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MODELING SIT TO STAND MOTION OF HUMANOID ROBOT USING TELESCOPIC INVERTED PENDULUM FOR PREDICTING STABLE MOTION

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A report submitted in partial fulfillment of the requirements for the degree of Bachelor of Mechatronic Engineering

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2013/2014

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I declare that this report entitle "Modeling Sit to Stand Motion of Humanoid Robot using Telescopic Inverted Pendulum for Predicting Stable Motion" is the result of my own research except as cited in the references. The report has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

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ABSTRACT

Sit to Stand (STS) motion is a challenging motion for any humanoid robot. Hence, development in humanoid robotics system is essential. In biomechanical fields, several models have been developed through observation of STS motion from human subjects. One of the models developed is based on telescopic inverted pendulum (TIP) concept which is an inverse kinematics concept. TIP model is the most suitable for STS trajectory generation since that TIP model focuses on motion of center of mass of human body in Cartesian space. However, the suitability of TIP model for representing the STS motion of humanoid robot is unknown. Furthermore, the resulting torque (Nm) from motion generated by TIP model has not been validated hence the accuracy of the robot when implementing the STS trajectory from the model is unknown. Therefore, the research on modeling STS motion of humanoid robot using TIP for predicting stable motion is proposed for the contribution to the development of humanoid robotics field as well as rehabilitation, prosthetic and exoskeleton robots. The objectives of this project is to model and validate sit to stand behavior using TIP model by simulation prove and to validate accuracy of TIP model in representing STS motion by comparing the output torque with three-link multi-segment robot. This project is carried out by MATLAB simulation in terms of the TIP model output and three-link multi-segment robot torque. The theoretical (based on three-link multi-segment robot) and actual (based on TIP model) torque (Nm) acted on COM are compared and analyze using statistical technique in terms of mean, percentage error and RMSE. Based on the analysis, there is high accuracy exist for TIP model in representing seat-off movement since the RMSE value is only 0.4207 and there is inaccurate for the seat-unloading phase. The deviation from actual value may be due to the model doesn't taking account of momentum components. Therefore, in future, these all parameters must be taken into account in order to increase the accuracy of TIP model in representing STS motion.

ABSTRAK

Pergerakan duduk dan berdiri (STS) merupakan pergerakan yang mencabar bagi semua jenis robot humanoid. Oleh itu, pembangunan dalam bidang robotik humanoid adalah penting. Dalam bidang-bidang biomekanik, beberapa model telah diciptakan melalui pemerhatian daripada pergerakan STS manusia. Salah satu model yang diciptakan adalah berdasarkan konsep teleskopik terbalik bandul (TIP) yang berkaitan dengan konsep kinematik songsang. Model TIP adalah antara yang paling sesuai untuk STS generasi trajektori kerana model ini menumpu kepada pergerakan pusat jisim (COM) tubuh manusia dalam ruang Cartesian. Walau bagaimanapun, kesesuaian model ini untuk mewakili pergerakan STS untuk robot humanoid masih tidak diketahui. Tambahan pula, torque daripada pergerakan yand terhasil oleh model TIP masih belum disahkan dan ketepatan robot apabila melaksanakan trajektori STS dari model itu tidak ketahui. Oleh itu, penyelidikan ke atas kestabilan pergerakan STS robot humanoid dijalankan untuk sumbangan kepada pembangunan bidang robotik humanoid. Objektif projek ini adalah untuk megesahkan kelakuan STS meggunakan model TIP dari segi simulasi dan eksperimen dan untuk megesahkan ketepatan model TIP dalam mewakili gerakan STS dengan membandingkan output torque dengan tiga-dimensi robot. Projek ini dilaksanakan dengan simulasi MATLAB dari segi output model TIP dan tiga-dimensi robot torque. Nilai torque teori (berdasarkan tiga-dimensi robot) dan nilai torque sebenar (berdasarkan model TIP) bertindak pada COM dibandingkan dan dianalisis dengan penggunaan kaedah statistic dari segi mean, peratusan error dan RMSE. Berdasarkan analisis, ketepatan untuk model TIP mewakili pergerakan bangun dari kerusi adalah tinggi dengan RMSE yang bernilai 0.4207 tetapi bagi pergerakan masa duduk, ketepatan adalah rendah. Sisihan dari nilai sebenar mungkin disebabkan oleh model TIP tidak mengambil kira komponen momentum. Oleh itu, pada masa akan dating, semua parameter tersebut mesti diambil kira untuk meningkatkan ketepatan model TIP dalam mewakili gerakan STS.

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CHAPTER 1

INTRODUCTION

The first chapter includes motivation, problem statement (involving research questions and hypothesis), project objectives, scope of the project, list of contribution of this project in the development of humanoid robotics field and outline of the dissertation.

1.1 Motivation

An excellent robot including humanoid robot should be able to perform anthropomorphic actions such as walking, running, jumping as well as sitting and standing up from a chair. The motion of sitting and standing up from a chair seems to be easier and is our common routine everyday but actually it is a challenging motion for elderly people and those with mobility disorders. The humanoid robots also facing difficulty in performing sit to stand (STS) motion since that they unable to maintain balance while performing the actions, unless they bolted to the floor.

The study related to stability of STS motion of humanoid robots is important for contribution in robotics field and preventing the robot from falling down when it moves from a large support position to much smaller one above the feet. From the research [3], precise COM control is a key to be successful in performing the STS motion. As the center of mass (COM) position is lower, its stability is increase. Hence, in this paper, we are modeling the stable STS motion of humanoid robot by investigating the displacement of COM position and couple vector using telescopic inverted pendulum model to prevent damage on a costly humanoid robot.

1.2 Problem Statement

Sit to Stand (STS) is a challenging motion for any humanoid robot. Hence, development in humanoid robotics system is essential. In biomechanical field, several models have been developed through observation of STS motion from human subjects. One of the models developed is based on telescopic inverted pendulum (TIP) concept which is an inverse kinematics concept. TIP is the most suitable for STS trajectory generation since that TIP focuses on motion of center of mass of human body in Cartesian space. However, the suitability of using the TIP model for humanoid STS motion is unknown. Furthermore, the resulting torque (Nm) from motion generated by TIP model has not been validated hence the accuracy of the robot when implementing the STS trajectory from the model is unknown.

1.2.1 Research Questions

- 1. Does the telescopic inverted pendulum model output suitable used to describe the behavior of STS motion of humanoid robots?
- 2. What are the relationship between the torque (Nm) and the TIP equation? Does the relationship predicted from TIP model able to stabilize and balance a humanoid robot when applying the STS trajectory?
- 3. How the cubic polynomial profile could be used to predict the STS trajectory?
- 4. What are the relationship between the torque (Nm) and the parameters of cubic polynomial profile?
- 5. How much the percentage of accuracy of TIP model in representing STS motion by making comparison with three-link multi-segment robot?

1.2.2 Hypothesis

- 1. The couple vectors predicted by the telescopic inverted pendulum model for STS trajectory profile will be the same with the couple vector produce by humanoid robot.
- 2. The torque magnitude and position of humanoid robots could be correlated to the couple vector in the TIP model.
- 3. The mathematical models created from TIP model will be able to apply in a humanoid robot for STS stability purpose although with different mass and configuration in any sitting condition.

1.3 Project Objectives

- To model and validate sit to stand behavior using telescopic inverted pendulum model by simulation prove.
- To validate accuracy of TIP model in representing STS motion by comparing the output torque with three-link multi-segment robot.

1.4 Scope of the Project

- i. The research is focused on model and validates sit to stand motion using telescopic inverted pendulum model.
- ii. The displacement of COM position is referred to the available sources.
- iii. The sit to stand motion is performed in selected configuration of sitting condition including height of seated position.
- iv. The performance of TIP model is discussed in terms of accuracy.
- v. The simulation and analysis is done by MATLAB software.
- vi. The results predicting from TIP model in terms of torque is compared with the output torque obtained from three-link multi-segment humanoid robot for accuracy estimation.

1.5 List of Contribution

Studies in STS contribute much in the development of humanoid robotics field as well as rehabilitation, prosthetic and exoskeleton robots. The development of robotics encourages the good impact to society, economy and nation. However, in Malaysia, the study related to robotics field only begins recently and thus require more attention and bring toward successful.

(a) Contribution to Society

Innovation of humanoid robots and exoskeleton robots will contribute to the improvement of working lifestyle in hazardous, dirty or toxic environments. The robots can be assist humans from performing dangerous tasks in unsatisfactory condition, repeated and demeaning but is compulsory for manufacturing. Furthermore, the research will also assist paraplegic or mobility disorders person in the development of rehabilitation and exoskeleton robots to enhance their sit to stand abilities and assist them to be a normal human.

(b) Contribution to Economy

The application of robotics in manufacturing sector will enhance Malaysia economy by increasing the efficiency of productivity. The automation technique enable the manufacturing carried out continuously since the robots able to perform repeated, hard and boring tasks for a long period. Besides that, the expert of Malaysia in robotics field can encourage the investment of foreigner and hence boost the growth of economy. Malaysia can also provide healthcare service of the robots.

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(c) Contribution to Nation

The exposure to robotics field will strengthen Malaysia technological capability and hence enhance Malaysia's reputation and put Malaysia to be in the same league with nation from developed countries such as Japan, United States and so on. This will lead to increasing of nation morale by having a good achievement to be proud of. In addition, foreign investors will realize Malaysia is a potential country and hence bring Malaysia for national development.

1.6 Outline of the Dissertation

- i. Chapter 1 describes engineering problem designated and goals to be achieved as well as limitations of the research work.
- ii. Chapter 2 describes published information related to sit to stand motion and performance indices used for evaluation of the model.
- iii. Chapter 3 describes method designed to evaluate the performance of sit to stand model in terms of accuracy.
- iv. Chapter 4 describes the findings obtained and analysis using statistical techniques as well as the interpretation of the result obtained with proof.
- v. Chapter 5 concludes the findings and describes the future research.

CHAPTER 2

THEORETICAL BACKGROUND AND LITERATURE REVIEW

This chapter highlights past studies related to sit to stand (STS) motion and also theoretical background that is necessary for the development of a stable STS motion trajectory of humanoid robot using telescopic inverted pendulum (TIP) model. There are few parts divided in this chapter. The first part focuses on the center of mass (COM) trajectory of STS motion. Accurate knowledge of robot COM position throughout the trajectory is important in planning a stable STS motion without falling. The second part is about the robotics background that is required for generating COM trajectory in developing a successful STS motion trajectory. The third part is followed by the design parameters involved in this STS research and the forth part is about the performance indices used to measure the performance of the TIP system generated. The fifth part highlights the comparison among the available solutions developed for representing STS motion. At last, this chapter is ended by summarization of the past studies (gap of knowledge).

2.1 Center of Mass (COM) Trajectory for STS Motion

Center of mass (COM) has been considered as an essential point for humanoid robot motion. A consistent COM trajectory allows the performing of a stable sit to stand motion. The COM trajectory tracks a path of COM through a sagittal plane of the human demonstrator. Figure 2.1 describes a human demonstration of a sit to stand action and a corresponding humanoid robot sit to stand action based on the human demonstration, according to one embodiment of the invention. As shown in the Figure 2.1, the COM of a human demonstrator and a humanoid robot varies from one support position (seated) to another (standing). The human demonstrator 101 starts in a seated position 100 on a chair 104a. In the seated position 100, the COM 103a of the human demonstrator 101 is above the chair 104a. When the human demonstrator 101 stands at 110 and 120 respectively, the COM 103a of the human demonstrator 101 lowers slightly to 103b, and then increases to 103c. In the standing position at 130, the COM 103a of the human demonstrator 101 is located at 103d, directly above the feet of the human demonstrator 101.

The humanoid robot 151 sit to stand action is emulating the COM trajectory of human demonstrator 101 so that it won't fall over. The identity of the COM trajectory 107a of the human demonstrator 101 and the COM trajectory 107b of the humanoid robot 151 allows the maintenance of sit to stand action in stable condition. [18]

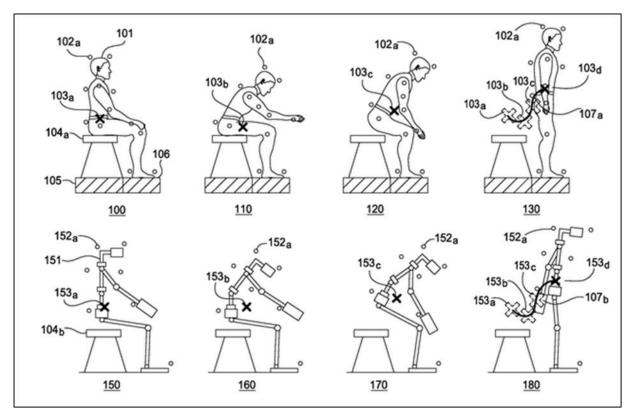


Figure 2.1: Procedure for generating humanoid robot STS motion trajectory from human demonstration [18]

2.2 Robotics Background for STS Motion

In this sub-section, theoretical robotics background that is required to develop a successful STS motion of humanoid robot is detailed up. The first part mentions about the trajectory generation that is required for creating COM trajectory profile of a stable STS motion using TIP model. The second part details out the inverse kinematics concept that is required for calculating the angle of hip, knee and ankle of a three-link humanoid robot in performing STS motion by considering the COM position. The third part focuses on the manipulator dynamic theorem that is needed to study the effect of force in developing a stable STS motion. The last part is an overview of system block diagram that shows the relationship between STS motion trajectory generated by TIP model and humanoid robot.

2.2.1 Trajectory Generation

Trajectory generation is related to the computation of desired motion of a manipulator to be smooth in multidimensional space. Trajectory refers to a time function of position, velocity, and acceleration for each degree of freedom. There are two methods to specify a trajectory or path through space which are joint-space schemes and Cartesian-space schemes. Joint-space schemes are defined as a path generation in which the path shapes (in space and in time) are described in terms of functions of joint angles while Cartesian-space schemes are described in terms of functions of cartesian coordinates.

Planning in joint-space schemes are the simplest and easiest ways to compute compared with Cartesian-space schemes since that each joint motion is calculated independently from other joints, there is basically no problem with singularities of the mechanism. Joint-space schemes are using application of cubic polynomials concept. Theoretically, position, velocity, and acceleration profile of the joint can be calculated using the equations of cubic polynomials as shown in the following:



Position:
$$\theta(t) = \theta_0 + \frac{3}{t_f^2} \left(\theta_f - \theta_0\right) t_2 - \frac{2}{t_f^3} \left(\theta_f - \theta_0\right) t^3$$
(2.1)

Velocity:
$$\dot{\theta}(t) = \frac{6}{t_f^2} \left(\theta_f - \theta_0\right) t - \frac{6}{t_f^3} \left(\theta_f - \theta_0\right) t^2$$
(2.2)

Acceleration: $\ddot{\theta}(t) = \frac{6}{t_f^2} (\theta_f - \theta_0) - \frac{12}{t_f^3} (\theta_f - \theta_0) t$

(2.3)

Where θ_0 = initial angle of the motion, θ_f = final angle of the motion, t_f = rising time of the motion, and t = variable time domain. [19, pp.201-225]

2.2.2 Inverse Kinematic Theorem

as:

Imagine a scenario of a robot that wants to perform STS motion regarding the current seat position. In order to stand up, the robot needs to bring its end effectors away from the seat. Given the whole body COM position of standing position, the robot needs to calculate the angles of each of its joint. Consider the three-link planar manipulator is introduced to solve the inverse kinematic solution. [19, pp.101-127]

The kinematic equation of the 3DOF robot is given as:

$${}^{B}_{W}T = {}^{0}_{3}T = \begin{bmatrix} c_{123} & -s_{123} & 0 & l_{1}c_{1} + l_{2}c_{12} \\ s_{123} & c_{123} & 0 & l_{1}s_{1} + l_{2}s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note: $c_{123} = \cos(\theta_1 + \theta_2 + \theta_3)$ and $s_{123} = \sin(\theta_1 + \theta_2 + \theta_3)$

The orientation and the position of the goal point with respect to the base are given

$${}^{B}_{W}T = \begin{bmatrix} c_{\phi} & -s_{\phi} & 0 & x \\ s_{\phi} & c_{\phi} & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Given that:

$$c_{\Phi} = c_{123}$$
 (2.4)

$$s_{\Phi} = s_{123}$$
 (2.5)

$$x = l_1 c_1 + l_2 c_{12} \tag{2.6}$$

$$y = l_1 s_1 + l_2 s_{12} \tag{2.7}$$

Square equation (2.6) and (2.7) and then add the result, equation (2.8) is obtained.

$$x^{2} + y^{2} = l_{1}^{2} + l_{2}^{2} + 2l_{1}l_{2}c_{2}$$
(2.8)

$$c_2 = \frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1 l_2} \tag{2.9}$$

Assuming the goal is in the workspace, s_2 is expressed as

$$s_2 = \pm \sqrt{1 - c_2^2} \tag{2.10}$$

Finally

$$\theta_2 = Atan2(s_2, c_2) \tag{2.11}$$

Having found θ_2 , equation (2.6) and (2.7) can be solved for θ_1 . Rewrite equation (2.6) and (2.7) in the form

$$x = k_1 c_1 - k_2 s_1 \tag{2.12}$$

$$y = k_1 s_1 + k_2 c_1 \tag{2.13}$$

Where

$$k_1 = l_1 + l_2 c_2 \tag{2.14}$$

$$k_2 = l_2 s_2 \tag{2.15}$$

In order to solve an equation of this form, a change of variables are performed.

$$r = +\sqrt{k_1^2 + k_2^2} \tag{2.16}$$

$$\gamma = Atan2(k_2, k_1) \tag{2.17}$$

$$k_1 = r \cos \gamma \tag{2.18}$$

$$k_2 = rsin\gamma \tag{2.19}$$

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Equation (2.14) and (2.15) can now be written as:

$$\frac{x}{r} = \cos\gamma\cos\theta_1 - \sin\gamma\sin\theta_1 \tag{2.20}$$

$$\frac{y}{r} = \cos\gamma\sin\theta_1 + \sin\gamma\cos\theta_1 \tag{2.21}$$

Rearranging those gives:

$$\cos(\gamma + \theta_1) = \frac{x}{r} \tag{2.22}$$

$$\sin(\gamma + \theta_1) = \frac{y}{r} \tag{2.23}$$

Using the arctangent we get:

$$\gamma + \theta_1 = Atan2\left(\frac{y}{r}, \frac{x}{r}\right) = Atan2(y, x)$$
(2.24)

$$\theta_1 = Atan^2(y, x) - Atan^2(k_2, k_1)$$
(2.25)

$$\theta_1 + \theta_2 + \theta_3 = Atan2(s_{\phi}, c_{\phi}) = \Phi$$
(2.26)

Finally

$$\theta_3 = \Phi - \theta_1 - \theta_2 \tag{2.27}$$

2.2.3 Manipulator Dynamics

The external forces or torques required for the STS motion of a manipulator can be described using Newton-Euler equations. If the linear and angular accelerations of the mass center of each link are available, then Newton-Euler equations as followed can be applied to compute the inertial force and torque acting at the center of mass of each link.

$$F_i = m \dot{v}_{C_i} \tag{2.28}$$

$$N_i = {}^{C_i} I \dot{\omega}_i + \omega_i \times {}^{C_i} I \omega_i \tag{2.29}$$

Where $\{C_i\}$ has its origin at the center of mass of the link and has the same orientation as the link frame, $\{i\}$. [19, pp. 165-192]

2.2.4 System Block Diagram

Figure 2.2 shows the relationship between the STS motion trajectory generator and the physical NAO humanoid robot. From the beginning, the desired angles will act as a source to the controller. A vector of joint torques, τ from the controller is then received by the robot. The manipulator's sensors in the block diagram allow the controller to read the vectors of joint angles. The feedback then is used to compute any error by finding the difference between the desired and the actual angles. As a result, the controller can compute actuator torques required to reduce the errors. [19, pp. 262-285]

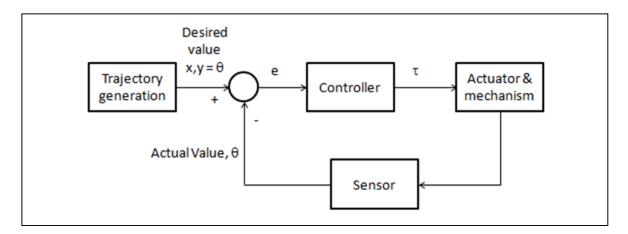


Figure 2.2: General block diagram of STS motion