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LAPORAN PROJEK
SARJANA MUDA

DEVELOPMENT OF AN ACTIVE BRAKING SYSTEM FOR A GO KART

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Bachelor of Mechatronics Engineering

May 2015

“I hereby declare that I have read through this report entitle “Development of an Active Braking System for a Go-kart” and found that it has comply the partial fulfilment for awarding the degree of Bachelor of Mechatronics Engineering.

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DEVELOPMENT OF AN ACTIVE BRAKING SYSTEM FOR A GO KART

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A report submitted in partial fulfilment of the requirements for the degree
of Mechatronics Engineering

Faculty of Electrical Engineering

UNIVERSITI TEKNIKAL MALAYSIA MELAKA

2014 / 2015

I declare that this report entitle “Development of an Active Braking System for a Go-kart” is the result of my own research except as cited in the references. The report has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

Signature :

Name :

Date :

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Abstract

This report presents the development of an active braking system for a go-kart. The active braking system is a noble invention where its benefit is significant to improve driving safety and to save human life during an emergency situation. The development of this electromechanical brake system is significant to the progress of the automobile technology towards the eco-friendly or green technology, as the electromechanical braking system mitigates the usage of hydraulic fluid; whereby common problems such as improper disposal of used hydraulic fluid or leaked hydraulic fluid pollutes our environment. Besides, driving safety of a vehicle greatly improves when active braking system such as anti-locking brakes, traction control system and vehicle stability control is implemented in a vehicle. The active braking system is implemented by designing an electromechanical braking system. The basic requirement of an active braking system is that actuators component in the system has to be electrically powered and controlled using electronic controllers. The electromechanical brake system consists of a linear actuated stepper motor as the brake actuator, an Arduino Uno R3 as the brake system microcontroller, IR coupling sensor as the wheel speed sensor and a potentiometer as the brake pedal, whereby certain conventional brake components such as brake disc and brake pads are being used as well. The linear actuated stepper motor must be able to provide strong thrust force and quick response time for a good braking performance. The brake caliper assembly has to be light weight and easy to assemble or disassemble for regular maintenance work. An IR sensor coupling is used to provide signal pulses for wheel speed measurement for the Anti-locking Brake System (ABS) and wheel slip control (TRC) that was implemented in the active braking system. To achieve the objective of this project, an active braking system for a go-kart will be developed and a test bench will be fabricated to test the completed electromechanical braking system.

ABSTRAK

Laporan ini membentangkan pembinaan sistem brek aktif untuk *go-kart*. Pembinaan sistem brek elektromekanikal ini mempunyai impak yang kuat terhadap kemajuan teknologi otomobil ke arah teknologi hijau, dimana cecair brek tidak digunakan dalam sistem brek elektromekanikal ini, mengurangkan masalah biasa seperti kebocoran cecair brek yang menyebabkan pencemaran alam sekitar. Bukan itu sahaja, keselamatan permanduan juga diperbaiki apabila teknologi aktif seperti *Anti-Locking Brake* dan *Traction Control* digunakan dalam sesuatu kenderaan. Sistem brek aktif ini adalah ciptaan yang penting dimana ia menaikkan taraf keselamatan pemandu kenderaan dan mampu menyelamatkan nyawa manusia semasa keadaan kecemasan. Sistem brek aktif ini dilaksanakan melalui rekaan cipta suatu sistem brek elektromekanikal. Keperluan asas untuk sesuatu system brek aktif ialah komponen penggerakannya mesti dikuasai elektrik dan dikawal oleh pengawal elektronik. Sistem brek elektromekanikal ini terdiri daripada satu *linear actuated stepper motor*, mikropengawal *Arduino*, satu set pengesan inframerah, dan pedal brek elektronik, dimana komponen brek biasa seperti cakera brek, pad brek juga digunakan. *Linear actuated stepper motor* perlu mempunyai kuasa cengkaman yang kuat untuk memberi prestasi baik untuk brek dan cepat balasan breknya . Pemasangan angkup brek perlu ringan dan mudah dipasang untuk mempermudah kerja pengelenggaraan. Set pengesan inframerah memberi isyarat nadi bagi menentukan kelajuan putaran roda untuk sistem *Anti-Locking Brake* dan *Traction Control* dalam system brek aktif ini. Untuk mencapai objektif projek ini, suatu sistem brek aktif untuk *go-kart* akan dibina dan diuji atas bangku ujian sistem brek elektromekanikal.

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CHAPTER 1

INTRODUCTION

1.1 Motivation

In recent years, electromechanical braking system (EMB) has gained worldwide popularity and had a strong development support by the automobile manufacturers. Motivation to research on the electromechanical braking system is to overcome certain problems caused by the hydraulic braking system.

Firstly, the United States Environmental Protection Agency (EPA) reported that there are 7.5 million liters of used hydraulic fluid that were improperly disposed yearly [20]. It is a significant improvement for the environment as the electromechanical brakes doesn't use any hydraulic fluid. Besides, leakage of the hydraulic fluid is also a common problem found in vehicles with hydraulic braking system.

Next, hydraulic braking system has long brake lines and excessive brake pipe valve components that often lead to the delay of hydraulic transmission time, weak dynamic response speed, and these disadvantages are responsible for the increased braking distance and reduced driving security [2].

Moreover, the electromechanical brakes have faster braking response due to the motor dynamics. Based on a journal in 2010 [4], the motor-driven brake system can apply the braking force more quickly and more accurately than conventional brake system.

Finally, the gaining popularity of go-karts in motorsports has brought an interest to develop a more advanced and satisfied driving experience. Therefore, it is essential to provide a safe go-kart platform for the amateurs and professionals at an affordable cost of utilizing the electromechanical braking system.

1.2 Problem Statement

When the project is in progress, I have found several problems related to the electromechanical brake system. Firstly, the selected linear actuated motor have a linear thrust force 58N only and it is inversely proportional to the rotational speed of the stepper motor. To achieve a good braking performance, the electromechanical brake must have two essential properties, which is strong clamping force and quick braking response time. Therefore, I have to make several experiments to achieve an optimum setting of linear trust force and motor rotational speed. At the same time, the brake assembly must be reliable, light-weight, easy to assemble and dissemble during maintenance work. Next, the Arduino Uno R3 controller has a processing speed of only 16Mghz, where by the speed of execution is limited by the hardware itself. I found that the Arduino Uno R3 can only handle one interrupt process at a time, although it has two interrupt pins at Digital Pin 2(Int0) & Digital Pin 3(Int1). The wheel speed sensor for anti-locking brake system (ABS) and wheel slip control or traction control (TSC) on both wheels can only be implemented if two Arduino Uno R3 controller is used simultaneously. Therefore, only the quarter wheel model and test bench will be tested in this research.

1.3 Objectives

1. To design, develop and fabricate an electromechanical braking system for a go-kart with anti-locking brake system (ABS).
2. To analyze the performance of the electromechanical braking system in terms of braking response time, braking distance, and active features which is anti-locking brake system (ABS) on a test bench.
3. To fabricate and implement the electromechanical braking system on a go-kart.

1.4 Scopes

1. The electromechanical braking system includes a potentiometer as the brake pedal to provide brake signal input, a brake caliper assembly for the brake actuator, brake disc for the spinning wheel and controller hardware as the brake control unit.
2. Arduino UNO R3 is used as the main controller and Arduino IDE programming and C++ control algorithm is used to control the electromechanical brake system.
3. The electromechanical brake system must show an improvement in braking performance under wheel lock condition with the ABS system activated, when full braking force is applied.
4. An electromechanical braking system test bench for only the front-right single wheel model will be developed for the purpose of this study.
5. A three phase AC motor will be used to rotate the disc brake to simulate a rotating wheel.

CHAPTER 2

LITERATURE REVIEW

2.0 Overview

This chapter elaborates the literature review based on research journals regarding the Electromechanical Brake System (EMB). The electromechanical brake system reviews were inclusive of design of the brake itself which consists of the components being used, types of actuators and mechanism, and types of motor. Besides, mathematical system modeling and physics of the electromechanical brake are compared were summarized. Not only that, algorithm and logic of the electromechanical brake controller from each of the system found in the research journals were explained. Lastly, system performance of the electromechanical brake system from certain analysis aspects was shown, where results were obtained from either simulations, actual tests or both.

2.1 Literature Review on Design

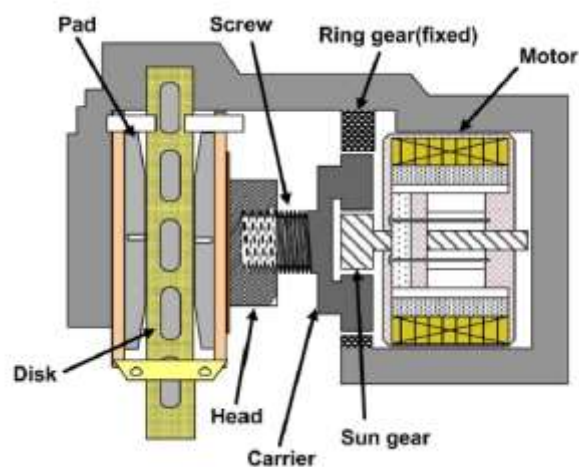


Figure 2.1.1: Electromechanical brake caliper using Servomotor [1]

In the paper done by [1], the author designed a “brake-by-wire” planetary reducing gear-type electromechanical brake (EMB) as shown in Figure 2.1.1. The electromechanical brake assembly consists of a motor that works together with a set of planetary reducing gear, and also a lead screw which converts rotational motion of the servo motor to a linear actuated motion. This linear actuated motion mimicked the piston in a hydraulic brake system. Clamping force act on the brake pads, which generates friction between the brake disc and brake pad to either slow down or stops the rotor from spinning. The maximum amount of clamping force was dependent on the torque of the servo motor, where it was rated at 2.86Nm. Power of servo motor was 300W and maximum rotation was 2000rpm. The sun and ring gears, or known as planetary reduction gear had a ratio of 10:1. The screw thread gear had a pitch of 2.5mm with a radius of 11.5mm and thread angle of 3.2°. The working principle of this EMB was as follow: When the driver pushes the brake pedal, a brake control unit (BCU) estimates the required braking force as per the driver demand. The brake control unit transmits the control signal to the motor driver to drive the motor. The rotational motion of the servo motor was converted to a linear motion through the lead screw, and provides the braking torque. The clamping force was regulated through the displacement of the head of lead screw; the more the head of lead screw travels, the greater the clamping force acts on the brake pad.

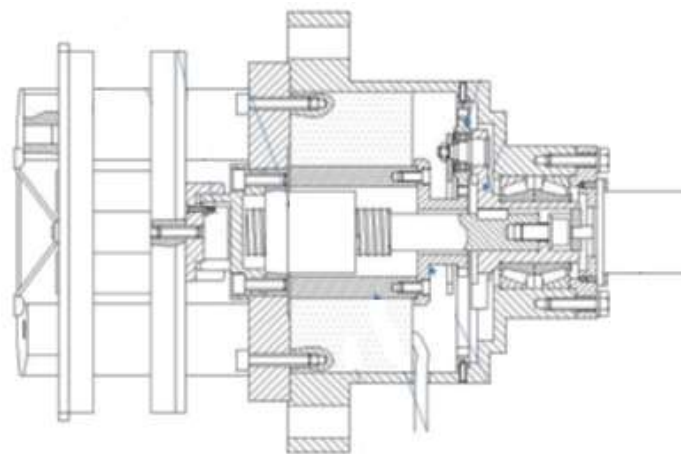


Figure 2.1.2: Electromechanical brake caliper using Brushless DC motor [2]

In the next paper done by [7], the author designed and simulated an electromechanical brake system which consists of a brushless DC motor and mechanism actuators as shown in Figure 2.1.2. Basic braking components were used

in this design, which were the brake pads, brake disc, calipers, and planetary gear sets. There was an addition of a speed sensor and a pressure sensor for closed-loop speed control, clamping force control and error correction. The linear actuated motion was produced through a ball screw and planetary gear set that was connected to the shaft of the brushless DC motor. The brushless DC motor was rated at 373W, with base armature voltage of 42V, base torque of 0.89Nm, and a maximum speed of 4000rpm. The lead of ball screw was 5mm while the planetary gear had a reduction ratio of 3.9:1. The operation of the system was as follow: When the controller unit receives a braking signal, the brushless DC motor rotates, and the rotation is reduced by the planetary gear sets, and the motor rotation is converted to a linear motion through the ball screw. The linear motion pushes the brake pad against the brake disc and produces the braking effect. When the braking signal ends, the brushless DC motor reverse its rotation direction and the ball screw retracts the brake pad away from the brake disc.

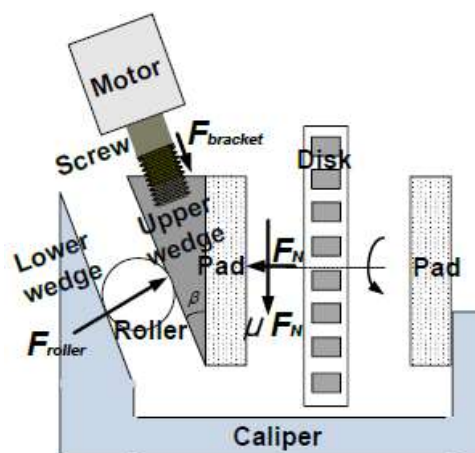


Figure 2.1.3: Wedge type electromechanical brake caliper using DC motor [14]

In the following paper done by [14], the author designed a wedge type electromechanical brake system as shown in Figure 2.1.3. This design has a diagonally mounted DC motor with 30kW power rating, 6500rpm maximum speed and 215Nm maximum torque. It uses the conventional brake disc and brake pads; however clamping motion of the wedge was at a diagonal direction, not horizontal when compared to the usual brake actuator motion. The working operation of this design was as follow: When the brake pedal was pushed, signal will be sent to the brake control unit to estimate the required clamping force. The brake control unit

controls the rotation of the DC motor, and the upper wedge moves towards the brake disc. When the upper wedge closes in, the roller pulls in the lower wedge. Brake pads made contact with the brake disc to provide the required braking friction.

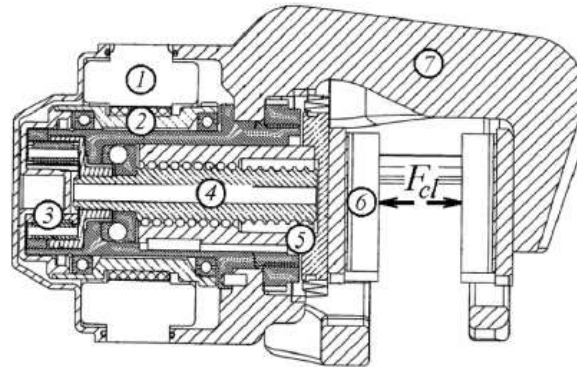


Figure 2.1.4: Electromechanical brake caliper with built-in linear actuated motor [16]

In the paper done by [16], the author designed an electromechanical brake which a built in linear-actuated DC motor, and used the ball screw to convert rotational motion of the motor to linear motion as shown in Figure 2.1.4. The linear motion acts on the inner brake pad (closer to the motor), and the outer brake pad (further from motor) were pulled in from the clamping motion. This was due to the floating characteristics of the brake caliper itself where the caliper was allowed to slide on a shaft.

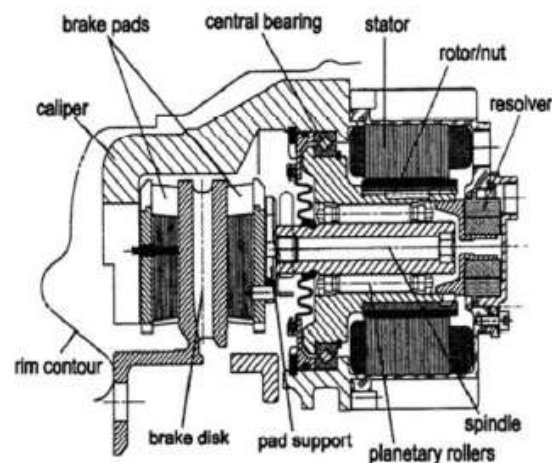


Figure 2.1.5: Electromechanical brake caliper using a Switch-Reluctance Motor (SRM) [17]

In the paper done by [17], the author designed an electromechanical brake which was controlled using “brake-by-wire” as shown in Figure 2.1.5. The driver’s foot pressure measured by a sensor on the brake pedal was communicated to microcontrollers that relay the signal to the electromechanical brake actuators situated at each wheel. The brake actuator then applies the required pressure at the brake pads to smoothly control the car’s speed. The motor used in this design was a Switch-Reluctance Motor (SRM). This SRM has a positioning and speed control embedded in the system. There was a clamping force sensor in this unit as well. The controller hardware that was being used for the brake control unit was a microcontroller.

2.2 Literature Review on System Modelling

In [1], the author modelled the system using a dynamic model of the EMB. The model includes motor, reduction gear, screw thread gear, and clamping force of caliper. The motor was modelled using standard equations for a permanent magnet DC motor in Eq.2.2.1, where V_a is armature voltage, R_a is armature resistance, i_a is armature current, L_a is armature inductance, K_{EMF} is back-emf voltage and ω_m is motor rotational speed.

$$\begin{aligned} V_a &= R_a i_a + K_{EMF} \omega_m + L_a \dot{i}_a \\ \dot{i}_a &= \frac{1}{L_a} (V_a - R_a i_a - K_{EMF} \omega_m) \end{aligned} \quad (\text{Eq.2.2.1})$$

It was assumed that the field current was constant; the motor torque T_m represented as in Eq.2.2.2, where K_{motor} is motor torque constant, J_m is angular inertia of motor and T_F is frictional torque.

$$\begin{aligned} T_m &= K_{motor} i_a \\ J_m \dot{\omega}_m + T_F + T_{load} &= K_{motor} i_a \end{aligned} \quad (\text{Eq.2.2.2})$$

The speed and torque ratio of the planetary reduction gear were represented by Eq.2.2.3, where Z_{sun} is the sun-gear teeth number, Z_{ring} is the ring-gear teeth number, ω_{sun} is the sun-gear speed which equals to the motor speed ω_m ; $\omega_{carrier}$ is the carrier speed, T_{sun} is the sun-gear torque, $T_{carrier}$ is the carrier torque, and GR is the reduction gear ratio.

$$\begin{aligned}\frac{Z_{\text{sun}}}{Z_{\text{sun}} + Z_{\text{ring}}}\omega_{\text{sun}} &= \frac{1}{GR}\omega_{\text{sun}} = \omega_{\text{carrier}} \\ \frac{Z_{\text{sun}} + Z_{\text{ring}}}{Z_{\text{sun}}}T_{\text{sun}} &= GRT_{\text{sun}} = T_{\text{carrier}}\end{aligned}\quad (\text{Eq.2.2.3})$$

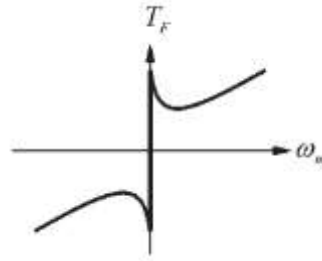


Figure 2.2.1: Motor torque friction model [1]

The motor friction torque, T_F model shown in Figure 2.2.1 above was expressed by Eq.2.2.4, where T_C is the Coulomb-friction torque, T_S is the stiction torque, b_v is coefficient of viscous friction, and ω_s is Stribeck velocity.

$$T_F = |T_C + (T_S - T_C)e^{-|\omega_m/\omega_s|^{\delta_S}} + b_v\omega_m \quad (\text{Eq.2.2.4})$$

For the screw-thread gear, it was modelled with a torsional spring damper. The forces acting on the screw was shown in Figure 2.2.2, and the rotational speed of the screw, $\dot{\theta}_{\text{screw}}$ was represented by Eq.2.2.5, where $\dot{\theta}_{\text{screw}}$ is the rotational displacement of the screw, r_{screw} is the average radius of the screw, and \dot{x}_{head} is the linear displacement of the head along the slope.

$$\dot{\theta}_{\text{screw}} = \frac{1}{GR}\omega_m - \frac{\cos \alpha}{r_{\text{screw}}}\dot{x}_{\text{head}} \quad (\text{Eq.2.2.5})$$

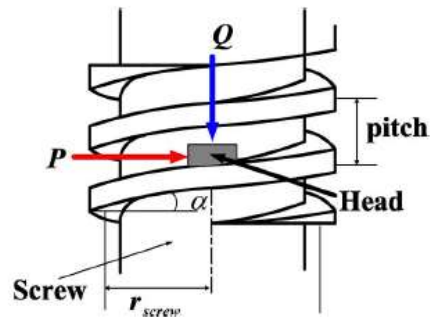


Figure 2.2.2: Diagram showing the screw radius, head and pitch of lead screw [1]

The force P acts on the head and was obtained from the rotational displacement of the screw θ_{screw} from Eq.2.2.6, where k_{screw} is the screw stiffness, and b_{screw} is the frictional coefficient.

$$P = \frac{k_{screw}\theta_{screw} + b_{screw}\dot{\theta}_{screw}}{r_{screw}} \quad (\text{Eq.2.2.6})$$

Static friction on the screw was described as shown in Figure 2.2.3 through the normal force as $F_s = \mu_s N$, where F_s is the static friction, μ_s is the static-friction coefficient and N is the normal force, which is represented by $N = Q \cos \alpha + P \sin \alpha$.

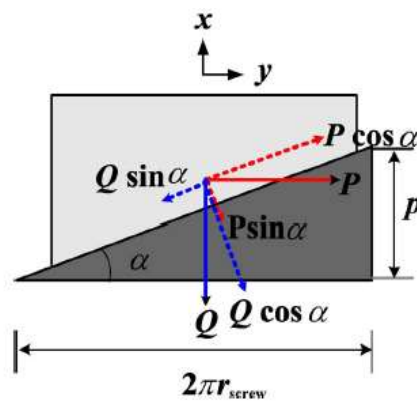


Figure 2.2.3: Static friction model [1]

The frictional force increases up to some point with the sliding velocity and decreases as the sliding velocity increases. The velocity where the frictional force begins to decrease was called the “Stribeck” velocity. After the region of decrease, the frictional force increases again with the sliding velocity. LuGre’s frictional force was represented by the following equations in Eq.2.2.7: