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ANALYTICAL MODELLING AND FREE VIBRATION ANALYSIS OF RECTANGULAR PLATE WITH AN ARBITRARILY ORIENTED SURFACE CRACK

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This thesis is submitted as a part of the fulfillment for the bestowal of Bachelor in Mechanical Engineering (Design & innovation) with honours.

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DECLARATION

"I hereby declare that the work in this report entitle
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SPECIAL TO

Mom, Puan Badariah bt Abu Bakar

A strong and gentle soul who taught me to trust in Allah, believe in hard work and that so much could be done with little. Thank you for guiding me as a person, teaching me that my job in life was to learn, to be happy, and to know and understand myself; only then could I know and understand others.

Dad, En Mohd Mokhtar b Saijon

For earning an honest living for our family, for supporting and encouraging me to believe in myself, and who supported me emotionally and financially. I always knew that you believed in me and wanted the best for me.

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Words cannot express my gratitude for everything you have done. Thank you for accompanying me on this project, and be the ones I tend to turn to first when I have something to discuss. Or just need a break.

ABSTRACT

In a world without borders, which aircraft is one of important transportation to travel anywhere, with low cost, less time and need energy. Nowadays, engineer study and make observation about dynamic behaviour of thin isotropic rectangular plates due to the safety of aircraft. In aerospace vehicles, lightweight material also one of the important characteristic due to the industrial needed which can reduce many materials and costing. However, as all known, the structure of light weight plate always very thin and the material is high modulus young which can produce to the unused high vibration. This research presents about vibration analysis for a thin isotropic plate containing an arbitrarily orientated surface crack. The content of this research activity is about the significantly enhanced crack model within the analytical model of the plate, in modal space, and taking the form of a specialised Duffing equation. To represent the vibrational response of the plate,the governing equation of motion of the healthy plate model with enhanced crack modelling is proposed based on Classical Plate Theory into which a developed crack model has been developed. The formulation of the angled crack is based on a simplified line-spring model, and the cracked plate is subjected to transverse harmonic excitation with arbitrarily chosen boundary conditions.The results from this thesis show that there is changes between healthy plate and cracked plate.The angle of cracked plate show the different neutral frequency obtain by final equation of cracked plate.

ABSTRAK

Pada masa kini,dalam dunia tanpa sempadan, pesawat udara merupakan salah satu pengangkutan yang penting untuk perjalanan kemana sahaja, dengan kos yang rendah, penjimatan masa dan tenaga. Jurutera telah membuat pemerhatian tentang tingkah laku dinamik plat segi empat tepat isotropi nipis untuk memenuhi keselamatan pesawat udara. Dalam kenderaan aeroangkasa, ringan merupakan salah satu ciri penting untuk industri dalam mengurangkan bahan dan kos. Walau bagaimanapun, seperti yang diketahui, struktur plat ringan sentiasa sangat nipis dan bahan ini adalah modulus tinggi muda yang boleh membawa kepada keadaan yang tidak diingini dalam getaran.Rujukan ilmiah ini membentangkan analisis getaran untuk plat isotropi nipis mengandungi permukaan retak..Persamaan Duffing digunakan untuk penyelidikan ini khas nya untuk membuat pengiraan model analisis plat retak. Teori Plat Klasik digunakan untuk persamaan yang mengawal gerakan model plat dengan pemodelan retak Penggubalan retak bersudut adalah berdasarkan kepada „Line Spring Model", dan plat retak ini tertakluk kepada harmonik pengujaan melintang dengan syarat kondisi.Keputusan dari kajian ini,dapati ada perubahan antara plat sihat dengan plat retak.Sudut retak pada plat retak juga memainkan peranan yang penting dalam pengumpulan data.

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LIST OF ABBREVIATIONS

HOQ	-	House of Quality
D		Flexural rigidity
E		Modulus of Elasticity
ν		Poisson‟s ratio
ρ		Density of the plate
w		Transverse deflection
n_x, n_y, n_{xy}, n_o		In-plane forces per unit length
h		Thickness of the plate
Q_x, Q_y		Forces per unit length acting on the plate element
M_x, M_y, M_{xy}, M_o		Bending moments per unit length
σ_x, σ_y, τ_{xy}		Bending and shear stresses
$\bar{M}_y$ $\bar{M}_{xy}$		Bending moment due to crack, per unit length
$\bar{n}_y$ $\bar{n}_{xy}$		In-plane force due to crack, per unit length
$\bar{\sigma}_{mn}$, $\bar{m}_{mn}$		Nominal tensile and bending stresses at the crack location
$\bar{\sigma}_{pq}$, $\bar{m}_{pq}$		Tangential tensile and bending stresses at the crack location
σ_{mn} and m_{mn}		Nominal tensile and bending stresses at the far sides of the plate
σ_{pq} and m_{pq}		Tangential tensile and bending stresses at the far sides of the plate
$\bar{n}_{mn}$		Force per unit length in the y-direction
$\bar{M}_{mn}$		Moment per unit length in the y-direction
$\bar{n}_{pq}$		Force per unit length in the x-y-direction

$\bar{M}_{pq}$	Moment per unit length in the *x*-*y*-direction
a	Half crack length
β	Crack orientation angle
CCFF	Clamped-clamped-free-free
SSSS	All edges simply supported
αtt, Ctt	Nondimensional stretching compliance coefficients
αbb, Cbb	Nondimensional bending compliance coefficients
$\alpha bt = \alpha tb$	Nondimensional stretching-bending compliance
$Cbt = Ctb$	Nondimensional stretching-bending compliance
X_i, Y_j	Characteristic or modal functions of the cracked plate
A_{ij}	Arbitrary amplitude
$\psi_{ij}(t)$	Time dependent modal coordinate
l_1, l_2	Lengths of the edges of the plate
$\lambda_{m,n}$, $\gamma_{m,n}$	Mode shape constants
m, n	Mode numbers
ε_x , ε_y	Middle surface strains
M_{ij}	Complex modal component-1
K_{ij}	Complex modal component-2
ω_{ij}	Natural frequency of the cracked plate
C_3 , γ_{ij}	Nonlinear cubic spring stiffness
b	Real amplitude
α	Phase angle

CHAPTER 1

INTRODUCTION

1.1 PROJECT BACKGROUND

In a world without borders, which aircraft is one of important transportation to travel anywhere, with low cost, less time and need energy. Nowadays, engineer study and make observation about dynamic behaviour of thin isotropic plates due to the safety of aircraft. In aerospace vehicles, lightweight material also one of the important characteristic due to the industrial needed which can reduce many materials and costing.

However, as all known, the structure of light weight plate always very thin and the material is high modulus young which can lead to the unwanted high vibration. Vibration can be known as oscillating, reciprocating, or other periodic motion of a rigid or elastic body or medium forced from a position or state of equilibrium. The sources of unwanted high vibration is from the gravity, the changing of pressure due to the ground surface to the atmosphere, changing of climate and strong wind upward.

In order to understand the behaviour of plate, there are some theories that will be use such as Kirchhoff plate theory known as classical plate theory and Mindlin-Reissner plate theory. This theory introduced by Thimoshenko and Woinowsky –Krieger in

1959.In this research, the boundary condition of plate will be in position clamped-clamped-free-free (CCFF) and semi supported-semi supported (SSSS) which are no external force is required. From the equation of an isotropic plate with an arbitrary oriented part-through surface crack at the centre by an arbitrarily located concentrated force will be extracted by include the position and angle. From derived equation of motion of the cracked plate, the parameter study will be carry on by considering different angle of crack and boundary conditions. Then, comparison study between the theoretical and numerical results will be performed.

1.1 PROBLEM STATEMENT

In this engineering problem, all moving and rotating object will produce vibration. Vibration can be produced by oscillating, reciprocating, or other periodic motion of a rigid or elastic body or medium forced from a position or state of equilibrium. However, the unwanted vibration which can lead damage to the structure, make danger and obstacles to the systems. As an engineer who put the safety as first priority, the detection of damage to thin plate structure at the initial stage of developed can be optimized system performance and safety.

At the same time, it will cut cost due to prevent from huge damage structure. Based on the literature review by researcher for the vibration analysis of a cracked plate, it can be seen as that most of the published papers have analysed vibrations in plates with part-through surface cracks, different geometry, boundary condition which has twenty one approved by researcher and part-through finite length cracks, all over part-through cracks, internal cracks, with different crack angle or location of the crack. However, there are limited papers or thesis have investigated the vibration analysis of the plate with consider the crack have different angle and different length.

There are many vibration methods in detection the crack publish in paper by researcher. There are same condition boundary but have different result by using analytical approach and Finite Element Analysis. For analytical approach, there are

many method uses such as First Order Multiple Scales Method while for Finite Element Method by using ABAQUS and CAE. However in this research, focus point on solving by using MATLAB software after produce equation of crack plate.

1.2 OBJECTIVE

The objective of this research is:

I. To develop mathematical model of an aircraft panel surface structure modelled as an isotropic thin plate with an arbitrarily orientated surface crack with arbitrarily chosen Boundary condition based on Classical Plate Theory.

II. To obtain the natural frequency of the cracked model and study the influence of the orientation of the crack on the vibration characteristic of the plate.

III. To verify the developed model via comparison of the results with different boundary condition, square geometry and rectangular geometry, the length of crack and angle of the crack.

1.3 SCOPE

I. Derive the equation of motion for the free vibration of cracked plate based on Classical Plate Theory.

II. Derive the equation of motion for vibration analysis on plate containing an arbitrarily oriented surface crack using Line Spring Model.

III. Obtain natural frequency of cracked and healthy plate by using MATLAB software.

IV. Compare the result between boundary condition, crack length and crack angle.

CHAPTER 2

LITERATURE REVIEW

2.1 PLATE STRUCTURE

Plate structures are used in almost every area of engineering field, including aerospace, naval architecture, civil engineering and electronic. These structures have diverse geometries and have to withstand a wide range of loading conditions. The basic elements for structural analysis are plates, beams, frames and shells. Flexible plates are initially structural element where the thickness is much smaller than the other dimensions. There are three kinds of plates which are thin plates with small deflections, thin plates with large deflections and thick plates by Timoshenko and Woinowsky-Krieger (1959). The definition of thin plates with small deflection is if the width of the plates small compare to its thickness, the theory of bending of the plate by lateral load can be developed by making some assumption which are :-

I. There is no deformation in the middle plane of the plate. The plane of plate remains neutral during bending.

II. Points of the plate lying initially on a normal to the middle plane of the plate remain on the normal to the middle surface of the plate after bending.

III. The normal stresses in the direction transverse to the plate can be disregarded.

From the first assumption given, the function of two coordinates in the plane of the plate which is all stress components expressed by deflection width of the plate. Width of the plate can be define by satisfy a linear partial differential equation with its boundary conditions. This equation can give information for calculating stress at any point of plates. The second assumption is equivalent of effect shear stress forces on the deflection of the plates. In additional case to lateral loads, the membrane forces acting in the middle plane of the plate, the first assumption be use, and its can be consider of the effect of the plate of the stress acting in the middle plane of the plate(Ismail.R,2012).

Thin plates with large deflection for the first assumption can be completely satisfied if the thin plate is bent to the developable surface. If the deflection plate are small compare its thickness, the stress of the middle plane are neglect. If the deflection is not too small which is larger than its thickness, the stress must be taken into consideration in derivation of differentiation equation. Kirchhoff plate theory or Classical plate theory and the Mindlin-Reissner plate theory are widely used solving or understanding the behaviour of plate structure in engineering problems. This two plate's theory have been accepted and applied by the researcher. This comprehensive background on plates has been provided by Timoshenko and Woinowsky-Krieger (1959) in their book "*Theory of Plate and Shells*".

This research focused on thin plate structures to develop a model which is light, reliable and efficient structures. The ratio of the thickness must be smaller less than 1/20 or 0.05 of span length. If the ratio is larger than 1/20 of span length, the transverse shear deformation must be considered and counted due to the definition of thick plate (Ugural, 1999). Plate which is originally flat, will develops shear force, bending and twisting moment to resist transverse loadings (Ventsel and Krauthammer, 2001).As all known, the plate which is light, thin and large structure can be lead to the high vibration. As an

example, put one plate which is large in dimension, thin and light in vertical position in the windy climax, sure the plate will produce vibration and sound. This vibration can detected by bare hand or more accurate by using vibration sensor. The effect of high vibration can cause structural fatigue, reduces safety of the element of component and reduce system effectiveness.

2.1.1 Free Vibration of Rectangular Plate

According to the book "*The Free Vibration of Rectangular Plates*" by A.W.Leissa from Federal Institute of Technology, Zurich, Switzerland ,there are twenty case exist which involve the possible combinations of clamped (C) ,simply supported (SS) and free (F) edge condition. Ritz method is employed 36 terms containing the product of beam function to analyze remaining 15 cases. For the last 15 cases, comparisons are made with Warburton's usefull approximate formula. From this comparison, the effects of changing Poisson's ratio have been studied. In 1909, Ritz used completely free problem to demonstrate and obtain upper bounds vibration frequencies by using extending Rayleigh Principle. Rectangular plates may be regarded as first approximation to wing and blades and occur as panels in many forms of engineering structures, thus the design stage is importance to consider natural frequency (Warburton, 1983)

2.1.2 Vibration Problems in Plates

There are many problems involving plate use in our daily day. As a result increasing of researcher since 1878 until 20^{th} century to solve the problem of vibration in plates whether it is free force or applied by forced. Free force is without membrane force

while for applied force, there have membrane force. All of importance in design of plate structure and many of the important things in these studies were documented in Leissa's Book (Zhou and Zang, 2006).The "*Theory of Sound*" by Rayleigh in 1894 contributions to modern vibration analysis through his physical insight, simple concept and energy principle. According to Szilard (2004),initial mathematical solution for free vibration problem based on membrane theory of plates formulated by Euler ,1776 and Bernoulli 1789,and in 1813,Lagrange developed the governing equation of free vibration analysis of plates.

Then in 1836, Navier derived differential equation for plates subjected to distributed static lateral loads and Kirchhoff (1887) obtained similar differential equation for plate problem through the use of different energy. In 1921, Timoshenko was applied corrections for rotary inertia and shear for case beams. In 1944 and 1945, Reissner improved equation for vibration of thick plates through complementary energy principle by including effect of shear and rotary inertia. Effect with a different approach by modified theorem and assumption by Mindlin in 1951; he managed to develop an equation.

Warburton, 1954 in his book "*The vibration of rectangular plates*", proposed the first collection of solution for rectangular plates. He managed to obtain approximate natural frequency formulas for plates with all boundary condition by using Rayleigh's method. Warburton make reference from "*Theory of sound*" by Rayleigh in 1894 and Leissa's thesis in 1973 "*The free vibration of rectangular plates*". There are 21 types of boundary condition problem which is free, freely-supported, and fixed edge boundary conditions. In one rectangular, there are four edges which are can be free-free-clamp-clamp, free-free-semi supported-semi supported and any other problem.

The book of "The vibration of rectangular plates" written by Leissa(1973), present comprehensive and accurate analytical results for the free vibration of rectangular plates. There is twenty one case exist which involve the possible combination of clamped and free edge conditions. Six cases having two opposite side simply-supported have exact characteristic equation. The Ritz method has thirty six terms containing beam function and analysts fifteen case. The effects of changing