level factors at most [11]. In the conclusion [11], the L9 orthogonal array in Taguchi methods can tune the feedback gain of the controller efficiently, which means that the RIP system able to stable at upright position. Yet, from A. Khashayar [12], the LQR controller could not set the pendulum position to zero degree, the reference angle at upright position, when it is at non zero initial condition. Thus, there were researchers suggesting to implement a double-PID controller into the system.

Fuzzy Logic Controller

Fuzzy logic control theory is very useful for systems with complicated structures such as non-linear and unstable of RIP system. The elements of fuzzy logic controller includes fuzzification, rule-based, inference mechanism and defuzzification. In paper [12], the fuzzy controller should use the real state variables, which are angular displacement of rotary arm, angular velocity of rotary arm, angular displacement of pendulum and angular velocity of pendulum, that are read by two encoders in the system. The fuzzy logic controller requires experts' previous experience about the operation of the real RIP system to create suitable experimental rules. The deviation on the angular displacement of pendulum from the reference angle is one of the input to the fuzzy logic controller, while the second input to the controller is angular velocity of the pendulum. These two inputs help the controller to diagnose how the motor decrease the angular position error to zero with rotating the rotary arm clockwise and anticlockwise repeatedly [13].

In paper [14], stabilization of RIP system is achieved by mapping linear optimal control law to the fuzzy inference system (FIS). A Mamdani FIS is designed which stabilizes the pendulum in the linear zone, emulating LQR control around the equilibrium point. The linear state feedback law is mapped to the rules of the fuzzy inference engine. The system consists of two fuzzy inference subsystems, one taking as inputs the angular position and speed of the pendulum arm, and other one taking as inputs the angular position and speed of the pendulum. The two output signals from both subsystems are then added to give a single control signal [14]. In the experiment, the observed performance of the system is smooth, and it is experimentally shown that the closed loop balancing system based on the fuzzy controller exhibits greater robustness to unmodeled dynamics and uncertain parameters than the LQR controller that it emulates [14]. Fuzzy logic controller requires complex linguistic expression. The linguistic expression which are the basic of the rule-base of the fuzzy logic controller must generated based on experts' whom have done many relevant experiments.

2.3 Summary and Discussion of the Review

In summary, the double-PID and LQR controller for stabilization control of RIP system are chosen in this project. The reason of proposing this design of controller is because, firstly, the PID controller is a common controller that used as in many applications, it has the effect of reducing overshoot and improving of transient response of a system. Secondly, the LQR controller has a good control effect with an easy and effective way of obtaining optimal feedback gain for RIP system stabilization.

CHAPTER 3

MODELLING AND CONTROLLER DESIGN

3.1 Experimental Setup

The RIP system setup requires two categories, which are host computer and Terasoft Electro-Mechanical Engineering Control System (EMECS). The EMECS is including three main components which are MicroBox 2000/2000C, servo-motor module, driver circuit and power supply. MicroBox 2000/2000C is a data acquisition unit to receive and send the signal via Ethernet cable connection with host computer. At the other end, the data acquisition unit interfaces with signal of rotary encoder of the system which has amplified by the driver circuit. According to Figure 3.1, the host computer is connected with each components in the system.

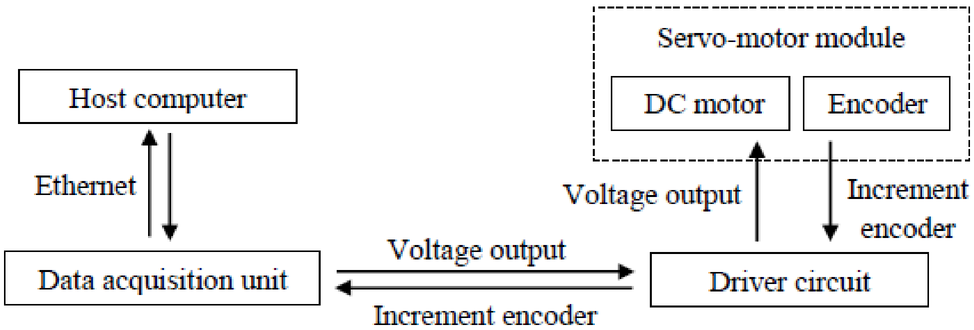

Figure 3.1: Connections between EMECS and host PC

The pendulum movement is actuated by the motor rotor indirectly where rotary arm is the connection link in between. The rotation of motor rotor result in angular displacement and its derivative, angular velocity of rotary arm. Then the change of the angular displacement and velocity each time the rotary arm moves, the kinetic energy and potential energy of the pendulum change instantly. The angular displacements of both rotary arm and pendulum are read by optical encoders which converts the analog displacement into digitized pulse signal. The digitize pulse signal is a voltage signal feeding into the MicroBox and the host PC.

The Electro-Mechanical Engineering Control System (EMECS) plant shown in Figure 3.2 was set up.

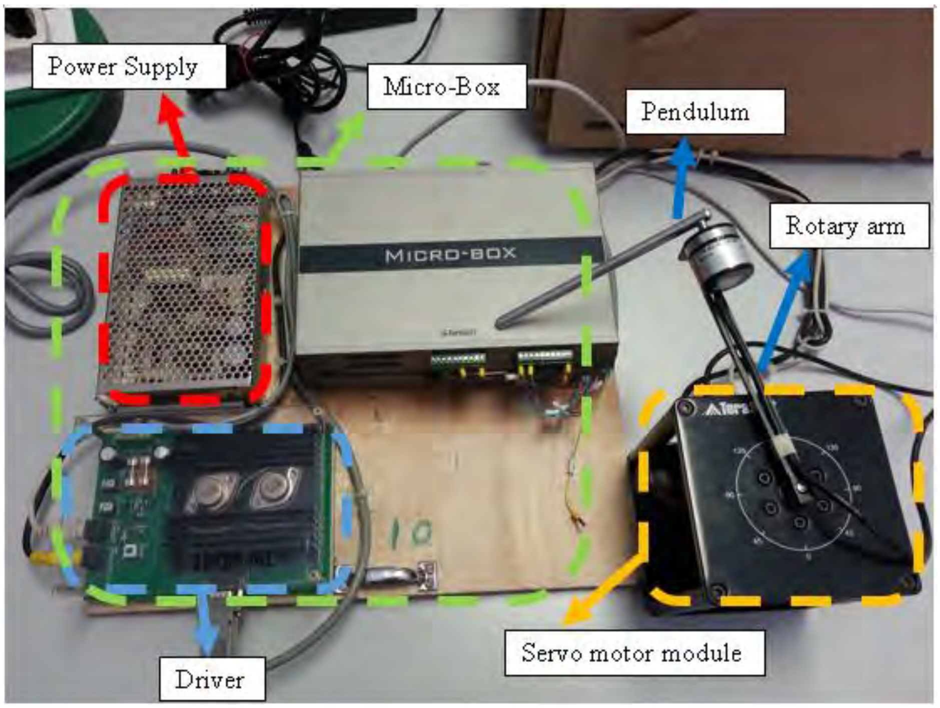

Figure 3.2: EMECS Plant

As to control the pendulum movement, polarity of the connection of servo motor module is considered. The different polarity of the connection of DC motor will result in different direction of rotation, either clockwise or anticlockwise. By interfacing between Simulink and the MicroBox, host PC can monitor all the signals sending from the MicroBox. All the signals are in form of digitized pulses which generated by the two rotary incremental encoders. Pin connections at MicroBox enable reading and sending digital or analog signals with the system. All the connections pin are shown in Appendix.

3.2 Modeling of RIP system

The study of system dynamics resides in modelling its behaviour. Systems models are simplified, abstracted structures used to predict the behaviour of the studied systems. Our interest is pointing towards the mathematical model used to predict certain aspects of the system response to the inputs. In mathematical notations a system model is described by a set of ordinary differential equations in terms of state variables and a set of algebraic equations that relate the state variable to other system variables [15].

Parameter of RIP System

The RIP system is an electro-mechanical engineering control system. The involved parameters consists of electrical and mechanical parameters. Table 3.1 shows the parameters of the whole system.

Table 3.1: System parameters for RIP system

Parameter	**SI unit**	**Symbol**	**Numerical Value**
Mass of arm	kg	m_1	0.056
Mass of pendulum	kg	m_2	0.022
Length of arm	m	l_1	0.16
Length of pendulum	m	l_2	0.16
Distance to centre of arm mass	m	c_1	0.08
Distance to centre of pendulum mass	m	c_2	0.08
Inertia of arm	kgm^2	J_1	0.00215058
Inertia of pendulum	kgm^2	J_2	0.00018773
Gravitational acceleration	m/s^2	g	9.81
Angular position of arm	°	α	-
Angular velocity of arm	rad/s	$\dot{\alpha}$	-
Angular position of pendulum	°	β	-
Angular velocity of pendulum	rad/s	$\dot{\beta}$	-
Viscous friction co-efficient of arm	kgm^2/s	C_1	0
Viscous friction co-efficient of pendulum	kgm^2/s	C_2	0
Motor torque constant	Nm/A	K_t	0.01826
Motor back-emf constant	Vs/rad	K_b	0.01826
Motor driver amplifier gain	V/count	K_u	850
Armature resistance	Ω	R_m	2.56204
Armature inductance	L	L_m	0.0046909