Universiti Teknikal Malaysia Melaka

Fakulti Kejuruteraan Elektrik

**FINAL YEAR PROJECT REPORT**

# TITLE: MODELING AND PSO-BASED LQR CONTROLLER DESIGN FOR COUPLED TANK SYSTEM



**FIONA SERINA DAUD**

**B011110074**

**BACHELOR OF ELECTRICAL ENGINEERING**

**(Control, Instrumentation and Automation)**

"I hereby declare that I have read through this report entitle "Modeling and PSO-based LQR Controller Design for Coupled Tank System" and found out that it has comply the partial fulfillment for awarding the degree of Bachelor of Electrical Engineering (Control, Instrumentation and Automation)"

Signature : ………………………………………….

Supervisor's Name : ………………………………………….

Date : ………………………………………….

# MODELING AND PSO-BASED LQR CONTROLLER DESIGN FOR COUPLED TANK SYSTEM

**FIONA SERINA DAUD**

**A report submitted in partial fulfillment of the requirements for the degree of Bachelor of Electrical Engineering (Control, Instrumentation and Automation)**

**Faculty of Electrical Engineering**

**UNIVERSITI TEKNIKAL MALAYSIA MELAKA**

**2014**

I declare that this report entitle "*Modeling and PSO-based LQR Controller Design for Coupled Tank System*" is the result of my own research except as cited in the references. The report has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.



Signature :………………………………………………

Name :………………………………………………

Date :………………………………………………

To my beloved father and mother

# ACKNOWLEDGEMENT

A major research project like this is never the work of anyone alone. The contributions of many different people, in their different ways, have made this possible. I would like to extend my appreciation especially to the following.

I would like to express my sincere gratitude to my supervisor Miss Nur Asmiza Binti Selamat for the continuous support on my final year project research, for her patience, motivation, enthusiasm, and immense knowledge. Her guidance helped me in all the time of research and completing this project. I could not have imagined having a better supervisor, mentor and guidance in completion of my final year project in Universiti Teknikal Malaysia Melaka.

Besides my supervisor, I would like to thank all the lecturers that helped me throughout the completion of the project and always willing to share and gives their best suggestions. Without their support and guidance, this project will not complete as what I wanted.

Moreover, I would like to express my gratitude to all my fellow friends who always there cheering me up and stood by me through the good and bad times. Completing this project would not have been possible without their helps and support.

Finally, I would like to thank my parents, three elder brothers and sister. They were always supporting me and encouraging me with their best wishes. My research would not have been possible without their encouragement and helps.

# ABSTRACT

Industrial application for liquid level and flow control is used tremendously in a process control industries such as the water treatment industries, paper making industries and petro-chemical industries. Level and flow control is important to ensure the quality and performance of the system can be maintained. This project is related on how to control the water level in the coupled-tank while both tanks is still regulating. It can be classified as Single-Input Single-Output (SISO) system as the process of this system only control one output using one input. Mathematical model can be obtained based on the experimental data collected from the coupled-tank system. The data will be loaded in a process called system identification where the mathematical representation or transfer function of the system can be determined. The validity of the transfer function will be verified before proceeds to controller design. Controller is used as a tool to improve the stability and performance of the transient response of the system. Hence, for this project LQR controller was selected as the controller to control and improve the transient response of the system. The LQR parameter is obtained using the optimization technique called Particle Swarm Optimization (PSO). To validate the result, system performance using LQR controller will be compared with PID controller.

# ABSTRAK

Penggunaan industri berkaitan dengan paras cecair dan kawalan aliran adalah amat diperlukan terutamanya dalam industri kawalan proses seperti industri rawatan air, industri membuat kertas dan industri petro-kimia. Pengawalan paras air dan kawalan aliran adalah penting untuk memastikan kualiti dan prestasi sesuatu sistem dapat dikekalkan. Projek ini berkaitan tentang bagaimana untuk mengawal paras air dalam tangki walaupun pengaliran air untuk tangki berkembar ini masih berjalan. Ia boleh diklasifikasikan sebagai sistem satu masukan satu keluaran (SISO) dimana ia hanya mengawal proses yang mempunyai satu keluaran menggunakan satu masukan. Model matematik boleh diperolehi berdasarkan data yang dikumpul daripada perlaksanaan eksperimen terhadap sistem tersebut. Data ini akan dimuatkan di dalam proses yang dipanggil pengenalan sistem di mana perwakilan matematik atau fungsi pindah sistem tersebut boleh ditentukan. Kesahihan fungsi pemindahan akan diperiksa sebelum beralih ke langkah seterusnya yang merupakan reka bentuk pengawal. Pengawal digunakan sebagai alat untuk meningkatkan kestabilan dan prestasi sesebuah sistem. LQR pengawal dipilih untuk mengawal dan meningkatkan prestasi sistem tersebut. Parameter LQR akan diperolehi dengan menggunakan teknik pengoptimuman yang dipanggil PSO. Untuk mensahihkan prestasi sistem, LQR pengawal akan dibandingkan dengan prestasi sistem daripada pengawal PID.

# TABLE OF CONTENTS

# LIST OF TABLES

# LIST OF FIGURES

\

# LIST OF APPENDICES

# CHAPTER 1

# INTRODUCTION

## 1.1 Background of Study

Coupled tank system plays important role in industrial application such as in petro-chemical industries, paper making industries, medical industries and water treatment industries. The coupled tank system consists of two vertical tanks which are joined together with an orifice and has an inlet liquid pumps and outlet valves. Each tank has its own inlet pump and output valve where the valve will regulate continuously. The coupled tank system can be configured as a Single-Input Single-Output (SISO) or as a Multiple-Input Multiple-Output (MIMO) system via manipulation of pumps input and sectional area of rotary valves. In industries, the liquid in the tank will go through several processes or mixing treatment whereby the level of liquid needs to be controlled and maintained. The flow between tanks must be continuously regulated. The basic control principle of the coupled tank system is to maintain the level of liquid in the tank when there is a process of inflow of liquid into the tank and output of liquid out of the tank.

## 1.2 Problem Statement

There are differences between real-time control and simulation control. Real-time is related to the surrounding where all of the parameters and condition need to be considered whereas simulation is an imitation of the operation of real process. All of the parameters can be controlled by human. To study the performance of the real-time implementation in a system, the coupled-tank system is chosen. In industries, there are problems faced related to the level of liquid in tank and the flow between the tanks. Most of the time, liquid level in the tanks needs to be controlled while regulating the flow in the tank. Common controller used to control the system is PID controller and Fuzzy Logic Control (FLC) controller. However in several cases, the conventional PID controller is not suitable for a controlled object with variable parameter or when there is existence of external disturbances in the system. One of the solutions to achieve high performance of the system is to apply state feedback controller to the system. There are several controllers that have been applied to control the system of the coupled tank. In this project, Linear Quadratic Regulator (LQR) controller will be selected as the state feedback controller for the system. The LQR controller approach will be tested to verify the improvement related to the performance of the system.

## 1.3 Objectives

The aim of this project is to obtain the modeling and controller design for coupled tank system. The main objectives of this project are:

1) To obtain transfer function of the coupled tank using system identification method
2) To implement LQR controller on coupled tank system and obtain its parameter using particle swarm optimization (PSO) technique
3) To compare the system performance of LQR and PID controller

## 1.4 Project Scopes

There are limitations to the completion of this project. The scopes of the project only cover:

1) Interface the coupled tank CTS-001 with DAQ card and use personal computer to stimulate data using software MATLAB
2) The experimental of coupled tank process is using single-input single-output (SISO) control system
3) System Identification used to generate the transfer function of the coupled tank
4) The parameter of LQR controller and PID controller will be determined using PSO optimization technique.

## 1.5 Project outlines

This report basically divided into five chapters:

**CHAPTER 1 Introduction**

This chapter allows the readers to visualize the basic aspects of the research done, such as the overview of the coupled tank system, problem statements, objectives and scopes of the project.

**CHAPTER 2 Literature Review**

This chapter reviews on the basic modeling for a coupled tank, previous controller used for the coupled tank system, several types of optimization techniques use for the system and other reviews that related to this project.



**CHAPTER 3 Design Methodology**

This chapter consists of the flow related to the study and methodology used for this project. The control principle for coupled tank system, modeling of the coupled tank and the design technique for PSO-based LQR controller will be explained in this chapter. The implementation of system identification to obtain the transfer function will also be included in this chapter.

**CHAPTER 4 Result and Discussion**

This chapter shows the characteristic of the system, the fitness function of each controller and the result of the transient response performance using the LQR and PID controller to the system.

**CHAPTER 5 Conclusion & Future Works**

This chapter consists of the conclusion based on the overall works and results. It also includes some future improvement that can be done in the system.

# CHAPTER 2

# LITERATURE REVIEW

## 2.1 Overview

This chapter will review on research which focuses on mathematical model, LQR controller, previous controller used for coupled tank, system identification and optimization method used for the system. Coupled tank system is common in process industries as it need to be pumped from one tank to another. Somehow, the liquid needs to be monitored and the flow and level of liquid need to be controlled although the tank need to be continuously regulated. There are many types of controller use to control the coupled tank system such as the LQR, PID, Fuzzy Logic and Slide Control. The controllers have similar objective which are to improve the system performance. System identification is a tool that produces a transfer function from data collected in the coupled tank. From the transfer function obtained using the system identification, the optimization method is applied to find the best value for Q and R matrix in easier way compared to the trial and error method.

## 2.2 Control Principle of Coupled Tank System

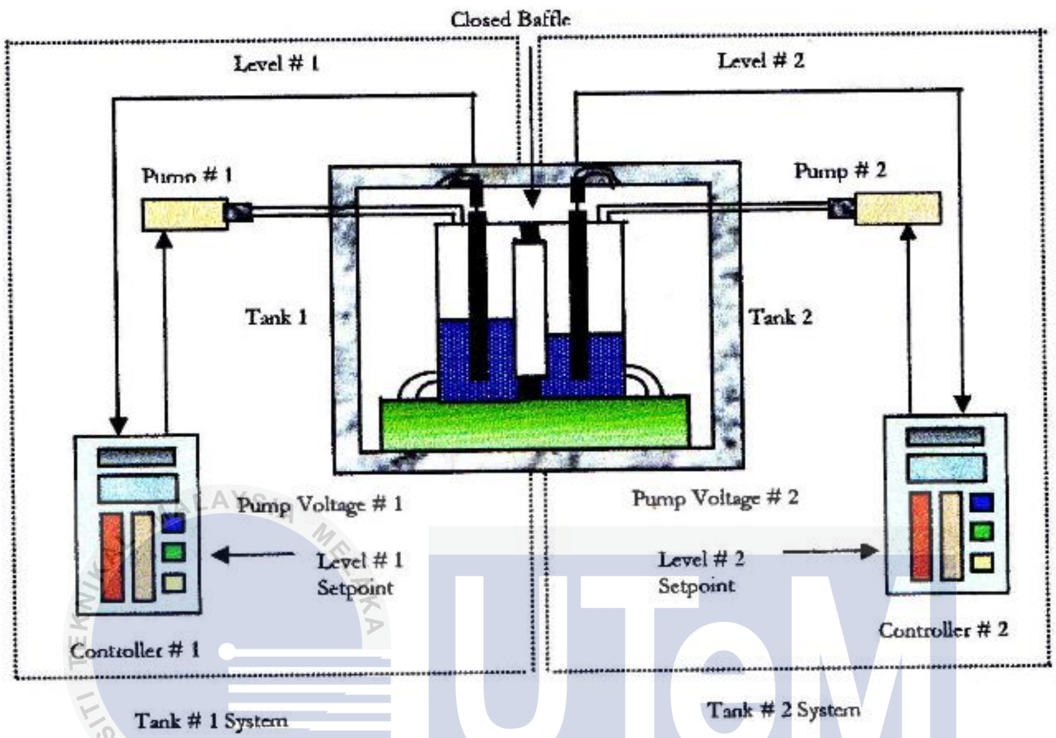


Figure 2.1: Schematic diagram for CTS-001 coupled tank [1]

Figure 2.1 shows the schematic diagram for CTS-001 coupled tank. The basic control principle of the coupled tank system is to maintain the water level in both tanks at a desired point value while the inflow and outflow of water tank keep regulating. Disturbance may be caused by variation in the rate flow from the baffle gap or the changes in the outlet of the tank. When disturbances occur, the water level in the tank will change and settle at a different steady-state level [1].

The control variable involve in the process control of this system is the water level in the coupled tank system. In order to control and maintain the water level at a desired point, the inlet flow rate needs to be adjusted. The pump voltage will become the device to adjust the inlet flow rate. The manipulated variable for this system is the input flow rate because it is the variable needs to maintain the process variable at the desired point.

The input flow rate needs to be adjusted in order to maintain the water level of the tank. For example, if the outflow rate is greater compared to the inflow rate, the inflow rate should be adjusted in order for the water level in the tank to increased and settled. For this project, SISO system will be used where the pump 1 will be used as the inlet and outlet 2 is chosen as the outlet for the system. Table 2.1 shows the condition of the system in a steady state manner for SISO.

Table 2.1: SISO steady state condition for coupled tank system

| **SYSTEM TYPE** | | **STEADY-STATE CONDITION** |
|---|---|---|
| **1st order** | Tank 1 | Inflow rate from the inlet equals to the outflow rate at the outlet |
| | Tank 2 | |
| **2nd order** | Tank 1 | Inflow rate at the inlet equals to the total value from the outflow of rate at the outlet and the outflow rate at the baffle gap |
| | Tank 2 | Total from the Inflow rate at the inlet with the inflow rate at the baffle gap equals to the outflow rate at the outlet |



From Table 2.1, 1st order system shows that the inflow rate from the inlet is equals to the outflow rate at the outlet. It is either to use tank1 or tank 2 because first order system only considered single energy storage element. It is not proper to combine other storage energy element to form a single equivalent energy storage element. For 2nd order system, both tank will be used because 2nd order system is the combination between tank1 and tank 2. Tank 1 and tank 2 will represent the two energy storage element that will be used for the system.

## 2.3 Mathematical Model

The description of a system or process can be illustrated using mathematical model. Model is a combination between mathematics and logical relationship that represents aspects of any situation in system. Mathematical model use the mathematical concepts and it describe the important relationships between variables. Using mathematical model approaches creates much simpler process than the real situation. The elements that are unimportant or irrelevant to the problem will be ignored in order to simplify the equation needed. Below is the dynamic model for the coupled tank control apparatus refer to [1]:

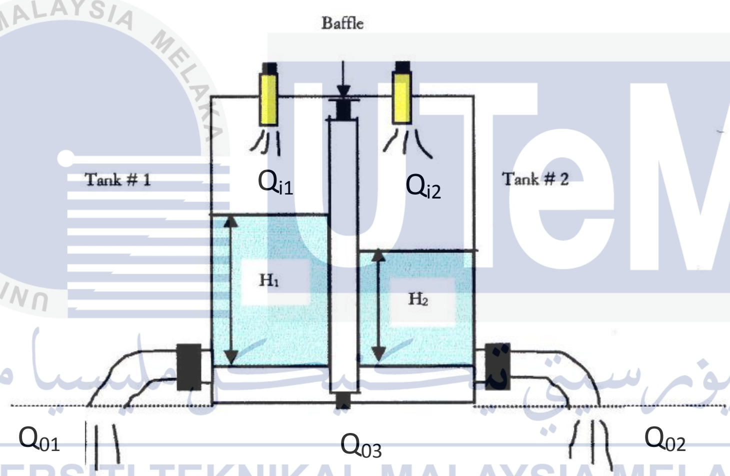




Figure 2.2: Diagram of the coupled tank control apparatus [1]

Considering a simple mass balance, the rate of change of fluid volume in each tank equals the net flow of fluid into the tank. Thus for each of Tank 1 and Tank 2 have:

$$A_1 \frac{dH_1}{dt} = Q_{i1} - Q_{01} - Q_{03} \tag{2.1}$$

$$A_2 \frac{dH_2}{dt} = Q_{i2} - Q_{02} - Q_{03} \tag{2.2}$$

Where $H_1, H_2$ = height of fluid in tank 1 and 2 respectively

$A_1, A_2$ = cross-sectional area of tank 1 and 2 respectively

$Q_{03}$ = flow rate of fluid between tanks

$Q_{i1}, Q_{i2}$ = pump flow rate into tank 1 and 2 respectively

$Q_{01}, Q_{02}$ = flow rate of fluid out of tank 1 and 2 respectively

Bernoulli's equation for steady, non-viscous, incompressible shows that the outlet flow in each tank is proportional to the square root of the height of water in the tank. The flow between the two tanks is proportional to the square root of the height differential. Thus

$$Q_{01} = \alpha_1 \sqrt{H_1} \quad (2.3)$$

$$Q_{02} = \alpha_2 \sqrt{H_2} \quad (2.4)$$

$$Q_{03} = \alpha_3 \sqrt{H_1 - H_2} \quad (2.5)$$

where $\alpha_1, \alpha_2, \alpha_3$ are proportionally constants which depend on the coefficients of discharge, the cross sectional area of each orifice and the gravitational constant. Combining equation (2.1) to (2.5), we have a set of non-linear state equations which describe the system dynamics of the coupled tank apparatus.

$$A_1 \frac{dH_1}{dt} = Q_{i1} - \alpha_1 \sqrt{H_1} - \alpha_3 \sqrt{H_1 - H_2} \quad (2.6)$$

$$A_2 \frac{dH_2}{dt} = Q_{i2} - \alpha_2 \sqrt{H_2} - \alpha_3 \sqrt{H_1 - H_2} \quad (2.7)$$

Suppose that for a set of inflows $Q_{i1}$ and $Q_{i2}$, the fluid level in the tanks is at some steady state levels $H_1$ and $H_2$. Consider small variations in each inflow, $q_1$ in $Q_{i1}$ and $q_2$ in $Q_{i2}$. Let the resulting perturbation in levels be $h_1$ and $h_2$ respectively. From equations (2.6) and (2.7), now have

$$A_1 \frac{d(H_1 + h_1)}{dt} = (Q_{i1} + q_1) - \alpha_1 \sqrt{(H_1 + h_1)} - \alpha_3 \sqrt{H_1 - H_2 + h_1 - h_2} \qquad (2.8)$$

$$A_2 \frac{d(H_2 + h_2)}{dt} = (Q_{i2} + q_2) - \alpha_2 \sqrt{(H_2 + h_2)} - \alpha_3 \sqrt{H_1 - H_2 + h_1 - h_2} \qquad (2.9)$$

Subtracting equation (2.6) and (2.7) from (2.8) and (2.9), we obtain

$$A_1 \frac{dh_1}{dt} = q_1 - \alpha_1 (\sqrt{H_1 + h_1} - \sqrt{H_1}) - \alpha_3 (\sqrt{(H_1 - H_2 + h_1 - h_2)} - \sqrt{H_1 - H_2}) \qquad (2.10)$$

$$A_2 \frac{dh_2}{dt} = q_2 - \alpha_2 (\sqrt{H_2 + h_2} - \sqrt{H_2}) + \alpha_3 (\sqrt{(H_1 - H_2 + h_1 - h_2)} - \sqrt{H_1 - H_2}) \qquad (2.11)$$



For small perturbations

$$\sqrt{H_1 + h_1} = \sqrt{H_1} (1 + \frac{h_1}{H_1})^{\frac{1}{2}} \approx \sqrt{H_1} (1 + \frac{h_1}{2H_1}) \qquad (2.12)$$

Therefore,

$$\sqrt{H_1 + h_1} - \sqrt{H_1} \approx \frac{h_1}{2H_1}$$

Similarly

$$\sqrt{H_2 + h_2} - \sqrt{H_2} \approx \frac{h_2}{2H_2}$$

And

$$(\sqrt{H_2 - H_1 + h_2 - h_1} - \sqrt{H_2 - H_1}) \approx \frac{h_2 - h_1}{2\sqrt{H_2 - H_1}}$$

With these approximations, equation (2.10) and (2.11) simplify to

$$A_1 \frac{dh_1}{dt} = q_1 - \frac{\alpha_1}{2\sqrt{H_1}} h_1 - \frac{\alpha_3}{2\sqrt{H_1 - H_2}} (h_1 - h_2) \qquad (2.13)$$

$$A_2 \frac{dh_2}{dt} = q_2 - \frac{\alpha_2}{2\sqrt{H_2}} h_2 - \frac{\alpha_3}{2\sqrt{H_1 - H_2}} (h_1 - h_2) \qquad (2.14)$$

In equations (2.13) and (2.14), note that the coefficients of the perturbations in level are functions of the steady state operation points $H_1$ and $H_2$. Note that the two equations can also be written in the form



$$A_1 \frac{dh_1}{dt} = q_1 - q_{01} - \frac{\alpha_3}{2\sqrt{H_1 - H_2}} (h_1 - h_2) \qquad (2.15)$$

$$A_2 \frac{dh_2}{dt} = q_2 - q_{02} - \frac{\alpha_3}{2\sqrt{H_1 - H_2}} (h_1 - h_2) \qquad (2.16)$$

where $q_{01}$ and $q_{02}$ represent perturbations in the outflow at the drain pipes. This would be appropriate in the case where outflow is controlled by attaching an external clamp for instance.

### 2.3.1 First-Order Single-Input Single-Output Plant

Consider the configuration where the baffle plate is completely depresses so that there is no flow between the two tanks. Equation (2.13) and (2.14) become

$$A_1 \frac{dh_1}{dt} = q_1 - \frac{\alpha_1}{2\sqrt{H_1}} h_1 \tag{2.17}$$

$$A_2 \frac{dh_2}{dt} = q_2 - \frac{\alpha_2}{2\sqrt{H_2}} h_2 \tag{2.18}$$

Effectively we have two independent first-order systems. Each first order differential equation relates the perturbation in level to the perturbation in inflow. The time constant of the first order model for tank 2 is

$$\text{Time constant, } \tau_2 = \frac{2A_2\sqrt{H_2}}{\alpha_1} \tag{2.19}$$

This shows the dependence on operation point of the plant dynamics. Furthermore it also shows that if the discharge proportionally constant is reduced, the time constant increased. The steady state gain of the model is also dependent on the operation point and is given as follows:

$$\text{Steady state gain} = \frac{2\sqrt{H_2}}{\alpha_2} \tag{2.20}$$

The perturbation in inflow is related in a dynamic fashion to the perturbation in voltage applied to the pump. The transient response for voltage to inflow is generally faster than the time constant of the tank dynamics and may be represented by a small dead time element. If the steady state flow rate is low and discontinuous, there may be further delay due to transport lag in the inflow tubing.

### 2.3.2 Second-Order Single Input Single Output Plant

For the second configuration, the baffle is raised slightly. The manipulated variable is the perturbation to tank 1 inflow. Taking the Laplace Transforms of equations (2.13) and (2.14) and assuming that initially all variables are at their steady state values,

$$A_1 s h_1(s) = q_1(s) - \left( \frac{\alpha_1}{2\sqrt{H_1}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}} \right) h_1(s) + \frac{\alpha_3}{2\sqrt{H_1 - H_2}} h_2(s) \qquad (2.21)$$

$$A_2 s h_2(s) = q_2(s) - \left( \frac{\alpha_2}{2\sqrt{H_2}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}} \right) h_2(s) + \frac{\alpha_3}{2\sqrt{H_1 - H_2}} h_1(s) \qquad (2.22)$$

This may be written as

$$(T_1 s + 1) h_1(s) = k_1 q_1(s) + k_{12} h_2(s) \qquad (2.23)$$

$$(T_2 s + 1) h_2(s) = k_2 q_2(s) + k_{21} h_1(s) \qquad (2.24)$$



where

$$T_1 = \frac{A_1}{\dfrac{\alpha_1}{2\sqrt{H_1}} + \dfrac{\alpha_3}{2\sqrt{H_1 - H_2}}} \qquad T_2 = \frac{A_2}{\dfrac{\alpha_2}{2\sqrt{H_2}} + \dfrac{\alpha_3}{2\sqrt{H_1 - H_2}}}$$

$$k_1 = \frac{1}{\dfrac{\alpha_1}{2\sqrt{H_1}} + \dfrac{\alpha_3}{2\sqrt{H_1 - H_2}}} \qquad k_2 = \frac{1}{\dfrac{\alpha_2}{2\sqrt{H_2}} + \dfrac{\alpha_3}{2\sqrt{H_1 - H_2}}}$$

$$k_{12} = \frac{\dfrac{\alpha_3}{2\sqrt{H_1 - H_2}}}{\dfrac{\alpha_1}{2\sqrt{H_1}} + \dfrac{\alpha_3}{2\sqrt{H_1 - H_2}}} \qquad k_{21} = \frac{\dfrac{\alpha_3}{2\sqrt{H_1 - H_2}}}{\dfrac{\alpha_2}{2\sqrt{H_2}} + \dfrac{\alpha_3}{2\sqrt{H_1 - H_2}}}$$

For second order configuration, $h_2$ is the process variable and $q_1$ is the manipulated variable. For simplicity, we first consider the case when $q_2$ is zero.

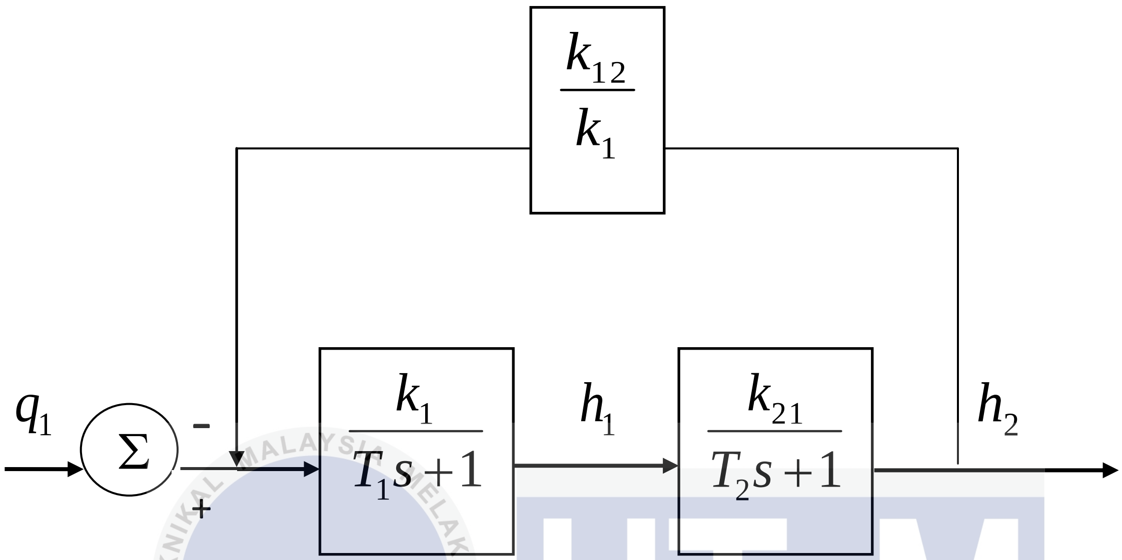


Figure 2.3 : Block diagram of 2nd order process

For equation (2.23) and (2.24) and Figure 2.3, the expressions for the gain, damping factor and natural frequency of the system can be obtained. The characteristic equation shown in equation (2.25)



Characteristic equation = $T_1 T_2 s^2 + (T_1 + T_2)s + 1 - k_{12}k_{21}$ (2.25)

Which yields a natural frequency and damping factor of

$$\omega_n = \sqrt{\frac{1 - k_{12}k_{21}}{T_1 T_2}} \tag{2.26}$$

$$\xi = \frac{T_1 + T_2}{\sqrt{T_1 T_2 (1 - k_{12}k_{21})}} \tag{2.27}$$

## 2.4 System Identification

System identification can be defined as a process to improve or develop the mathematical representation of system using the experimental data collected. There are two types of method used for the system identification which are the frequency domain identification methods and time domain identification methods. System identification need to produce a mathematical model for any system using statistical method by collect data from measured input-output.

Classification of the system using system identification technique involved non-parametric and parametric methods. Non-parametric methods consist of transient response analysis, correlation analysis, frequency response analysis and spectral analysis. There are several techniques involved in parametric methods which are the least square method, pseudorandom binary sequence (PRBS) analysis, prediction error method, instrumental variable method and forgetting factor technique.

H.Gupta and O.P.Verna in [2] used Pseudo Random Binary Signal (PRBS) method where the input and output data of the second order transfer function is computed using System Identification toolbox in MATLAB. The transfer function obtained from the SI can represent the behavior of the system. H.M.Shariff in [3] proposed that Random Gaussian Signal (RGS) is better compared to PRSB in a nonlinear system. RGS is capable to exhibit the nonlinear behavior of the system where PRSB is insufficient to excite the system dynamic and fail to exhibit the nonlinearity behavior in nonlinear system process.

## 2.5 Linear Quadratic Regulator (LQR)

LQR is a state feedback controller that gives the best possible performance with respect to some given measure of performance. The performance measure is a quadratic function composed of state vector and control output [4]. LQR is one of the optimal control techniques, which takes into account the states of the dynamic system and control input to make the optimal control decision [5]. Optimal can be defined as to provide the smallest possible error to its input. Q and R matrix of LQR are usually selected by trial and error. The LQR is well-known design technique that provides practical feedback gains. It is also a tool use for complex system that need high robust in the performance requirement. In real time, LQR problem will be solved using the MATLAB. The data collected need to be run in the System Identification to obtain the transfer function of the system. The challenge undergoing this type of controller is on how to obtain the weight matrices from the system.

### 2.5.1 Performance of LQR Controller



LQR controller can improve the performance of the system depends on how the system works. Some general advantages can be obtained when using LQR controller is such as the precision, speed, recovery time, efficiency and safety. There are several systems that used LQR controller to improve the performance of the system.

Paper [6] found that from simulation, LQR controller give the best performance compared to fuzzy logic controller in pitch control system in an aircraft. It shows that LQR controller has the best performance in term of rising time, settling time and percent of steady state error. The result obtained show that LQR controller is good in eliminating the error from the system which is almost tend to be zero value. Fuzzy logic controller gives the best range in the percent of overshoot. From both performance, it shows that LQR controller provide higher ability in controlling the pitch angle compared to the fuzzy logic controller.

C.Chitu, J,Lackner, M.Horn, H.Waser and M.Kohlbock in [7] proposed that LQR controller is suitable in designing an electrical power steering system. By using LQR controller to the system, it offer better stability in frequency, robustness and closed loop stable step responses during parameters variation.

In 2011[8] compared the performance of LQR controller and fuzzy logic controller in an aircraft roll control System. Aircraft roll control System is based on designing an autopilot that can control the roll angle of an aircraft. From the observation, LQR gives faster response compared to fuzzy logic controller in terms of rise time. Although both controller follow the desired angle, LQR controller follow the reference input by producing very small steady-state error compare to fuzzy logic controller. It shows that LQR controller can handle the effect of disturbances in the system.

The implementation of LQR controller is also used in 4-leg voltage sources inverter. 4-leg inverter is utilized in three-phase four-wire power converter due to its superior performance characteristics such as relatively low DC bus voltage and switching loss, and capability to handle unbalanced load current [9]. A. Mohammadbagher and M.Yaghoobi in [9] stated that LQR controller and PID controller are suitable to utilize the 4-Leg inverter due to both can give zero steady-state error, fast response and no overshoots at the transient response. However, the result obtained proves that the LQR method is better than PID controller in terms of its faster response.



Based on [10], LQR using PSO optimization can gives better results in showing the choice of matrix Q and R for the controller. The best weight matrix Q and R of LQR controller has been very difficult to be determined without using any optimization technique to the controller. By applying PSO as the optimization technique for the system, it gives better results in finding the weight matrix for LQR controller and obtained the best performance despite from differences value of weight matrix.

From the analysis, LQR can be widely used in various industries regardless on the operation of the system. It gives many advantages to the system not only for linear but also non-linear system. Therefore, by using LQR as the controller for coupled tank system, it can improve the performance and allows a better output compared to uncompensated system.

## 2.6 Types of Controller

There are several types of controller used to control the coupled tank system in the industries. Generally, the objective of all the controllers in controlling the liquid level is to maintain the level set point at a specific value and be able to accept new set point values dynamically. The examples of controllers are Linear Quadratic Regulator (LQR), PID, Fuzzy Logic, Sliding Mode and Direct Model Reference Adaptive Control.

### 2.6.1 Proportional Integral Derivative Controller (PID)



The proportional-integral-derivative (PID) method is a kind of feedback controller which is generally based on the error between desired set point and measured process value. The error is then used to adjust some input to the process in order to define its set point. Three parameters (P,I and D) must be designed in the PID controller and each parameter has an effect on the error [4]. Both of the transient and steady-state responses are taken care of with these three parameter functionality. PID controller gives the most efficient solution to problem which related to control system. In [11] proposed PID controller for coupled tank-system because PID controller can achieve the desired output response with given specification. It gives high efficiency of control algorithm and gives a good performance with affordable costs. O.D.Kieran in [12] state that PID controller using tuning method Genetic Algorithm (GA) gives better performance compared to conventional PID. The system for PID controller with tuning method can be tune without approximated model, gives less overshoot and less settling time.

### 2.6.2 Fuzzy Logic Controller (FLC)

M.Abid in [13] used FLC as the controller for the coupled tank system. It is due to the fact that for any system, FLC is not based on mathematical model where it can be applied to a non-linear system such as delay, saturation and dead zones. FLC is a control system based on fuzzy logic mathematical system which are use to analyze the analog input values in terms of logical variables of continuous values taken between 1 and 0. Fuzzy logic is a form of logic that uses approximate method instead extract from the actual value. It emulates the ability to use approximate data to find solutions.

FLC consists of linguistic rules (i.e IF-THEN rule) that can be constructed using the knowledge of experts in the given field of interest [2]. Based on several researchers, FLC can be considered as flexile controller since it is easy to be controlled so that the performance of the system can be improved. In [13] proposed that FLC is better compared to PID controller. By research, FLC gives a better response because the settling time of the system is shorter and the system is more stable compared to PID controller. One of the advantage using the FLC is it improves the non-linearity of the system. L.Liang in [14] compared the Fuzzy-PID controller with the conventional PID and FLC. Fuzzy-PID controller improves the lumping lag and non-linear characteristic of the system. For FLC, it helps to improve the response speed whereas the PID controller will eliminate the steady-state error of the system.

### 2.6.3 Sliding Mode Controller (SMC)

Sliding mode control is a type of variable structure control where the dynamics of a nonlinear system is changed by switching discontinuously on time on a predetermined sliding surface with a high speed, non linear feedback [15]. The SMC usually use when it related to non-linear and time varying systems. SMC comes with simple procedure to design the controller for any linear or non-linear plants and can be considered as one of a controller use to produce a robust system. The main advantage of SMC is that, it is robust to plant uncertainties and insensitive to disturbances acting on the system [16].

In [15], S.Aydin and S. Tokat proposed that SMC give a robust behavior to the coupled-tank system even in the presence of noise and able to control non-linear system with different set point given by the time varying commanded input signals. H. Abbas, S. Asghar and S. Qamar in [16] had suggested that SMC is better compared to PID controller due to its performance to the system. Using SMC allow to improve the non-linearity of the system as well as to gives a good robustness to the system [17] proposed that using SMC guarantee the asymptotic stability of the close loop system. The system robustness is not guaranteed until the slide mode is reached.



### 2.6.4 Direct Model Reference Adaptive Control (DMRAC)

DMRAC can be classify as fixed controlled because it usually not capable to control a process which the system parameter vary and when there is existence of disturbance during the operation. DMRAC provides an attractive adaptive control approach where it only required the plant and reference model outputs and reference model states to be able for measurement. Other characteristic of DMRAC is that the order of the reference model can be made lower than the order of the plant to be controlled. In [18] proposed DMRAC as the controller of the couple-tank system using non-linear plant model. From the comparison of DMRAC with PID

(Ziegler-Nichors tuning method), it shows that DMRAC improve the non-linearity of the system and able to maintain the transient response with or without the presence of disturbance in the system.

Table 2.2: Comparison Of Coupled Tank Water Level Control Method

| Paper/Journal | Controller/Method | Performance |
|---|---|---|
| **Real-time Adaptive PID Control of a Non-linear Process based on Genetic Optimization (2002)** | PID controller using GA tuning method | • Gives better performance compared to conventional PID.<br>• Can be tuned without approximated model<br>• Gives less overshoot and less settling time. |
| **Fuzzy logic control of coupled liquid tank system (2005)** | Fuzzy Logic controller (compare to PID Controller) | • The settling time of the system is shorter<br>• System is more stable |
| **Simulation of Direct Model Reference Adaptive Control (DMRAC) on a Coupled-Tank system using Nonlinear Plant Model (2007)** | Direct Model Reference Adaptive Control Compare to PID (using Ziegler-Nichors tuning method) | • Improve the non-linearity of the system<br>• Able to maintain the transient response with or without the presence of disturbance in the system. |

| | | |
|---|---|---|
| **Sliding mode control of a coupled tank system with a state varying sliding surface parameter (2008)** | Sliding Mode Control controller | • Give a robust behavior to the system even in the presence of noise<br>• Able to control non-linear system with different set point given by the time varying commanded input signals |
| **The application of Fuzzy-PID controller in Coupled-tank Liquid Level Control System (2011)** | Fuzzy-PID controller (compare to conventional PID and FLC controller) | • Improves the lumping lag and non-linear characteristic of the system<br>• FLC only helps to improve the response speed whereas the PID controller will eliminate the steady-state error of the system |
| **A design of Fuzzy PID Controller based on ARM7TDMI for coupled-tanks process (2012)** | Fuzzy-PID controller | • Gives high efficiency of control algorithm<br>• Gives a good performance with affordable costs |
| **Sliding Mode Control for coupled tank liquid level** | Sliding Mode Control controller | • Allow to improve the non-linearity of the |

| | | |
|---|---|---|
| **control (2012)** | (compare to PID Controller) | system<br>• Gives a good robustness to the system. |
| **Sliding mode control of coupled tanks system: Theory and an application (2013)** | Sliding Mode Control controller | • SMC guarantee the asymptotic stability of the close loop system. The system robustness is not guaranteed until the slide mode is reached. |

The performance of each controller can be observed from Table 2.2. PID controller using an optimization technique gives better performance compared to the conventional PID. Fuzzy logic controller gives a stable system and improvement in transient response when being compared with PID controller. When there is disturbance in a non-linear system, Direct Model Reference Adaptive Control (DMRAC), Fuzzy-PID controller and Sliding Mode Control controller can improve the system and gives better performance to the transient response. It shows that each controller gives different improvement related to the stability and transient response of the coupled tank system.

## 2.7 Optimization Technique

Optimization theory deals with algorithm to approach proximity of the maximum probability and allow it to evaluate the function in a finite number of points. Below are several techniques of optimization to control a system.

Table 2.3: Types of optimization technique

| OPTIMIZATION METHOD | DESCRIPTION |
|---|---|
| FIREFLY ALGORITHM | Firefly algorithm is inspired by the behavior of the fireflies. It is a nature-inspired metaheuristic optimization algorithm. The application that use firefly algorithm is for digital image compression and image processing, rigid image registration problems and antenna design. |
| PARTICLE SWARM OPTIMIZATION (PSO) | PSO is a method of optimization technique that based on the movement and intelligence of swarm. It was developed in 1995 by James Kennedy and Russell Eberhart based on the social behaviors of bird flocking or fish schooling. In [16], J.S Cheng proved that although it provides a simple program, it also helps to improve the quality of the system and more accurate. |
| GENETIC ALGORITHM (GA) | GA is a method of solving an optimization problem related to both constrain and unconstrained problems based on a natural selection process that similar to biological evolution. There is limitation for GA in the complexity of the system. GA does not scale well with complexity and it is extremely difficult to use the technique on problems such as designing plane or house. The application of GA |

| | |
|---|---|
| | can be used in bioinformatics, pahrmacometrics, physics, manufacturing and economics. |

Optimization techniques are useful in finding the optimum solution or unconstrained maximum or minimum of continuous and differentiable functions. Each gain or weight matrices in any system need an exact value in order to gives the best performance that can be applied to the system. Therefore by using optimization techniques, it helps to find the best solution or unconstrained maximum or minimum of continuous and differentiable functions for the system.

## 2.8 Summary of review

Literature review has been presented in terms of different aspects. The first part that had been discussed is the coupled tank and modeling of the system. Previous research related to types of controller used for coupled tank system was discussed and the output performance using each controller is stated. The performance of this research (LQR) had been discussed to see its behavior and outstanding characteristics to different types of system. The comparison of the performance using LQR controller in different system shows that LQR controller is a controller that can be used in any system despite having non-linear system and presence of disturbance in the system.

# CHAPTER 3

# METHODOLOGY

## 3.1 Overview

This chapter will review on the method or approach that will be used to complete this project. There are several component need to be considered in order to achieve the objective of the overall project. The main components for this project are based on modeling the system and the design of the controller for the system. The controller that will be used for this project is LQR controller using the PSO optimization technique.  The operation of the coupled tank was discussed in this chapter in order to have a clear understanding about the coupled tank.

## 3.2 Project Flow Chart

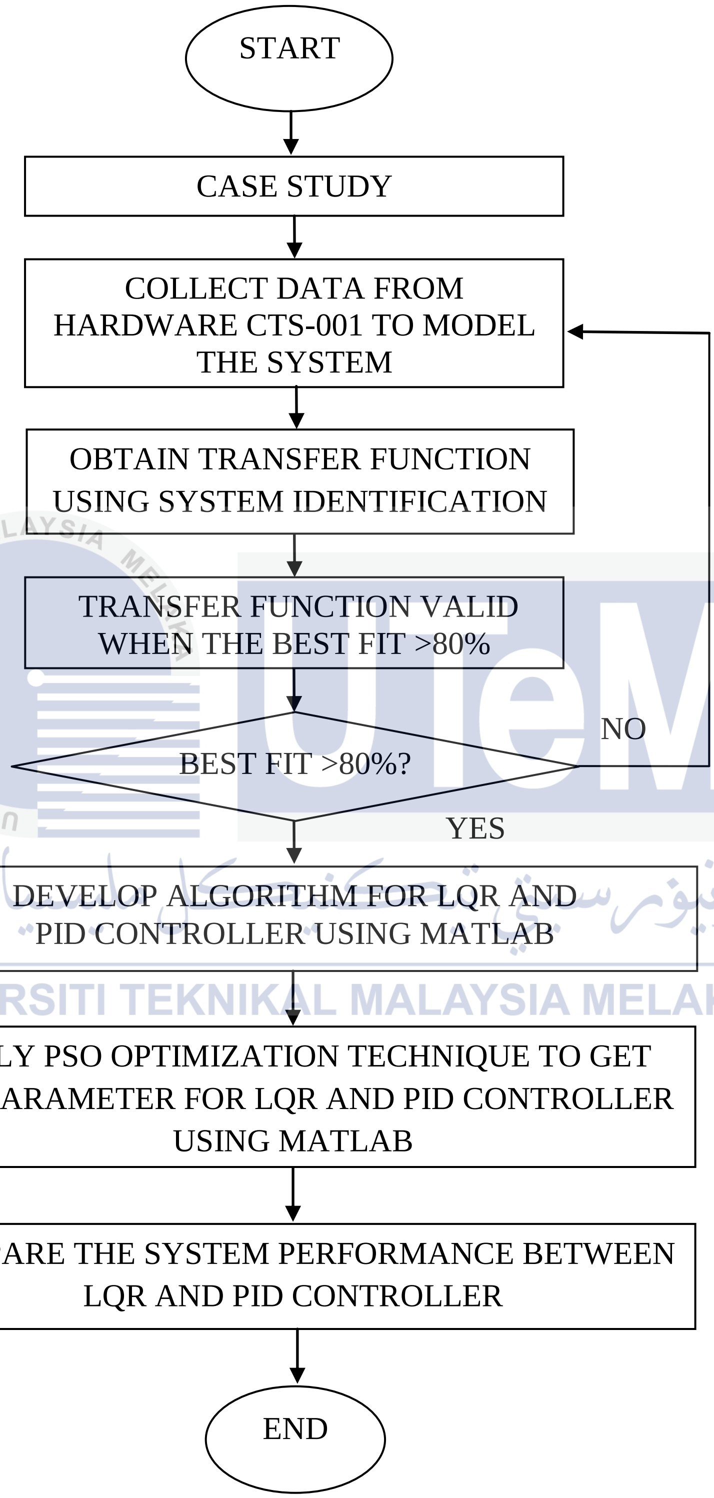


Figure 3.1: Flow chart for project development

Figure 3.2 shows the overall progress for modeling and controller design of coupled tank system. The project is related to both simulation and hardware. The critical part on the hardware is to extract the data from the tank and maintain the functionality of each component of the system. For software, the modeling for the system must be obtained and need to be verified using the system identification in MATLAB. The correct input and output need to be considered in order to find the appropriate transfer function to the system. If the mathematical model is valid, then the LQR controller will be included in the system. The suitable value of matrices Q and R will be verified using the PSO optimization technique to get best performance for the system.

## 3.3 Integrate Hardware and Software

Modeling of the coupled tank consists of the integration between hardware and software. The instrument used for the coupled tank is CTS-001. CTS-001 is a computer-controlled coupled tank system used for liquid level control. CTS-001 consists of coupled tank apparatus CT-001, a software package developed using LABWINDOWS/CVI environment and a National Instrument (NI) data acquisition card. LABWINDOWS/CVI development tools and NI data acquisition card are use for implementation of virtual instrumentation in an experiment [1].



The coupled tank CTS-001 can be used as a liquid level control system. The coupled tank liquid level control system apparatus is interfaced to the computer via the NI data acquisition card. NI data acquisition card is use to measure the actual physical conditions and converting the resulting samples into digital numeric values that can be manipulated by a computer. A software package is developed using LABWINDOWS/CVI for the purpose of analysis data, data acquisition and monitoring the liquid level. The software is used for the purpose of sending and receiving the information to the coupled tank liquid level control system.

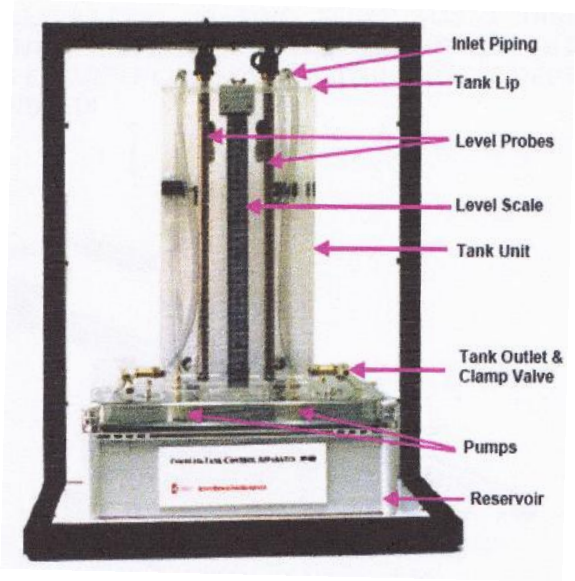


Figure 3.2: Coupled tank control apparatus model CTS-001 [1]

### 3.3.1 Coupled Tank Model (Operation)

Water is pumped into the tank lips at the top of each tank by two independent pumps. The water level in each tank is visible on the attached scale at the front of the tanks. Each tank is fitted with an outlet located at the side near the base. The amount of water which returns to the reservoir is approximately proportional to the water entered the tank since the return tube at the base of the tank functions as a pseudo-linear hydraulic resistance. The return tune is constructed of flexible tubing so that resistance may be increased by the use of a screw-type clamp [1].

The level of water in each tank is measured using a capacitive-type probe together with an associated electronics. The capacitance changes as the water level changes. The changes in the capacitance determine the change in frequency of an oscillator and this change is electronically converted to a DC voltage in the range of 0 to +5 Volts (DC). The zero level has been calibrated (approximately 20 mm on the scale) to represent the rest point of the water level when the tank is nearly empty. When the tank is in full state, +5 Volts is calibrated at the level of the opening to the rear overflow stand-pipes. This occurs at approximately 300mm on the scale.

The two pumps at the rear of the unit are controlled by Pulse Width Modulation (PWM) circuits using power MOSFET devices. The input signal to each pump circuit is in a PWM waveform (can be generated by microcontroller or an external DC voltage) in the range of 0 to +5 Volts.

An internal baffle controls the leakage between the two tanks. Turn the wing-nut on the top of the tank will allow the baffle to be raised in a small amount which sufficient to provide a useful range of inter-tank resistance. A spring will return the baffle to the closed position when the wing-nut is released.

### 3.3.2 Calibration of Coupled Tank

Calibration can be defined as one of the primary processes used to maintain the accuracy of the instrument. It is important to know the measurement and calibration of the component characteristic. The information is needed to determine the relationship between the input and output of the component and to identify the presence of non-linearities in the system which leads to inconsistency in the experimental result.



There are three important components of the apparatus that need serious analysis and experimentation [1]. The components are:

i. Tank dynamic characteristic
ii. Level sensor characteristics
iii. Pump characteristics

The understanding of the characteristic for each component is important in order to relate with the theoretical aspect of the function of the components and to understand the relationship between the system parameters and its effect on the system response.

Below is the relationship between the input voltage, $Y_1$ and the water level of the coupled tank A, $H_1$:

$$k_1 = \frac{Y_1}{H_1} \quad , \; where\ k_1 = gain\ of\ level\ sensor (V/cm)$$

The relationship between the water level and the corresponding voltage for both tank A and B are shown in Figure 3.3and Figure 3.4. The gain of level sensor after linearization for Tank A is 0.195V/cm whereas for tank B is 0.174V/cm.

The relationship between the input voltage to the pump $U_1$ and the pump flow rate into the tank $Q_1$ can be determined by :

$$k_p = \frac{Q_1}{U_1} \qquad ,where\ k_p = gain\ of\ pump(\frac{cm3}{sec}/V)$$



It is shown in the Figure 3.5 and Figure3.6 on the relationship of voltage and flow rate for both tank. The controlled inflow for Tank A is $10\ \frac{cm3}{sec}/V$ whereas for Tank B is $20\ \frac{cm3}{sec}/V$.

Both of the gain, $k_x$ and $k_p$ is important to obtain the relationship between both parameter in the tank. The calibration on input voltage which gives changes to the water level and pump flow rate is important so that the relationship between voltage and water level and voltage with pump flow rate can be monitored and maintained.

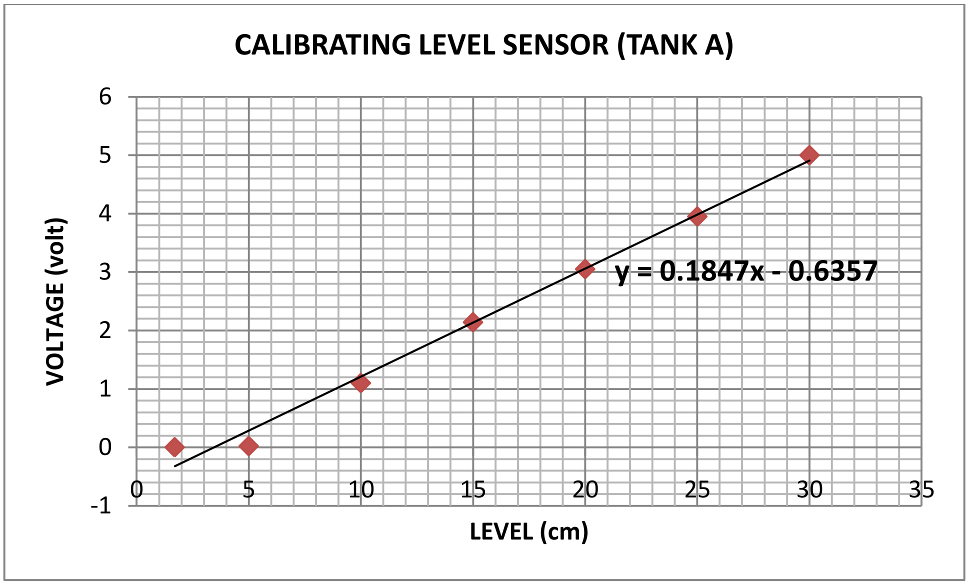


Figure 3.3: The relationship between the water level and the corresponding voltage in Tank A

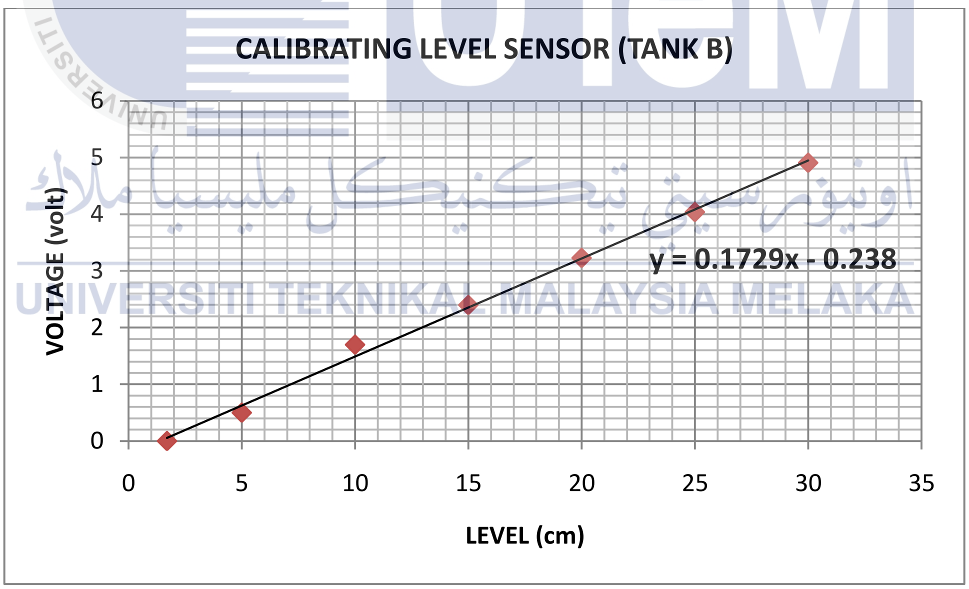


Figure 3.4: The relationship between the water level and the corresponding voltage in Tank B

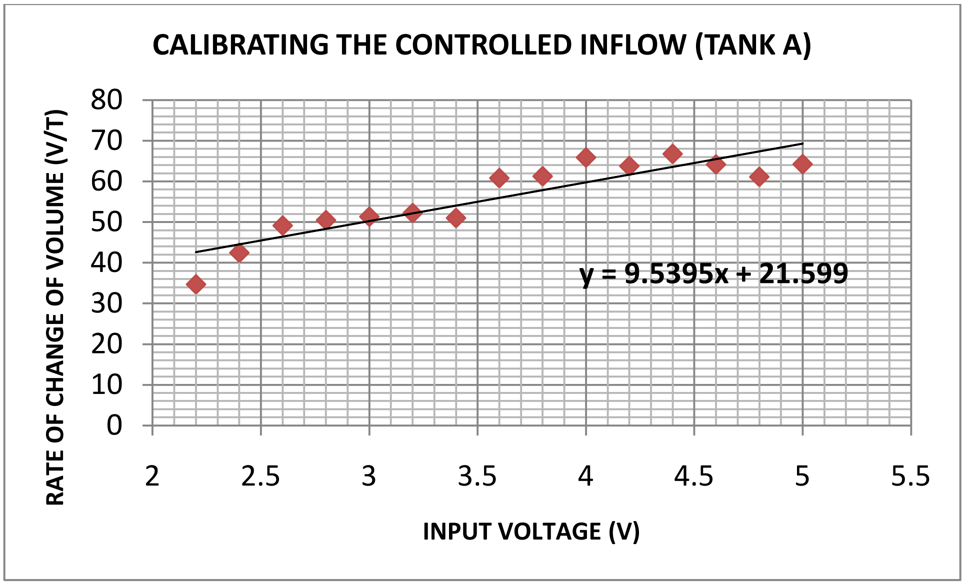


Figure 3.5: The relationship between the pump voltage and the corresponding voltage in Tank A

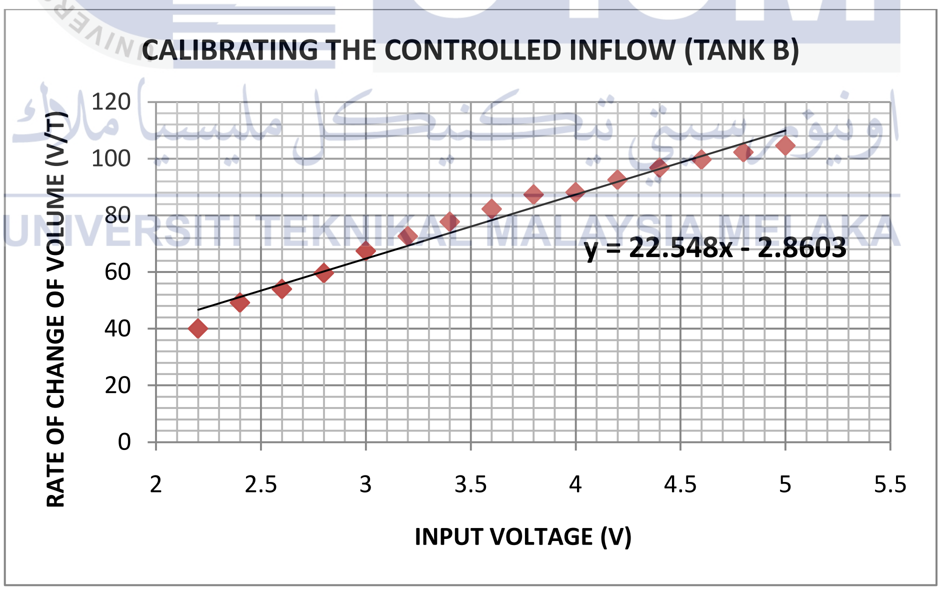




Figure 3.6: The relationship between the pump voltage and the corresponding voltage in Tank B

## 3.4 Modeling of Coupled Tank

Modeling is needed to obtain the proper transfer function for the coupled tank system. To obtain the transfer function, several methods need to be done. The data need to be collected on the coupled tank where the voltages become the input whereas the levels of water become the output for the system. The component of the hardware as well as the interface device needs to perform well so that appropriate output can be obtained for the next stage of modeling. The input data used for this project is Pseudorandom Binary Sequence **(**PRBS). Once the data is collected, it will be loaded in process called system identification.

### 3.4.1 Pseudorandom Binary Sequence (PRBS)

PRBS is an input that is widely applied in the linear system identification. The Auto-correlation function (ACF) of a PRBS provides the best and useful approximations to periodic white noise [19]. PRBS are commonly used as the forcing function in the testing for statistical system. The advantages of using PRBS as an input is that the signal can be easily generated and applied to the system.



PRBS comes with two types which are the QRC (Quadratic Residue Code) and MLS (Maximum Length Sequences). Type of PRBS used for this project is MLS. The characteristics are it has periodic sequence, deterministic signal and generate using n-bit shift register with feedback through a logic function called “exclusive or” (XOR) function.

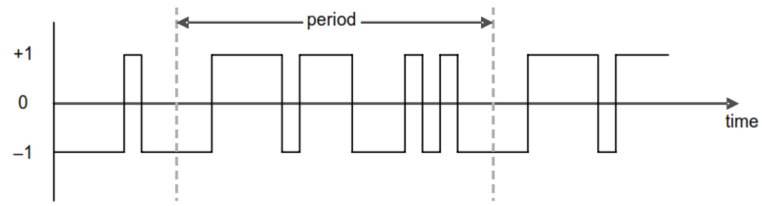


Figure 3.7: General form of PRBS waveform [19]

Shift register with suitable feedback is a useful tool to generate the PRBS waveform. The length of the period sequence MLS can be determine using the formula $N=2^n-1$ where n represent the number of stages in the shift register. Once the number of stage is choose, it will be evaluated mathematically and organized in a tabular form. The frequency for both PRBS waveform and clock frequency for the shift register will be the same.

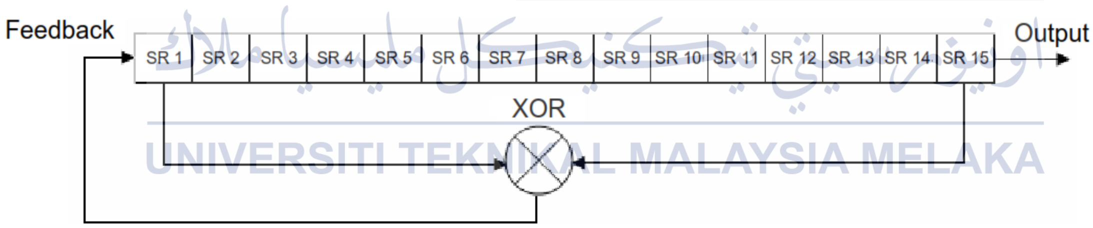




Figure 3.8: Shift register for generation of PRBS [19]

#### 3.4.1.1 PRBS Stages

Integer choose for the input, n=3. Therefore the length of sequence for the stages is $N = 2^n - 1 = 2^3 - 1 = 7$ bits.

Table 3.1: PRBS input table

| SR1 | SR2 | SR3 | SR1 ⊕ SR3 |
|---|---|---|---|
| 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 |



The sequence is 0100111 in a repetitive manner at the clock frequency. The amount of cycles chose for this project is about 152 cycles with 1069 of collected data. The two possible states for PRBS is +4V and 0V. The change is referring from the deterministic pseudo random manner referring to Table 3.1. The feedback value, SR1 ⊕ SR3 will become the input for the system. Figure 3.9 shows the result obtained using the PRBS method which is used for the input in the system identification method. It is simulated using the CAIRO CTS001 software.

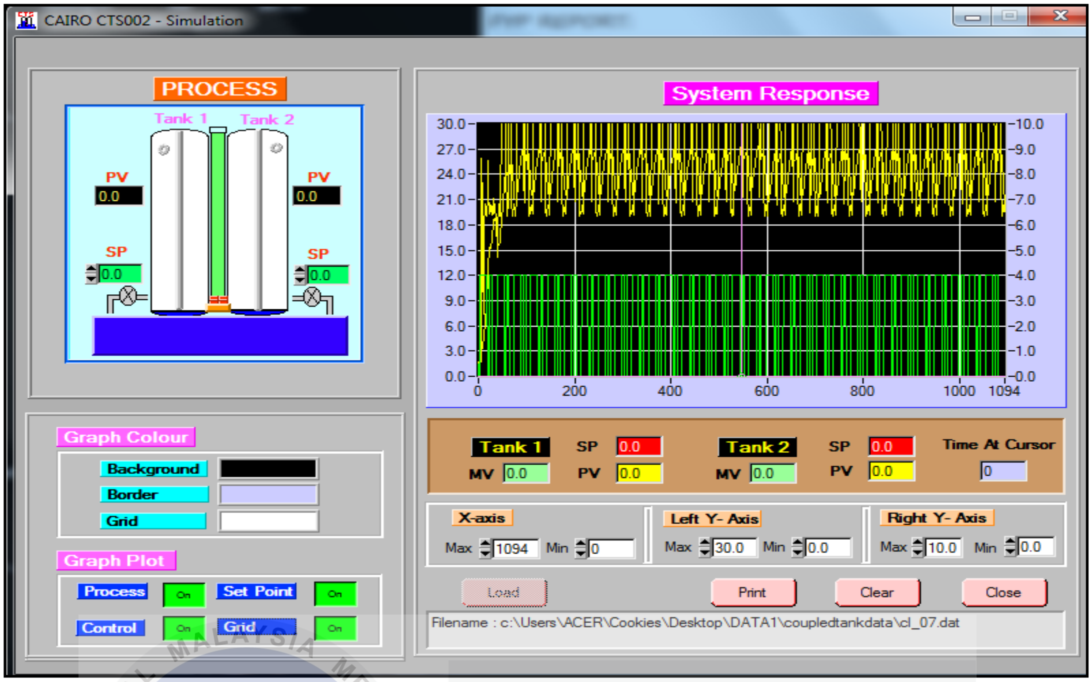


Figure 3.9: Data obtained from CAIRO CTS-001

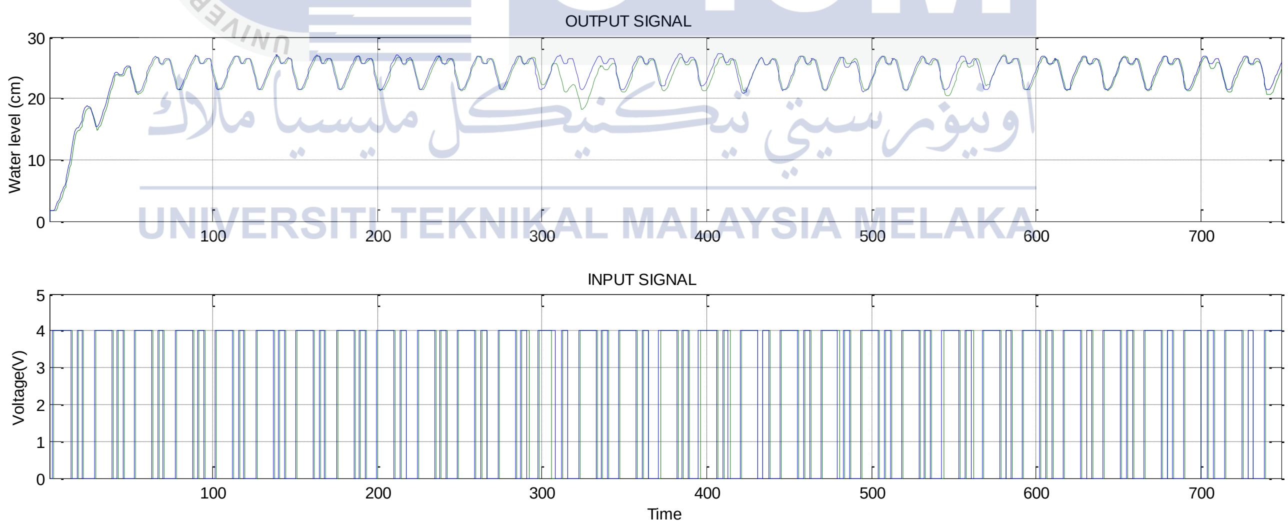


Figure 3.10: Input for the system identification

### 3.4.2 System Identification

The main purpose of on applying system identification method is to determine a transfer function of a system from observed data. The input and output of the system need to be determined in order to get the appropriate transfer function for the system. System identification method was used to infer a model in the transfer function and it can conclude that the best and valid result was a model with the best fit over than 80 percent [20]. Figure 3.11 shows the flow chart on how the procedure to conduct the system identification.

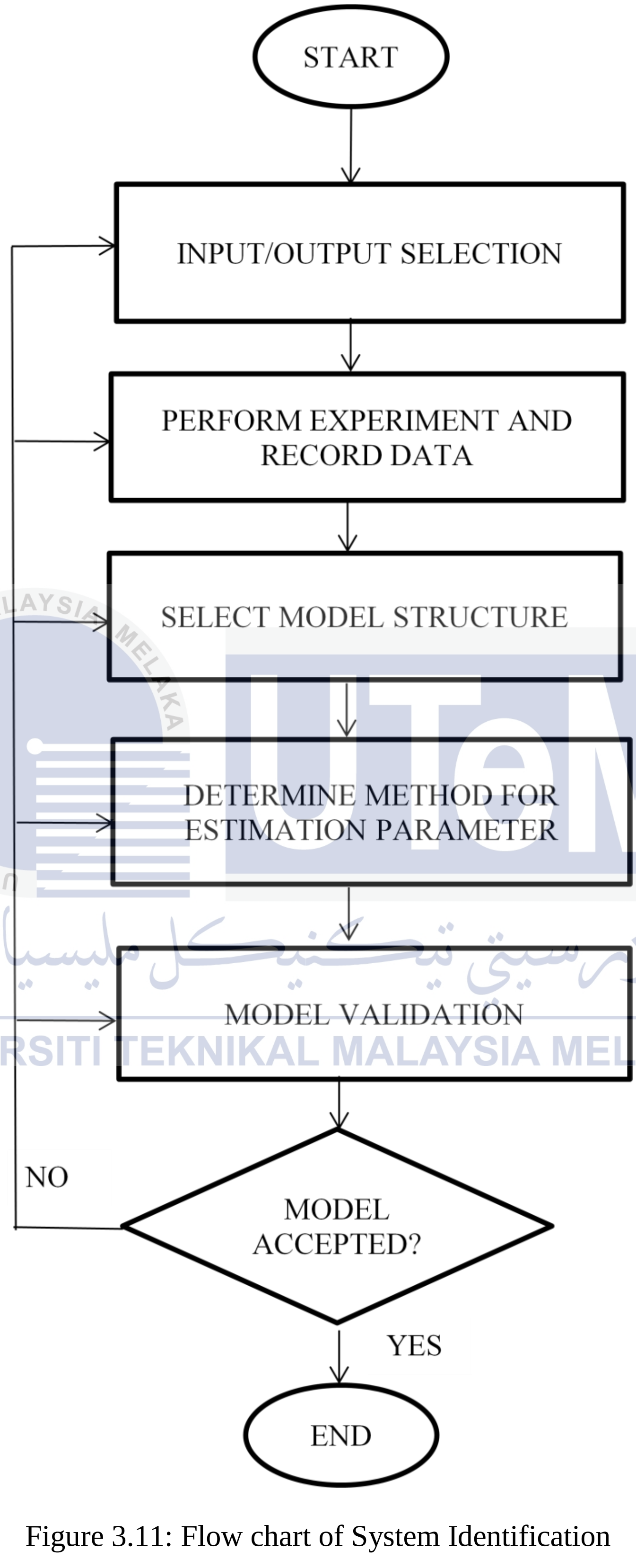


Figure 3.11: Flow chart of System Identification

Below shows the procedure or steps on how to obtain the mathematical model of the system once the data is collected from the coupled tank.

STEP 1:

The input decided for this project is the voltage applied to the coupled tank and the output is the water level in the tank. Once the experiment data is collected, the data is then loaded to the workspace as a variable. In the "*ident"* window, select the popup menu import data and select time domain data. Change the sampling time to 0.7 s based on the sampling time selected while undergoing the experiment. Use the same step to load the second input and output for the system.

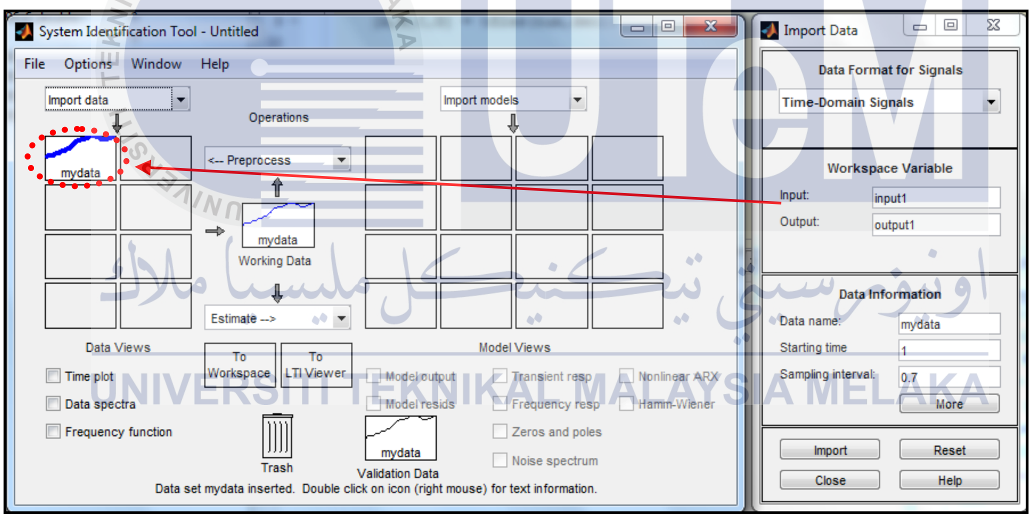


Figure 3.12: Import 1st data from workspace

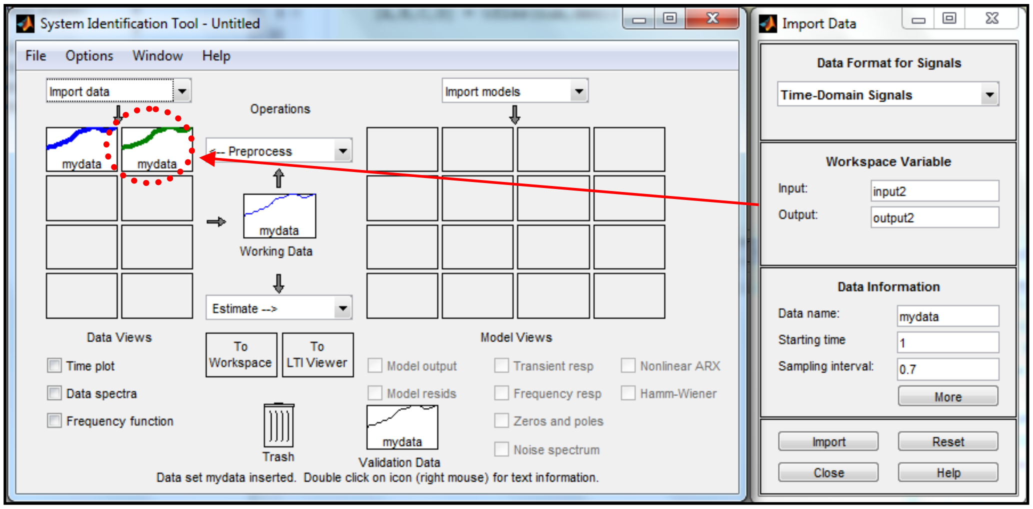


Figure 3.13: Import 2$^{nd}$ data from workspace

STEP 2:

In the data board of the "*ident*" figure, select a first data set in the data views to work with for estimation by dragging it to the Working Data icon. Then, the second data set is used for validation purposes should be dragged and dropped onto the Validation Data icon.



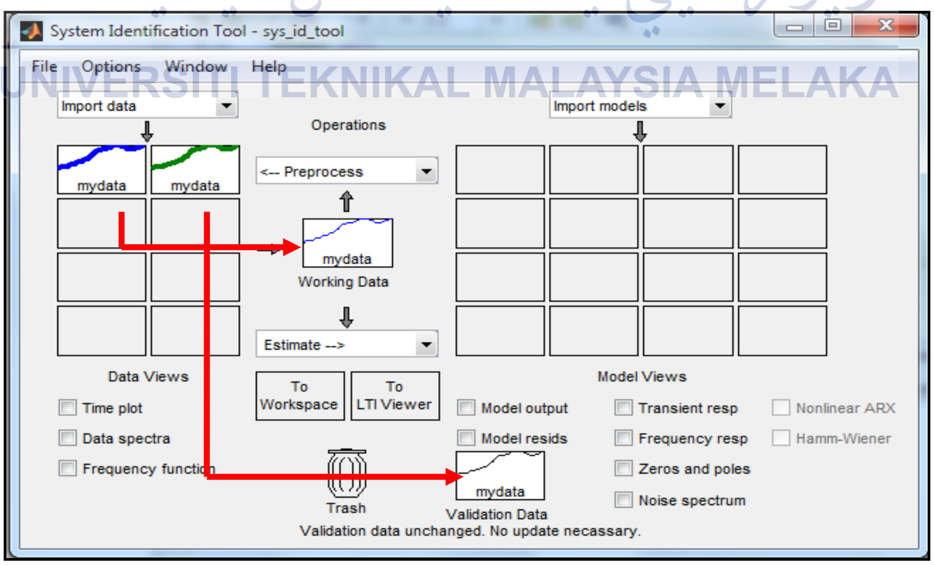


Figure 3.14: Save data for Working Data and Validation Data

STEP 3:

To compute the transfer function, select the transfer function models from the Estimation popup menu. New dialog will open and enter the information in the transfer function model dialog and then generate the model by pressing the estimate button. The model will be computed and inserted into the Model board as an icon.

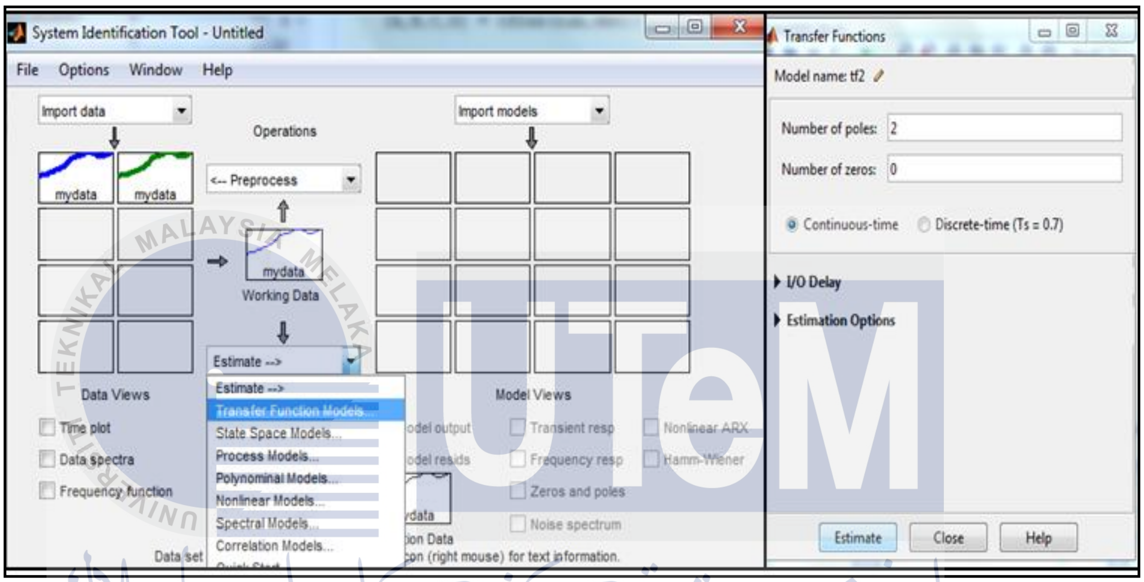


Figure 3.15: Select estimation as transfer function models

STEP 4:

Once the model is inserted, click the model output to measure the best fit of the transfer function model. Double click on the '*tf1*' icon to get the calculated transfer function obtained.

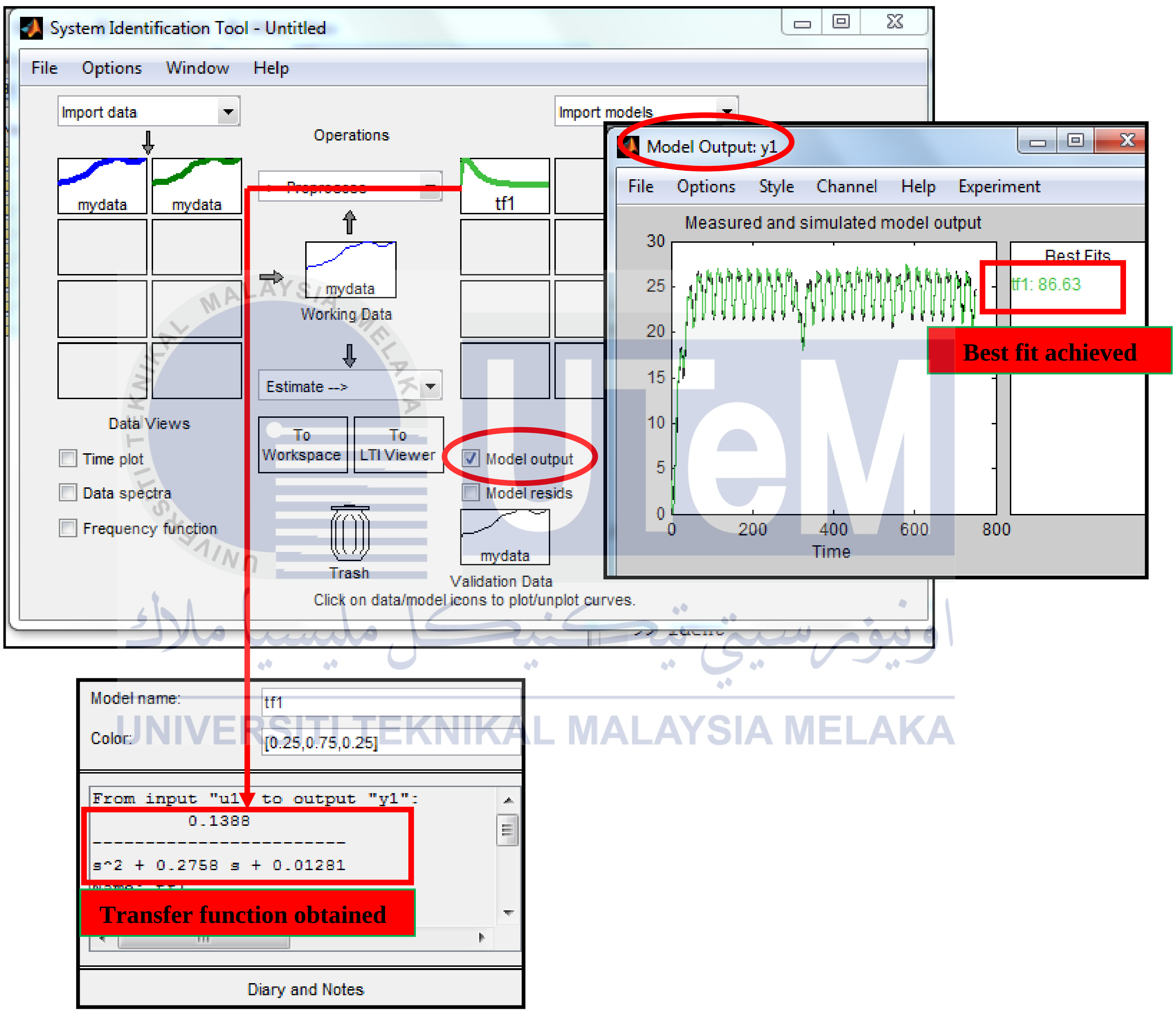




Figure 3.16: Best fit percentage and transfer function obtained

## 3.5 Design LQR and PID Controller

LQR Controller and PID Controller will be used to control the coupled tank system. The parameter for the controller can be obtained using MATLAB algorithm (coding) with chosen optimization technique. The result for both controllers will be showed in the next chapter.

### 3.5.1 LQR Controller

LQR is a method that uses state space approach to design and control a system. The two matrices Q and R are selected to control the system in order to maintain the performance of the system. The objective in design using LQR Controller is to select the best gain, K that minimizes the performance of the performance index, J. The schematic for this type of controller is shown in Figure 3.17.

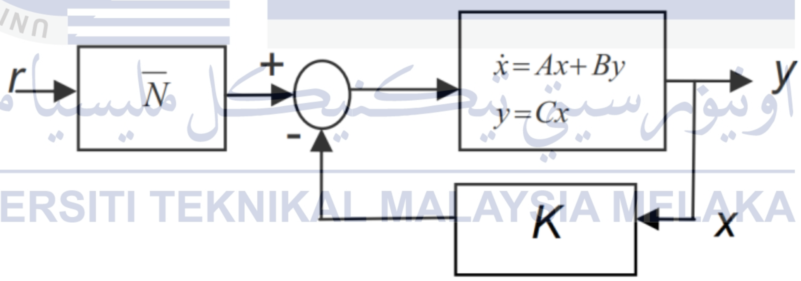


Figure 3.17: Schematic diagram for LQR

Below shows the state variable form to represent a system:

$\dot{x} = Ax + Bu$

The initial condition is $x(0)$. We assume that all the state are measurable and seek to find a state-variable feedback (SVFB) control

$u = -Kx + v$

The closed-loop system using this control should be

$$\dot{x} = (A - BK)x + Bv = A_c x + Bv \tag{3.1}$$

With $A_c$ the closed-loop plant matrix and $v(t)$ the new command input

This is the formula to find the performance index of a system

$$Performance\ index(PI), \qquad J = \frac{1}{2}\int_0^\infty x^T\, Qx + u^T Ru\; dt \tag{3.2}$$

Subtitute the SVFB control into this yields

$$Performance\ index(PI), \qquad J = \frac{1}{2}\int_0^\infty x^T\, (Q + K^T RK)x\; dt \tag{3.3}$$

These are the procedures on how to find the LQR feedback, K:

i) Select design parameter matrices Q and R
ii) Solve the Algebraic Riccati Equation (ARE) for P
iii) Find LQR feedback, K using $K = R^{-1}B^T$



Q is a positive semi-definite n x n matrix and R is positive definite m x m. In general, choose large value for Q means that it keeps J to become small. Larger Q allow the poles in the closed-loop to become more further to the left side of the s-plane so that the state decay become faster to achieve zero. Similar to Q, having large value for R means that the control input $u(t)$ will be reduce in order to keep J small. Increment value for R shows less amount control effort is used. It gives the poles to move slower which resulting larger value of state $x(t)$.

In order to find the feedback, K value of P need to be obtained. Given the constant matrix P

$$\frac{d}{dt}(x^T P x) = -x^T (Q + K^T R K) x \tag{3.4}$$

Substitute the equation (3.4) into (3.3) gives

$$J = -\frac{1}{2}\int_0^{\infty} \frac{d}{dt}(x^T P x) dt = \frac{1}{2} x^T(0) P x(0) \tag{3.5}$$

Equation (3.5) shows that J is now independent of K. x(t) will become zero as time, t goes to infinity due to assumption the system assumed to give a stable response.

Substitute the closed loop equation (3.1) in (3.4) and differentiate the equation to become

$$\dot{x}^T P x + P\dot{x} + x^T Q x + x^T K^T R K x = 0$$
$$x^T {A_c}^T P x + x^T P A_c x + x^T Q x + x^T K^T R K x = 0$$
$$x^T \left({A_c}^T P + P A_c + Q + K^T R K\right) x = 0$$

It has been assumed that the external control $v(t)$ is equal to zero. Now not e that the best equation has to hold for every $x(t)$. Therefore, the term in brackets must be identically equal to zero. Thus, proceeding one secs that

$$(A - BK)^T P + P(A - BK) + Q + K^T R K = 0$$
$$A^T P + PA + Q + K^T R K - K^T B^T P - PBK = 0$$

This is a matrix quadratic equation. Exactly as for the scalar case, one may complete the squares. Though this procedure is a bit complicated for matrices, suppose we select

$$K = R^{-1}B^{T}P \tag{3.6}$$

Then, there results

$$A^{T}P + PA + Q + (R^{-1}B^{T}P)^{T}R(R^{-1}B^{T}P) - (R^{-1}B^{T}P)^{T}B^{T}P - PB(R^{-1}B^{T}P) = 0$$
$$A^{T}P + PA + Q - PBR^{-1}B^{T}P = 0 \tag{3.7}$$

Where (3.7) called the Algebraic Riccati Equation (ARE).

The value of ARE can be perform using Matlab Routine called *lqr(A,B,Q,R)*. Once the value of P obtained, the feedback can be determined and the performance of the system can be analyze. The result of the system will be shown in the next chapter.

In order to reduce the steady state error of the system output, a value of constant gain, Nbar should be added after the reference [21]. Nbar can be found using the m-file code function. The value of parameter obtained for LQR controller will be determine using the PSO.



### 3.5.2 PID Controller

The Proportional-Integral-Dervative (PID) control gives the simplest and yet the most efficient solution to various real-world control problems [21]. The controller will take the measured value from the plant and compares it with the value of the reference set point. Figure 3.18 shows the block diagram for PID controller.

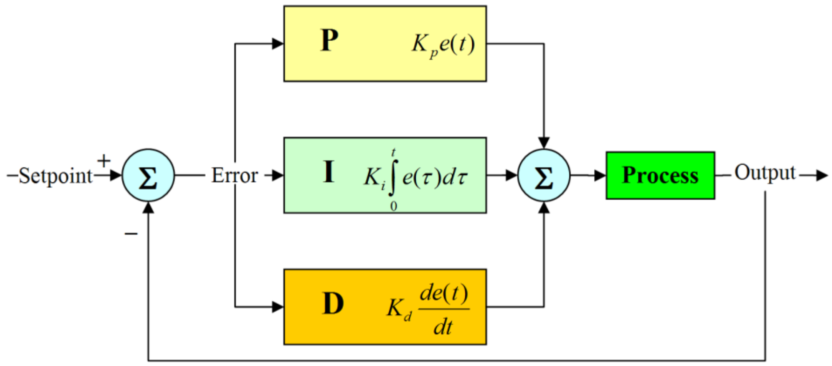


Figure 3.18: Schematic diagram for PID

The transfer function for PID controller is

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s$$

These are several steps on how to design n a PID controller in order to obtain the desired response.

i) Obtain the open loop response and specify the parameter needs to be improved

ii) Select proportional gain, Kp to improve the rise time

iii) Select integral control, Ki to eliminate the steady-state error

iv) Select derivative gain, Kd to improve the overshoot

v) Adjust the gains of Kp, Ki, and Kd until it meet the desired overall response.

In this project, the weight value of Kp, Ki and Kd are determined by using one of the optimization technique, PSO. The responses using the selected value will be discussed in the next chapter.

### 3.6 Particle Swarm Optimization (PSO) technique

PSO is an optimization technique that the working principle is based on the movement and intelligence of swarm such as birds flocking and fish schooling. The sharing of information will happen when they are searching for their food. In PSO concept, the “particle” in the search space can be related to the “bird”. The swarm is modeled as particles in multi-dimensional space. The swarms of particle communicate through adjustment of position and velocity.

Below are the general processes on how the PSO works in a system for each execution:

1) Initialize a group of particles including the random positions, velocities and acceleration of particles
2) Evaluate the fitness of each particle
3) Compare the individual fitness of each particle with previous save data. If it is better, update as a new data
4) Update the velocity and position for each particle
5) Repeat step 2 onwards until the best stopping criteria is met

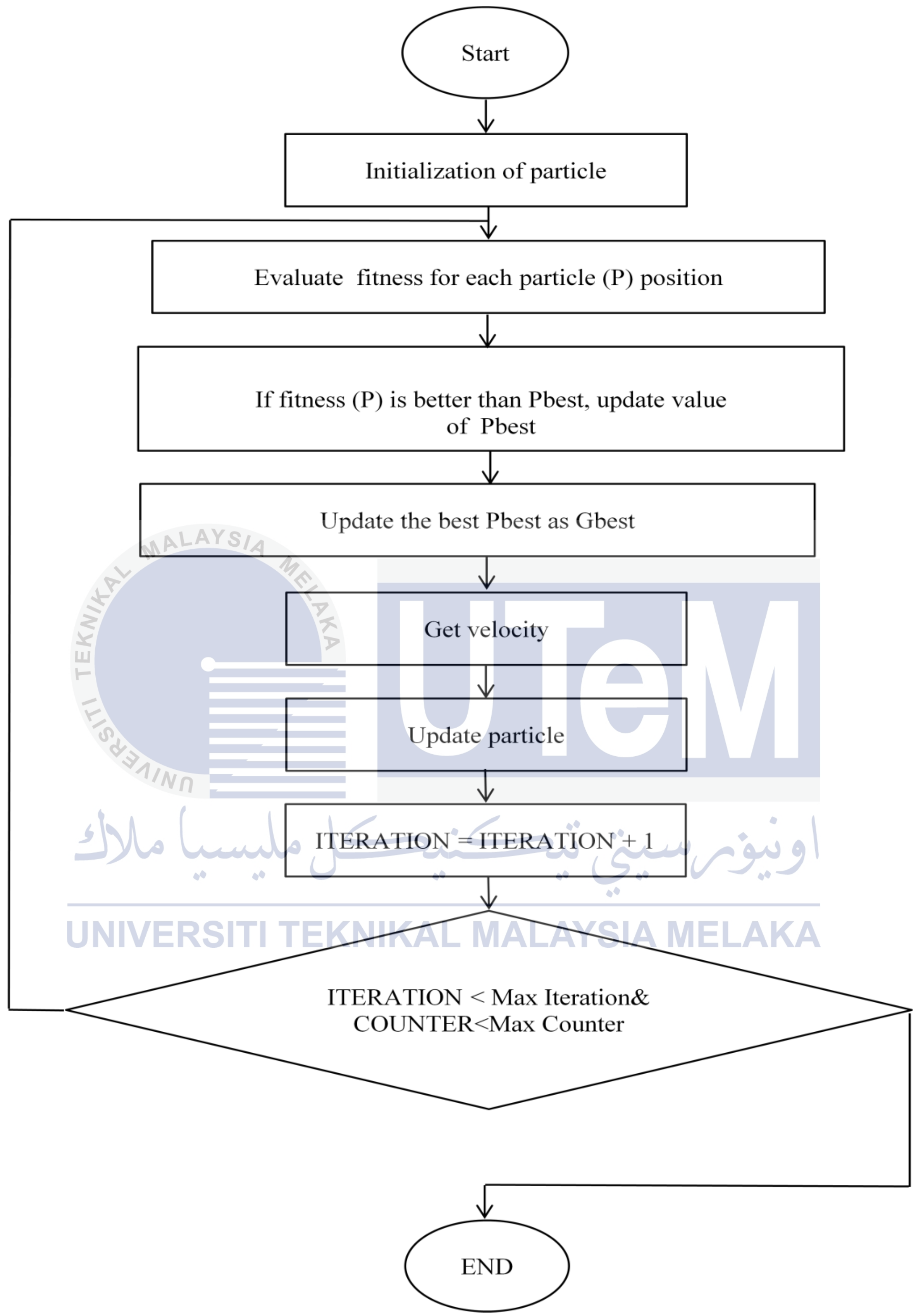


Figure 3.19: Flow chart of PSO technique

Figure 3.19 shows the flow of PSO technique for better understanding on how the optimization technique works in a system. There are several parameter that needs to be initialize before execute the PSO algorithm. Each parameter is simulated using various numbers of criterions. The changing of the criterion will influence the performance respectively. The best performance once the simulation had been done will be chosen as the set value of initialization in PSO. Table 3.2 shows the set value of initialization in PSO.

Table 3.2: Set value of initialization in PSO

| **INITIALIZATION** | |
|---|---|
| Number of particles | 20 |
| Number of counter | 9 |
| Number of iteration | 100 |
| Search Range | 0-100 |
| **VELOCITY INITIALIZATION** | |
| $c_1, c_2$ | 2 |
| Maximum velocity | $\pi/1000$ |
| Maximum weight | 0.9 |
| Minimum weight | 0.4 |



Refer to Figure 3.19, the initialization depends on the number of tuning parameters. For this project, there will be three tuning parameters for each controller which is Q1, Q2 and R for LQR controller and Kp, Ki and Kd for PID controller. A set of particle will be initialized based on the number of tuning parameter. The iteration will be based on random values with a range of search set which is from 0 to 100. For this project, there will be two stopping criteria. The stopping criteria operate either when 9 constant fitness values repeated during the range of iteration or  when the iteration reach the maximum range which is 100 iterations.

The initialization for iteration is evaluated based on the equation (3.8). The fitness of each parameter is evaluated during particle initialization.

$$initialization = range_{\min} + (range_{\max} - range_{\min})\text{X}random\_number \tag{3.8}$$

The fitness is evaluated in order to obtain the particle best, Pbest. Pbest in each iteration will be updated based on the better performance of performance index. The best performance index out of all the Pbest will be updated as the global best, Gbest. The value of Gbest also be updated until the stopping criteria meet. These processes are done to find the optimal parameter for both LQR and PID controller.

Next stage is to evaluate the velocity and position of the particle. An inertia weight is added to the velocity equation (3.10) in order to find the new velocity. The weight will act as a mechanism of exploration abilities of the swarm. The new particle position is obtained by summing the current position with the new velocity value as in equation (3.11). The steps will be repeated until it meets the stopping criteria.



$$w_k = w_{\max} - (iteration \times (w_{\max} - w_{\min}) / \max\_iteration) \tag{3.9}$$

$$V_{i,k+1} = w_k V_{i,k} + c_1 r_1 (P_{Pbest,i,k} - X_{i,k}) + c_2 r_2 (P_{Gbest,i,k} - X_{i,k}) \tag{3.10}$$

$$X_{i,k+1} = X_{i,k} + V_{i,k+1} \tag{3.11}$$

Where

| | |
|---|---|
| $v_i^k$ | = Velocity of $i^{th}$ individual at iteration *k* |
| $w_k$ | = Inertia weight at iteration *k* |
| $c_1$, $c_2$ | = Acceleration factor between 0 and 2 |
| $r_1$, $r_1$ | = Random numbers between 0 and 1 |
| $x_i^k$ | = Position of the $i^{th}$ individual at iteration *k* |
| $P_{Pbest,i,k}$ | = Best position of the $i^{th}$ individual at iteration *k* |
| $P_{Gbest,i,k}$ | = Best position of group until iteration *k* |

This optimization technique can be applied once the system is cascade with the LQR controller. The main function of implementing the PSO in this system is to ease the system to find the parameter value of matrices Q and R for LQR controller instead of using conventional techniques like the try and error method. PSO is a type of stochastic technique that once it is run, it will not gives the same value. To validate the best value of Q and R, the results after stimulate the PSO technique to the system will gives is merely constant output. If the output for few simulations is similar, it shows that the value of Q and R can be used for the system. The details about the value for Q and R will be discussed more in Chapter 4.

# CHAPTER 4

# RESULT AND DISCUSSION

## 4.1 Overview

This chapter presents the verification of transfer function using the system identification method. It includes the simulation results for coupled tank system using the same optimization technique, PSO for different controller which are PID and LQR controller. The results are divided into several parts to gives brief information regarding the system. The characteristic of the system is shown in the first section whereas it gives brief information for the uncompensated system. The parameter for PSO is selected in order to get the optimal value for performance index. Performance index chosen for this project is ITSE. The ITSE of both controller is discussed for the next section and the parameter obtained using the PSO is determined. The performances of the system using both controllers are observed based on the rise time, settling time, steady state error and overshoot. The performance of controllers also being compared when there is presence of noise in order to monitor the robustness of the controller. Other criteria included to compare the performances of the system are from the standard deviation and average taken from the ITSE value for both controllers.

## 4.2 Transfer function verification

This section will discuss about the transfer function obtained for the coupled tank system using the system identification method. System identification is used to obtain the transfer function from the collected data. The transfer function obtained need to be validated based on best fit before being used as the plant for the system. In [20] stated that when the best fit is above 80 percent the model is valid and can be used to represent the characteristic of system. Based on the result obtained using the system identification method, it shows that the transfer function obtained can be used because it achieved the limit of the best fit which is 86.83%. Figure 4.1 shows the best fit percentage and Figure 4.2 shows the transfer function obtained from the collected data.

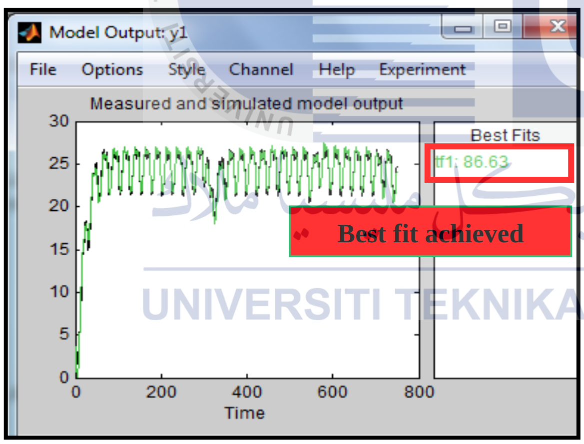


Figure 4.1: Best fit percentage

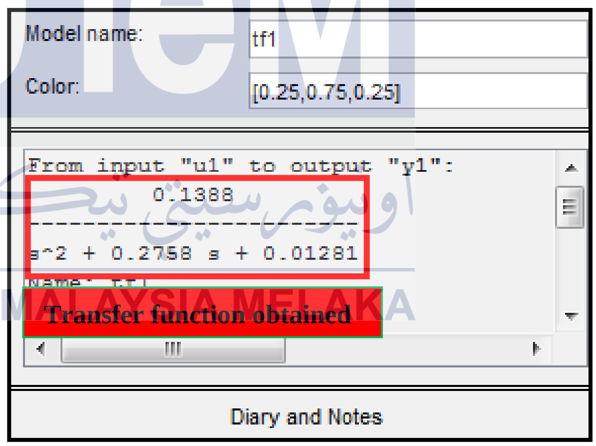


Figure 4.2: Transfer function obtained

## 4.3 Characteristic of system

This section will discuss about the characteristic of the coupled tank system for uncompensated condition. Transfer function obtained from the coupled tank system that will be used to determine the performance of the system is

$$G = \frac{0.1338}{s^2 + 0.2758s + 0.01281}$$

Figure 4.3 shows the response for open loop and Table 4.1 shows the performance response regarding the transient response of the system in open loop condition. The input set for this system is in step input with value 1. From the observation, the system took longer time to settle down. This is due to no feedback use in open loop system to compensated any error or correct any disturbance if any. It also shows that the system does not have an overshoot whereas the performance of the system gives slow response for the transient response.

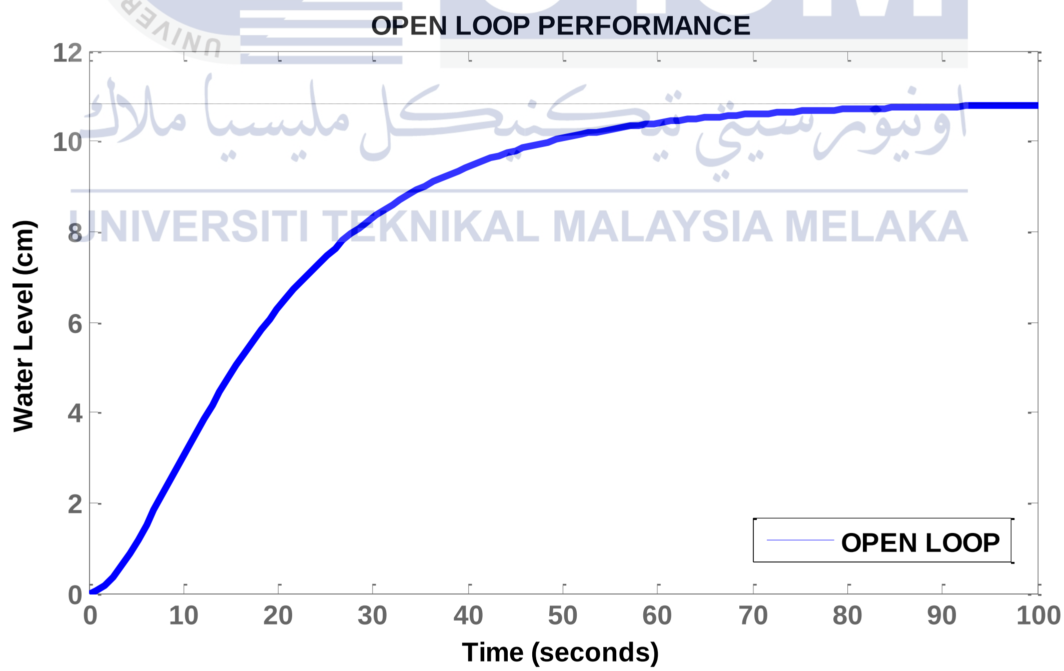


Figure 4.3: Open loop step response

Table 4.1: Transient response performance for open loop system

| Output | Open Loop System |
|---|---|
| Settling time, Ts | 71.5648s |
| Rise time,Tr | 39.4574s |
| Overshoot,%OS | 0 |
| Steady-state error, Ess | 9.8279 |

Figure 4.4 shows the closed-loop response and Table 4.2 shows the performance of the closed loop system. The graph shows the presence of overshoot and the system did settle much faster compared to the open loop response. The details regarding the transient response is shown in Table 4.2.



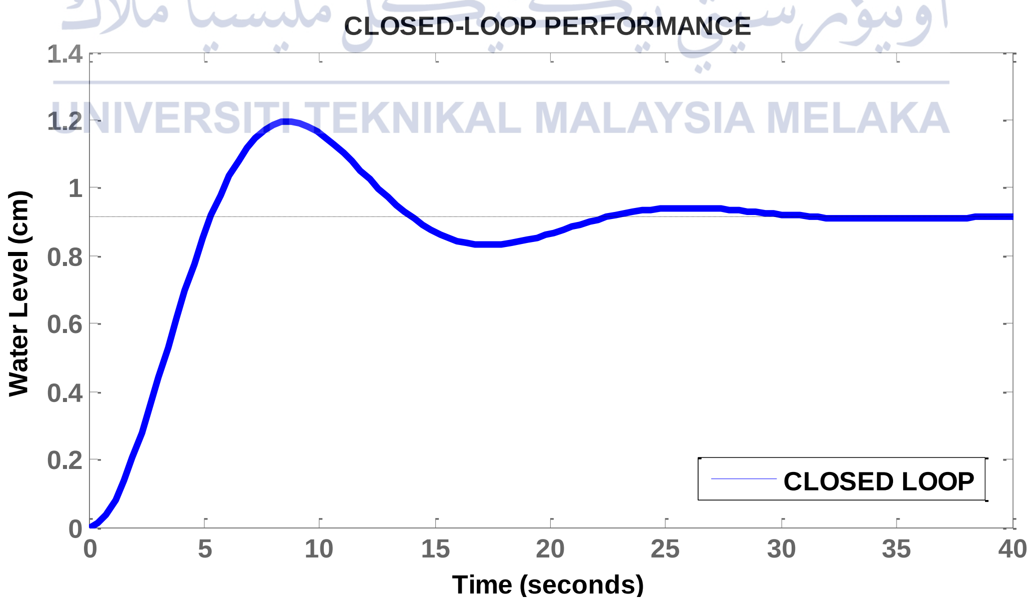


Figure 4.4: Closed Loop step response

Table 4.2: Transient response performance for closed loop system

| Output | Closed Loop System |
|---|---|
| Settling time, Ts | 0.8309s |
| Rise time,Tr | 3.5927s |
| Overshoot,%OS | 30.3961 |
| Steady-state error, Ess | 0.084 |

## 4.4 Selection of PSO parameter

This section will discuss on the selection for the parameter in PSO technique. There are two parameter selected for PSO in order to obtain the optimal parameter for LQR and PID controller which are number of particles and number of iteration. A good controller should have a closed loop that gives stable and fast response and has the lowest possible control effort. Control effort can be found based on the performance index of the controller. Type of performance index selected for this project is Integral Time-weighted Squared Error (ITSE) since it can provide controllers with a high load disturbance rejection and minimize the system overshoot while maintain the robustness of the system [22]. There are several types of performance index such as ITAE, ISE and IAE. Somehow ITSE is well known as the popular performance criterion used for control system design.

### 4.4.1 Number of particles

The result shows the value of ITSE when there are changes in the number of particles. The number of particles was increased in order to observe the response of the ITSE value. Table 4.3 shows the ITSE value when the number of particle is increase for both controller and Figure 4.5 shows the graph of ITSE versus number of particles.

Table 4.3: ITSE value when number of particles increases

| | Performance index, ITSE | | | | |
|---|---|---|---|---|---|
| **Number of particles** | **10** | **20** | **30** | **40** | **50** |
| **LQR** | 0.614 | 0.4028 | 0.8595 | 1.0782 | 0.9513 |
| **PID** | 0.8708 | 0.8233 | 0.8278 | 0.8721 | 0.8272 |

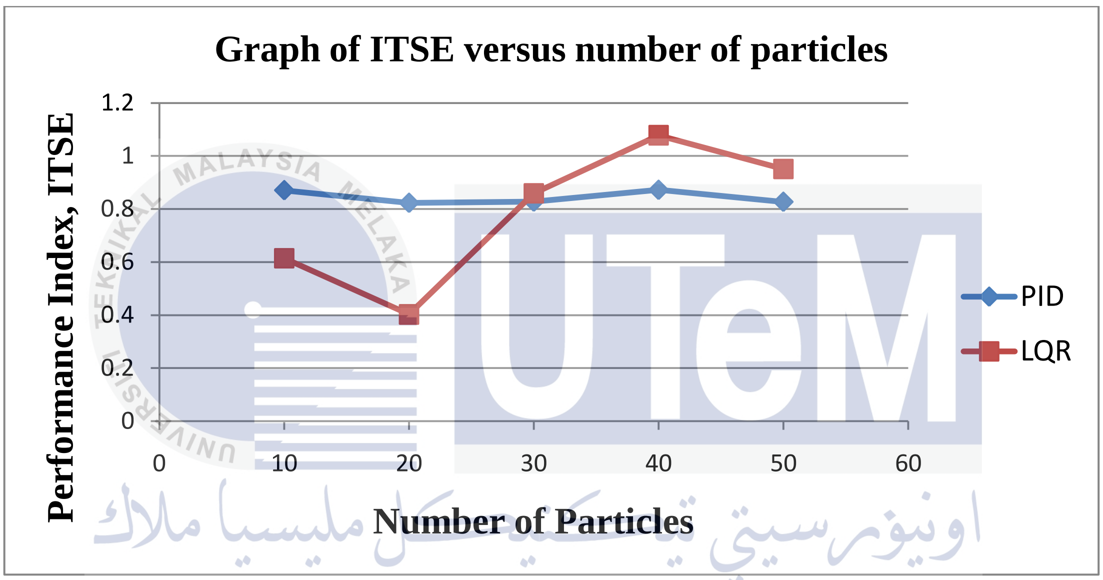


Figure 4.5: Graph of ITSE versus number of particles

Refer to Figure 4.5, it shows that different number of particles will give different value of ITSE. The ITSE value for PID controller is within the same range whereas the value for LQR is differs for each number of particles. In [24] state that good tracking response will give small value of ITSE. Based on Figure 4.5, it shows that when the number of particle is 20, it gives the minimum value of ITSE for LQR and PID controller. Therefore, for this project the number of particles used will be 20 in order to obtain optimal value for ITSE.

### 4.4.2 Number of iteration

The range for the number of iterations needs to be selected before undergoing the optimization process. There are two stopping criteria selected for this project in order to ease the execution process. Table 4.4 shows the characteristic of the stopping criteria for the iteration.

Table 4.4: Stopping criteria characteristics

| STOPPING CRITERIA | DESCRIPTION |
|---|---|
| 1 | Constant fitness value repeated 9 times |
| 2 | Range for iteration only 0 to 100 |

Based on Table 4.4, the selected stopping criteria happen when 9 constant fitness values repeated during the range of iteration. Fitness is also known as the performance index, ITSE of the system. Minimum value of ITSE gives better optimal tuning for the parameter of the controller. When similar ITSE value repeats 9 times, it can be concluded that it reaches the minimum value for ITSE. The second stopping criterion is the range selected for the iteration from 0 to 100. Based on [24], it stated that the performance in design and optimization process can be improved by increasing the number of iterations. More number of iterations gives better in the performance but it takes time to run each of the iteration. For this project, having two stopping criteria will help to improve the time management as well as maintaining the performance of the system.

## 4.5 PSO for LQR and PID

The result shows the value of ITSE obtained using PSO for both LQR and PID. The parameter of PSO used is referring to the selected parameter obtained in previous topic. In this project, the number of execution conducted for each simulation is 20 times. There will be 10 simulations executed and the optimal ITSE from each simulation is compared to choose the parameter of the controller.

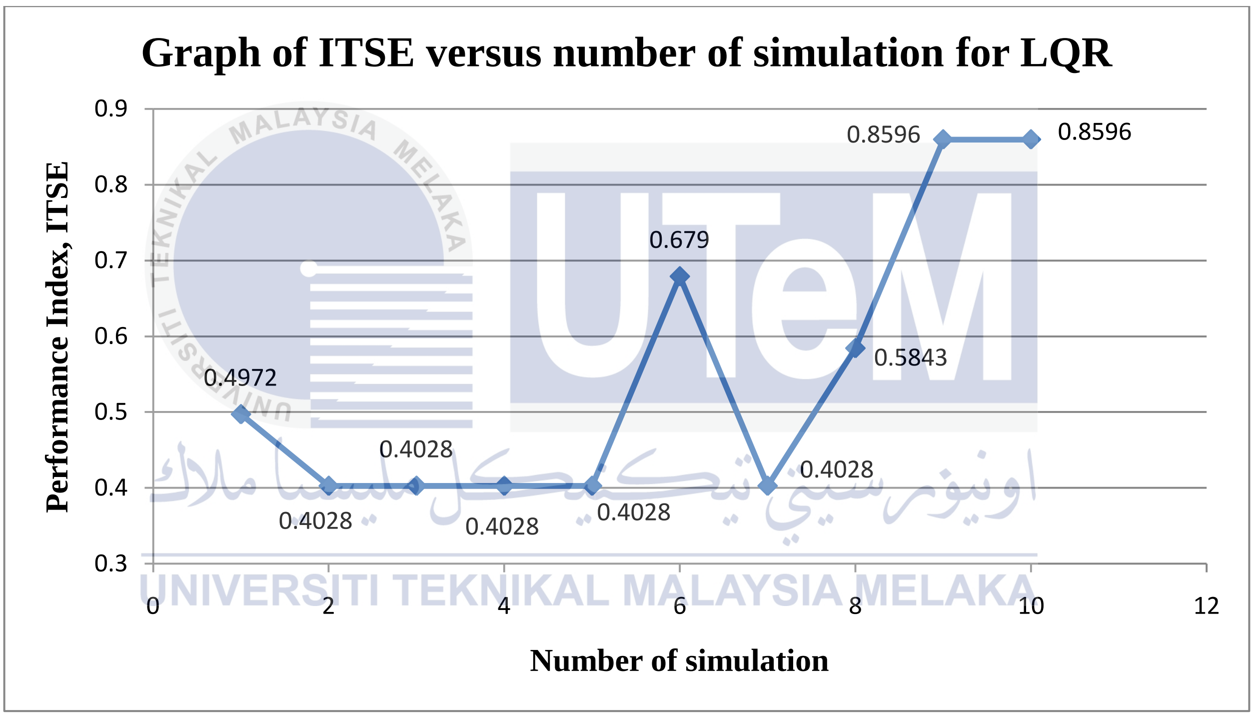


Figure 4.6: Graph of ITSE versus number of simulation for LQR

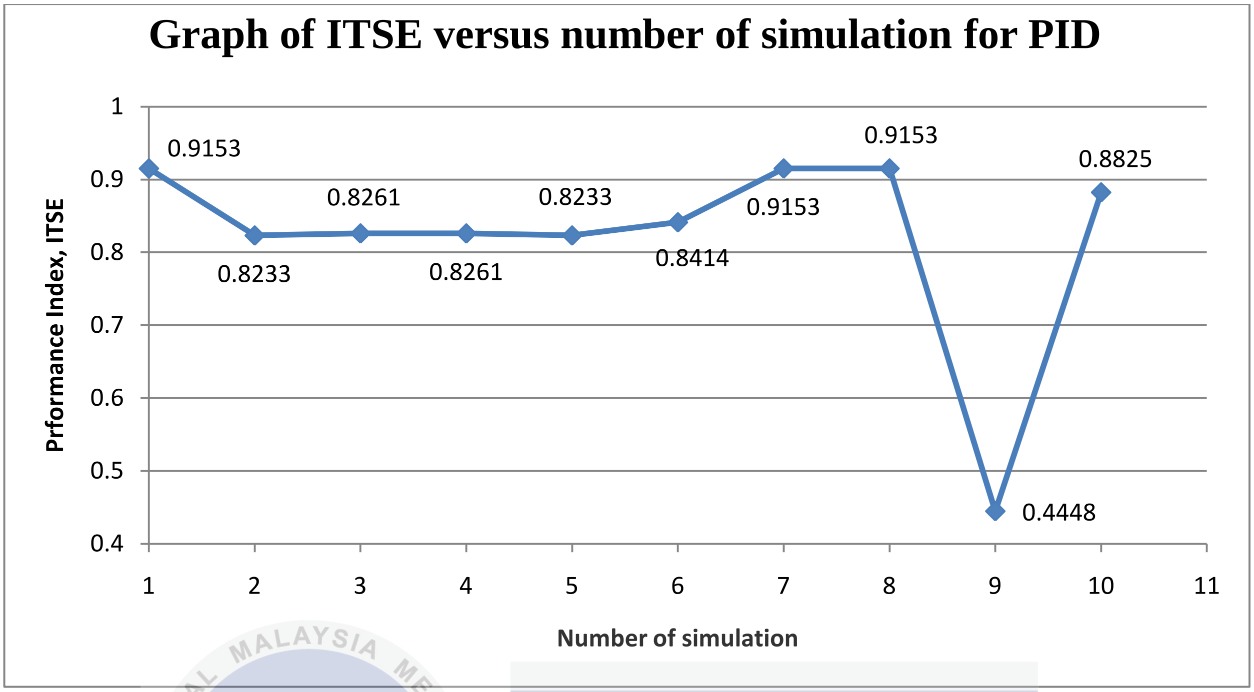


Figure 4.7: Graph of ITSE versus number of simulation for PID

Figure 4.6 and 4.7 shows the optimal ITSE in each of the 10 simulations for LQR and PID. As mention in [24], small value of ITSE gives good tracking response. It is supported by the paper [25] which stated that good set parameters can yield a good system response and result in a minimization of performance index. Refer to Figure 4.6, it shows that the best value of ITSE for LQR are 0.4028 where the value are same for 4 consecutive times of simulation which is on the 2$^{nd}$, 3$^{rd}$, 4$^{th}$, 5$^{th}$ and 7$^{th}$ of simulation. Each of the parameter and responses in LQR are the same for the 2$^{nd}$, 3$^{rd}$, 4$^{th}$, 5$^{th}$ and 7$^{th}$ run of simulation because it has the same value of ITSE. The chosen simulation that can be used to compare with PID is either one of the simulation that have same ITSE value. In this project, the simulation use for LQR is during the 2$^{nd}$ simulation. For PID controller, Figure 4.7 shows the ITSE value for each simulation using the controller. The optimal ITSE value obtained from PID is 0.4448 where it happened on the 9$^{th}$ run of simulation. Table 4.5 and Table 4.6 shows the number of simulation that gives the best ITSE value for both controller. Each of the run gives different value for parameter in LQR and PID. For LQR, the best value of ITSE happened in the 2$^{nd}$ simulation whereas the best value of ITSE for PID is during the 9$^{th}$ simulation.

Table 4.5: LQR data with 20 times execution using PSO at 2nd simulation

| RUN | ITSE | Q1 | Q2 | R |
|---|---|---|---|---|
| 1 | 1.6425 | 3.5711 | 84.9129 | 0.0093 |
| 2 | 2.3989 | 11.2283 | 78.4427 | 0.0029 |
| 3 | 2.2306 | 5.6932 | 45.0323 | 0.0058 |
| 4 | 1.5796 | 3.0389 | 75.3200 | 0.0070 |
| 5 | 1.9681 | 3.4630 | 45.1123 | 0.0001 |
| 6 | 10.6190 | 1.28771 | 18.6643 | 0.0048 |
| **7** | **0.4028** | **0.0326** | **64.0389** | **0.0000735** |
| 8 | 2.1538 | 4.8454 | 44.2740 | 0.00789 |
| 9 | 11.9389 | 5.9490 | 16.9227 | 0.00684 |
| 10 | 1.4350 | 0.8668 | 92.8280 | 0.00520 |
| 11 | 0.9936 | 1.7968 | 70.1511 | 0.000157 |
| 12 | 1.6063 | 1.9760 | 60.1611 | 0.00607 |
| 13 | 1.3417 | 0.9290 | 19.8328 | 0.000206 |
| 14 | 2.1614 | 0.8474 | 41.6031 | 0.00439 |
| 15 | 1.9653 | 1.1330 | 28.2843 | 0.00589 |
| 16 | 1.6430 | 2.6659 | 55.0467 | 0.00563 |
| 17 | 1.3766 | 2.0850 | 72.8717 | 0.00398 |
| 18 | 1.4296 | 1.0384 | 59.3550 | 0.00301 |
| 19 | 1.4379 | 3.0121 | 72.0036 | 0.00376 |
| 20 | 2.2273 | 5.9441 | 46.03362 | 0.00318 |

Table 4.6: PID data with 20 times execution using PSO at 9$^{nd}$ simulation

| RUN | ITSE | Kp | Kd | Ki |
|---|---|---|---|---|
| 1 | 1.1745 | 97.8339 | 90.4369 | 0.0005 |
| 2 | 1.0747 | 77.4712 | 35.2163 | 0.0051 |
| 3 | 1.2311 | 95.9761 | 73.2211 | 0.0089 |
| 4 | 0.4448 | 91.5044 | 48.9905 | 0.0033 |
| 5 | 1.2154 | 88.0966 | 29.2925 | 0.0068 |
| 6 | 0.8486 | 92.1423 | 44.8657 | 0.0063 |
| **7** | 1.2465 | 85.1271 | 46.2528 | 0.0099 |
| 8 | 1.0386 | 85.6046 | 35.5595 | 0.0085 |
| 9 | 1.13596 | 66.4955 | 34.4451 | 0.0074 |
| 10 | 1.1456 | 93.8688 | 31.0075 | 0.0064 |
| 11 | 0.9470 | 84.9391 | 40.1363 | 0.0005 |
| 12 | 1.1606 | 81.0063 | 31.7704 | 0.0072 |
| 13 | 1.2485 | 89.0906 | 28.2652 | 0.0031 |
| 14 | 1.3654 | 84.6521 | 50.2333 | 0.0022 |
| 15 | 1.0361 | 88.7511 | 35.3346 | 0.0053 |
| 16 | 1.0856 | 94.2661 | 33.0896 | 0.0038 |
| 17 | 1.1019 | 85.1538 | 33.4391 | 0.0008 |
| 18 | 1.4146 | 91.7292 | 85.3263 | 0.0035 |
| 19 | 0.9798 | 98.8466 | 36.8410 | 0.0052 |
| 20 | 1.2130 | 59.1021 | 32.0705 | 0.0062 |

### 4.5.1 Statistical estimation from ITSE

Other method to verify the performance of the system is by using statistical estimation method. The important aspect for statistical estimation method is that it determines the tendency on the average for the statistics to assume values that are close to parameter interest. Small value of standard deviation gives better estimation where it can be a method to measure the performance of the system because small standard deviation gives accurate data. The estimation calculated to get the value for average and standard deviation is from the minimum ITSE for each simulation. The statistical estimation for LQR and PID is shown in Table 4.7. The LQR and PID show that the standard deviations given are small.

Table 4.7: Comparison of statistical estimation

| Statistical Estimation | LQR | PID |
|---|---|---|
| **Average** | 0.54937 | 0.82134 |
| **Standard Deviation** | 0.18875 | 0.13843 |



### 4.5.2 ITSE for LQR and PID

The result shows the value of fitness function obtained using PSO for both LQR and PID. The parameter stated is based on the selected execution data for PSO. Figure 4.8 shows for the fitness for LQR controller and Figure 4.9 shows the fitness for PID controller.

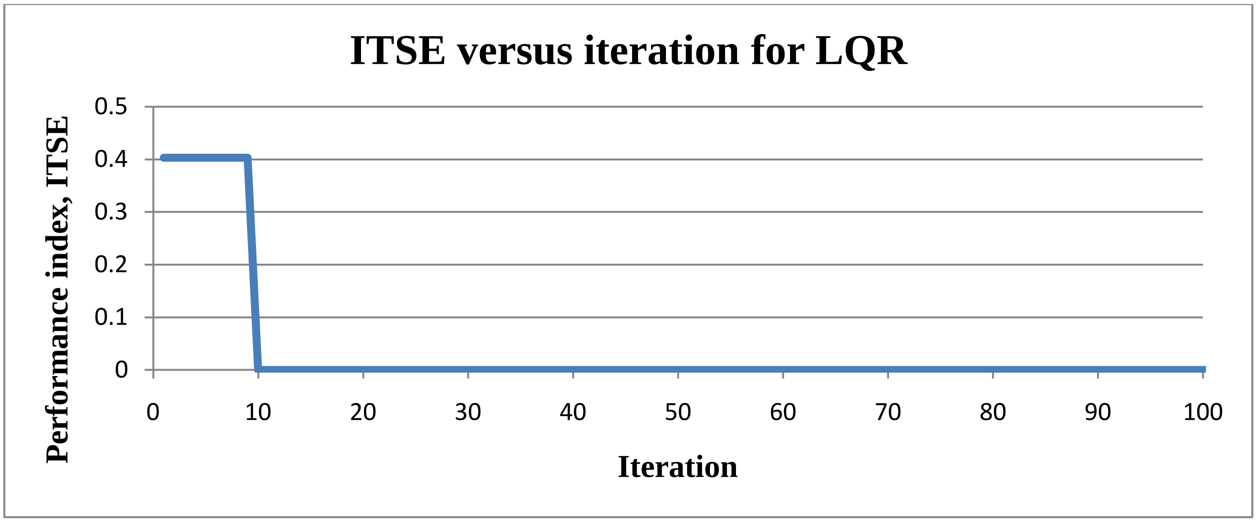


Figure 4.8: Graph of ITSE versus iteration for LQR

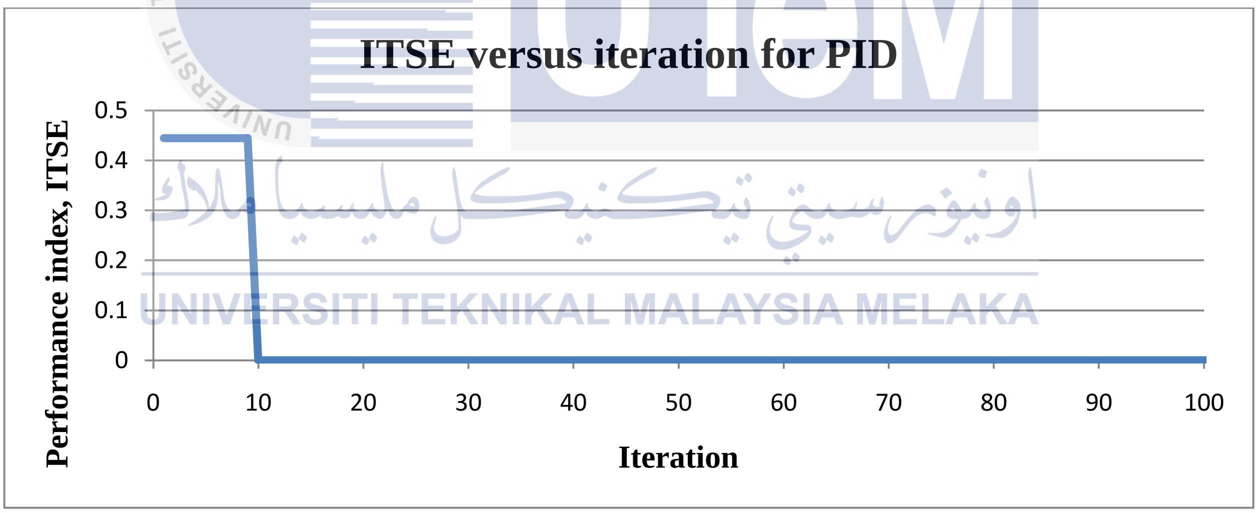




Figure 4.9: Graph of ITSE versus iteration for PID

Refer to Figure 4.8 and Figure 4.9, PSO proves that it gives an optimum fitness function for LQR and PID. Both of the controllers undergo the first stopping criteria where when constant fitness value repeated 9 times it will stop and choose as the optimal ITSE value. The stopping value for the ITSE is differs for both controllers. For PID the ITSE value is

0.4448 whereas LQR is 0.4028. The value for the ITSE will be evaluated based on each particle. If the fitness is better for the next particle, it will update is as a new data. Once the fitness value maintain, it shows that the iteration will gives the most minimum ITSE for the system and the data obtained will be used to select the best value for parameter in LQR and PID.

## 4.6 Parameter for PID and LQR

This section will discuss about the parameter obtained for PID and LQR using PSO optimization technique. The least ITSE obtained from the iteration will be chosen as the best performance index for the system and the data weighted gain for the selected execute number will be used as the best parameter for the controller. A set of good control parameters can produces a good step response that will result in the performance criteria where it minimize in the time response data. These performance criteria include the overshoot, rise time, settling time, and steady-state error. The parameter need to be obtained for LQR controller is Q1, Q2 and R where for PID the parameters needed are Kp, Ki and Kd.



Table 4.8: Parameter value for PID and LQR

| **LQR** | | | |
|---|---|---|---|
| **ITSE : 0.4028** | | | |
| Parameter | **Q1** | **Q2** | **R1** |
| | 0.0326 | 64.0389 | 0.0000735 |
| **PID** | | | |
| **ITSE : 0.4448** | | | |
| Parameter | **Kp** | **Ki** | **Kd** |
| | 91.5044 | 0.0033 | 48.9905 |

Table 4.8 shows the value of parameter for LQR and PID based on the based value of performance index obtained after 20 run done. It shows that the ITSE for LQR is smaller compared to PID. Smaller value of ITSE is better for a system so that the total energy used for the system can be minimizes as possible.

## 4.7 Comparison between LQR and PID

This section shows the performance of the system response using LQR controller and PID controller. The parameter for both LQR and PID was obtained using the same optimization technique, PSO in order to ease the user to find the gain value for the controllers. The comparison between LQR and PID controller responses are shown in Figure 4.10, Figure 4.11 and Table 4.9.

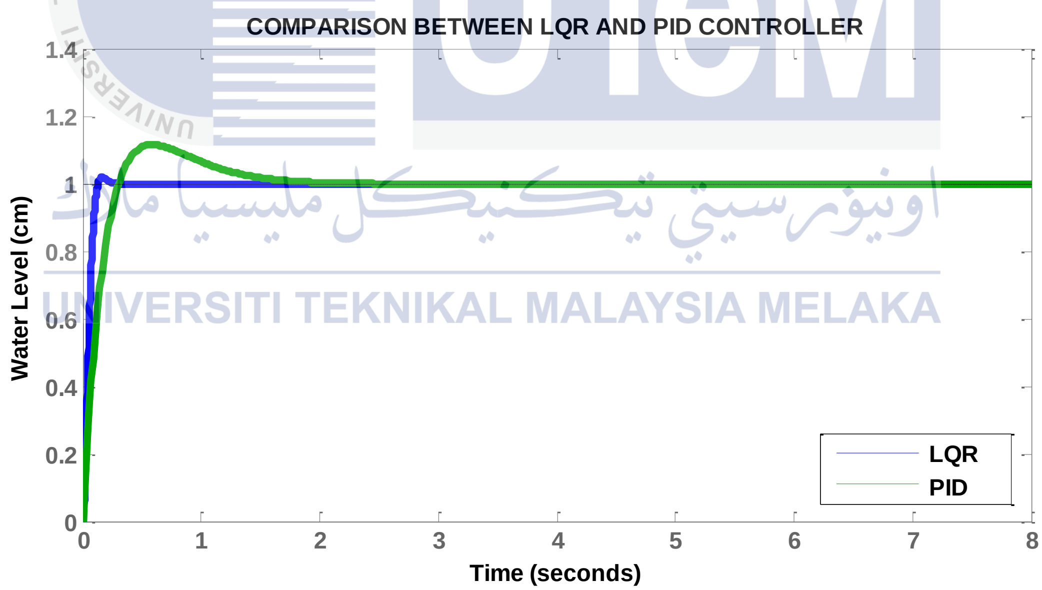


Figure 4.10: Responses for LQR and PID controller

From the observation on Figure 4.10, it shows that LQR controller gives the best performance compared to PID controller in terms of the transient responses. For LQR controller, it gives less overshoot compared to the PID controller. It also shows that the system using LQR as the controller meet the steady state condition at short period of time. In order to investigate and verify the performance using different controllers, time response data for both controllers was taken and shown in Table 4.9 and Figure 4.11.

Table 4.9: Comparison between LQR and PID

| **OUTPUT** | **CONTROLLER** | |
|---|---|---|
| | **LQR** | **PID** |
| **Rise time, Tr (s)** | 0.0792 | 0.22 |
| **Settling time, Ts (s)** | 0.1200 | 1.46 |
| **Overshoot, OS(%)** | 1.83 | 11.6 |
| **Steady-state error, Ess** | 0 | 0 |



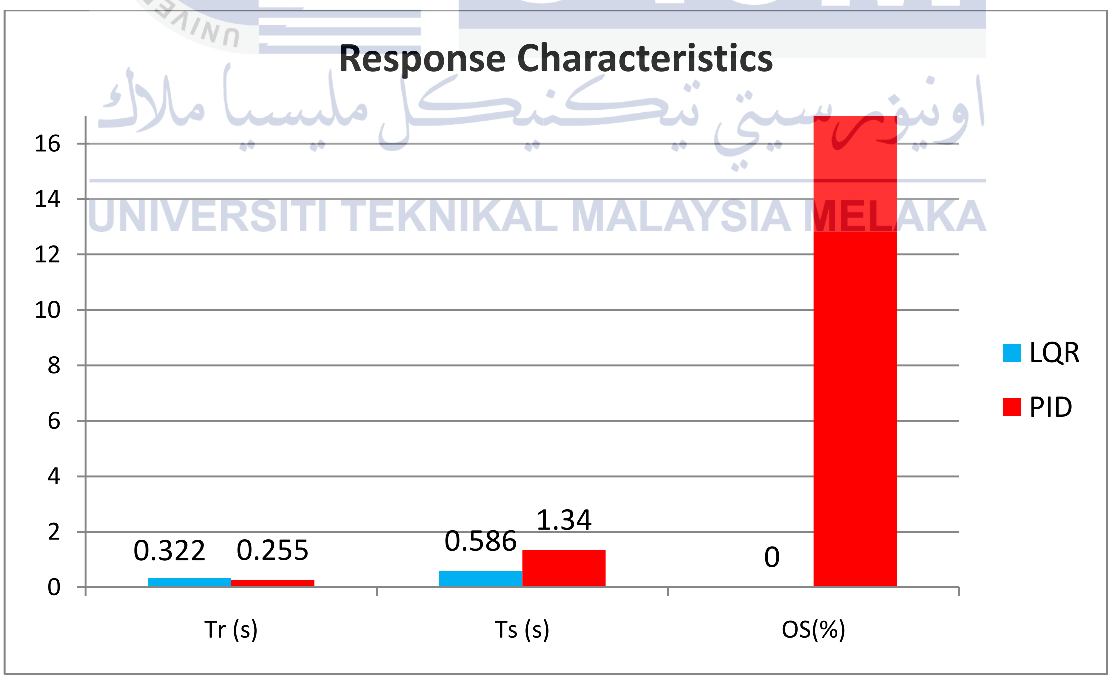


Figure 4.11: Performance comparison between LQR and PID controller

Refer to Table 4.8, it shows that the transient response for LQR controller is much better than PID controller. For the rise time, Tr LQR responses faster than PID where it rises at 0.0792 s which is 0.1408 s faster than the PID. In terms of settling time, Ts the time taken for LQR to settle is shorter which is at 0.12 s whereas for PID it took longer time at 1.46 s. For PID controller, it gives quite high overshoot reading for merely 11% while on the contrary LQR gives less overshoot which is 1.83%. Both of the controller gives better response where is gives zero steady-state error for the system.

## 4.8 Robustness of LQR and PID using PSO

Robustness is an important criteria needed by any controller because it shows the ability of the controller to withstand or resist by any disturbance or noise that might occurred. LQR and PID controller with PSO optimization technique were tested with the presence of disturbance to the system in order to compare the robustness between the controllers. Unit step disturbance with value $\frac{1}{s}$ is added as a disturbance for the system.



### 4.8.1 ITSE for LQR and PID (with disturbance)

The result shows the value of ITSE obtained using PSO for both LQR and PID when there is disturbance added to the system. Figure 4.12 shows for the fitness for LQR controller and Figure 4.13 shows the fitness for PID controller.

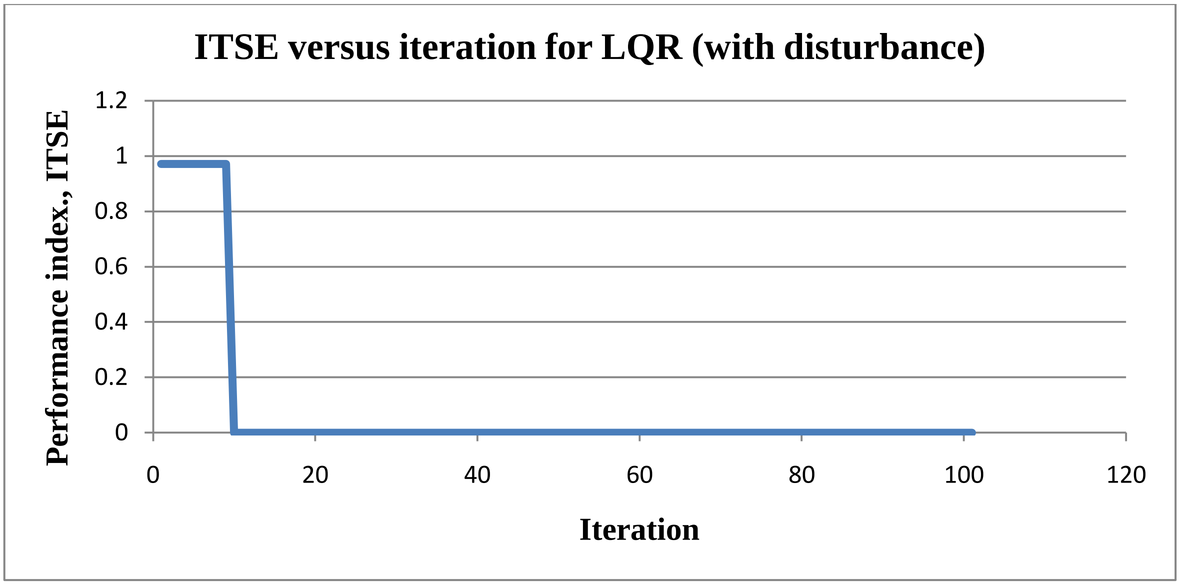


Figure 4.12: Graph of ITSE versus iteration for LQR (with disturbance)

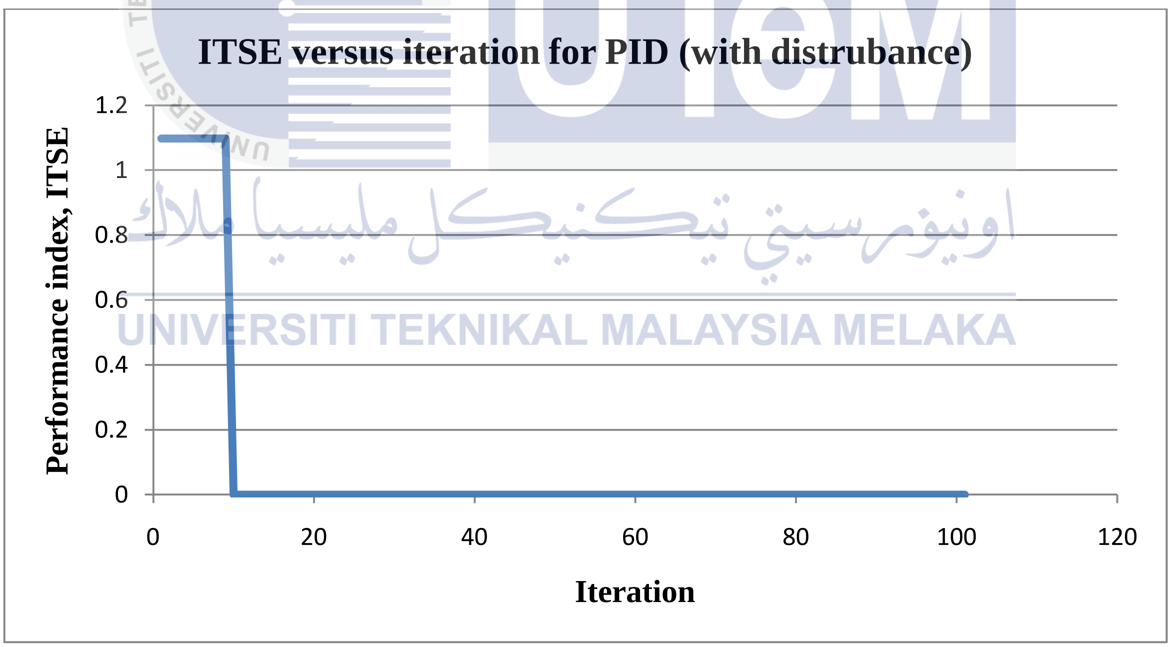


Figure 4.13: Graph of ITSE versus iteration for PID (with disturbance)

Refer to Figure 4.12 and Figure 4.13, even though disturbance is injected in the system, PSO proves that it can gives an optimum fitness function for LQR and PID. Small value of ITSE gives better performance to the system. ITSE value for PID is 1.0976 whereas LQR is 0.9718. The values of ITSE for both controllers are slightly increase compared to system without disturbance.

### 4.8.2 Parameter for PID and LQR (with disturbance)

This section will discuss about the parameter obtained for PID and LQR using PSO optimization technique when there is disturbance. The parameter obtained will be used to compare the system performance for both controllers.

Table 4.10: Parameter value for PID and LQR (with disturbance)

<table>
<tr><th colspan="4">LQR</th></tr>
<tr><td colspan="4">ITSE : 0.9718</td></tr>
<tr><td rowspan="2">Parameter</td><td>Q1</td><td>Q2</td><td>R1</td></tr>
<tr><td>2.0924</td><td>98.8731</td><td>0.00087</td></tr>
<tr><th colspan="4">PID</th></tr>
<tr><td colspan="4">ITSE : 1.0976</td></tr>
<tr><td rowspan="2">Parameter</td><td>Kp</td><td>Ki</td><td>Kd</td></tr>
<tr><td>94.4418</td><td>0.0094</td><td>32.6307</td></tr>
</table>



Table 4.10 shows the value of parameter for LQR and PID. Disturbance is injected in this system in order to compare the performance of LQR and PID as well as to check the robustness of each controller. It shows that the ITSE for LQR is 0.9718 which is smaller compared to ITSE value of PID, 1.0976.

## 4.9 Comparison between LQR and PID (with disturbance)

This section shows the performance of the system response using LQR controller and PID controller. The comparison between LQR and PID controller responses are shown in Figure 4.14 and Table 4.11.

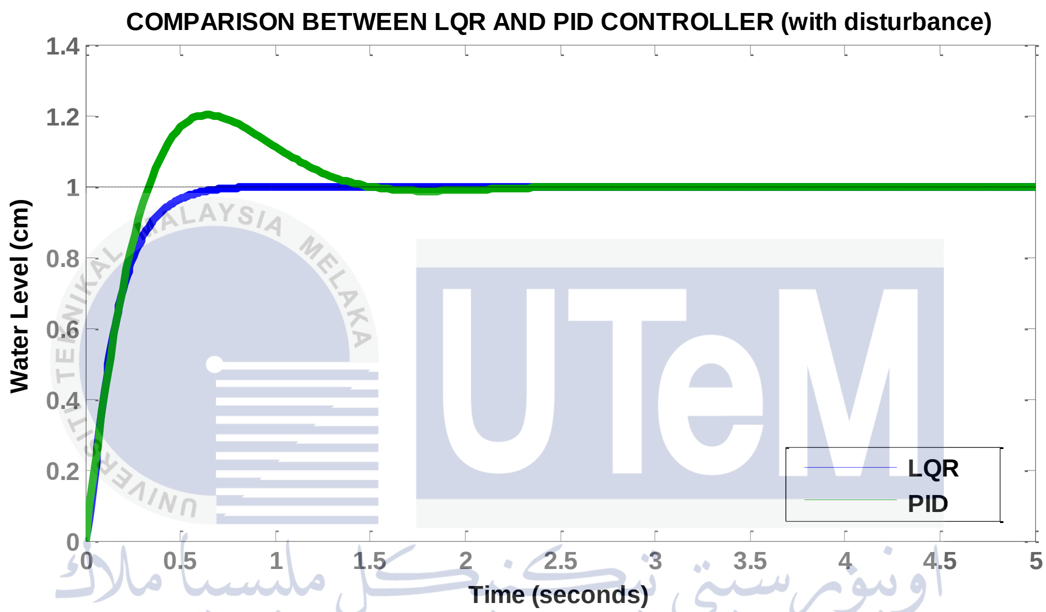




Figure 4.14: Responses for LQR and PID controller (with disturbance)

From the observation on Figure 4.10, it shows that LQR controller gives the best performance compared to PID controller in terms of the transient responses. For LQR controller, it gives zero overshoot compared to the PID controller. It also shows that the system using LQR will meet the steady state condition at short period of time. Even though the rise time taken for PID is faster compared to LQR, the settling time taken for LQR is faster which 0.586 s. To observe and verify the performance using different controllers, time response data for both controllers was taken and shown in Table 4.11 and Figure 4.15.

Table 4.11: Comparison between LQR and PID (with disturbance)

| OUTPUT | CONTROLLER | |
|---|---|---|
| | LQR | PID |
| **Rise time, Tr (s)** | 0.322 | 0.255 |
| **Settling time, Ts (s)** | 0.586 | 1.34 |
| **Overshoot, OS(%)** | 0 | 20.1 |
| **Steady-state error, Ess** | 0 | 0 |

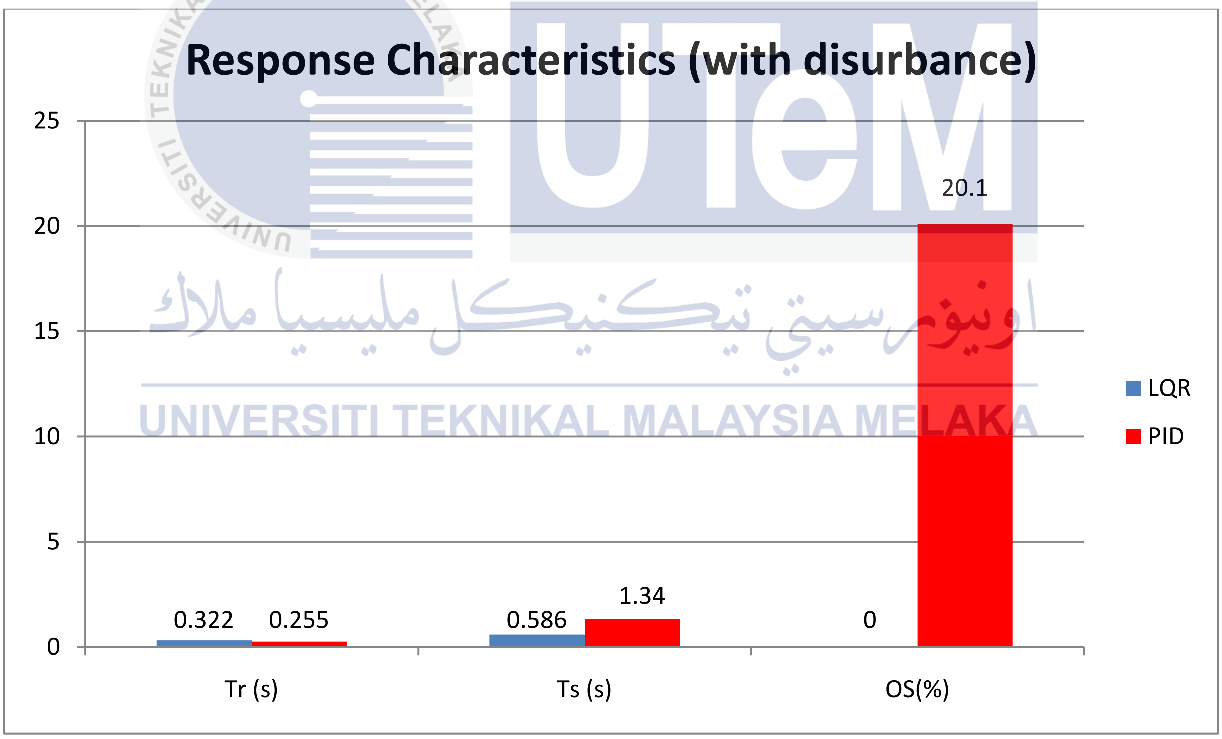




Figure 4.15: Performance comparison between LQR and PID controller (with disturbance)

Based on Figure 4.14, Figure 4.15 and Table 4.11, both of the controllers can withstand the disturbance injected to the system and are able to stabilize the system as faster as possible. From Figure 4.14, the overshoot for PID when there is disturbance is higher compared to system without disturbance. It takes slower time for LQR to response at the beginning but the controller manage to gives a steady-state error performance in a short time compared to PID. After few simulation had been done to compare the performance between LQR and PID controller, it shows that LQR controller and PID controller are competent to be used in the coupled tank system to maintain the water level in the tank. However, the results proved that using the approach of LQR controller gives better performance than PID controller in terms of transient response, robustness and can have optimal value for the performance index, ITSE.

# CHAPTER 5 CONCLUSION

# CONCLUSION

## 5.1 Overview

This section consists of the conclusion of the project and future work that can be used to improved the result of the project.

## 5.2 Conclusion

From the research done and result presented in Chapter 4, there are several conclusion can be made for this project. The mathematical model for the coupled tank system can be determined using system identification method and the validation process is based on the best fit obtained.

Secondly, it shows that by using the PSO optimization technique, it eases the user to obtain the weight parameter for both controllers. Having PSO can validate the comparison for the response of the system because it has a constant variable that can be compared instead of using the conventional method such as trial and error method which does not gives firm validation in terms of the selection of parameter and number of particle used. The selection of the best parameter can be determined by referring to the data collected at the most minimum Integral Time-weighted Squared Error (ITSE) because less error gives small amount of energy used for system and it allow the system to give response which similar with the given input.

In general, LQR controller and PID controller are competent to be used in the coupled tank system to maintain the water level in the tank. However, the results shows and prove that using the approach of LQR controller gives better performance than PID controller in terms of faster response, less overshoot, optimal ITSE, robustness and statistical estimation .

## 5.2 Recommendation for Future Work

Several actions need to be done in the future in order to produce a quality performance for the coupled tank system. Replacement of new coupled tank module CTS-001 is needed so that it can gives better performance of the tank during collecting data and produce less error which can be found from the malfunction of certain component in the hardware itself.

Different optimization technique such as Genetic Algorithm (GA) and Firefly Algorithm (FA) can also be used to find the best value for parameter in LQR and PID. Different optimization techniques will give variant response due to different characteristic and concept of the algorithm.

# REFERENCES


[1] "Coupled-tank Liquid Level Computer-controlled Laboratory Teaching Package Experimental and Operation Service Manual". Augmenred Innovation Sdn. Bhd Center For Artificial Intelligence and Robotic(CAIRO).*Lab manual*

[2] H.Gupta, O.P.Verma. "Intelligent Controller for Coupled Tank System" ISSN:2277 128X.Volume 2. Issue 4. April 2012

[3] H.M.Shariff, M.Hezri, F.Rahiman, M.Tajjudin. "Non linear System Identification : Comparison between PRBS and Random Gaussian Perturbation on Steam Distallation Pilot Plant". 2013 IEEE 3rd International Conference on System Engineering and Technology. pp.269-274. August 2013

[4] A.Mohammadbagher, M.Yaghoobi. "Comparison performance between PID and LQR controller for 4-leg voltage sources inverter". 2011 International Conference on Circuit, System and Simulation.Volume 7. pp.230-234. 2011

[5] L.B.Prasad, B.Tyagi, H.O.Gupta. "Optimal control of nonlinear inverted pendulum dynamical system with disturbance input using PID controller and LQR". IEEE International Conference on Control System, Computing and Engineering. pp.540-545. 2012

[6] N.Wahid,M.F.Rahmat. "Pitch Control System using LQR and Fuzzy Logic Controller" IEEE Symposium on Industrial Electronics and Aplications (ISIEA 2010).pp.389-394.2010

[7] C.Chitu, J,Lackner, M.Horn,H.Waser, M.Kohlbock, "A robust and optimal LQR controller design for Electric Power Steering System" Nonlinear Dynamics and Synchronization (INDS) & 16th Int'l Symposium on Theoretical Electrical Engineering (ISTET).pp.1-5.2011

[8] M.A.Usta, O.Akyazi, A.S.Akpinar. "Aircraft Roll Control System using LQR and Fuzzy Logic Controller".Innovations in Intelligent Systems and Applications (INISTA). pp.223-227.2011

[9] A.Mohammadbagher, M.Yaghoobi. "Comparison performance between PID and LQR controller for 4-leg voltage sources inverter". 2011 International Conference on Circuit, System and Simulation.Volume 7. pp.230-234.2011

[10] X. Xiong, Z. Wan. "The simulation of double inverted pendulum control based on particle swarm optimization LQR algorithm". Software Engineering and Service Sciences (ICSESS) IEEE International Conference.pp.253-256.2010

[11] R.Jaisue,J.Chaoraingern,V.Tipsuwanport,A.Numsonran."A design of Fuzzy PID Controller based on ARM7TDMI for coupled-tanks process". Control, Automation and Systems (ICCAS).12th International Conference.pp.610-613.2012

[12] O.D.Kieran."Real-time Adaptive PID Control of a Non-linear Process based on Genetic Optimization".Dublin City University.August 2002.

[13] M.Abid."Fuzzy Logic Control of Coupled Liquid Tank System". Information and Communication Technologies. ICICT. 1st International Conference.pp.144-147.2005

[14] O.D.Kieran."Real-time Adaptive PID Control of a Non-linear Process based on Genetic Optimization".Dublin City University.August 2002.

[15] S.Aydin, S.Tokat. "Sliding mode control of a coupled tank system with a state varying sliding surface parameter". Variable Structure Systems.VSS08.International workshop. pp.355-360.2008

[16] H.Abbas,S.Asghar,S.Qamar.“Sliding Mode Control for Coupled Tank liquid level control”.10$^{th}$ International Conference on frontiers of Information Technology. International Islamic University. pp.325-330.2012

[17] M.T.Alam, P.Charan, Q.Alam, S.Purwar. “Sliding Mode Control of Coupled Tanks System: Theory and an application”. ISSN 2250-2459.ISO 9001:2008 Certified Journal.Volume 3.Issue 8.August 2013

[18] M.N.Mahyuddin, M.R.Arshad, Z.Mohamed. “Simulation of Direct Model Reference Adaptive Control on a Coupled-Tank System using Nonlinear Plant Model”. International Conference on Control, Instrumentation and Mechatronics Engineering (CIM’07).2007

[19] M.F.Rahmat, S.M.Rozali “Modeling and Controller design for a Coupled-Tank Liquid Level system: Analysis & Comparison”. Journal Teknologi (48)D. pp.113-141.Jun 2008

[20] M.S.M.Aras, M.F.Basar,N.Hasim,M.N.Kamaruddin,H.I.Jaafar “Development and Modeling of Water Tank System Using System Identification Method”. International Journal of Engineering and Advanced Technology (IJEAT).pp.278-283.August 2013

[21] A.N.K.Nasir, M.A.Ahmad,M.F.Rahmat. “Performance Comparison Between LQR and PID controller for an Inverted Pendulum System”. International Conference on Power Control and Optimization,Chiang May.Thailand. pp.18-20.July 2008

[22] J.Patra,P.S.Khuntia,S.Samal. “Analysis and Comparison of different performance index factor for Conventional PID & GA plus PID Controller”. International Journal of Emerging Technologies in Computational and Applied Sciences (IJETCAS). pp.242-250. 2003

[23] S.M.Rozali,M.F.Rahmat,A.B.Husain.“Performance Comparison of Particle Swarm Optimization and Gravitational Search Algorithm to the Designed of Controller for Nonlinear System”. Hindawi Publishing Corporation Journal of Applied Mathematics Volume 2014pp.1-9. 2014

[24] ]A.Jalilvand, A.Kimiyaghalam,A.Ashouri,H.Kord. “Optimal Tuning of PID Controller arameters on a DC Motor Based on Advanced Particle Swarm Optimization Algorithm”.International Journal on “Tenchical and Physical Problems of Engineering”(IJTPE) Issue 9.pp.10-17

[25] A.Kumar,R.Gupta.”Tuning of PID Controller using PSO Algorith and Compare Results of Integral Errors for AVR System”. International Journal of Innovative Research & Development Vol 2 Issue 4.pp.58-68.April 2013

# APPENDIX A

**Table A1**: 1st simulation LQR data with 20 times execution using PSO

| RUN | ITSE | Q1 | Q2 | R |
|---|---|---|---|---|
| 2.13050627998622 | 17 | 4.79274204430698 | 74.3396131994710 | 5.78125973957944e-05 |
| 1.69590483497912 | 17 | 0.491481647083236 | 38.5761870299283 | 0.00552476959795654 |
| 0.679099522075348 | 13 | 0.898068737876157 | 80.6088144091522 | 0.000127805271264323 |
| 2.35337899754909 | 7 | 11.2433991109230 | 80.2168704201640 | 0.00344245877858750 |
| 3.54141652680822 | 16 | 14.0842806740166 | 73.6026684234095 | 0.000608208070124958 |
| 2.04997980778278 | 11 | 4.80055578809437 | 46.6454408149075 | 0.00570436641604506 |
| 1.06930165954766 | 10 | 0.342823337294396 | 88.0378863578350 | 0.00447469525923488 |
| 11.1624071160574 | 15 | 12.9071638093653 | 68.3828546263153 | 0.00711262900804189 |
| 0.497174368373740 | 15 | 0.279828764891865 | 80.1457982084609 | 7.50256973377383e-05 |
| 2.49147542927778 | 3 | 0.573727589254591 | 20.2760171859482 | 0.00834872050634867 |
| 2.35436533978933 | 18 | 12.1082032693739 | 84.3984758906507 | 0.00552534389218350 |
| 1.56831243776666 | 12 | 0.917565642354878 | 81.9583374175061 | 0.00594013225881150 |
| 1.90669129502014 | 15 | 8.44061757182513 | 91.8379745324556 | 0.00666690737526759 |
| 1.53520122905759 | 13 | 2.04819932988941 | 35.8293534908822 | 0.00145330976472437 |
| 1.55455375037506 | 5 | 4.41719617426647 | 95.3388382496468 | 0.00731940004658055 |
| 1.77861081670062 | 3 | 6.17225639989654 | 75.4450691410559 | 0.00302664862755968 |
| 1.03916676119619 | 18 | 0.386108251862960 | 81.2128492547820 | 0.000503316989113276 |
| 1.66224939744029 | 16 | 6.47454883862451 | 99.4110611092764 | 0.00631545668381895 |
| 1.49049288644050 | 12 | 1.20592196038944 | 84.7864536455246 | 0.00372179849793528 |
| 1.84250315396077 | 9 | 6.12440822585023 | 81.5411633829385 | 0.00958533271440733 |

**Table A2**: 2nd simulation LQR data with 20 times execution using PSO

| RUN | ITSE | Q1 | Q2 | R |
|---|---|---|---|---|
| 1.64252862896314 | 8 | 3.57116785741896 | 84.9129305868777 | 0.00933993247757551 |
| 2.39893633950533 | 15 | 11.2283962156027 | 78.4427890743913 | 0.00291570317906931 |
| 2.23066860172050 | 7 | 5.69328854352480 | 45.0323812733453 | 0.00582470301736287 |
| 1.57967646345843 | 17 | 3.03890236611514 | 75.3200532708854 | 0.00700042669529619 |
| 1.96811664521068 | 2 | 3.46300613124667 | 45.1123791403174 | 0.000137945567780947 |
| 10.6190979800645 | 17 | 1.28771628002986 | 18.6643440738731 | 0.00485230456949770 |
| 0.402756076662918 | 20 | 0.0326096340118931 | 64.0389224633762 | 7.35569277648585e-05 |
| 2.15381103187116 | 11 | 4.84545523824337 | 44.2740383821396 | 0.00789846368465099 |
| 11.9389225926296 | 5 | 5.94902029010737 | 16.9227529964748 | 0.00684679742407259 |
| 1.43507573988711 | 17 | 0.866849408275416 | 92.8280818433486 | 0.00520219495030670 |
| 0.993611398851545 | 19 | 1.79688396310653 | 70.1511447274707 | 0.000157523069350503 |
| 1.60637373042458 | 12 | 1.97606465104744 | 60.1611188055368 | 0.00607371066617602 |
| 1.34179022909105 | 11 | 0.929021743182679 | 19.8328389991404 | 0.000206801420908972 |
| 2.16148694804115 | 17 | 0.847423194155961 | 41.6031807168918 | 0.00439118205411470 |
| 1.96539027002886 | 8 | 1.13300513836901 | 28.2843484575170 | 0.00589571290675808 |
| 1.64308611012115 | 9 | 2.66598923540069 | 55.0467405320711 | 0.00563050714329366 |
| 1.37665857222925 | 7 | 2.08506707549020 | 72.8717839188495 | 0.00398517860314893 |
| 1.42960615891802 | 18 | 1.03843695834612 | 59.3550306259397 | 0.00301630950131320 |
| 1.43798297970720 | 12 | 3.01212458382484 | 72.0036195118740 | 0.00376215712094936 |
| 2.22730364406355 | 11 | 5.94410432308502 | 46.0336271903498 | 0.00318710525919819 |

**Table A3**: 3rd simulation LQR data with 20 times execution using PSO

| RUN | ITSE | Q1 | Q2 | R |
|---|---|---|---|---|
| 1.64252862896314 | 8 | 3.57116785741896 | 84.9129305868777 | 0.00933993247757551 |
| 2.39893633950533 | 15 | 11.2283962156027 | 78.4427890743913 | 0.00291570317906931 |
| 2.23066860172050 | 7 | 5.69328854352480 | 45.0323812733453 | 0.00582470301736287 |
| 1.57967646345843 | 17 | 3.03890236611514 | 75.3200532708854 | 0.00700042669529619 |
| 1.96811664521068 | 2 | 3.46300613124667 | 45.1123791403174 | 0.000137945567780947 |
| 10.6190979800645 | 17 | 1.28771628002986 | 18.6643440738731 | 0.00485230456949770 |
| 0.402756076662918 | 20 | 0.0326096340118931 | 64.0389224633762 | 7.35569277648585e-05 |
| 2.15381103187116 | 11 | 4.84545523824337 | 44.2740383821396 | 0.00789846368465099 |
| 11.9389225926296 | 5 | 5.94902029010737 | 16.9227529964748 | 0.00684679742407259 |
| 1.43507573988711 | 17 | 0.866849408275416 | 92.8280818433486 | 0.00520219495030670 |
| 0.993611398851545 | 19 | 1.79688396310653 | 70.1511447274707 | 0.000157523069350503 |
| 1.60637373042458 | 12 | 1.97606465104744 | 60.1611188055368 | 0.00607371066617602 |
| 1.34179022909105 | 11 | 0.929021743182679 | 19.8328389991404 | 0.000206801420908972 |
| 2.16148694804115 | 17 | 0.847423194155961 | 41.6031807168918 | 0.00439118205411470 |
| 1.96539027002886 | 8 | 1.13300513836901 | 28.2843484575170 | 0.00589571290675808 |
| 1.64308611012115 | 9 | 2.66598923540069 | 55.0467405320711 | 0.00563050714329366 |
| 1.37665857222925 | 7 | 2.08506707549020 | 72.8717839188495 | 0.00398517860314893 |
| 1.42960615891802 | 18 | 1.03843695834612 | 59.3550306259397 | 0.00301630950131320 |
| 1.43798297970720 | 12 | 3.01212458382484 | 72.0036195118740 | 0.00376215712094936 |
| 2.22730364406355 | 11 | 5.94410432308502 | 46.0336271903498 | 0.00318710525919819 |

**Table A4**: 4th simulation LQR data with 20 times execution using PSO

| RUN | ITSE | Q1 | Q2 | R |
|---|---|---|---|---|
| 1.64252862896314 | 8 | 3.57116785741896 | 84.9129305868777 | 0.00933993247757551 |
| 2.39893633950533 | 15 | 11.2283962156027 | 78.4427890743913 | 0.00291570317906931 |
| 2.23066860172050 | 7 | 5.69328854352480 | 45.0323812733453 | 0.00582470301736287 |
| 1.57967646345843 | 17 | 3.03890236611514 | 75.3200532708854 | 0.00700042669529619 |
| 1.96811664521068 | 2 | 3.46300613124667 | 45.1123791403174 | 0.000137945567780947 |
| 10.6190979800645 | 17 | 1.28771628002986 | 18.6643440738731 | 0.00485230456949770 |
| 0.402756076662918 | 20 | 0.0326096340118931 | 64.0389224633762 | 7.35569277648585e-05 |
| 2.15381103187116 | 11 | 4.84545523824337 | 44.2740383821396 | 0.00789846368465099 |
| 11.9389225926296 | 5 | 5.94902029010737 | 16.9227529964748 | 0.00684679742407259 |
| 1.43507573988711 | 17 | 0.866849408275416 | 92.8280818433486 | 0.00520219495030670 |
| 0.993611398851545 | 19 | 1.79688396310653 | 70.1511447274707 | 0.000157523069350503 |
| 1.60637373042458 | 12 | 1.97606465104744 | 60.1611188055368 | 0.00607371066617602 |
| 1.34179022909105 | 11 | 0.929021743182679 | 19.8328389991404 | 0.000206801420908972 |
| 2.16148694804115 | 17 | 0.847423194155961 | 41.6031807168918 | 0.00439118205411470 |
| 1.96539027002886 | 8 | 1.13300513836901 | 28.2843484575170 | 0.00589571290675808 |
| 1.64308611012115 | 9 | 2.66598923540069 | 55.0467405320711 | 0.00563050714329366 |
| 1.37665857222925 | 7 | 2.08506707549020 | 72.8717839188495 | 0.00398517860314893 |
| 1.42960615891802 | 18 | 1.03843695834612 | 59.3550306259397 | 0.00301630950131320 |
| 1.43798297970720 | 12 | 3.01212458382484 | 72.0036195118740 | 0.00376215712094936 |
| 2.22730364406355 | 11 | 5.94410432308502 | 46.0336271903498 | 0.00318710525919819 |

**Table A5**: 5th simulation LQR data with 20 times execution using PSO

| RUN | ITSE | Q1 | Q2 | R |
|---|---|---|---|---|
| 1.64252862896314 | 8 | 3.57116785741896 | 84.9129305868777 | 0.00933993247757551 |
| 2.39893633950533 | 15 | 11.2283962156027 | 78.4427890743913 | 0.00291570317906931 |
| 2.23066860172050 | 7 | 5.69328854352480 | 45.0323812733453 | 0.00582470301736287 |
| 1.57967646345843 | 17 | 3.03890236611514 | 75.3200532708854 | 0.00700042669529619 |
| 1.96811664521068 | 2 | 3.46300613124667 | 45.1123791403174 | 0.000137945567780947 |
| 10.6190979800645 | 17 | 1.28771628002986 | 18.6643440738731 | 0.00485230456949770 |
| 0.402756076662918 | 20 | 0.0326096340118931 | 64.0389224633762 | 7.35569277648585e-05 |
| 2.15381103187116 | 11 | 4.84545523824337 | 44.2740383821396 | 0.00789846368465099 |
| 11.9389225926296 | 5 | 5.94902029010737 | 16.9227529964748 | 0.00684679742407259 |
| 1.43507573988711 | 17 | 0.866849408275416 | 92.8280818433486 | 0.00520219495030670 |
| 0.993611398851545 | 19 | 1.79688396310653 | 70.1511447274707 | 0.000157523069350503 |
| 1.60637373042458 | 12 | 1.97606465104744 | 60.1611188055368 | 0.00607371066617602 |
| 1.34179022909105 | 11 | 0.929021743182679 | 19.8328389991404 | 0.000206801420908972 |
| 2.16148694804115 | 17 | 0.847423194155961 | 41.6031807168918 | 0.00439118205411470 |
| 1.96539027002886 | 8 | 1.13300513836901 | 28.2843484575170 | 0.00589571290675808 |
| 1.64308611012115 | 9 | 2.66598923540069 | 55.0467405320711 | 0.00563050714329366 |
| 1.37665857222925 | 7 | 2.08506707549020 | 72.8717839188495 | 0.00398517860314893 |
| 1.42960615891802 | 18 | 1.03843695834612 | 59.3550306259397 | 0.00301630950131320 |
| 1.43798297970720 | 12 | 3.01212458382484 | 72.0036195118740 | 0.00376215712094936 |
| 2.22730364406355 | 11 | 5.94410432308502 | 46.0336271903498 | 0.00318710525919819 |

**Table A6**: 6th simulation LQR data with 20 times execution using PSO

| RUN | ITSE | Q1 | Q2 | R |
|---|---|---|---|---|
| 10.3526970997094 | 3 | 0.233952635947434 | 28.7286478806351 | 0.00821270082456773 |
| 11.9159124091207 | 1 | 17.4842882290990 | 50.4104425083411 | 0.00495986198002481 |
| 3.47251803959533 | 20 | 21.3516159682177 | 75.3660319785068 | 0.00712053280195641 |
| 10.4228791027415 | 13 | 0.455235101842423 | 18.7157388132182 | 0.00651350077499493 |
| 1.91884983245839 | 8 | 1.61486059952399 | 97.6485753531106 | 0.00898779900812044 |
| 2.09381153567298 | 6 | 1.06912757564572 | 27.1813176584230 | 0.00660901376287224 |
| 3.01909995732341 | 20 | 15.1344442724013 | 70.2000864717242 | 0.00401099593319308 |
| 1.76128881799014 | 9 | 7.14151059626476 | 91.6302518159905 | 0.00511799279781735 |
| 1.73769270967662 | 3 | 7.46163046084833 | 94.9703558363172 | 0.00232460098891134 |
| 1.94178346640768 | 14 | 1.94818069331700 | 46.3906572590082 | 0.00955899329988603 |
| 1.33980789867631 | 11 | 2.51144668707560 | 95.9525722467023 | 0.00472885982775281 |
| 1.66773816261396 | 20 | 5.68106388188665 | 98.9177444702064 | 0.00903987910149907 |
| 1.96106958190221 | 13 | 8.10836184616291 | 83.6279274593202 | 0.00696701232571157 |
| 1.80891249662930 | 8 | 5.85047725184698 | 70.9125413153822 | 0.00427374369741027 |
| 2.79704829701106 | 8 | 15.7043535595677 | 79.0027567695258 | 0.00717189844036508 |
| 3.31434707875594 | 13 | 17.6474452701465 | 64.2324736042979 | 0.00929251555874401 |
| 1.32861445107277 | 18 | 0.869710592320649 | 53.0961615183720 | 0.00214766479950175 |
| 1.29346445111219 | 16 | 2.61736390277365 | 86.1773105080071 | 0.00335173380192581 |
| 0.679099522075348 | 13 | 0.898068737876157 | 80.6088144091522 | 0.000127805271264323 |
| 2.02273655275533 | 15 | 3.60513411442917 | 43.5982639673742 | 0.00893169888499464 |

**Table A7**: 7th simulation LQR data with 20 times execution using PSO

| RUN | ITSE | Q1 | Q2 | R |
|---|---|---|---|---|
| 1.64252862896314 | 8 | 3.57116785741896 | 84.9129305868777 | 0.00933993247757551 |
| 2.39893633950533 | 15 | 11.2283962156027 | 78.4427890743913 | 0.00291570317906931 |
| 2.23066860172050 | 7 | 5.69328854352480 | 45.0323812733453 | 0.00582470301736287 |
| 1.57967646345843 | 17 | 3.03890236611514 | 75.3200532708854 | 0.00700042669529619 |
| 1.96811664521068 | 2 | 3.46300613124667 | 45.1123791403174 | 0.000137945567780947 |
| 10.6190979800645 | 17 | 1.28771628002986 | 18.6643440738731 | 0.00485230456949770 |
| 0.402756076662918 | 20 | 0.0326096340118931 | 64.0389224633762 | 7.35569277648585e-05 |
| 2.15381103187116 | 11 | 4.84545523824337 | 44.2740383821396 | 0.00789846368465099 |
| 11.9389225926296 | 5 | 5.94902029010737 | 16.9227529964748 | 0.00684679742407259 |
| 1.43507573988711 | 17 | 0.866849408275416 | 92.8280818433486 | 0.00520219495030670 |
| 0.993611398851545 | 19 | 1.79688396310653 | 70.1511447274707 | 0.000157523069350503 |
| 1.60637373042458 | 12 | 1.97606465104744 | 60.1611188055368 | 0.00607371066617602 |
| 1.34179022909105 | 11 | 0.929021743182679 | 19.8328389991404 | 0.000206801420908972 |
| 2.16148694804115 | 17 | 0.847423194155961 | 41.6031807168918 | 0.00439118205411470 |
| 1.96539027002886 | 8 | 1.13300513836901 | 28.2843484575170 | 0.00589571290675808 |
| 1.64308611012115 | 9 | 2.66598923540069 | 55.0467405320711 | 0.00563050714329366 |
| 1.37665857222925 | 7 | 2.08506707549020 | 72.8717839188495 | 0.00398517860314893 |
| 1.42960615891802 | 18 | 1.03843695834612 | 59.3550306259397 | 0.00301630950131320 |
| 1.43798297970720 | 12 | 3.01212458382484 | 72.0036195118740 | 0.00376215712094936 |
| 2.22730364406355 | 11 | 5.94410432308502 | 46.0336271903498 | 0.00318710525919819 |

**Table A8**: 8th simulation LQR data with 20 times execution using PSO

| RUN | ITSE | Q1 | Q2 | R |
|---|---|---|---|---|
| 2.31512997210434 | 15 | 11.7759567037476 | 84.7341975079693 | 0.00911654660214633 |
| 1.51648751012963 | 8 | 4.71949179827239 | 80.0776380326191 | 0.00199669740701472 |
| 2.73085298321044 | 2 | 28.8583527595549 | 90.5849603652388 | 0.000594564782137067 |
| 0.933612857838172 | 8 | 0.0437389474680749 | 71.7890447513429 | 0.00460761778096094 |
| 0.584341571067338 | 18 | 0.824142178228771 | 98.3648529472140 | 7.85390411311771e-05 |
| 10.6270606686989 | 1 | 0.739024383663835 | 10.4398041333002 | 0.00844262672615013 |
| 11.7261768727976 | 18 | 25.6019183250404 | 83.1759042261132 | 0.00440929254506972 |
| 1.03804239602261 | 13 | 0.153256667436530 | 84.9506164346575 | 0.00674776713801600 |
| 1.71978774233919 | 8 | 5.42621255347137 | 85.5381671773636 | 0.00821237341474924 |
| 2.63691353791934 | 20 | 11.8367824584735 | 66.6678921252530 | 0.00517094851989823 |
| 1.94822461415401 | 6 | 0.914253572140877 | 54.6717715544212 | 0.00422475186044116 |
| 2.61201288524622 | 2 | 9.05538406603882 | 52.4272588604043 | 0.00343131770743085 |
| 3.11553579419432 | 19 | 18.8135006055736 | 86.9214930064311 | 0.00355015394677996 |
| 1.36936126857922 | 17 | 0.231097831368277 | 32.4739695141863 | 0.00360970359531321 |
| 10.4094814094835 | 5 | 1.32888871220480 | 62.5554325656073 | 0.00959460857221243 |
| 1.19194944852670 | 16 | 2.15698431417624 | 88.5524475051786 | 0.00259052085734989 |
| 2.31512997210434 | 15 | 11.7759567037476 | 84.7341975079693 | 0.00911654660214633 |
| 1.51648751012963 | 8 | 4.71949179827239 | 80.0776380326191 | 0.00199669740701472 |
| 2.73085298321044 | 2 | 28.8583527595549 | 90.5849603652388 | 0.000594564782137067 |
| 0.933612857838172 | 8 | 0.0437389474680749 | 71.7890447513429 | 0.00460761778096094 |

**Table A9**: 9th simulation LQR data with 20 times execution using PSO

| RUN | ITSE | Q1 | Q2 | R |
|---|---|---|---|---|
| 10.3526970997094 | 3 | 0.233952635947434 | 28.7286478806351 | 0.00821270082456773 |
| 11.9159124091207 | 1 | 17.4842882290990 | 50.4104425083411 | 0.00495986198002481 |
| 3.47251803959533 | 20 | 21.3516159682177 | 75.3660319785068 | 0.00712053280195641 |
| 10.4228791027415 | 13 | 0.455235101842423 | 18.7157388132182 | 0.00651350077499493 |
| 1.91884983245839 | 8 | 1.61486059952399 | 97.6485753531106 | 0.00898779900812044 |
| 2.09381153567298 | 6 | 1.06912757564572 | 27.1813176584230 | 0.00660901376287224 |
| 3.01909995732341 | 20 | 15.1344442724013 | 70.2000864717242 | 0.00401099593319308 |
| 1.76128881799014 | 9 | 7.14151059626476 | 91.6302518159905 | 0.00511799279781735 |
| 1.73769270967662 | 3 | 7.46163046084833 | 94.9703558363172 | 0.00232460098891134 |
| 1.94178346640768 | 14 | 1.94818069331700 | 46.3906572590082 | 0.00955899329988603 |
| 1.33980789867631 | 11 | 2.51144668707560 | 95.9525722467023 | 0.00472885982775281 |
| 1.66773816261396 | 20 | 5.68106388188665 | 98.9177444702064 | 0.00903987910149907 |
| 1.96106958190221 | 13 | 8.10836184616291 | 83.6279274593202 | 0.00696701232571157 |
| 1.80891249662930 | 8 | 5.85047725184698 | 70.9125413153822 | 0.00427374369741027 |
| 2.79704829701106 | 8 | 15.7043535595677 | 79.0027567695258 | 0.00717189844036508 |
| 3.31434707875594 | 13 | 17.6474452701465 | 64.2324736042979 | 0.00929251555874401 |
| 1.32861445107277 | 18 | 0.869710592320649 | 53.0961615183720 | 0.00214766479950175 |
| 1.29346445111219 | 16 | 2.61736390277365 | 86.1773105080071 | 0.00335173380192581 |
| 0.859527973696827 | 8 | 0.112685950778124 | 91.8375788830026 | 0.00344131392363552 |
| 2.02273655275533 | 15 | 3.60513411442917 | 43.5982639673742 | 0.00893169888499464 |

**Table A10**: 10th simulation LQR data with 20 times execution using PSO

| RUN | ITSE | Q1 | Q2 | R |
|---|---|---|---|---|
| 10.3526970997094 | 3 | 0.233952635947434 | 28.7286478806351 | 0.00821270082456773 |
| 11.9159124091207 | 1 | 17.4842882290990 | 50.4104425083411 | 0.00495986198002481 |
| 3.47251803959533 | 20 | 21.3516159682177 | 75.3660319785068 | 0.00712053280195641 |
| 10.4228791027415 | 13 | 0.455235101842423 | 18.7157388132182 | 0.00651350077499493 |
| 1.91884983245839 | 8 | 1.61486059952399 | 97.6485753531106 | 0.00898779900812044 |
| 2.09381153567298 | 6 | 1.06912757564572 | 27.1813176584230 | 0.00660901376287224 |
| 3.01909995732341 | 20 | 15.1344442724013 | 70.2000864717242 | 0.00401099593319308 |
| 1.76128881799014 | 9 | 7.14151059626476 | 91.6302518159905 | 0.00511799279781735 |
| 1.73769270967662 | 3 | 7.46163046084833 | 94.9703558363172 | 0.00232460098891134 |
| 1.94178346640768 | 14 | 1.94818069331700 | 46.3906572590082 | 0.00955899329988603 |
| 1.33980789867631 | 11 | 2.51144668707560 | 95.9525722467023 | 0.00472885982775281 |
| 1.66773816261396 | 20 | 5.68106388188665 | 98.9177444702064 | 0.00903987910149907 |
| 1.96106958190221 | 13 | 8.10836184616291 | 83.6279274593202 | 0.00696701232571157 |
| 1.80891249662930 | 8 | 5.85047725184698 | 70.9125413153822 | 0.00427374369741027 |
| 2.79704829701106 | 8 | 15.7043535595677 | 79.0027567695258 | 0.00717189844036508 |
| 3.31434707875594 | 13 | 17.6474452701465 | 64.2324736042979 | 0.00929251555874401 |
| 1.32861445107277 | 18 | 0.869710592320649 | 53.0961615183720 | 0.00214766479950175 |
| 1.29346445111219 | 16 | 2.61736390277365 | 86.1773105080071 | 0.00335173380192581 |
| 0.859527973696827 | 8 | 0.112685950778124 | 91.8375788830026 | 0.00344131392363552 |
| 2.02273655275533 | 15 | 3.60513411442917 | 43.5982639673742 | 0.00893169888499464 |

# APPENDIX B

**Table B1**: 1st simulation PID data with 20 times execution using PSO

| RUN | ITSE | Kp | Kd | Ki |
|---|---|---|---|---|
| 0.915332177062245 | 5 | 98.2803785467415 | 40.2215401149021 | 0.00620671947199578 |
| 1.10389164885255 | 18 | 54.0137248285902 | 31.1078223677466 | 0.000712345719245837 |
| 0.933912014628606 | 10 | 84.6335053710108 | 40.8032078475375 | 0.00462018163773892 |
| 1.07398437424774 | 12 | 97.6600180815677 | 49.2823765088584 | 0.00400882866391995 |
| 1.13819386013663 | 18 | 80.5854925343397 | 32.6407493384841 | 0.00549879145125724 |
| 1.24143193575552 | 2 | 71.9634196252383 | 30.2720909803246 | 0.00459022394078519 |
| 1.16761833988691 | 6 | 75.7091720499107 | 40.5166107614247 | 0.00701901080171245 |
| 1.21385271126024 | 4 | 79.5725747119935 | 30.1673395748036 | 0.00607461688382013 |
| 1.77523350596745 | 7 | 73.2389489575974 | 63.0979032965214 | 0.00468206154981374 |
| 0.971610281942150 | 16 | 77.9175608038320 | 39.4380691499402 | 0.00882325966968808 |
| 0.975999431020841 | 8 | 82.8290546442714 | 42.4438570653092 | 0.00727096156947871 |
| 1.22023043540366 | 12 | 93.8228061805316 | 89.8685021667879 | 0.00694859162934643 |
| 1.02161702018015 | 13 | 81.6371614467867 | 36.8223331504681 | 0.00426215425372259 |
| 1.21467679794895 | 17 | 98.7389049157127 | 61.3833349365827 | 0.00652081525715332 |
| 0.948782290611945 | 3 | 99.8338675978874 | 38.2374042868206 | 0.00190079981771239 |
| 1.00088613638324 | 19 | 91.4863270983913 | 45.6575905786543 | 0.00904682764693900 |
| 1.01377228296562 | 5 | 88.8381930317520 | 36.2928550409516 | 0.00453713606901237 |
| 0.971598733541770 | 6 | 90.9974779700008 | 38.2116496292373 | 0.00701392065887907 |
| 0.968981015915279 | 4 | 94.6937919773885 | 37.9529164774708 | 0.00134520896003630 |
| 1.03659187252856 | 8 | 80.5002814152127 | 41.7867094621429 | 0.00602931412462276 |

**Table B2**: 2nd simulation PID data with 20 times execution using PSO

| RUN | ITSE | Kp | Kd | Ki |
|---|---|---|---|---|
| 1.15647335482744 | 5 | 52.8211424704260 | 30.5692769990855 | 0.00739280254762875 |
| 0.906044539684614 | 17 | 91.2117657314611 | 41.6459473524070 | 0.00807085189423006 |
| 1.30967165138232 | 6 | 89.9203226479877 | 70.3167083832757 | 0.00118644271606670 |
| 0.997381366668315 | 4 | 90.4698837087829 | 37.0727447823453 | 0.00332590772150525 |
| 1.06562619855524 | 9 | 90.3550000738955 | 34.2821481001107 | 0.00860961047764190 |
| 1.24780319649858 | 20 | 86.9682122114768 | 28.4508386312122 | 0.00543879301907192 |
| 0.823310595627441 | 19 | 97.5864941272797 | 46.0038955714545 | 0.00230658286428795 |
| 1.01119192594198 | 7 | 77.3890818820416 | 40.5875296908503 | 0.00169975617447305 |
| 1.06535628692059 | 13 | 64.3829289284005 | 35.5222181484638 | 0.00431216283186782 |
| 1.04396237096534 | 10 | 73.7360003137001 | 36.9465082205785 | 0.000464948106723218 |
| 0.869513813284712 | 19 | 96.6669626096769 | 43.1641925278091 | 0.00316551619078046 |
| 0.937822598352616 | 8 | 89.8430534366899 | 39.9465401239338 | 0.000448706676361251 |
| 1.43884818776678 | 15 | 74.1508793506826 | 24.8519447759337 | 0.00314130421565196 |
| 1.69897839847459 | 13 | 79.9568175683155 | 89.9448986707184 | 0.00447909799279623 |
| 1.21045113605426 | 7 | 63.9317637971839 | 32.3312188236256 | 0.000200173514303332 |
| 0.928299178443933 | 9 | 92.0250120869436 | 40.2093350779071 | 0.00331769141440340 |
| 1.20320437093854 | 13 | 95.6155259063233 | 89.1131916756042 | 0.00850977110359181 |
| 1.07815187050563 | 3 | 92.7550908072563 | 33.5232257897426 | 0.00593616421522111 |
| 1.12102901023037 | 16 | 63.9201332946748 | 34.5827026504082 | 0.00341161890654572 |
| 1.15214607581600 | 15 | 97.0609226812872 | 30.5609709089961 | 0.00709003755319626 |

**Table B3**: 3rd simulation PID data with 20 times execution using PSO

| RUN | ITSE | Kp | Kd | Ki |
|---|---|---|---|---|
| 1.07720325100888 | 5 | 96.7067682728587 | 33.1965083837651 | 0.00532423013200547 |
| 1.10811225831367 | 20 | 87.9155250451676 | 32.8791959416614 | 0.00469187667996306 |
| 1.65429340311114 | 17 | 79.8046039211713 | 77.5815002122993 | 0.00165558118300414 |
| 1.20846225744043 | 14 | 99.0070221120794 | 60.3683207308325 | 4.14628036949649e-05 |
| 1.12395371872820 | 5 | 72.9471796973848 | 34.3078836103585 | 0.00471018252235055 |
| 1.09201299529800 | 18 | 99.5538538888431 | 32.4239412704775 | 0.00620049521203031 |
| 1.27421610307189 | 6 | 93.0339625173405 | 70.0458014281184 | 0.00289147339288430 |
| 0.984998254148300 | 12 | 92.3939436979279 | 37.4377316925528 | 0.00890649389365567 |
| 1.30228667660919 | 20 | 91.9794386606826 | 65.4101864865892 | 0.00650939929186731 |
| 0.910371590250838 | 15 | 82.7797923417378 | 42.0019373684164 | 0.00548655591447403 |
| 1.56217536407998 | 2 | 86.3200030289753 | 97.5211963586364 | 0.00419248298809311 |
| 1.00365542123090 | 6 | 84.3321367500631 | 45.5271908328320 | 0.00303935941960058 |
| 1.16199411688386 | 11 | 84.5718968752979 | 31.3129492135257 | 0.00927684879038271 |
| 1.01521052497679 | 10 | 91.0337201696738 | 36.0012137408718 | 0.00772051665992024 |
| 1.10626281320571 | 19 | 87.9198020265814 | 32.9479080362526 | 0.00610258861897323 |
| 1.30402916704629 | 3 | 90.9895922582920 | 58.0275704183840 | 0.00581239256755058 |
| 1.04026161229894 | 4 | 84.8946701796894 | 35.5664865636337 | 0.00604026370901761 |
| 0.970646662985338 | 6 | 84.5073102285536 | 38.8607827166408 | 0.00520976393026460 |
| 1.32641460677607 | 5 | 88.0042235575298 | 74.9072793815861 | 0.00250735531523539 |
| 0.826108164234755 | 8 | 99.5187219256502 | 45.6170585635518 | 0.00653447452614668 |

**Table B4**: 4th simulation PID data with 20 times execution using PSO

| RUN | ITSE | Kp | Kd | Ki |
|---|---|---|---|---|
| 1.30402916704629 | 3 | 90.9895922582920 | 58.0275704183840 | 0.00581239256755058 |
| 1.04026161229894 | 4 | 84.8946701796894 | 35.5664865636337 | 0.00604026370901761 |
| 0.970646662985338 | 6 | 84.5073102285536 | 38.8607827166408 | 0.00520976393026460 |
| 1.32641460677607 | 5 | 88.0042235575298 | 74.9072793815861 | 0.00250735531523539 |
| 0.826108164234755 | 8 | 99.5187219256502 | 45.6170585635518 | 0.00653447452614668 |
| 1.07720325100888 | 5 | 96.7067682728587 | 33.1965083837651 | 0.00532423013200547 |
| 1.10811225831367 | 20 | 87.9155250451676 | 32.8791959416614 | 0.00469187667996306 |
| 1.65429340311114 | 17 | 79.8046039211713 | 77.5815002122993 | 0.00165558118300414 |
| 1.20846225744043 | 14 | 99.0070221120794 | 60.3683207308325 | 4.14628036949649e-05 |
| 1.12395371872820 | 5 | 72.9471796973848 | 34.3078836103585 | 0.00471018252235055 |
| 1.09201299529800 | 18 | 99.5538538888431 | 32.4239412704775 | 0.00620049521203031 |
| 1.27421610307189 | 6 | 93.0339625173405 | 70.0458014281184 | 0.00289147339288430 |
| 0.984998254148300 | 12 | 92.3939436979279 | 37.4377316925528 | 0.00890649389365567 |
| 1.30228667660919 | 20 | 91.9794386606826 | 65.4101864865892 | 0.00650939929186731 |
| 0.910371590250838 | 15 | 82.7797923417378 | 42.0019373684164 | 0.00548655591447403 |
| 1.56217536407998 | 2 | 86.3200030289753 | 97.5211963586364 | 0.00419248298809311 |
| 1.00365542123090 | 6 | 84.3321367500631 | 45.5271908328320 | 0.00303935941960058 |
| 1.16199411688386 | 11 | 84.5718968752979 | 31.3129492135257 | 0.00927684879038271 |
| 1.01521052497679 | 10 | 91.0337201696738 | 36.0012137408718 | 0.00772051665992024 |
| 1.10626281320571 | 19 | 87.9198020265814 | 32.9479080362526 | 0.00610258861897323 |

**Table B5**: 5th simulation PID data with 20 times execution using PSO

| RUN | ITSE | Kp | Kd | Ki |
|---|---|---|---|---|
| 1.15647335482744 | 5 | 52.8211424704260 | 30.5692769990855 | 0.00739280254762875 |
| 0.906044539684614 | 17 | 91.2117657314611 | 41.6459473524070 | 0.00807085189423006 |
| 1.30967165138232 | 6 | 89.9203226479877 | 70.3167083832757 | 0.00118644271606670 |
| 0.997381366668315 | 4 | 90.4698837087829 | 37.0727447823453 | 0.00332590772150525 |
| 1.06562619855524 | 9 | 90.3550000738955 | 34.2821481001107 | 0.00860961047764190 |
| 1.24780319649858 | 20 | 86.9682122114768 | 28.4508386312122 | 0.00543879301907192 |
| 0.823310595627441 | 19 | 97.5864941272797 | 46.0038955714545 | 0.00230658286428795 |
| 1.01119192594198 | 7 | 77.3890818820416 | 40.5875296908503 | 0.00169975617447305 |
| 1.06535628692059 | 13 | 64.3829289284005 | 35.5222181484638 | 0.00431216283186782 |
| 1.04396237096534 | 10 | 73.7360003137001 | 36.9465082205785 | 0.000464948106723218 |
| 0.869513813284712 | 19 | 96.6669626096769 | 43.1641925278091 | 0.00316551619078046 |
| 0.937822598352616 | 8 | 89.8430534366899 | 39.9465401239338 | 0.000448706676361251 |
| 1.43884818776678 | 15 | 74.1508793506826 | 24.8519447759337 | 0.00314130421565196 |
| 1.69897839847459 | 13 | 79.9568175683155 | 89.9448986707184 | 0.00447909799279623 |
| 1.21045113605426 | 7 | 63.9317637971839 | 32.3312188236256 | 0.000200173514303332 |
| 0.928299178443933 | 9 | 92.0250120869436 | 40.2093350779071 | 0.00331769141440340 |
| 1.20320437093854 | 13 | 95.6155259063233 | 89.1131916756042 | 0.00850977110359181 |
| 1.07815187050563 | 3 | 92.7550908072563 | 33.5232257897426 | 0.00593616421522111 |
| 1.12102901023037 | 16 | 63.9201332946748 | 34.5827026504082 | 0.00341161890654572 |
| 1.15214607581600 | 15 | 97.0609226812872 | 30.5609709089961 | 0.00709003755319626 |

**Table B6**: 6th simulation PID data with 20 times execution using PSO

| RUN | ITSE | Kp | Kd | Ki |
|---|---|---|---|---|
| 1.20812153063093 | 1 | 78.8378056543791 | 30.4329708740363 | 0.00107395906801673 |
| 1.19384395604742 | 7 | 84.0241885483054 | 30.3252594209591 | 0.00305797571073910 |
| 1.24174283922803 | 1 | 90.2474317875537 | 49.9805716950041 | 0.000702155656388018 |
| 1.04721692798378 | 15 | 98.3201716881450 | 34.2279072216798 | 0.00147776644230518 |
| 1.13210085411936 | 8 | 84.0048375849638 | 32.4317554652739 | 0.00241057721476690 |
| 1.30727332286695 | 14 | 90.2306029370276 | 76.3949765615099 | 0.00771072589331176 |
| 1.25995782940471 | 16 | 92.8857013509894 | 54.6013121481733 | 0.00269604158937481 |
| 0.841403101101359 | 14 | 93.6481605404138 | 45.1932544987495 | 0.00393909901183189 |
| 1.13277490690168 | 1 | 85.4667121557888 | 32.2421065681174 | 0.00885593595116055 |
| 0.821371926633897 | 9 | 96.5046497874918 | 46.2783376299243 | 0.000906846869228576 |
| 1.23155040402889 | 12 | 96.6090759086536 | 71.4990405735479 | 0.00963165880993126 |
| 1.10295041488477 | 10 | 93.3382099001871 | 47.7140202708439 | 0.00514839859946411 |
| 1.23895436006614 | 11 | 94.7175485283580 | 77.4981845736920 | 0.00556321566147681 |
| 1.05125583145369 | 15 | 88.1478140360003 | 35.1765670003233 | 0.00611605810696489 |
| 0.883188552657564 | 10 | 90.0018512892489 | 43.0320859818843 | 0.00615461362781956 |
| 1.24871522545423 | 2 | 95.3588725282839 | 70.4513024105141 | 0.00608156047046982 |
| 1.18250170106892 | 18 | 50.5039692235510 | 29.3476269913170 | 0.00198827509876005 |
| 1.21610079890613 | 12 | 97.5654112196514 | 78.8533740678686 | 0.00661915805895335 |
| 0.951428417562421 | 11 | 94.1474050846086 | 38.8617520318246 | 0.00301930541981647 |
| 1.31150035143023 | 12 | 93.0306075043110 | 26.3967307898329 | 0.00916220725479049 |

**Table B7**: 7th simulation PID data with 20 times execution using PSO

| RUN | ITSE | Kp | Kd | Ki |
|---|---|---|---|---|
| 1.08469704194832 | 5 | 95.7135532316410 | 48.5344232796305 | 0.00800280468888800 |
| 0.915332177062236 | 10 | 98.2803785467415 | 40.2215401149021 | 0.00620671947199578 |
| 1.20189725021255 | 7 | 96.2399988440098 | 53.5035689329621 | 0.00963870129971715 |
| 0.933912014628605 | 15 | 84.6335053710108 | 40.8032078475375 | 0.00462018163773892 |
| 1.07398437424773 | 17 | 97.6600180815677 | 49.2823765088584 | 0.00400882866391995 |
| 1.28709891850712 | 15 | 93.3028445838600 | 60.2840145045761 | 0.00377492173215029 |
| 1.24143193575552 | 7 | 71.9634196252383 | 30.2720909803246 | 0.00459022394078519 |
| 1.16761833988686 | 11 | 75.7091720499107 | 40.5166107614247 | 0.00701901080171245 |
| 1.21122358231993 | 4 | 82.0797777917659 | 29.9744044177577 | 0.00787678917413547 |
| 1.13846007091383 | 2 | 94.6492027734031 | 49.2401555341857 | 0.00748213626946423 |
| 1.18621693203822 | 19 | 97.9418665339702 | 54.2737495350093 | 0.00912387045873848 |
| 0.976944502020496 | 13 | 82.8227714589642 | 42.4438570653092 | 0.00727096156947871 |
| 1.22029584825117 | 17 | 93.8165229952245 | 89.8685021667879 | 0.00694859162934643 |
| 1.02161702018019 | 18 | 81.6371614467867 | 36.8223331504681 | 0.00426215425372259 |
| 0.924417161869255 | 19 | 83.8977751393686 | 41.3120650889348 | 0.00402175572445444 |
| 1.04236570888722 | 17 | 97.6665808333917 | 34.4823793786986 | 0.00277363250387578 |
| 1.00088613638320 | 4 | 91.4863270983913 | 45.6575905786543 | 0.00904682764693900 |
| 1.19612660487719 | 20 | 73.5222023154627 | 31.5513806358532 | 0.00623999736711295 |
| 1.07583404658366 | 17 | 64.8298403281148 | 35.5406549619715 | 0.00850430024041377 |
| 1.30631003210400 | 12 | 84.2635291068734 | 47.4580273630282 | 0.00231697349362769 |

**Table B8**: 8th simulation PID data with 20 times execution using PSO

| RUN | ITSE | Kp | Kd | Ki |
|---|---|---|---|---|
| 1.08469704194832 | 5 | 95.7135532316410 | 48.5344232796305 | 0.00800280468888800 |
| 0.915332177062236 | 10 | 98.2803785467415 | 40.2215401149021 | 0.00620671947199578 |
| 1.20189725021255 | 7 | 96.2399988440098 | 53.5035689329621 | 0.00963870129971715 |
| 0.933912014628605 | 15 | 84.6335053710108 | 40.8032078475375 | 0.00462018163773892 |
| 1.07398437424773 | 17 | 97.6600180815677 | 49.2823765088584 | 0.00400882866391995 |
| 1.28709891850712 | 15 | 93.3028445838600 | 60.2840145045761 | 0.00377492173215029 |
| 1.24143193575552 | 7 | 71.9634196252383 | 30.2720909803246 | 0.00459022394078519 |
| 1.16761833988686 | 11 | 75.7091720499107 | 40.5166107614247 | 0.00701901080171245 |
| 1.21122358231993 | 4 | 82.0797777917659 | 29.9744044177577 | 0.00787678917413547 |
| 1.13846007091383 | 2 | 94.6492027734031 | 49.2401555341857 | 0.00748213626946423 |
| 1.18621693203822 | 19 | 97.9418665339702 | 54.2737495350093 | 0.00912387045873848 |
| 0.976944502020496 | 13 | 82.8227714589642 | 42.4438570653092 | 0.00727096156947871 |
| 1.22029584825117 | 17 | 93.8165229952245 | 89.8685021667879 | 0.00694859162934643 |
| 1.02161702018019 | 18 | 81.6371614467867 | 36.8223331504681 | 0.00426215425372259 |
| 1.07583404658366 | 17 | 64.8298403281148 | 35.5406549619715 | 0.00850430024041377 |
| 1.30631003210400 | 12 | 84.2635291068734 | 47.4580273630282 | 0.00231697349362769 |
| 0.830067194733704 | 4 | 96.7089955616805 | 45.6431052401802 | 0.00248028735394078 |
| 1.24112452290046 | 18 | 94.9166549892944 | 55.8129496006456 | 0.00493938082298926 |
| 1.32757885996058 | 14 | 79.6237985892597 | 26.9544370110098 | 0.00947640967815361 |
| 1.01691892068057 | 2 | 80.7956742006842 | 37.1110907913607 | 0.00644795140349993 |

**Table B9**: 9th simulation PID data with 20 times execution using PSO

| RUN | ITSE | Kp | Kd | Ki |
|---|---|---|---|---|
| 1.17454763392508 | 3 | 97.8339997178853 | 90.4369997699204 | 0.000506720447047913 |
| 1.07474344941367 | 20 | 77.4712998857865 | 35.2163270018136 | 0.00512223371546208 |
| 1.23116508792791 | 1 | 95.9761365552909 | 73.2211716723420 | 0.00890659928728320 |
| 0.444757325067024 | 8 | 91.5044145562169 | 48.9905266202631 | 0.00333150412873305 |
| 1.21540304023698 | 3 | 88.0966493222462 | 29.2925228739747 | 0.00687498148462686 |
| 0.848654734436945 | 2 | 92.1423182004146 | 44.8657482885443 | 0.00633298051262497 |
| 1.24651318366238 | 18 | 85.1271718368989 | 46.2528736384405 | 0.00997156944139304 |
| 1.03861258200384 | 8 | 85.6046767175743 | 35.5595314722047 | 0.00858591106738562 |
| 1.13596466547372 | 7 | 66.4955137801371 | 34.4451420432576 | 0.00747392219930520 |
| 1.14569660069054 | 12 | 93.8688144592610 | 31.0075171374584 | 0.00647692942398235 |
| 0.947060048102284 | 9 | 84.9391131039101 | 40.1363356399627 | 0.000540217297625327 |
| 1.16065324289709 | 18 | 81.0063345396748 | 31.7704564805953 | 0.00722769819557812 |
| 1.24858274733247 | 18 | 89.0906249220551 | 28.2652356756046 | 0.00312765168675816 |
| 1.36540314602014 | 19 | 84.6521002876562 | 50.2333107869623 | 0.00229017973439221 |
| 1.03616234386614 | 15 | 88.7511831587790 | 35.3346685381884 | 0.00537995038758771 |
| 1.08568929426208 | 9 | 94.2661847536928 | 33.0896680438102 | 0.00383982170293399 |
| 1.10190706227826 | 6 | 85.1538029829808 | 33.4391903509099 | 0.000832733290864999 |
| 1.41465879270878 | 12 | 91.7292371624791 | 85.3263095903909 | 0.00357131779377951 |
| 0.979877441972247 | 8 | 98.8466060884898 | 36.8410550754731 | 0.00524902566263428 |
| 1.21303596384876 | 13 | 59.1021843611335 | 32.0705647246493 | 0.00626913840883664 |

**Table B10**: 10th simulation PID data with 20 times execution using PSO

| RUN | ITSE | Kp | Kd | Ki |
|---|---|---|---|---|
| 1.16863560557504 | 20 | 97.8472981294412 | 91.6219973540328 | 0.00214962550281133 |
| 1.01338338188516 | 12 | 97.2249049737748 | 35.7900566414077 | 0.00529803261456162 |
| 1.26638464216479 | 4 | 94.9821521487403 | 66.3944754912512 | 0.000244887486440572 |
| 1.37091235083280 | 18 | 77.8227695791411 | 44.0080646743295 | 0.00108656643039646 |
| 1.05199926089631 | 3 | 78.8176959601822 | 35.8388189988566 | 0.000654116562343879 |
| 1.27176182230189 | 14 | 89.5750991098924 | 88.4605205899003 | 0.00767812253758924 |
| 1.20119340518868 | 20 | 98.7772330188790 | 73.4502765263386 | 0.00393813240523759 |
| 0.882490918141863 | 20 | 92.6845916227868 | 42.8197382345942 | 0.00179239793681866 |
| 1.20350238526057 | 20 | 94.9097091190128 | 29.1361095065801 | 0.00792624197259553 |
| 1.12072785341025 | 18 | 84.6072337127116 | 32.7792461567966 | 0.00101186218440120 |
| 1.11943286565719 | 19 | 97.9991295672341 | 50.7743384674421 | 0.00167289880425806 |
| 1.25503931046333 | 18 | 90.9563732704761 | 89.1888730503773 | 0.00707361073646754 |
| 1.00547799944289 | 14 | 90.4632020123964 | 36.4924030776797 | 0.00926528994863122 |
| 1.05697282284750 | 19 | 94.6191407417707 | 47.4948301717518 | 0.00530595846528246 |
| 1.57232751116488 | 1 | 69.7223619971650 | 41.4548827549362 | 0.00819469946630990 |
| 1.09310539202358 | 12 | 91.3718043498100 | 33.0847574038206 | 0.00882655125338894 |
| 1.25369074457501 | 14 | 95.3027499261002 | 63.6771825034230 | 0.00879149229341179 |
| 1.21281971957729 | 13 | 69.3206133924253 | 31.6078240256175 | 0.00864601581052142 |
| 0.930715070100440 | 13 | 94.5892316769698 | 39.7931871923518 | 0.00358528040099439 |
| 1.27233472929652 | 7 | 91.8361995105866 | 75.8586080738727 | 0.00238554685356440 |

# APPENDIX C

**Table C1 : Gantt Chart**

| | FYP 1 | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | W1 | W2 | W3 | W4 | W5 | W6 | W7 | W8 | W9 | W10 | W11 | W12 | W13 | W14 |
| **TITLE SELECTION** | | | | | | | | | | | | | | |
| **FYP BRIEFING** | | | | | | | | | | | | | | |
| **LITERATURE REVIEW** | | | | | | | | | | | | | | |
| **HARDWARE (DATA COLLECTION)** | | | | | | | | | | | | | | |
| **SOFTWARE (SIMULATION)** | | | | | | | | | | | | | | |
| **REPORT PROGRESS (SUBMIT)** | | | | | | | | | | | | | | |
| **SEMINAR 1 (PRESENTATION)** | | | | | | | | | | | | | | |
| | FYP 2 | | | | | | | | | | | | | |
| **HARDWARE (DATA COLLECTION)** | | | | | | | | | | | | | | |
| **DESIGN CONTROLLER** | | | | | | | | | | | | | | |
| **FULL REPORT** | | | | | | | | | | | | | | |
| **SEMINAR II (FINAL PRESENTATION)** | | | | | | | | | | | | | | |

# FYP REPORT

*by* Fiona Serina Daud



| | | | |
|---|---|---|---|
| FILE | 10-FULLREPORT.DOCX (3.5M) | | |
| TIME SUBMITTED | 26-MAY-2014 03:30AM | WORD COUNT | 9953 |
| SUBMISSION ID | 430199598 | CHARACTER COUNT | 54118 |

# CHAPTER 1

# INTRODUCTION

## 1.1 Background of Study

Coupled tank system plays important role in industrial application such as in petro-chemical industries, paper making industries, medical industries and water treatment industries. The coupled tank system consists of two vertical tanks which are joined together with an orifice and has an inlet liquid pumps and outlet valves. Each tank has its own inlet pump and output valve where the valve will regulate continuously. The coupled tank system can be configured as a single-input single-output (SISO) or as a multiple-input multiple-output (MIMO) system via manipulation of pumps input and sectional area of rotary valves. In industries, the liquid in the tank will go through several processes or mixing treatment whereby the level of liquid needs to be controlled and maintained. The flow between tanks must be continuously regulated. The basic control principle of the coupled tank system is to maintain the level of liquid in the tank when there is a process of inflow of liquid into the tank and output of liquid out of the tank.

## 1.2 Problem Statement

There are differences between real-time control and simulation control. Real-time is related to the surrounding where all of the parameters and condition need to be considered whereas simulation is an imitation of the operation of real process. All of the parameters can be controlled by human. To study the performance of the real-time implementation in a system, the coupled-tank system is chosen. In industries, there are problems faced related to the level of liquid in tank and the flow between the tanks. Most of the time, liquid level in the tanks needs to be controlled while regulating the flow in the tank. Common controller use to control the system is PID controller and Fuzzy Logic Control (FLC) controller. However in several cases, the conventional PID controller is not suitable for a controlled object with variable parameter or when there is existence of external disturbances in the system. One of the solutions to achieve high performance of the system is to apply state feedback controller to the system. There are several controllers that have been applied to control the system of the coupled tank. In this project, Linear Quadratic Regulator (LQR) controller will be selected as the state feedback controller for the system. The LQR controller approach will be tested to verify the improvement related to the performance of the system.

## 1.3 Objectives

The aim of this project is to obtain the modeling and controller design for coupled tank system. The main objectives of this project are:

1) To obtain transfer function of the coupled tank using system identification method
2) To implement LQR controller on coupled tank system and obtain its parameter using particle swarm optimization (PSO) technique
3) To compare the system performance of LQR and PID controller

## 1.4 Project Scopes

There are limitations to the completion of this project. The scopes of the project only cover:

1) Interface the coupled tank CTS-001 with DAQ card and use personal computer to stimulate data using software MATLAB
2) The experimental of coupled tank process is using single-input single-output (SISO) control system
3) System Identification used to generate the transfer function of the coupled tank
4) The parameter of LQR controller and PID controller will be determined using PSO optimization technique.

## 1.5 Project outlines

This report basically divided into five chapters:

### CHAPTER 1 Introduction

This chapter allows the readers to visualize the basic aspects of the research done, such as the overview of the coupled tank system, problem statements, objectives and scopes of the project.

### CHAPTER 2 Literature Review

This chapter reviews on the basic modeling for a coupled tank, previous controller used for the coupled tank system, several types of optimization techniques use for the system and other reviews that related to this project.



### CHAPTER 3 Design Methodology

This chapter consists of the flow related to the study and methodology used for this project. The control principle for coupled tank system, modeling of the coupled tank and the design technique for PSO-based LQR controller will be explained in this chapter. The implementation of system identification to obtain the transfer function will also be included in this chapter.

2

## CHAPTER 4 Result and Discussion

This chapter shows the characteristic of the system, the fitness function of each controller and the result of the transient response performance using the LQR and PID controller to the system.

## CHAPTER 5 Conclusion & Future Works

This chapter consists of the conclusion based on the overall works and results. It also includes some future improvement that can be done in the system.

# CHAPTER 2

# LITERATURE REVIEW

## 2.1 Overview

This chapter will review on research which focuses on mathematical model, LQR controller, previous controller used for coupled tank, system identification and optimization method used for the system. Coupled tank system is common in process industries as it need to be pump from one tank to another. Somehow, the liquid needs to be monitored and the flow and level of liquid need to be controlled although the tank need to be continuously regulated. There are many types of controller use to control the coupled tank system such as the LQR, PID, Fuzzy Logic and Slide Control. The controllers have similar objective which are to improve the system performance. System identification is a tool that produces a transfer function from data collected in the coupled tank. From the transfer function obtained using the system identification, the optimization methods is applied to find the best value for Q and R matrix in easier way compare using the trial and error method.

## 2.2 Control Principle of Coupled Tank System

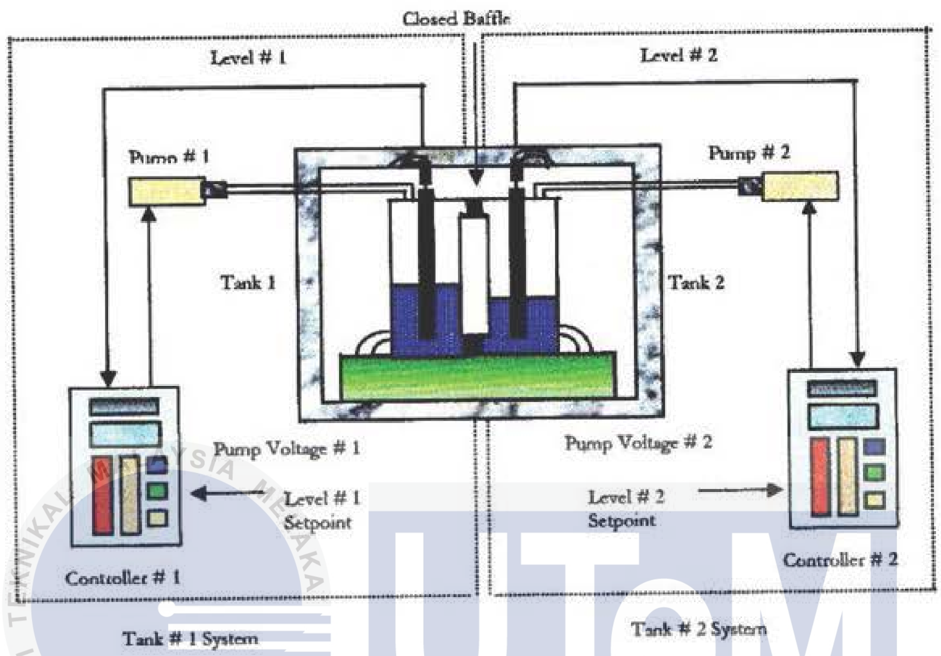


Figure 2.1: Schematic diagram for CTS-001 coupled tank [1]

Figure 2.1 shows the schematic diagram for CTS-001 coupled tank. The basic control principle of the coupled tank system is to maintain the water level in both tanks at a desired point value while the inflow and outflow of water tank keep regulating. Disturbance may be caused by variation in the rate flow from the baffle gap or the changes in the outlet of the tank. If there is disturbances occur, the water level in the tank will change and settle at a different steady-state level [1].

The control variable involve in the process control of this system is the water level in the coupled tank system. In order to control and maintain the water level at a desired point, the inlet flow rate needs to be adjusted. The pump voltage will become the device to adjust the inlet flow rate. The manipulated variable for this system is the input flow rate because it is the variable needs to maintain the process variable at the desired point.

The input flow rate needs to be adjusted in order to maintain the water level of the tank. For example, if the outflow rate is greater compared to the inflow rate, the inflow rate should be adjusted in order for the water level in the tank to increased and settled. For this project, SISO system will be used where the pump 1 will be used as the inlet and outlet 2 is chosen as the outlet for the system. Table 2.1 shows the condition of the system in a steady state manner for SISO.

Table 2.1: SISO steady state condition for coupled tank system

<table>
<tr><th colspan="2">SYSTEM TYPE</th><th>STEADY-STATE CONDITION</th></tr>
<tr><td rowspan="2">1<sup>st</sup> order</td><td>Tank 1</td><td rowspan="2">Inflow rate from the inlet equals to the outlet rate at the outlet</td></tr>
<tr><td>Tank 2</td></tr>
<tr><td rowspan="2">2<sup>nd</sup> order</td><td>Tank 1</td><td>Inflow rate at the inlet equals to the total value from the outflow of rate at the outlet and the outflow rate at the baffle gap</td></tr>
<tr><td>Tank 2</td><td>Total from the Inflow rate at the inlet with the inflow rate at the baffle gap equals to the outflow rate at the outlet</td></tr>
</table>

## 2.3 Mathematical Model

The description of a system or process can be illustrated using mathematical model. Model is a combination between mathematics and logical relationship that represents aspects of any situation in system. Mathematical model use the mathematical concepts and it describe the important relationships between variables. Using mathematical model approaches creates much simpler process than the real situation. The elements that are unimportant or irrelevant to the problem will be ignored in order to simplify the equation needed. Below is the dynamic model for the coupled tank control apparatus refer to [1]:

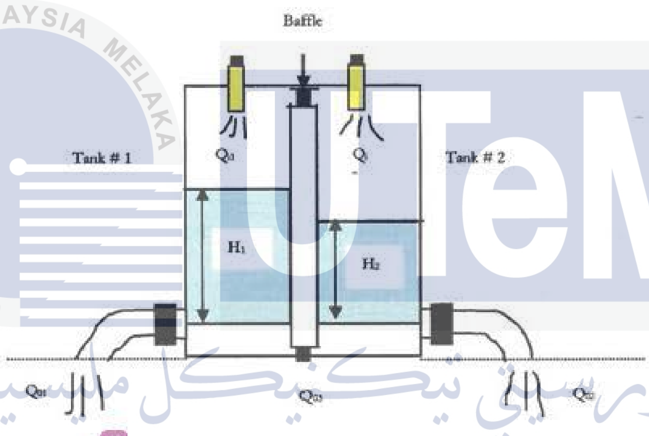


Figure 2.2: Diagram of the coupled tank control apparatus [1]

Considering a simple mass balance, the rate of change of fluid volume in each tank equals the net flow of fluid into the tank. Thus for each of Tank 1 and Tank 2 have:

$$A_1 \frac{dH_1}{dt} = Q_{i1} - Q_{01} - Q_3 \quad (1)$$

$$A_2 \frac{dH_2}{dt} = Q_{i2} - Q_{02} - Q_3 \quad (2)$$

Where $H_1, H_2$ = height of fluid in tank 1 and 2 respectively

$A_1, A_2$ = cross-sectional area of tank 1 and 2 respectively

$Q_1$ = flow rate of fluid between tanks

$Q_{i1}, Q_{i2}$ = pump flow rate into tank 1 and 2 respectively

$Q_{01}, Q_{02}$ = flow rate of fluid out of tank 1 and 2 respectively

Bernoulli's equation for steady, non-viscous, incompressible shows that the outlet flow in each tank is proportional to the square root of the head of water in the tank. The flow between the two tanks is proportional to the square root of the head differential. Thus

$$Q_{01} = \alpha_1 \sqrt{H_1} \quad (3)$$

$$Q_{02} = \alpha_2 \sqrt{H_2} \quad (4)$$

$$Q_3 = \alpha_3 \sqrt{H_1 - H_2} \quad (5)$$

where $\alpha_1, \alpha_2, \alpha_3$ are proportionally constants which depend on the coefficients of discharge, the cross sectional area of each orifice and the gravitational constant. Combining equation (1) to (5), we have a set of non-linear state equations which describe the system dynamics of the coupled tank apparatus.



$$A_1 \frac{dH_1}{dt} = Q_{i1} - \alpha_1 \sqrt{H_1} - \alpha_3 \sqrt{H_1 - H_2} \quad (6)$$

$$A_2 \frac{dH_2}{dt} = Q_{i2} - \alpha_2 \sqrt{H_2} - \alpha_3 \sqrt{H_1 - H_2} \quad (7)$$

Suppose that for a set of inflows $Q_{i1}$ and $Q_{i2}$, the fluid level in the tanks is at some steady state levels $H_1$ and $H_2$. Consider small variations in each inflow, $q_1$ in $Q_{i1}$ and $q_2$ in $Q_{i2}$. Let the resulting perturbation in levels be $h_1$ and $h_2$ respectively. From equations (6) and (7), now have

$$A_1\frac{d(H_1+h_1)}{dt}=(Q_{i1}+q_1)-\alpha_1\sqrt{(H_1+h_1)}-\alpha_3\sqrt{H_1-H_2+h_1-h_2} \quad (8)$$

$$A_2\frac{d(H_2+h_2)}{dt}=(Q_{i2}+q_2)-\alpha_2\sqrt{(H_2+h_2)}-\alpha_3\sqrt{H_1-H_2+h_1-h_2} \quad (9)$$

Subtracting equation (6) and (7) from (8) and (9), we obtain

$$A_1\frac{dh_1}{dt}=q_1-\alpha_1(\sqrt{H_1+h_1}-\sqrt{H_1})-\alpha_3(\sqrt{(H_1-H_2+h_1-h_2})-\sqrt{H_1-H_2}) \quad (10)$$

$$A_2\frac{dh_2}{dt}=q_2-\alpha_2(\sqrt{H_2+h_2}-\sqrt{H_2})+\alpha_3(\sqrt{(H_1-H_2+h_1-h_2})-\sqrt{H_1-H_2}) \quad (11)$$

For small perturbations

$$\sqrt{H_1+h_1}=\sqrt{H_1}(1+\frac{h_1}{H_1})^{\frac{1}{2}}\approx\sqrt{H_1}(1+\frac{h_1}{2H_1}) \quad (12)$$

Therefore,

$$\sqrt{H_1+h_1}-\sqrt{H_1}\approx\frac{h_1}{2H_1}$$

Similarly

$$\sqrt{H_2 + h_2} - \sqrt{H_2} \approx \frac{h_2}{2H_2}$$

And

$$(\sqrt{H_2 - H_1 + h_2 - h_1} - \sqrt{H_2 - H_1}) \approx \frac{h_2 - h_1}{2\sqrt{H_2 - H_1}}$$

With these approximations, equation (10) and (11) simplify to

$$A_1 \frac{dh_1}{dt} = q_1 - \frac{\alpha_1}{2\sqrt{H_1}} h_1 - \frac{\alpha_3}{2\sqrt{H_1 - H_2}}(h_1 - h_2) \qquad (13)$$

$$A_2 \frac{dh_2}{dt} = q_2 - \frac{\alpha_2}{2\sqrt{H_2}} h_2 - \frac{\alpha_3}{2\sqrt{H_1 - H_2}}(h_1 - h_2) \qquad (14)$$

In equations (13) and (14), note that the coefficients of the perturbations in level are functions of the steady state operation points $H_1$ and $H_2$. Note that the two equations can also be written in the form



$$A_1 \frac{dh_1}{dt} = q_1 - q_{01} - \frac{\alpha_3}{2\sqrt{H_1 - H_2}}(h_1 - h_2) \qquad (15)$$

$$A_2 \frac{dh_2}{dt} = q_2 - q_{02} - \frac{\alpha_3}{2\sqrt{H_1 - H_2}}(h_1 - h_2) \qquad (16)$$

where $q_{01}$ and $q_{02}$ represent perturbations in the outflow at the drain pipes. This would be appropriate in the case where outflow is controlled by attaching an external clamp for instance.

### 2.3.1 First-Order Single-Input Single-Output Plant

Consider the configuration where the baffle plate is completely depresses so that there is no flow between the two tanks. Equation (13) and (14) become

$$A_1 \frac{dh_1}{dt} = q_1 - \frac{\alpha_1}{2\sqrt{H_1}} h_1 \tag{17}$$

$$A_2 \frac{dh_2}{dt} = q_2 - \frac{\alpha_2}{2\sqrt{H_2}} h_2 \tag{18}$$

Effectively we have two independent first-order systems. Each first order differential equation relates the perturbation in level to the perturbation in inflow. The time constant of the first order model for tank 2 is

$$\text{Time constant, } \tau_2 = \frac{2A_2\sqrt{H_2}}{\alpha_1} \tag{19}$$

This shows the dependence on operation point of the plant dynamics. Furthermore it also shows that if the discharge proportionally constant is reduced, the time constant increased. The steady state gain of the model is also dependent on the operation point and is given as follows:



$$\text{Steady state gain} = \frac{2\sqrt{H_2}}{\alpha_2} \tag{20}$$

The perturbation in inflow is related in a dynamic fashion to the perturbation in voltage applied to the pump. The transient response for voltage to inflow is generally faster than the time constant of the tank dynamics and may be represented by a small dead time element. If the steady state flow rate is low and discontinuous, there may be further delay due to transport lag in the inflow tubing.

### 2.3.2 Second-Order Single Input Single Output Plant

For the second configuration, the baffle is raised slightly. The manipulated variable is the perturbation to tank 1 inflow. Taking the Laplace Transforms of equations (13) and (14) and assuming that initially all variables are at their steady state values,

$$A_1 s h_1(s) = q_1(s) - \left( \frac{\alpha_1}{2\sqrt{H_1}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}} \right) h_1(s) + \frac{\alpha_3}{2\sqrt{H_1 - H_2}} h_2(s) \tag{21}$$

$$A_2 s h_2(s) = q_2(s) - \left( \frac{\alpha_2}{2\sqrt{H_2}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}} \right) h_2(s) + \frac{\alpha_3}{2\sqrt{H_1 - H_2}} h_1(s) \tag{22}$$

This may be written as

$$(T_1 s + 1) h_1(s) = k_1 q_1(s) + k_{12} h_2(s) \tag{23}$$

$$(T_2 s + 1) h_2(s) = k_2 q_2(s) + k_{21} h_1(s) \tag{24}$$

where

$$T_1 = \frac{A_1}{\frac{\alpha_1}{2\sqrt{H_1}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}}} \qquad T_2 = \frac{A_2}{\frac{\alpha_2}{2\sqrt{H_2}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}}}$$

$$k_1 = \frac{1}{\frac{\alpha_1}{2\sqrt{H_1}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}}} \qquad k_2 = \frac{1}{\frac{\alpha_2}{2\sqrt{H_2}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}}}$$

$$k_{12} = \frac{\frac{\alpha_3}{2\sqrt{H_1 - H_2}}}{\frac{\alpha_1}{2\sqrt{H_1}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}}} \qquad k_{21} = \frac{\frac{\alpha_3}{2\sqrt{H_1 - H_2}}}{\frac{\alpha_2}{2\sqrt{H_2}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}}}$$

For second order configuration, $h_2$ is the process variable and $q_1$ is the manipulated variable. For simplicity, we first consider the case when $q_2$ is zero.

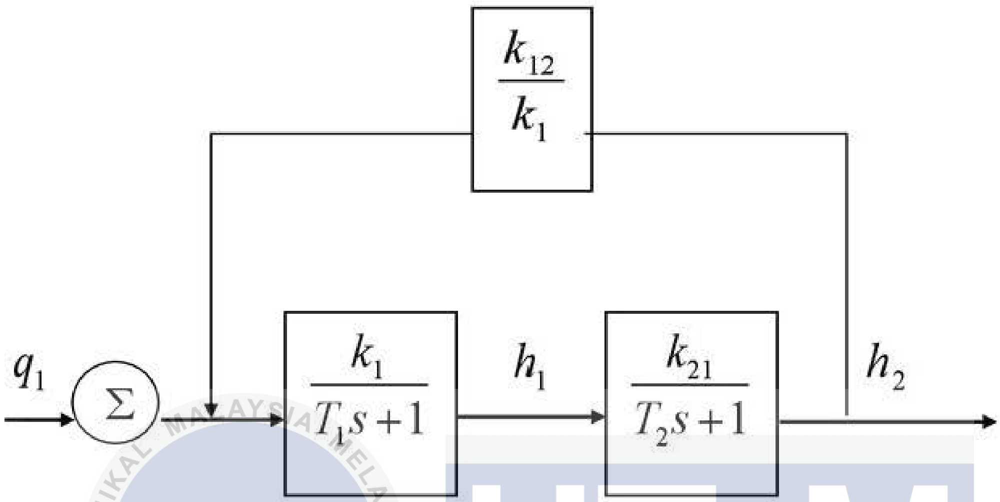


Figure 2.3 : Block diagram of 2nd order process

For equation (23) and (24) and Figure 2.3, the expressions for the gain, damping factor and natural frequency of the system can be obtained. The characteristic equation shown in equation (25)



$$\text{Characteristic equation} = T_1 T_2 s^2 + (T_1 + T_2)s + 1 - k_{12}k_{21} \tag{25}$$

Which yields a natural frequency and damping factor of

$$\omega_n = \sqrt{\frac{1 - k_{12}k_{21}}{T_1 T_2}} \tag{26}$$

$$\xi = \frac{T_1 + T_2}{\sqrt{T_1 T_2 (1 - k_{12}k_{21})}} \tag{27}$$

## 2.4 System Identification

System identification can be defined as a process to improve or develop the mathematical representation of system using the experimental data collected. There are two types of method used for the system identification which are the frequency domain identification methods and time domain identification methods. System identification need to produce a mathematical model for any system using statistical method by collect data from measured input-output.

Classification of the system using system identification technique involved non-parametric and parametric methods. Non-parametric methods consist of transient response analysis, correlation analysis, frequency response analysis and spectral analysis. There are several techniques involved in parametric methods which are the least square method, pseudorandom binary sequence (PRBS) analysis, prediction error method, instrumental variable method and forgetting factor technique.

H.Gupta and O.P.Verna in [2] used Pseudo Random Binary Signal (PRBS) method where the input and output data of the second order transfer function is computed using System Identification toolbox in MATLAB. The transfer function obtained from the SI can represent the behavior of the system. H.M.Shariff in [3] proposed that Random Gaussian Signal (RGS) is better compared to PRSB in a nonlinear system. RGS is capable to exhibit the nonlinear behavior of the system where PRSB is insufficient to excite the system dynamic and fail to exhibit the nonlinearity behavior in nonlinear system process.

## 2.5 Linear Quadratic Regulator (LQR)

LQR is a state feedback controller that gives the best possible performance with respect to some given measure of performance. The performance measure is a quadratic function composed of state vector and control output [4]. LQR is one of the optimal control techniques, which takes into account the states of the dynamic system and control input to make the optimal control decision [5]. Optimal can be defined as to provide the smallest possible error to its input. Q and R matrix of LQR are usually selected by trial and error. The LQR is well-known design technique that provides practical feedback gains. It is also a tool use for complex system that need high robust in the performance requirement. In real time, LQR problem will be solved using the MATLAB. The data collected need to be run in the System Identification to obtain the transfer function of the system. The challenge undergoing this type of controller is on how to obtain the weight matrices from the system.

### 2.5.1 Performance of LQR Controller

LQR controller can improve the performance of the system depends on how the system works. Some general advantages can be obtained when using LQR controller is such as the precision, speed, recovery time, efficiency and safety. There are several systems that used LQR controller to improve the performance of the system.

Paper [6] found that from simulation, LQR controller give the best performance compared to fuzzy logic controller in pitch control system in an aircraft. It shows that LQR controller has the best performance in term of rising time, settling time and percent of steady state error. The result obtained show that LQR controller is good in eliminating the error from the system which is almost tend to be zero value. Fuzzy logic controller gives the best range in the percent of overshoot. From both performance, it shows that LQR controller provide higher ability in controlling the pitch angle compared to the fuzzy logic controller.

C.Chitu, J,Lackner, M.Horn, H.Waser and M.Kohlbock in [7] proposed that LQR controller is suitable in designing an electrical power steering system. By using LQR controller to the system, it offer better stability in frequency, robustness and closed loop stable step responses during parameters variation.

In 2011[8] compared the performance of LQR controller and fuzzy logic controller in an aircraft roll control System. Aircraft roll control System is based on designing an autopilot that can control the roll angle of an aircraft. From the observation, LQR gives faster response compared to fuzzy logic controller in terms of rise time. Although both controller follow the desired angle, LQR controller follow the reference input by producing very small steady-state error compare to fuzzy logic controller. It shows that LQR controller can handle the effect of disturbances in the system.

The implementation of LQR controller is also used in 4-leg voltage sources inverter. 4-leg inverter is utilized in three-phase four-wire power converter due to its superior performance characteristics such as relatively low DC bus voltage and switching loss, and capability to handle unbalanced load current [9]. A. Mohammadbagher and M.Yaghoobi in [9] stated that LQR controller and PID controller are suitable to utilize the 4-Leg inverter due to both can give zero steady-state error, fast response and no overshoots at the transient response. However, the result obtained proves that the LQR method is better than PID controller in terms of its faster response.



Based on[10], LQR using PSO optimization can gives better results in showing the choice of matrix Q and R for the controller. The best weight matrix Q and R of LQR controller has been very difficult to be determined without using any optimization technique to the controller. By applying PSO as the optimization technique for the system, it gives better results in finding the weight matrix for LQR controller and obtained the best performance despite from differences value of weight matrix.

From the analysis, LQR can be widely used in various industries regardless on the operation of the system. It gives many advantages to the system not only for linear but also non-linear system. Therefore, by using LQR as the controller for coupled tank system, it can improve the performance and allows a better output compared to uncompensated system.

## 2.6 Types of Controller

There are several types of controller used to control the coupled tank system in the industries. Generally, the objective of all the controllers in controlling the liquid level is to maintain the level set point at a specific value and be able to accept new set point values dynamically. The examples of controllers are Linear Quadratic Regulator (LQR), PID, Fuzzy Logic, Sliding Mode and Direct Model Reference Adaptive Control.

### 2.6.1 Proportional Integral Derivative Controller (PID)

The proportional-integral-derivative (PID) method is a kind of feedback controller which is generally based on the error between desired set point and measured process value. The error is then used to adjust some input to the process in order to define its set point. Three parameters (P,I and D) must be designed in the PID controller and each parameter has an effect on the error [4]. Both of the transient and steady-state responses are taken care of with these three parameter functionality. PID controller gives the most efficient solution to problem which related to control system. In [11] proposed PID controller for coupled tank-system because PID controller can achieve the desired output response with given specification. It gives high efficiency of control algorithm and gives a good performance with affordable costs. O.D.Kieran in [12]state that PID controller using tuning method Genetic Algorithm (GA) gives better performance compared to conventional PID. The system for PID controller with tuning method can be tune without approximated model, gives less overshoot and less settling time.

### 2.6.2 Fuzzy Logic Controller (FLC)

M.Abid in [13] used FLC as the controller for the coupled tank system. It is due to the fact that for any system, FLC is not based on mathematical model where it can be applied to a non-linear system such as delay, saturation and dead zones. FLC is a control system based on fuzzy logic mathematical system which are use to analyze the analog input values in terms of logical variables of continuous values taken between 1 and 0. Fuzzy logic is a form of logic that uses approximate method instead extract from the actual value. It emulates the ability to use approximate data to find solutions.

FLC consists of linguistic rules (i.e IF-THEN rule) that can be constructed using the knowledge of experts in the given field of interest [2]. Based on several researchers, FLC can be considered as flexile controller since it is easy to be controlled so that the performance of the system can be improved. In [13] proposed that FLC is better compared to PID controller. By research, FLC gives a better response because the settling time of the system is shorter and the system is more stable compared to PID controller. One of the advantage using the FLC is it improves the non-linearity of the system. L.Liang in [14] compared the Fuzzy-PID controller with the conventional PID and FLC. Fuzzy-PID controller improves the lumping lag and non-linear characteristic of the system. For FLC, it helps to improve the response speed whereas the PID controller will eliminate the steady-state error of the system.

### 2.6.3 Sliding Mode Controller (SMC)

Sliding mode control is a type of variable structure control where the dynamics of a nonlinear system is changed by switching discontinuously on time on a predetermined sliding surface with a high speed, non linear feedback [15]. The SMC usually use when it related to non-linear and time varying systems. SMC comes with simple procedure to design the controller for any linear or non-linear plants and can be considered as one of a controller use to produce a robust system. The main advantage of SMC is that, it is robust to plant uncertainties and insensitive to disturbances acting on the system [16].

In [15], S.Aydin and S. Tokat proposed that SMC give a robust behavior to the coupled-tank system even in the presence of noise and able to control non-linear system with different set point given by the time varying commanded input signals. H. Abbas, S. Asghar and S. Qamar in [16] had suggested that SMC is better compared to PID controller due to its performance to the system. Using SMC allow to improve the non-linearity of the system as well as to gives a good robustness to the system [17] proposed that using SMC guarantee the asymptotic stability of the close loop system. The system robustness is not guaranteed until the slide mode is reached.



### 2.6.4 Direct Model Reference Adaptive Control (DMRAC)

DMRAC can be classify as fixed controlled because it usually not capable to control a process which the system parameter vary and when there is existence of disturbance during the operation. DMRAC provides an attractive adaptive control approach where it only required the plant and reference model outputs and reference model states to be able for measurement. Other characteristic of DMRAC is that the order of the reference model can be made lower than the order of the plant to be controlled. In [18] proposed DMRAC as the controller of the couple-tank system using non-linear plant model. From the comparison of DMRAC with PID

(Ziegler-Nichors tuning method), it shows that DMRAC improve the non-linearity of the system and able to maintain the transient response with or without the presence of disturbance in the system.

Table 2.2: Comparison Of Coupled Tank Water Level Control Method

| Paper/Journal | Controller/Method | Performance |
|---|---|---|
| **Real-time Adaptive PID Control of a Non-linear Process based on Genetic Optimization (2002)** | PID controller using GA tuning method | • Gives better performance compared to conventional PID.<br>• Can be tune without approximated model<br>• Gives less overshoot and less settling time. |
| **Fuzzy logic control of coupled liquid tank system (2005)** | Fuzzy Logic controller (compare to PID Controller) | • The settling time of the system is shorter<br>• System is more stable |
| 14 **Simulation of Direct Model Reference Adaptive Control (DMRAC) on a Coupled-Tank system using Nonlinear Plant Model (2007)** | Direct Model Reference Adaptive Control Compare to PID (using Ziegler-Nichors tuning method) | • Improve the non-linearity of the system<br>• Able to maintain the transient response with or without the presence of disturbance in the system. |

| | | |
|---|---|---|
| **Sliding mode control of a coupled tank system with a state varying sliding surface parameter (2008)** | Sliding Mode Control controller | • Give a robust behavior to the system even in the presence of noise<br>• Able to control non-linear system with different set point given by the time varying commanded input signals |
| **The application of Fuzzy-PID controller in Coupled-tank Liquid Level Control System (2011)** | Fuzzy-PID controller (compare to conventional PID and FLC controller) | • Improves the lumping lag and non-linear characteristic of the system<br>• FLC only helps to improve the response speed whereas the PID controller will eliminate the steady-state error of the system |
| **A design of Fuzzy PID Controller based on ARM7TDMI for coupled-tanks process (2012)** | Fuzzy-PID controller | • Gives high efficiency of control algorithm<br>• Gives a good performance with affordable costs |
| **Sliding Mode Control for** | Sliding Mode Control | • Allow to improve the |

| | | |
|---|---|---|
| **coupled tank liquid level control (2012)** | controller (compare to PID Controller) | non-linearity of the system<br>• Gives a good robustness to the system. |
| **Sliding mode control of coupled tanks system: Theory and an application (2013)** | Sliding Mode Control controller | • SMC guarantee the asymptotic stability of the close loop system. The system robustness is not guaranteed until the slide mode is reached. |

The performance of each controller can be observed from Table 2.2. PID controller using an optimization technique gives better performance compared to the conventional PID. Fuzzy logic controller gives a stable system and improvement in transient response when being compared with PID controller. When there is disturbance in a non-linear system, Direct Model Reference Adaptive Control (DMRAC), Fuzzy-PID controller and Sliding Mode Control controller can improve the system and gives better performance to the transient response. It shows that each controller gives different improvement related to the stability and transient response of the coupled tank system.

## 2.7 Optimization Technique

Optimization theory deals with algorithm to approach proximity of the maximum probability and allow it to evaluate the function in a finite number of points. Below are several techniques of optimization to control a system.

Table 2.3: Types of optimization technique

| OPTIMIZATION METHOD | DESCRIPTION |
|---|---|
| FIREFLY ALGORITHM | Firefly algorithm is inspired by the behavior of the fireflies. It is a nature-inspired metaheuristic optimization algorithm. The application that use firefly algorithm is for digital image compression and image processing, rigid image registration problems and antenna design. |
| PARTICLE SWARM OPTIMIZATION (PSO) | PSO is a method of optimization technique that based on the movement and intelligence of swarm. It was developed in 1995 by James Kennedy and Russell Eberhart based on the social behaviors of bird flocking or fish schooling. In [16],J.S Cheng proved that although it provides a simple program, it also helps to improve the quality of the system and more accurate. |
| GENETIC ALGORITHM (GA) | GA is a method of solving an optimization problem related to both constrain and unconstrained problems based on a natural selection process that similar to biological evolution. There is limitation for GA in the complexity of the system. GA does not scale well with complexity and it is extremely difficult to use the technique on problems such as designing plane or house. The application of GA |

| | |
|---|---|
| | can be used in bioinformatics, pahrmacometrics, physics, manufacturing and economics. |

Optimization techniques are useful in finding the optimum solution or unconstrained maximum or minimum of continuous and differentiable functions. Each gain or weight matrices in any system need an exact value in order to gives the best performance that can be applied to the system. Therefore by using optimization techniques, it helps to find the best solution or unconstrained maximum or minimum of continuous and differentiable functions for the system.

## 2.8 Summary of review

Literature review has been presented in terms of different aspects. The first part that had been discussed is the coupled tank and modeling of the system. Previous research related to types of controller used for coupled tank system was discussed and the output performance using each controller is stated. The performance of this research (LQR) had been discussed to see its behavior and outstanding characteristics to different types of system. The comparison of the performance using LQR controller in different system shows that LQR controller is a controller that can be used in any system despite having non-linear system and presence of disturbance in the system.

# CHAPTER 3

# METHODOLOGY

## 3.1 Overview

This chapter will review on the method or approach that will be used to complete this project. There are several component need to be considered in order to achieve the objective of the overall project. The main components for this project are based on modeling the system and the design of the controller for the system. The controller that will be used for this project is LQR controller using the PSO optimization technique. The operation of the coupled tank was discussed in this chapter in order to have a clear understanding about the coupled tank.

### 3.2 Project Flow Chart

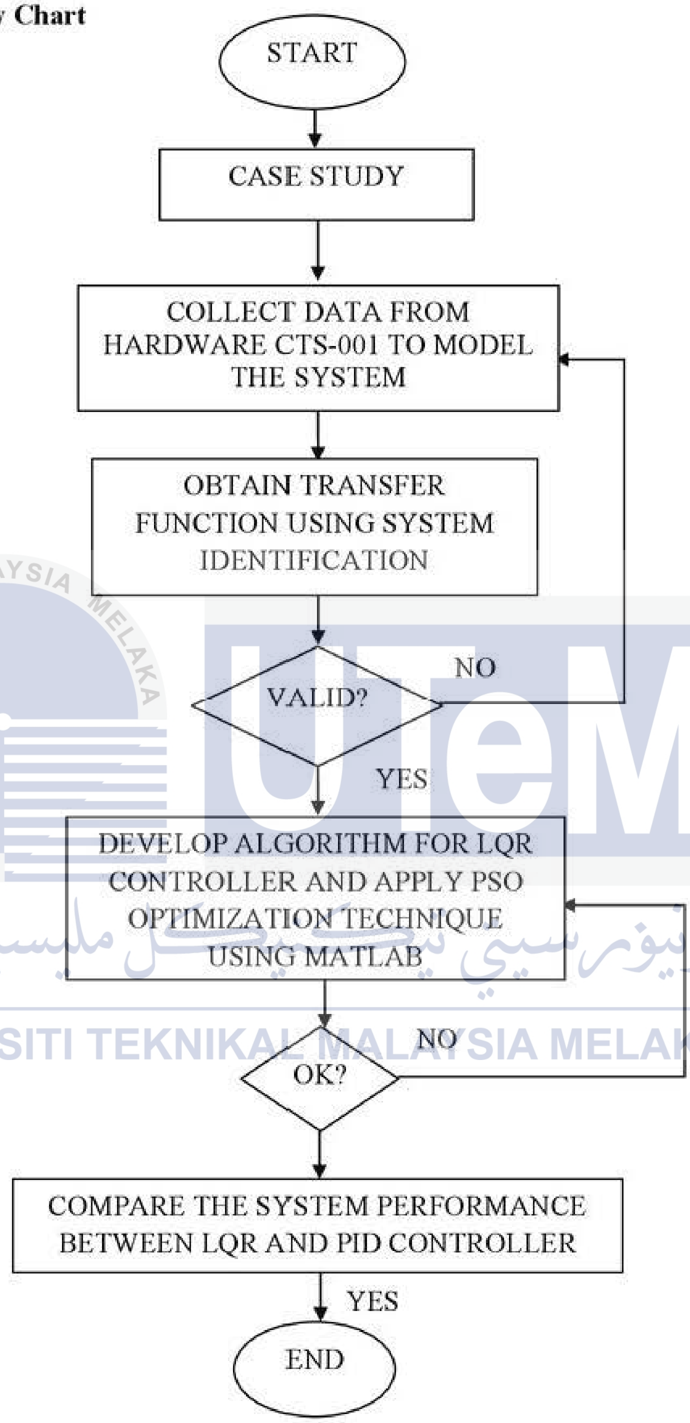


Figure 3.1: Flow chart for project development

Figure 3.2 shows the overall progress for modeling and controller design of coupled tank system. The project is related to both simulation and hardware. The critical part on the hardware is to extract the data from the tank and maintain the functionality of each component of the system. For software, the modeling for the system must be obtained and need to be verified using the system identification in MATLAB. The correct input and output need to be consider in order to find the appropriate transfer function to the system. If the mathematical model is valid, then the LQR controller will be included in the system. The suitable value of matrices Q and R will be verified using the PSO optimization technique to get best performance for the system.

### 3.3 Integrate Hardware and Software

Modeling of the coupled tank consists of the integration between hardware and software. The instrument used for the coupled tank is CTS-001. CTS-001 is a computer-controlled coupled tank system used for liquid level control. CTS-001 consists of coupled tank apparatus CT-001, a software package developed using LABWINDOWS/CVI environment and a National Instrument (NI) data acquisition card. LABWINDOWS/CVI development tools and NI data acquisition card are use for implementation of virtual instrumentation in an experiment [1].

The coupled tank CTS-001 can be used as a liquid level control system. The coupled tank liquid level control system apparatus is interfaced to the computer via the NI data acquisition card. NI data acquisition card is use to measure the actual physical conditions and converting the resulting samples into digital numeric values that can be manipulated by a computer. A software package is developed using LABWINDOWS/CVI for the purpose of analysis data, data acquisition and monitoring the liquid level. The software is used for the purpose of sending and receiving the information to the coupled tank liquid level control system.

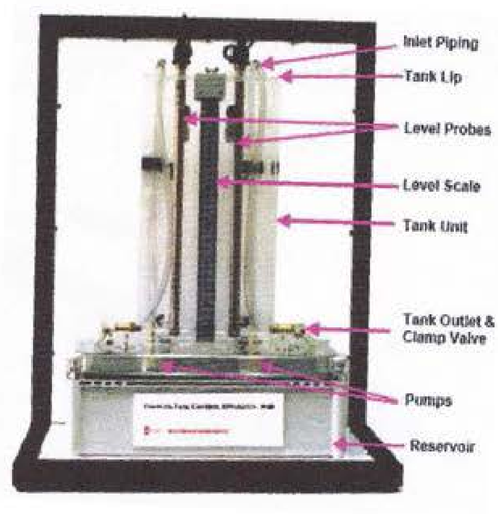


Figure 3.2: Coupled tank control apparatus model CTS-001 [1]

### 3.3.1 Coupled Tank Model (Operation)

Water is pumped into the tank lips at the top of each tank by two independent pumps. The water level in each tank is visible on the attached scale at the front of the tanks. Each tank is fitted with an outlet located at the side near the base. The amount of water which returns to the reservoir is approximately proportional to the water entered the tank since the return tube at the base of the tank functions as a pseudo-linear hydraulic resistance. The return tune is constructed of flexible tubing so that resistance may be increased by the use of a screw-type clamp [1].

The level of water in each tank is measured using a capacitive-type probe together with an associated electronics. The capacitance changes as the water level changes. The changes in the capacitance determine the change in frequency of an oscillator and this change is electronically converted to a DC voltage in the range of 0 to +5 Volts (DC). The zero level has been calibrated (approximately 20mm on the scale) to represent the rest point of the water level when the tank is nearly empty. When the tank is in full state, +5 Volts is calibrated at the level of the opening to the rear overflow stand-pipes. This occurs at approximately 30mm on the scale.

The two pumps at the rear of the unit are controlled by Pulse Width Modulation (PWM) circuits using power MOSFET devices. The input signal to each pump circuit is in a PWM waveform (can be generated by microcontroller or an external DC voltage) in the range of 0 to +5 Volts.

An internal baffle controls the leakage between the two tanks. Turn the wing-nut on the top of the tank will allow the baffle to be raised in a small amount which sufficient to provide a useful range of inter-tank resistance. A spring will return the baffle to the closed position when the wing-nut is released.

### 3.3.2 Calibration of Coupled Tank

Calibration can be defined as one of the primary processes used to maintain the accuracy of the instrument. It is important to know the measurement and calibration of the component characteristic. The information is needed to determine the relationship between the input and output of the component and to identify the presence of non-linearities in the system which leads to inconsistency in the experimental result.



There are three important components of the apparatus that need serious analysis and experimentation [1]. The components are:

i. Tank dynamic characteristic
ii. Level sensor characteristics
iii. Pump characteristics

The understanding of the characteristic for each component is important in order to relate with the theoretical aspect of the function of the components and to understand the relationship between the system parameters and its effect on the system response.

Below is the relationship between the input voltage, $Y_1$ and the water level of the coupled tank A, $H_1$:

$$k_1 = \frac{Y_1}{H_1} \quad , \; where \; k_1 = gain \; of \; level \; sensor (V/cm)$$

The relationship between the water level and the corresponding voltage for both tank A and B are shown in Figure 3.3and Figure 3.4. The gain of level sensor after linearization for Tank A is 0.195V/cm whereas for tank B is 0.174V/cm.

The relationship between the input voltage to the pump $U_1$ and the pump flow rate into the tank $Q_1$ can be determined by :

$$k_p = \frac{Q_1}{U_1} \quad ,where \; k_p = gain \; of \; pump (\frac{cm3}{sec}/V)$$

It is shown in the Figure 3.5 and Figure3.6 on the relationship of voltage and flow rate for both tank. The controlled inflow for Tank A is $10 \frac{cm3}{sec}/V$ whereas for Tank B is $20 \frac{cm3}{sec}/V$.

Both of the gain, $k_x$ and $k_p$ is important to obtain the relationship between both parameter in the tank. The calibration on input voltage which gives changes to the water level and pump flow rate is important so that the relationship between voltage and water level and voltage with pump flow rate can be monitored and maintained.

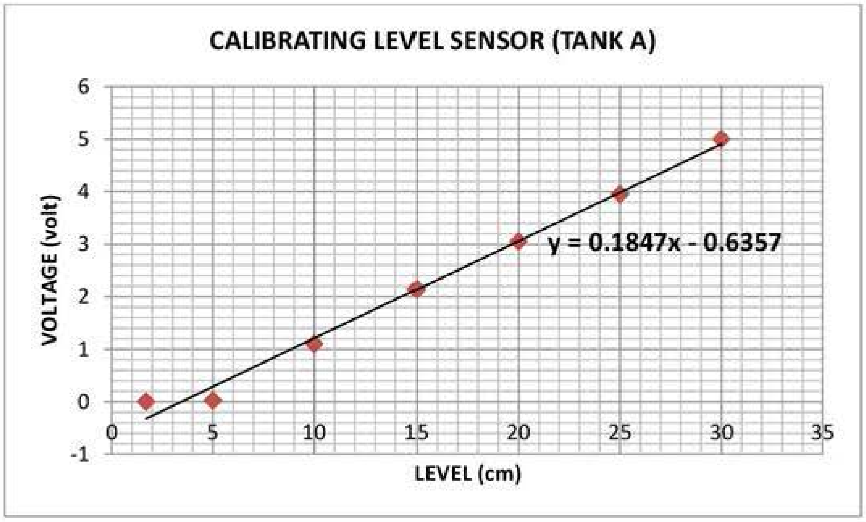


Figure 3.3: The relationship between the water level and the corresponding voltage in Tank A

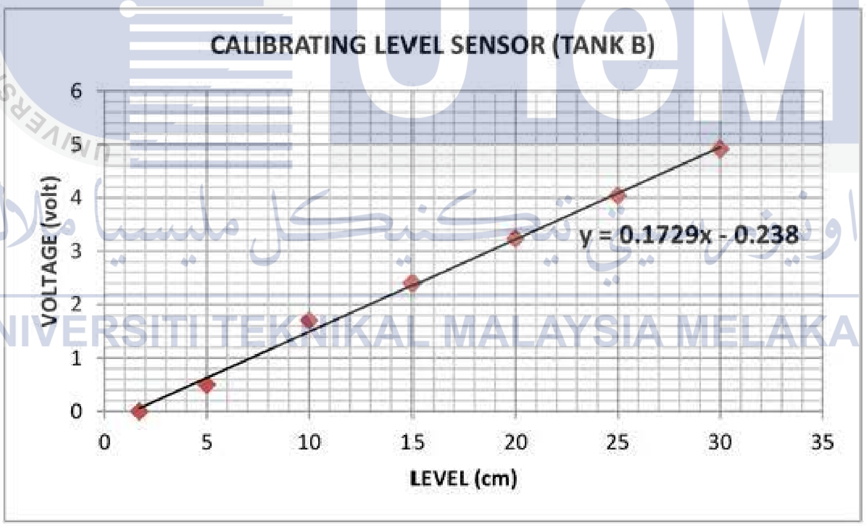


Figure 3.4: The relationship between the water level and the corresponding voltage in Tank B

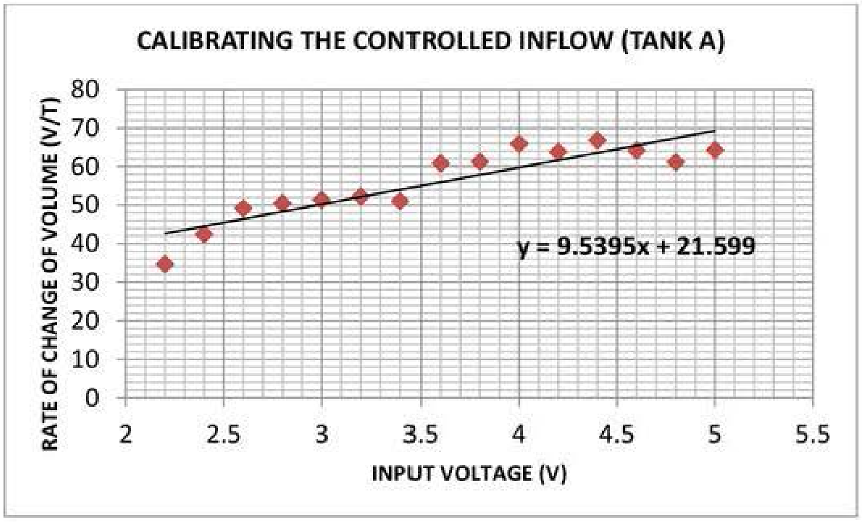


Figure 3.5: The relationship between the pump voltage and the corresponding voltage in Tank A

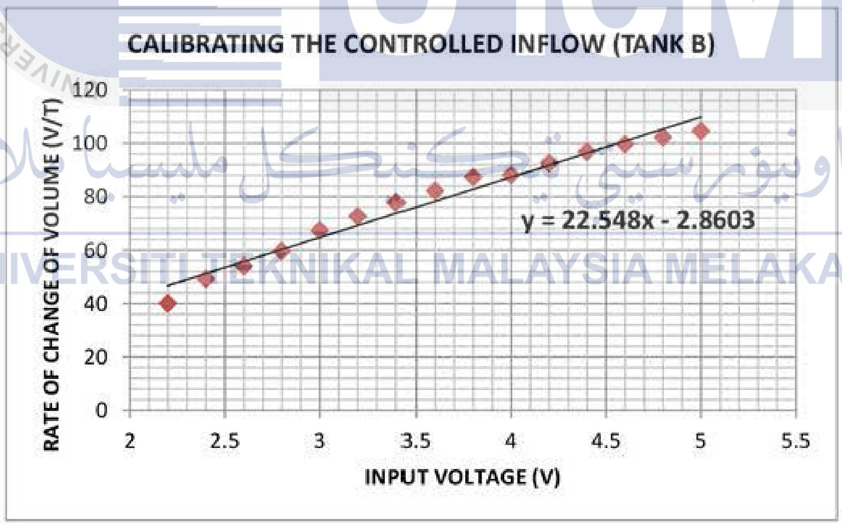


Figure 3.6: The relationship between the pump voltage and the corresponding voltage in Tank B

## 3.4 Modeling of Coupled Tank

Modeling is needed to obtain the proper transfer function for the coupled tank system. To obtain the transfer function, several methods need to be done. The data need to be collected on the coupled tank where the voltages become the input whereas the levels of water become the output for the system. The component of the hardware as well as the interface device needs to perform well so that appropriate output can be obtained for the next stage of modeling. The input data used for this project is Pseudorandom Binary Sequence (PRBS). Once the data is collected, it will be loaded in process called system identification.

### 3.4.1 Pseudorandom Binary Sequence (PRBS)

PRBS is an input that is widely applied in the linear system identification. The Auto-correlation function (ACF) of a PRBS provides the best and useful approximations to periodic white noise [19]. PRBS are commonly used as the forcing function in the testing for statistical system. The advantages of using PRBS as an input is that the signal can be easily generated and applied to the system.

PRBS comes with two types which are the QRC (Quadratic Residue Code) and MLS (Maximum Length Sequences). Type of PRBS used for this project is MLS. The characteristics are it has periodic sequence, deterministic signal and generate using n-bit shift register with feedback through a logic function called "exclusive or" (XOR) function.

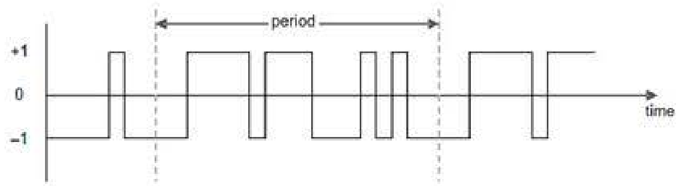


Figure 3.7: General form of PRBS waveform [19]

Shift register with suitable feedback is a useful tool to generate the PRBS waveform. The length of the period sequence MLS can be determine using the formula $N=2^n-1$ where n represent the number of stages in the shift register. Once the number of stage is choose, it will be evaluated mathematically and organized in a tabular form. The frequency for both PRBS waveform and clock frequency for the shift register will be the same.

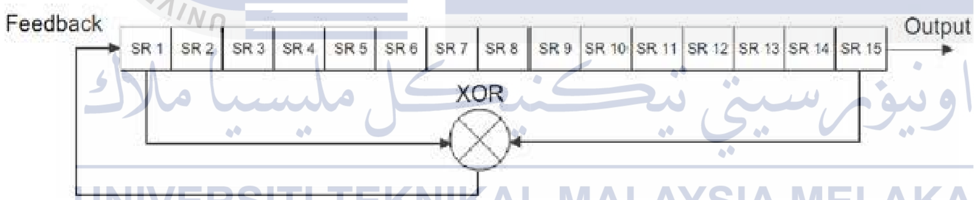




Figure 3.8: Shift register for generation of PRBS[19]

### 3.4.1.1 PRBS Stages

Integer choose for the input, n=3. Therefore the length of sequence for the stages is $N = 2^n - 1 = 2^3 - 1 = 7$ bits.

Table 3.1: PRBS input table

| SR1 | SR2 | SR3 | SR1 ⊕ SR3 |
|---|---|---|---|
| 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 |



The sequence is 0100111 in a repetitive manner at the clock frequency. The amount of cycles chose for this project is about 152 cycles with 1069 of collected data. The two possible states for PRBS is +4V and 0V. The change is referring from the deterministic pseudo random manner referring to Table 3.1. The feedback value, SR1 ⊕ SR3 will become the input for the system. Figure 3.9 shows the result obtained using the PRBS method which is used for the input in the system identification method. It is simulated using the CAIRO CTS001 software.

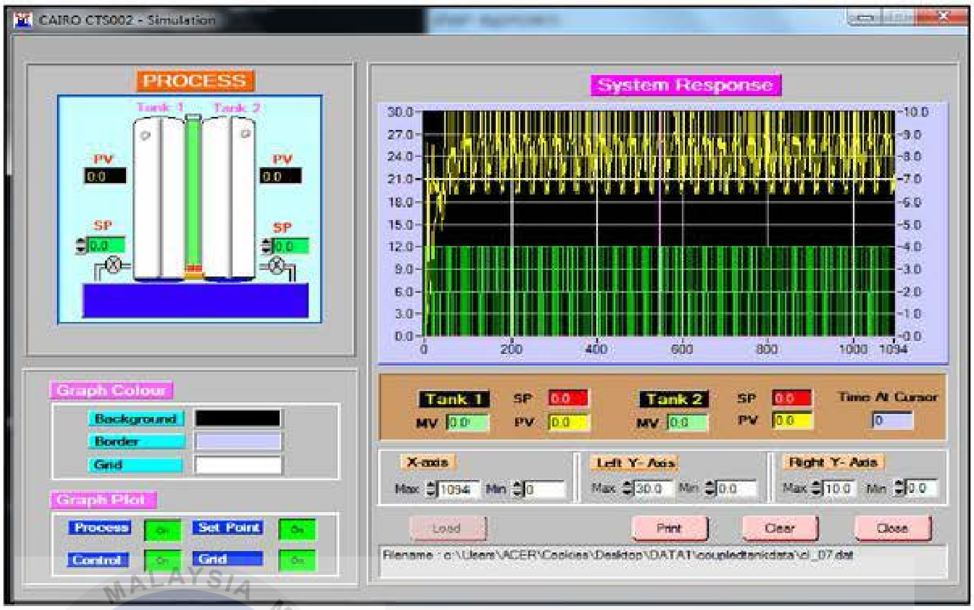


Figure 3.9: Data obtained from CAIRO CTS-001

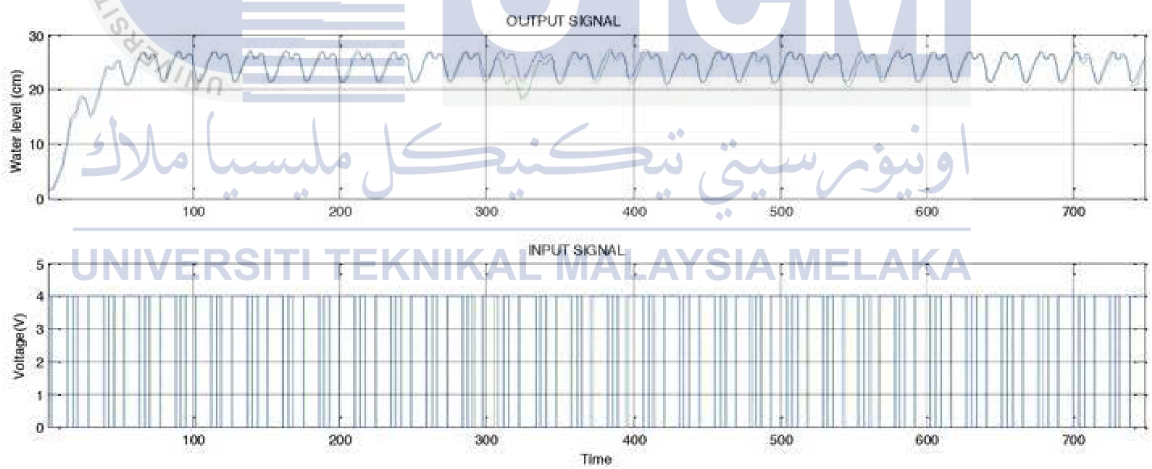


Figure 3.10: Input for the system identification

### 3.4.2 System Identification

The main purpose of on applying system identification method is to determine a mathematical model of a system from observed data. The input and output of the system need to be determined in order to get the appropriate mathematical model for the system. System identification method was used to infer a model in the transfer function and it can conclude that the best and valid result was a model with the best fit over than 80 percent [20]. Figure 3.11 shows the flow chart on how the procedure to conduct the system identification.

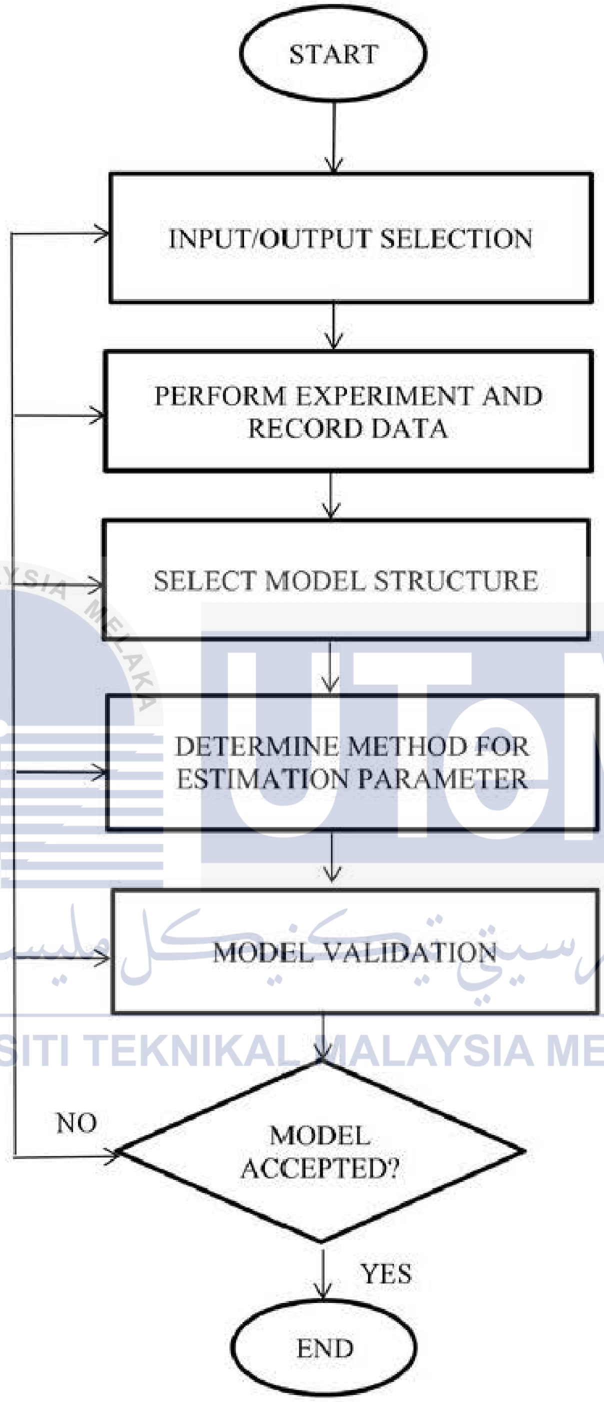


Figure 3.11: Flow chart of System Identification

Below shows the procedure or steps on how to obtain the mathematical model of the system once the data is collected from the coupled tank.

STEP 1:

The input decided for this project is the voltage applied to the coupled tank and the output is the water level in the tank. Once the experiment data is collected, the data is then loaded to the workspace as a variable. In the "*ident*" window, select the popup menu import data and select time domain data. Change the sampling time to 0.7s based on the sampling time selected while undergoing the experiment. Use the same step to load the second input and output for the system.

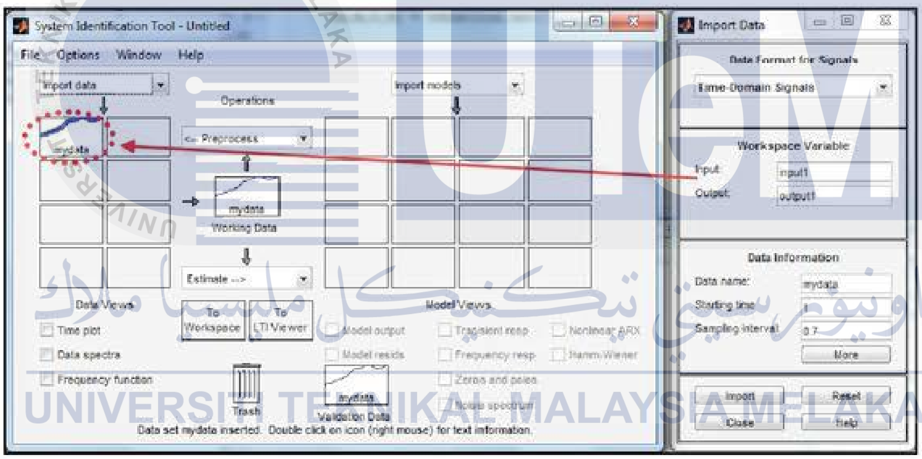


Figure 3.12: Import 1st data from workspace

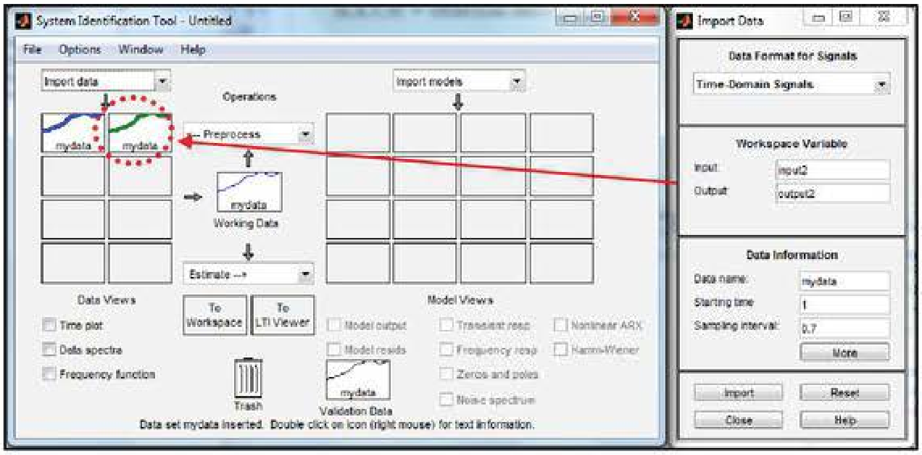


Figure 3.13: Import 2$^{nd}$ data from workspace

STEP 2:

In the data board of the *"ident"* figure, select a first data set in the data views to work with for estimation by dragging it to the Working Data icon. Then, the second data set is used for validation purposes should be dragged and dropped onto the Validation Data icon.



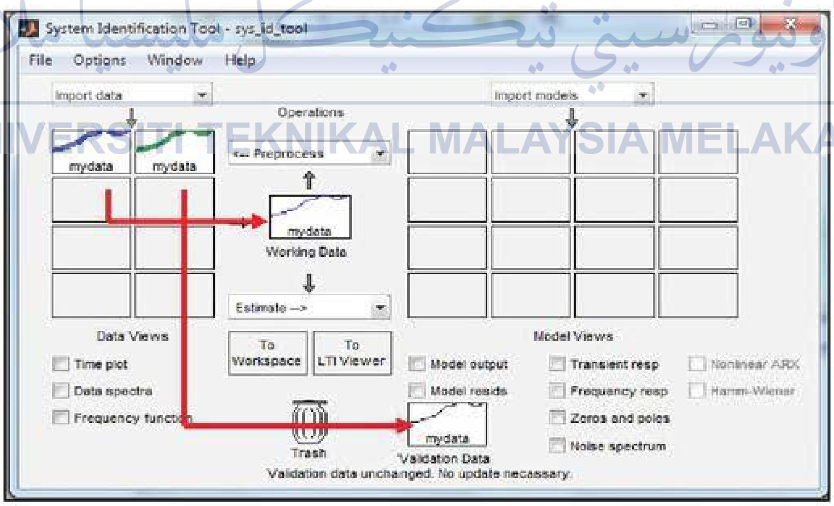


Figure 3.14: Save data for Working Data and Validation Data

STEP 3:

To compute the transfer function, select the transfer function models from the Estimation popup menu. New dialog will open and enter the information in the transfer function model dialog and then generate the model by pressing the estimate button. The model will be computed and inserted into the Model board as an icon.

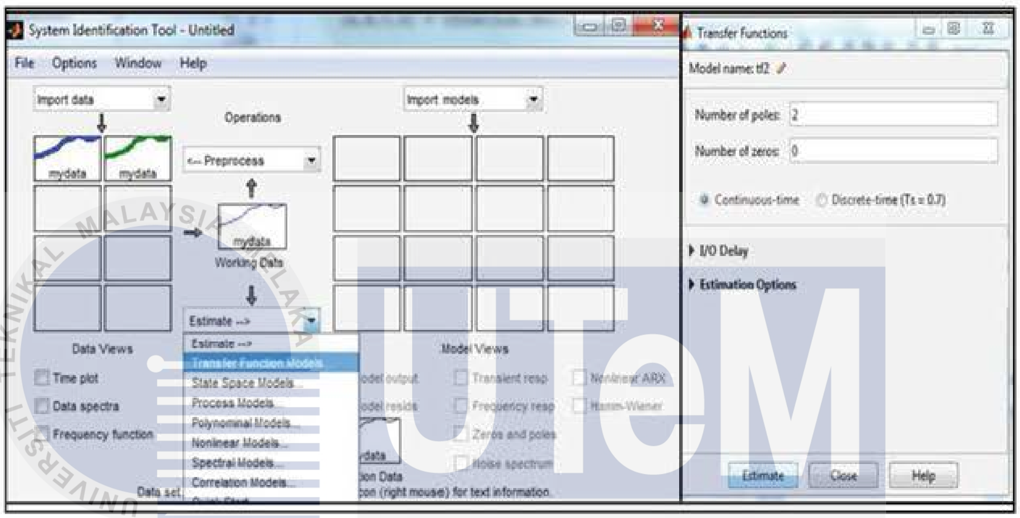


Figure 3.15: Select estimation as transfer function models

STEP 4:

Once the model is inserted, click the model output to measure the best feet of the transfer function model. Double click on the '*tf1*' icon to get the calculated transfer function obtained.

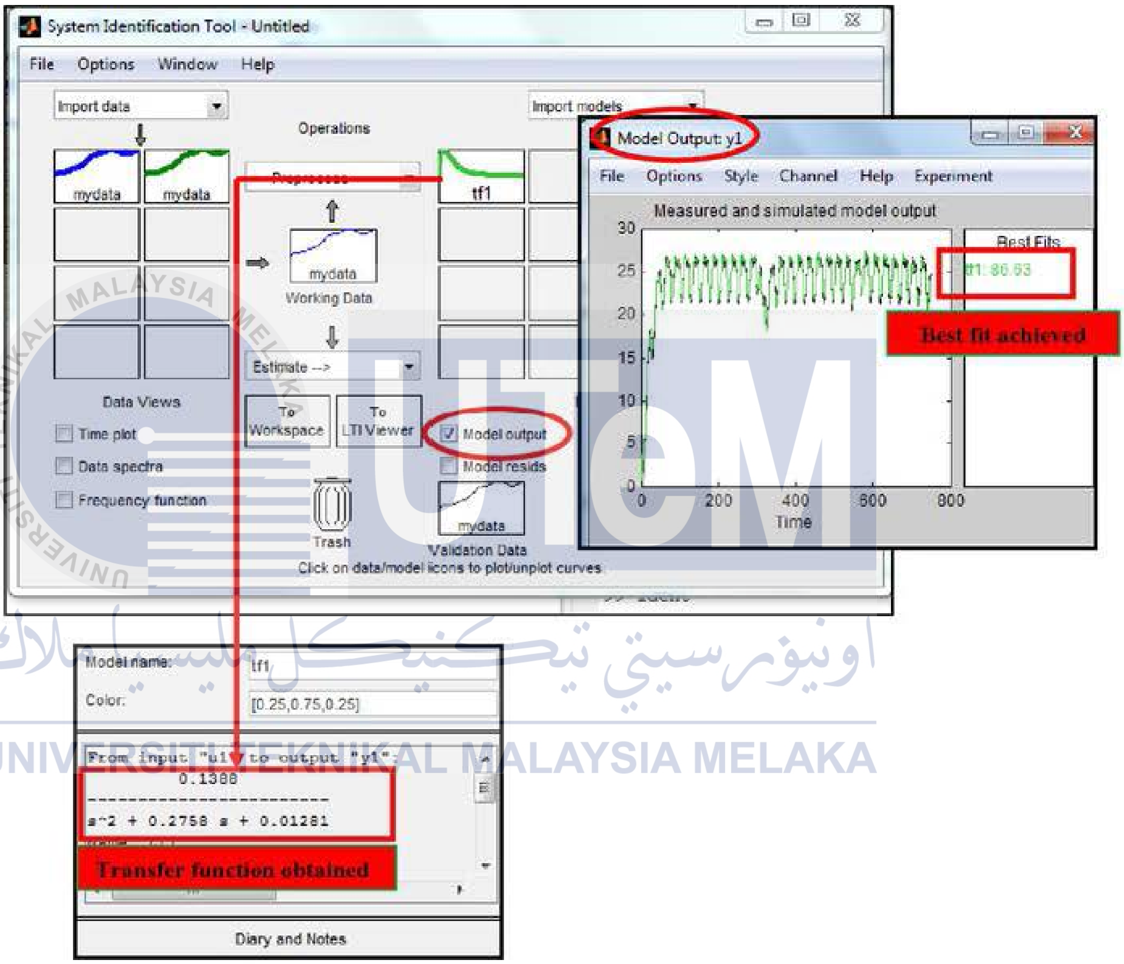


Figure 3.16: Best fit percentage and transfer function obtained

## 3.5 Design LQR and PID Controller

LQR Controller and PID Controller will be used to control the coupled tank system. The parameter for the controller can be obtained using MATLAB algorithm (coding) with chosen optimization technique. The result for both controllers will be showed in the next chapter.

### 3.5.1 LQR Controller

LQR is a method that uses state space approach to design and control a system. The two matrices Q and R are selected to control the system in order to maintain the performance of the system. The objective in design using LQR Controller is to select the best gain, K that minimizes the performance of the performance index, J. The schematic for this type of controller is shown in Figure 3.17.

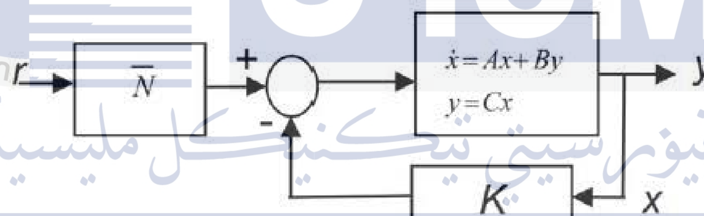


Figure 3.17: Schematic diagram for LQR

Below shows the state variable form to represent a system:

$\dot{x} = Ax + Bu$

The initial condition is $x(0)$. We assume that all the state are measurable and seek to find a state-variable feedback (SVFB) control

$u = -Kx + v$

The closed-loop system using this control should be

$$\dot{x} = (A - BK)x + Bv = A_c x + Bv \tag{28}$$

With $A_c$ the closed-loop plant matrix and $v(t)$ the new command input

This is the formula to find the performance index of a system

$$Performance\ index(PI), \quad J = \frac{1}{2}\int_0^\infty x^T Qx + u^T Ru\, dt \tag{29}$$

Subtitute the SVFB control into this yields

$$Performance\ index(PI), \quad J = \frac{1}{2}\int_0^\infty x^T (Q + K^T RK)x\, dt \tag{30}$$

These are the procedures on how to find the LQR feedback, K:

i) Select design parameter matrices Q and R
ii) Solve the Algebraic Riccati Equation (ARE) for P
iii) Find LQR feedback, K using $K = R^{-1}B^T$

Q is a positive semi-definite n x n matrix and R is positive definite m x m. In general, choose large value for Q means that it keeps J to become small. Larger Q allow the poles in the closed-loop to become more further to the left side of the s-plane so that the state decay become faster to achieve zero. Similar to Q, having large value for R means that the control input $u(t)$will be reduce in order to keep J small. Increment value for R shows less amount control effort is used. It gives the poles to move slower which resulting larger value of state $x(t)$.

In order to find the feedback, K value of P need to be obtained. Given the constant matrix P

$$\frac{d}{dt}(x^T P x) = -x^T(Q + K^T R K)x \tag{31}$$

Substitute the equation (31) into (30) gives

$$J = -\frac{1}{2}\int_0^{\infty} \frac{d}{dt}(x^T P x)dt = \frac{1}{2}x^T(0)Px(0) \tag{32}$$

Equation (32) shows that J is now independent of K. x(t) will become zero as time, t goes to infinity due to assumption the system assumed to give a stable response.

Substitute the closed loop equation (28) in (31) and differentiate the equation to become

$$\dot{x}^T P x + P\dot{x} + x^T Q x + x^T K^T R K x = 0$$
$$x^T {A_c}^T P x + x^T P A_c x + x^T Q x + x^T K^T R K x = 0$$
$$x^T\left({A_c}^T P + P A_c + Q + K^T R K\right)x = 0$$

10

It has been assumed that the external control $v(t)$ is equal to zero. Now not e that the best equation has to hold for every $x(t)$. Therefore, the term in brackets must be identically equal to zero. Thus, proceeding one secs that

$$(A - BK)^T P + P(A - BK) + Q + K^T R K = 0$$
$$A^T P + PA + Q + K^T R K - K^T B^T P - PBK = 0$$

This is a matrix quadratic equation. Exactly as for the scalar case, one may complete the squares. Though this procedure is a bit complicated for matrices, suppose we select

$$K = R^{-1}B^{T}P \tag{33}$$

Then, there results

$$A^{T}P + PA + Q + (R^{-1}B^{T}P)^{T}R(R^{-1}B^{T}P) - (R^{-1}B^{T}P)^{T}B^{T}P - PB(R^{-1}B^{T}P) = 0$$
$$A^{T}P + PA + Q - PBR^{-1}B^{T}P = 0 \tag{34}$$

Where (34) called the Algebraic Riccati Equation (ARE).

The value of ARE can be perform using Matlab Routine called *lqr(A,B,Q,R)*. Once the value of P obtained, the feedback can be determined and the performance of the system can be analyze. The result of the system will be shown in the next chapter.

In order to reduce the steady state error of the system output, a value of constant gain, Nbar should be added after the reference [21]. Nbar can be found using the m-file code function.

### 3.5.2 PID Controller

The Proportional-Integral-Dervative (PID) control gives the simplest and yet the most efficient solution to various real-world control problems [21]. The controller will take the measured value from the plant and compares it with the value of the reference set point. Figure 3.18 shows the block diagram for PID controller.

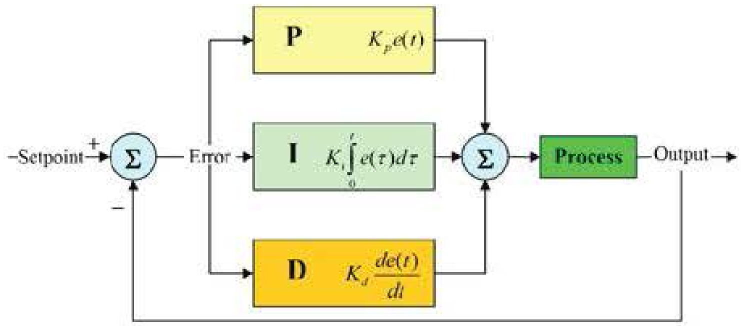


Figure 3.18: Schematic diagram for PID

The transfer function for PID controller is

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s$$

These are several steps on how to design n a PID controller in order to obtain the desired response.

i) Obtain the open loop response and specify the parameter needs to be improved

ii) Select proportional gain, Kp to improve the rise time

iii) Select integral control, Ki to eliminate the steady-state error

iv) Select derivative gain, Kd to improve the overshoot

v) Adjust the gains of Kp, Ki, and Kd until it meet the desired overall response.

In this project, the weight value of Kp, Ki and Kd are determined by using one of the optimization technique, PSO. The responses using the selected value will be discussed in the next chapter.

### 3.6 Particle Swarm Optimization (PSO) technique

PSO is an optimization technique that the working principle is based on the movement and intelligence of swarm such as birds flocking and fish schooling. The sharing of information will happen when they are searching for their food. In PSO concept, the "particle" in the search space can be related to the "bird". The swarm is modeled as particles in multi-dimensional space. The swarms of particle communicate through adjustment of position and velocity. Therefore, PSO will be used as the optimization technique for the system.

Below are the general processes on how the PSO works in a system:

1) Initialize a group of particles including the random positions, velocities and acceleration of particles
2) Evaluate the fitness of each particle
3) Compare the individual fitness of each particle with previous save data. If it is better, update as a new data
4) Update the velocity and position for each particle
5) Repeat step 2 onwards until the best stopping criteria is met



This optimization technique can be applied once the system is cascade with the LQR controller. The main function of implementing the PSO in this system is to help the system to find the best value of matrices Q and R for LQR controller instead of using conventional techniques like the try and error method. PSO is a type of stochastic technique that once it is run, it will not gives the same value. To validate the best value of Q and R, the results after stimulate the PSO technique to the system will gives is merely constant output. If the output for few simulations is similar, it shows that the value of Q and R can be used for the system. The details about the value for Q and R will be discussed more in Chapter 4.

# CHAPTER 4

# RESULT AND DISCUSSION

## 4.1 Overview

This chapter presents the simulation result for coupled tank system using the same optimization technique, PSO for different controller. It consists of result for two types of controller which are PID and LQR controller. The results are divided into several parts to gives brief information regarding the system. The characteristic of the system is shown in the first section whereas it gives brief information for the uncompensated system. The fitness function of both controller is discussed for the next section and the parameter obtained using the PSO is determined. The performances of the system using both controllers are observed based on the rise time, settling time, steady state error and overshoot.

## 4.2 Characteristic of system

This section will discuss about the characteristic of the coupled tank system for uncompensated condition. Figure 4.1 shows the response for open loop and Table 4.1 shows the performance response regarding the transient response of the system in open loop condition. The input set for this system is in step input with value 1. From the observation, the system took longer time to settle down. This is due to no feedback use in open loop system to compensated any error or correct any disturbance if any. It also shows that the system does not have an overshoot whereas the performance of the system gives slow response for the transient response.

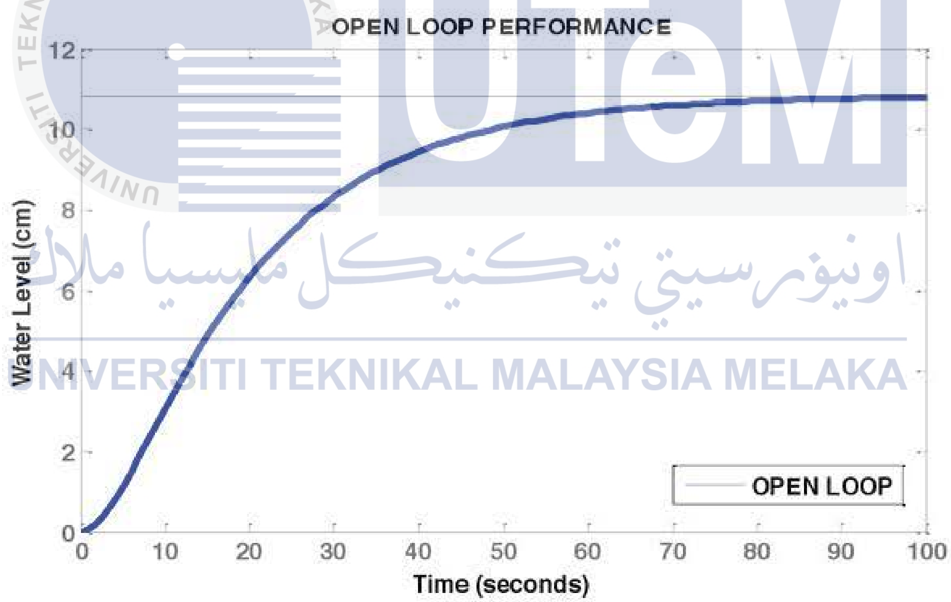


Figure 4.1: Open loop step response

Table 4.1 : Transient response performance for open loop system

| Output | Open Loop System |
|---|---|
| Settling time, Ts | 71.5648 |
| Rise time,Tr | 39.4574 |
| Overshoot,%OS | 0 |
| Steady-state error, Ess | 9.8279 |

Figure 4.2 shows the closed-loop response and Table 4.2 shows the performance of the closed loop system. The graph shows the presence of overshoot and the system did settle much faster compared to the open loop response. The details regarding the transient response is shown in Table 4.2.

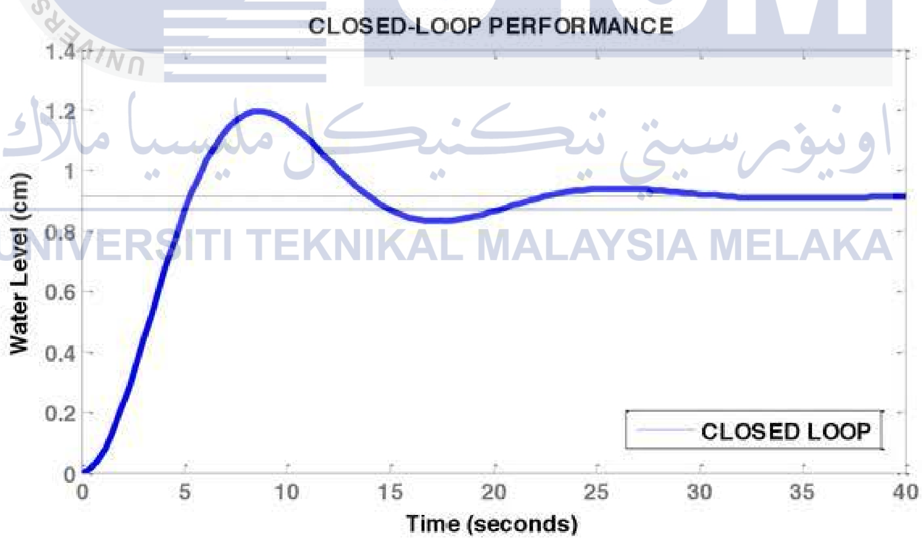


Figure 4.2: Closed Loop step response

Table 4.2 : Transient response performance for closed loop system

| Output | Closed Loop System |
|---|---|
| Settling time, Ts | 0.8309 |
| Rise time,Tr | 3.5927 |
| Overshoot,%OS | 30.3961 |
| Steady-state error, Ess | 0.084 |

## 4.3 PSO for LQR and PID

The result shows the value of fitness function obtained using PSO for both LQR and PID. The parameter stated is based on the selected execution data for PSO. Figure 4.3 shows for the fitness for LQR controller and Figure 4.4 shows the fitness for PID controller.

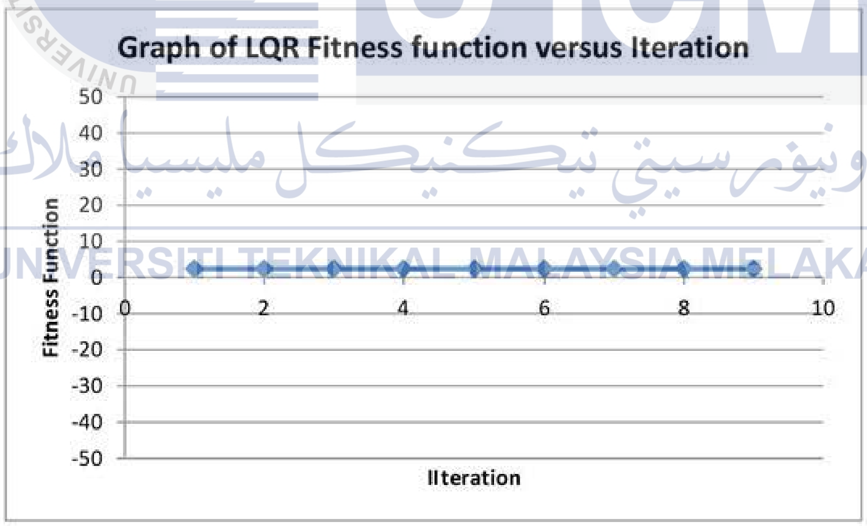


Figure 4.3: Graph of LQR fitness function versus iteration

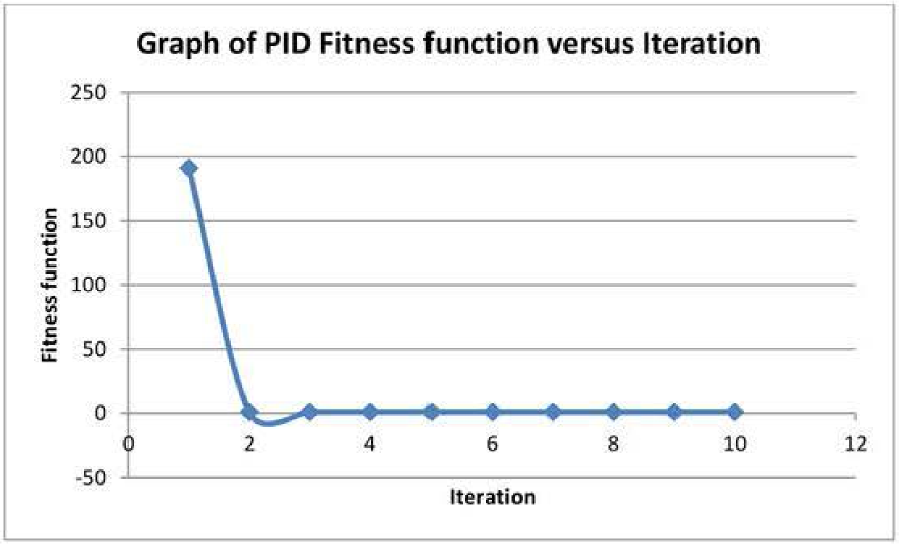


Figure 4.4: Graph of PID fitness function versus iteration

Refer to Figure 4.3 and Figure 4.4, PSO proves that it gives an optimum fitness function for LQR and PID. The stopping value for the fitness function is differs for both controllers. The value for the fitness function will be evaluated based on each particle. If the fitness is better for the next particle, it will update is as a new data. Once the fitness value maintain, it shows that the iteration will gives the most minimum performance index for the system and the data obtained will be used to select the best value for parameter in LQR and PID.



## 4.4 Parameter for PID and LQR

A good controller should have a closed loop that gives stable and fast response and has the lowest possible control effort. Control effort can be found based on the performance index of the controller. Type of performance index selected for these project is Integral Time-weighted Squared Error (ITSE) since it can provide controllers with a high load disturbance rejection and minimize the system overshoot while maintain the robustness of the system [22].

There are several types of performance index such as ITAE, ISE and IAE. Somehow ITSE is well known as the popular performance criterion used for control system design.

The number of execution needed to be run for the controller is 20 times. The least ITSE obtained from the iteration will be choose as the best performance index for the system and the data weighted gain for the selected execute number will be used as the best parameter for the controller. A set of good control parameters can produces a good step response that will result in the performance criteria where it minimize in the time response data. These performance criteria include the overshoot, rise time, settling time, and steady-state error. The parameter need to be obtained for LQR controller is Q1, Q2 and R where for PID the parameters needed are Kp, Ki and Kd.



Table 4.3: Parameter value for PID and LQR

| | | | |
|---|---|---|---|
| **LQR** | | | |
| **ITSE : 0.4028** | | | |
| | Q1 | Q2 | R1 |
| Parameter | 0.03260 | 64.0389 | 0.0000735 |
| **PID** | | | |
| **ITSE : 0.9153** | | | |
| | Kp | Ki | Kd |
| Parameter | 98.2804 | 0.0062 | 40.2215 |

Table 4.3 shows the value of parameter for LQR and PID based on the based value of performance index obtained after 20 executions done. It shows that the ITSE for LQR is smaller compared to PID. Smaller value of ITSE is better for a system so that the total energy used for the system can be minimizes as possible.

## 4.5 COMPARISON BETWEEN LQR AND PID

This section shows the performance of the system response using LQR controller and PID controller. The parameter for both LQR and PID was obtained using the same optimization technique, PSO in order to get the best value of gain for the controllers. The outputs are shown in the Figure 4.5and Figure 4.6 while Figure 4.7 shows the combination for both responses.

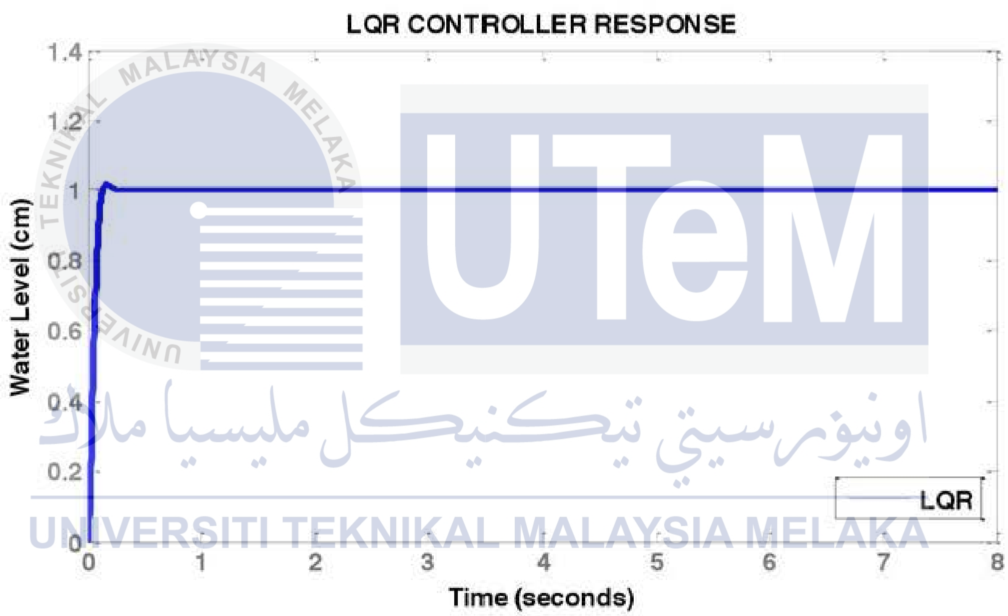


Figure 4.5: Response for LQR Controller

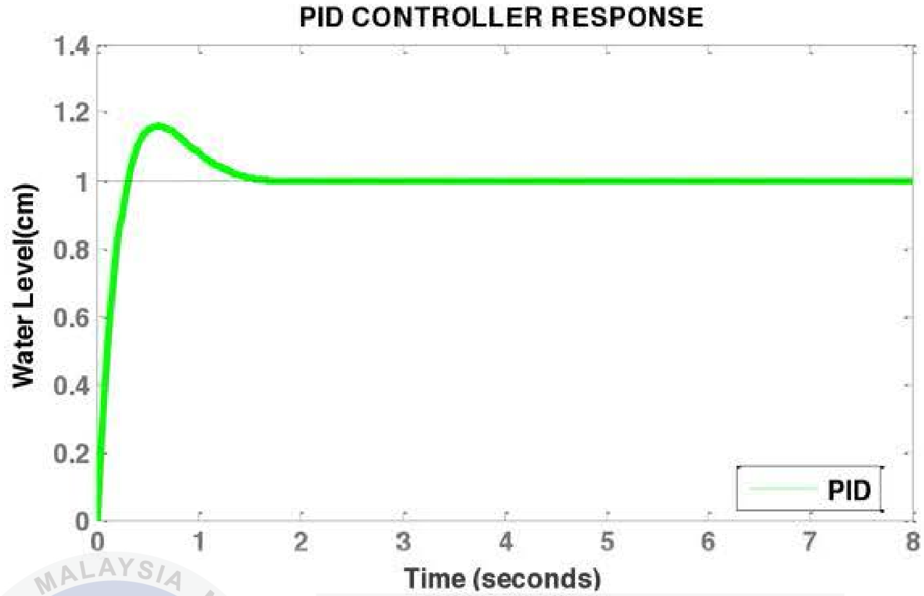


Figure 4.6: Response for PID Controller

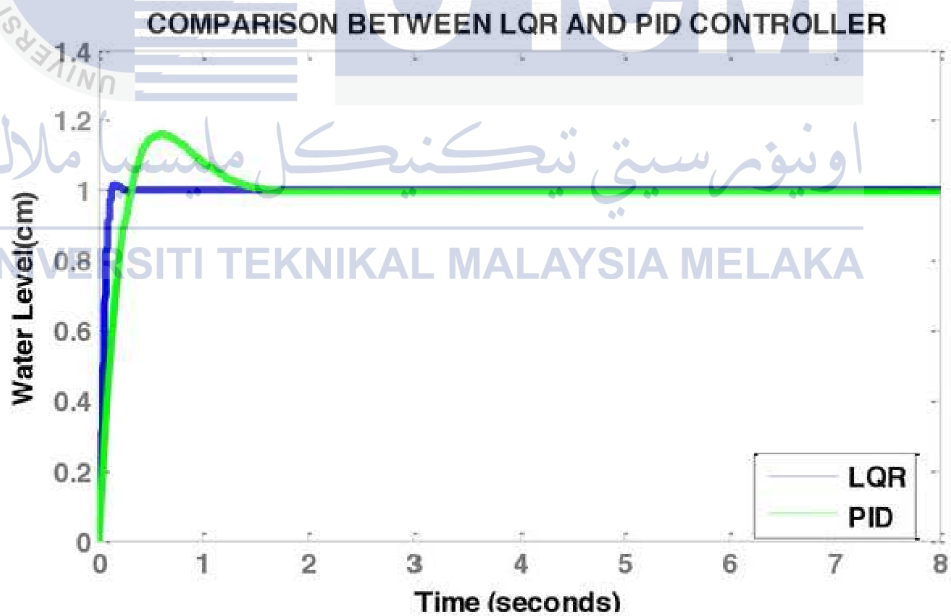


Figure 4.7: Responses for LQR and PID controller

Figure 4.5 and Figure 4.6 shows the responses for LQR and PID controller based on the same optimization technique. From the observation on Figure 4.7, it shows that LQR controller gives the best performance compared to PID controller in terms of the transient responses. For LQR controller, it gives a less overshoot compared to the PID controller. It also shows that the system using LQR as the controller meet the steady state condition at short period of time. In order to investigate and verify the performance using different controllers, time response data for both controllers was taken and shown in Table 4.4 and Figure 4.8.

Table 4.4: Comparison between LQR and PID

| OUTPUT | CONTROLLER | |
|---|---|---|
| | **LQR** | **PID** |
| **Rise time,Tr (s)** | 0.0792 | 0.2338 |
| **Settling time, Ts (s)** | 0.1200 | 1.3538 |
| **Overshoot, OS(%)** | 1.83 | 16.0060 |
| **Steady-state error, Ess** | 0 | 0 |

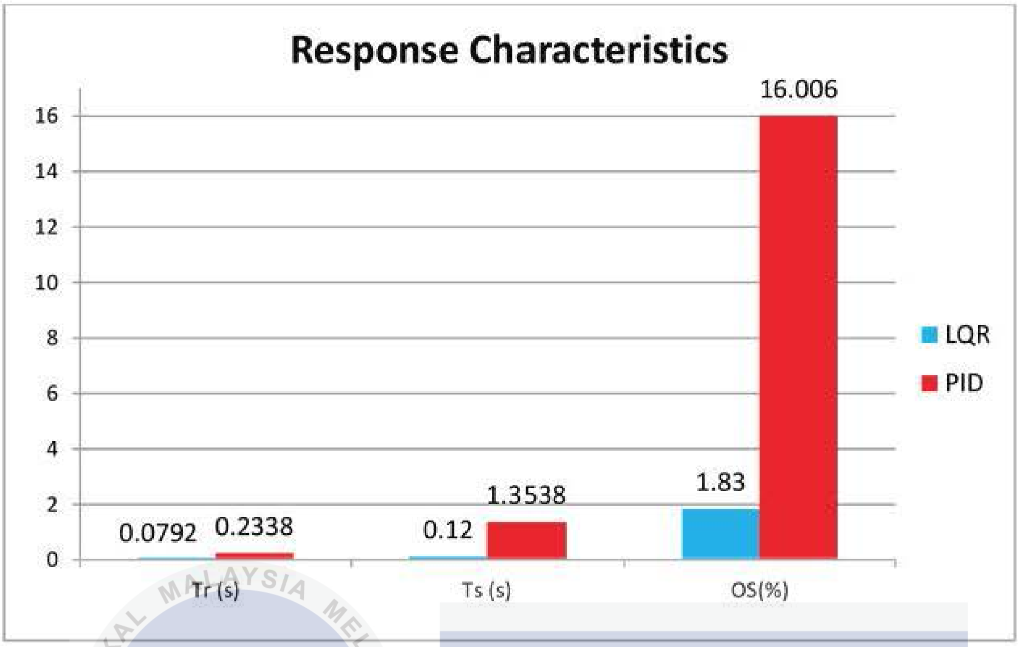


Figure 4.8 : Performance comparison between LQR and PID controller

Refer to Table 4.4, it shows that the transient response for LQR controller is much better than PID controller. For the rise time, Tr LQR responses faster than PID where it rises at 0.0792s which is 0.1546s faster than the PID. In terms of settling time, Ts the time taken for LQR to settle is shorter which is at 0.12s whereas for PID it took longer time at 1.3538s. For PID controller, it gives quite high overshoot reading for merely 16% while on the contrary LQR gives less overshoot which is 1.83%. Both of the controller gives better response where is gives zero steady-state error for the system. From Figure 4.7, it shows that LQR controller and PID controller are competent to be used in the coupled tank system to maintain the water level in the tank. However, the results proved that using the approach of LQR controller gives better performance than PID controller in terms of faster response and less overshoot.

# CHAPTER 5 CONCLUSION

## CONCLUSION

### 5.1 Overview

This section consists of the conclusion of the project and future work that can be used to improved the result of the project.

## 5.2 Conclusion

From the research done and result presented in Chapter 4, there are several conclusion can be made for this project. The mathematical model for the coupled tank system can be determined using system identification method and the validation process is based on the best fit obtained.

Secondly, it shows that by using the PSO optimization technique, it gives a better response and best weight parameter for both controllers. Having PSO can validate the comparison for the response of the system because it has a constant variable that can be compared instead of using the conventional method such as trial and error method which does not gives firm validation in terms of the selection of parameter. The selection of the best parameter can be determined by referring to the data collected at the most minimum Integral Time-weighted Squared Error (ITSE) because less error gives small amount of energy used for system and it allow the system to give response which similar with the given input.

In general, LQR controller and PID controller are competent to be used in the coupled tank system to maintain the water level in the tank. However, the results shows and prove that using the approach of LQR controller gives better performance than PID controller in terms of faster response and less overshoot.

## 5.2 Recommendation for Future Work

Several actions need to be done in the future in order to produce a quality performance for the coupled tank system. Replacement of new coupled tank module CTS-01 is needed so that it can gives better performance of the tank during collecting data and produce less error which can be found from the malfunction of certain component in the hardware itself. Changing the number of execution for every controller can be applied for the future work in order to observe the effectiveness of the response once the value of execution changed.

Different optimization technique such as Genetic Algorithm (GA) and Firefly Algorithm (FA) can also be used to find the best value for parameter in LQR and PID. Different optimization techniques will give variant response due to different characteristic and concept of the algorithm.

## REFERENCES

[1] "Coupled-tank Liquid Level Computer-controlled Laboratory Teaching Package Experimental and Operation Service Manual". Augmenred Innovation Sdn. Bhd Center For Artificial Intelligence and Robotic(CAIRO).*Lab manual*

[2] H.Gupta, O.P.Verma. "Intelligent Controller for Coupled Tank System" ISSN:2277 128X.Volume 2. Issue 4. April 2012

[3] H.M.Shariff, M.Hezri, F.Rahiman, M.Tajjudin. "Non linear System Identification : Comparison between PRBS and Random Gaussian Perturbation on Steam Distallation Pilot Plant". 2013 IEEE 3rd International Conference on System Engineering and Technology. pp.269-274. August 2013

[4] A.Mohammadbagher, M.Yaghoobi. "Comparison performance between PID and LQR controller for 4-leg voltage sources inverter". 2011 International Conference on Circuit, System and Simulation.Volume 7. pp.230-234. 2011

[5] L.B.Prasad, B.Tyagi, H.O.Gupta. "Optimal control of nonlinear inverted pendulum dynamical system with disturbance input using PID controller and LQR". IEEE International Conference on Control System, Computing and Engineering. pp.540-545. 2012

[6] N.Wahid,M.F.Rahmat. "Pitch Control System using LQR and Fuzzy Logic Controller" IEEE Symposium on Industrial Electronics and Aplications (ISIEA 2010).pp.389-394.2010

[7] C.Chitu, J,Lackner, M.Horn,H.Waser, M.Kohlbock, "A robust and optimal LQR controller design for Electric Power Steering System" Nonlinear Dynamics and Synchronization (INDS) & 16th Int'l Symposium on Theoretical Electrical Engineering (ISTET).pp.1-5.2011

[8] M.A.Usta, O.Akyazi, A.S.Akpinar. "Aircraft Roll Control System using LQR and Fuzzy Logic Controller".Innovations in Intelligent Systems and Applications (INISTA). pp.223-227.2011

[9] A.Mohammadbagher, M.Yaghoobi. "Comparison performance between PID and LQR controller for 4-leg voltage sources inverter". 2011 International Conference on Circuit, System and Simulation.Volume 7. pp.230-234.2011

[10] X. Xiong, Z. Wan. "The simulation of double inverted pendulum control based on particle swarm optimization LQR algorithm". Software Engineering and Service Sciences (ICSESS) IEEE International Conference.pp.253-256.2010

[11] R.Jaisue,J.Chaoraingern,V.Tipsuwanport,A.Numsonran."A design of Fuzzy PID Controller based on ARM7TDMI for coupled-tanks process". Control, Automation and Systems (ICCAS).12th International Conference.pp.610-613.2012

[12] O.D.Kieran."Real-time Adaptive PID Control of a Non-linear Process based on Genetic Optimization".Dublin City University.August 2002.

[13] M.Abid."Fuzzy Logic Control of Coupled Liquid Tank System". Information and Communication Technologies. ICICT. 1st International Conference.pp.144-147.2005

[14] O.D.Kieran."Real-time Adaptive PID Control of a Non-linear Process based on Genetic Optimization".Dublin City University.August 2002.

[15] S.Aydin, S.Tokat. "Sliding mode control of a coupled tank system with a state varying sliding surface parameter". Variable Structure Systems.VSS08.International workshop. pp.355-360.2008

[16] H.Abbas,S.Asghar,S.Qamar."Sliding Mode Control for Coupled Tank liquid level control".10th International Conference on frontiers of Information Technology. International Islamic University. pp.325-330.2012

[17] M.T.Alam, P.Charan, Q.Alam, S.Purwar. "Sliding Mode Control of Coupled Tanks System: Theory and an application". ISSN 2250-2459.ISO 9001:2008 Certified Journal.Volume 3.Issue 8.August 2013

[18] M.N.Mahyuddin, M.R.Arshad, Z.Mohamed. "Simulation of Direct Model Reference Adaptive Control on a Coupled-Tank System using Nonlinear Plant Model". International Conference on Control, Instrumentation and Mechatronics Engineering (CIM'07).2007

[19] M.F.Rahmat, S.M.Rozali "Modeling and Controller design for a Coupled-Tank Liquid Level system: Analysis & Comparison". Journal Teknologi (48)D. pp.113-141.Jun 2008

[20] M.S.M.Aras, M.F.Basar,N.Hasim,M.N.Kamaruddin,H.I.Jaafar "Development and Modeling of Water Tank System Using System Identification Method". International Journal of Engineering and Advanced Technology (IJEAT).pp.278-283.August 2013

[21] A.N.K.Nasir, M.A.Ahmad,M.F.Rahmat. "Performance Comparison Between LQR and PID controller for an Inverted Pendulum System". International Conference on Power Control and Optimization,Chiang May.Thailand. pp.18-20.July 2008

[22] J.Patra,P.S.Khuntia,S.Samal. "Analysis and Comparison of different performance index factor for Conventional PID & GA plus PID Controller". International Journal of Emerging Technologies in Computational and Applied Sciences (IJETCAS). pp.242-250. 2003

# FYP REPORT

ORIGINALITY REPORT

| 20% | 17% | 6% | % |
|---|---|---|---|
| SIMILARITY INDEX | INTERNET SOURCES | PUBLICATIONS | STUDENT PAPERS |

PRIMARY SOURCES

| | | |
|---|---|---|
| 1 | lrd.yahooapis.com<br>Internet Source | 3% |
| 2 | umpir.ump.edu.my<br>Internet Source | 3% |
| 3 | www.ipcsit.com<br>Internet Source | 2% |
| 4 | www.kri.com.sg<br>Internet Source | 2% |
| 5 | Rubiyah Yusof. "An alternative laboratory package for teaching control engineering", 2009 3rd IEEE International Conference on E-Learning in Industrial Electronics (ICELIE), 11/2009<br>Publication | 1% |
| 6 | Abbas, Hur, Sajjad Asghar, and Shahid Qamar. "Sliding Mode Control for Coupled-Tank Liquid Level Control System", 2012 10th International Conference on Frontiers of Information Technology, 2012.<br>Publication | 1% |

| | | |
|---|---|---|
| 7 | eprints.utm.my<br>Internet Source | 1% |
| 8 | www.ijetae.com<br>Internet Source | 1% |
| 9 | www.ijeat.org<br>Internet Source | 1% |
| 10 | www.ijettjournal.org<br>Internet Source | 1% |
| 11 | www.itu.dk<br>Internet Source | 1% |
| 12 | ijarcsse.com<br>Internet Source | 1% |
| 13 | Lal Bahadur Prasad. "Optimal control of nonlinear inverted pendulum dynamical system with disturbance input using PID controller & LQR", 2011 IEEE International Conference on Control System Computing and Engineering, 11/2011<br>Publication | 1% |
| 14 | urrg.eng.usm.my<br>Internet Source | 1% |
| 15 | Usta, M. Ali, Omur Akyazi, and A. Sefa Akpinar. "Aircraft roll control system using LQR and fuzzy logic controller", 2011 International Symposium on Innovations in Intelligent Systems and | 1% |

Applications, 2011.
Publication

| | | | |
|---|---|---|---|
| 16 | www.researchgate.net<br>Internet Source | 1% |

| | | | |
|---|---|---|---|
| EXCLUDE QUOTES | OFF | EXCLUDE MATCHES | < 1% |
| EXCLUDE BIBLIOGRAPHY | OFF | | |