NUMERICAL MODELLING OF

SIMPLE HARMONIC OSCILLATOR

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NUMERICAL MODELLING OF SIMPLE HARMONIC OSCILLATOR

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DECLARATION

"I hereby declare that the work in this report is my own except for summaries and quotations which have been duly acknowledged"

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ABSTRACT

Numerical method is a technique whereby mathematical problems are formulated so that they can be solved with arithmetic and logical operations. Harmonic oscillator is a physical system that is bound to a position of stable equilibrium by a restoring force or torque proportional to the linear or angular displacement from this position. If such a body is disturbed from its equilibrium position and released, and if damping can be neglected, the resulting vibration will be simple harmonic motion, with no overtones. The behaviour of the simple harmonic oscillator will change whenever the parameters involved are changed and the prediction of the changes can be predicted using MATLAB software. The calculation of the simple harmonic motion can be obtained from finite difference scheme equation. When the equation of finite difference scheme of simple harmonic oscillator is obtained, the parameters and equation are keyed in into MATLAB language. The parameters related are changed based on certain range and the graph simple harmonic oscillator is analyzed and the behaviour of simple harmonic oscillation is obtained. In this project, the program had been completed and verified by using MATLAB software.

ABSTRAK

Kaedah berangka adalah salah satu teknik penyelesaian masalah matematik yang digubal untuk menyelesaikan pemasalahan aritmetik dengan menggunakan operasi logik ringkas. Pengayun harmonik adalah sistem fizikal yang dikawal kedudukan keseimbangannya yang stabil dengan daya ataupun daya kilas yang berkadar dengan kadar sesaran linear ataupun sudut dari kedudukan asal. APabila sesuatu badan terganggu dari kedudukan seimbangnya lalu dilepaskan serta sekiranya redaman diabaikan, getaran yang terhasil akan menjadi pergerakan harmonik mudah tanpa redaman. Tingkah laku pengayun harmonik mudah akan berubah apabila parameter yang terlibat diubah. Keadaan perubahan tersebut boleh dilakukan menggunakan pengiraan yang dimasukkan di dalam perisian MATLAB. Parameter yang digunakan dalam jalankerja pengiraan diperolehi daipada persamaan skim pembezaan terhingga. Apabila persamaan skim pembeza terhingga pengayun harmonik mudah telah didapati, parameter dan persamaan dimasukkan ke dalam bahasa MATLAB. Parameter yang berkaitan ditukar berdasarkan pelbagai nilai tertentu dan graf pengayun harmonik mudah dianalisa dan tingkah laku ayunan harmonik mudah diperolehi. Dalam projek ini, program penuh telah dihasilkan dan disahkan meggunakan perisiam MATLAB.

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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

A model is a representation of the construction and working of some system of interest. A model is similar to but simpler than the system it represents. One purpose of a model is to enable the analyst to predict the effect of changes to the system.

On the one hand, a model should be a close approximation to the real system and incorporate most of its salient features. A good model is a judicious trade off between realism and simplicity.

As written by Agarwal (2011), Mathematical and Computer Modelling provides a medium of exchange for the diverse disciplines utilizing mathematical or computer modelling as either a theoretical or working tool.

A simulation of a system is the operation of a model of the system. The model can be reconfigured and experimented. Usually, this is impossible, too expensive or impractical to do in the system it represents. Simulation is used before an existing system is altered or a new system built, to reduce the chances of failure to meet specifications, to eliminate unforeseen bottlenecks, to prevent under or over-utilization of resources, and to optimize system performance, Claude (2012).

Based on the process of simulations, there are three types of models (Hensen, 2012):

1) Analytical/empirical models

Analytical and empirical based models are the closed form solution. These mathematical expressions may be derived based on experimental data, laws of physic and statistics. The advantage of the analytical types of models is they are easy to implement into standard engineering tools or spreadsheets.

2) Experimental models

Experimental models may either address the entire application or may be a model for a small part of the system. The main advantage of experimental models is that they yield results close to the real world application. However, it must be pointed out that the result generated from an experimental model is derived through the means of subjecting a simplified structure that corresponds to a certain characteristic of the application with a set of controlled parameters.

3) Numerical models

Numerical models technique is the youngest one of the three types of models. Numerical models are often the type of models that are dealt with as 'simulations', since numerical models are often implemented into expensive expert tools where the process from initial input to advanced colour plots of the solution is fairly incomprehensible . One of the main advantages in the use of numerical modelling is studies and optimization techniques are easy to apply.

Results derived from all types of models may be used as a part of a simulation process and the results from the models may be used to enhance the outcome of a different type of model. An overview of the relations between the types of models and simulation is seen in Figure 1.1.

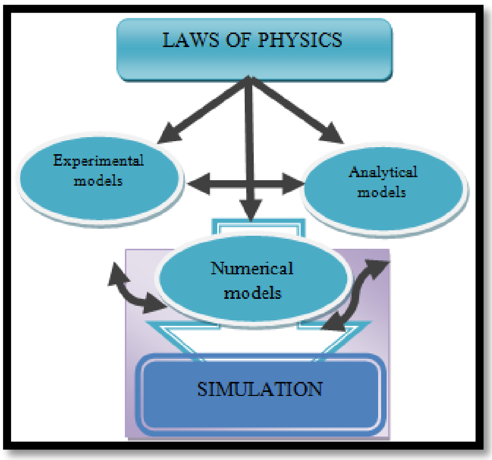

Figure 1.1: Relationship between Types of Models and Simulation

1.2 SIMPLE HARMONIC OSCILLATOR

Michael (2009) explained that a simple harmonic oscillator (SHO) is a mass connected to some elastic object of negligible mass that is fixed at the other end and constrained so that it may only move in one dimension. This simplified model approximates many systems that vibrate or oscillate: drum heads, guitar strings, and the quantum mechanical descriptions of an atom. The differential equation describing a simple harmonic equation is as below:

$$\frac{d^2x}{dt^2} = -\omega_o^2 x$$

or equivalently

$$\left(\frac{d^2}{dt^2} + \omega_0^2\right) x(t) = 0$$

Here, ω_o is the natural frequency of the oscillator and $x(t)$ define its position. The second order equation can be broken into two first order equations as follows:

$$\frac{dx}{dt} = v$$

$$\frac{dv}{dt} = -\omega^2 x$$

To develop the idea of a harmonic oscillator the most common example of harmonic oscillation is a mass on a spring. For a given spring with constant k, the spring always puts a force on the mass to return it to the equilibrium position. Recall also that the magnitude of this force is always given by:

$$F(x) = -kx$$

where the equilibrium point is denoted by $x = 0$. In other words, the more the spring is stretched or compressed, the harder the spring pushes to return the block to its equilibrium position.

This equation is only valid if there is no other force acting on the block. If there is friction between the block and the ground, or air resistance, the motion is not simple harmonic, and the force on the block cannot be described by the previous equation.

Provided below by Douglas (2003) are some examples of simple oscillator

1) A mass on a spring

Figure 1.2 shows that when the spring is compressed, it pushes back on the mass. When the spring is extended, it pulls on the mass. The system is an oscillator because the push-pull of the spring is a restoring force and the mass supplies the inertia. An example of such system is a car (mass) and its shock absorbers (springs). Along with springs, shock absorbers also have high friction dampers that quickly slow any oscillation down.

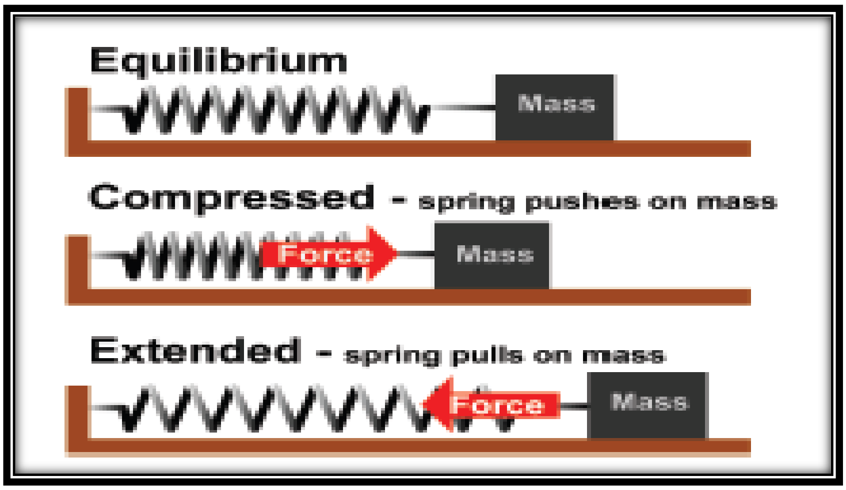

Figure 1.2: Mass on Spring during Extension and Compression

2) A vibrating string

Figure 1.3 shows Rubber band stretched between two rods. If the middle of the rubber band is pulled to the side, it will move back toward being straight when it is released. Stretching the rubber band to the side creates a restoring force. When the rubber band is released, inertia carries it past being straight and it vibrates. Vibrating strings tend to move much faster than springs and pendulums.

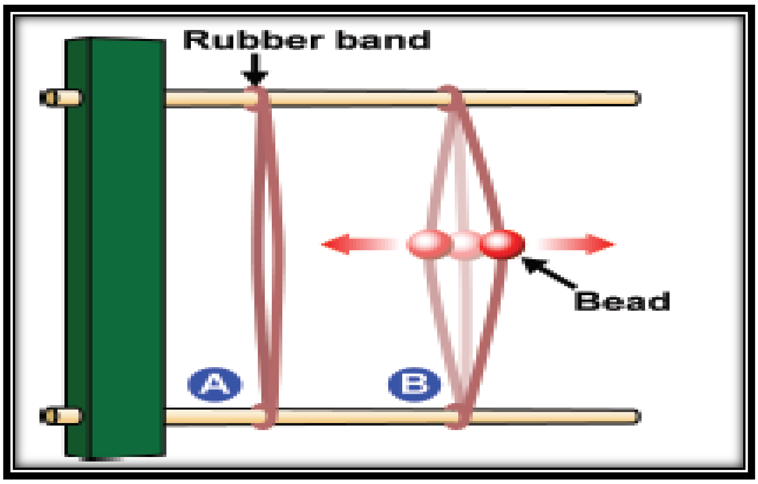

Figure 1.3: A Stretched Rubber Band

1.3 PROBLEM STATEMENT

Simple harmonic motion (SHM) is a type of periodic motion where the restoring force is directly proportional to the displacement. Simple harmonic motion can be easily understood via aid of SHM graph as shown in Figure 1.4. It often happen in any situation such as a system consisting of a mass attached to a spring, a molecule inside a solid, and a swing on a playground. SHM depends on many parameters such as displacement, velocity and frequency. Whenever any of these parameters change, the behaviour and characteristic of simple harmonic oscillator will also change. The prediction of this change will assist in many system designs. This can be achieved by using numerical modelling.

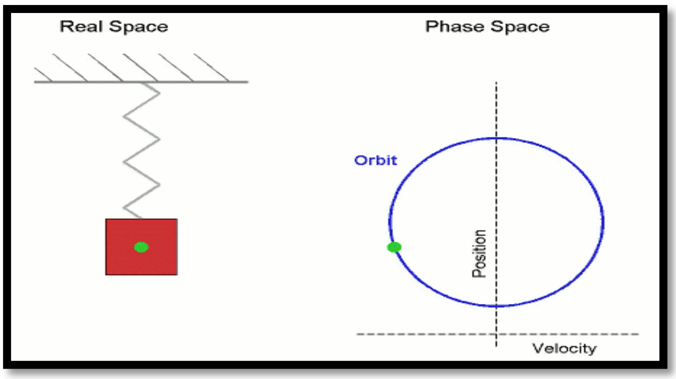

Figure 1.4: Simple Harmonic Oscillation

1.4 OBJECTIVES OF PROJECT

The objectives of the project are as follows:

1. To establish a clear understanding of the behaviour of simple harmonic oscillator through numerical modelling.
2. To investigate the characteristics of simple harmonic oscillator through the variation of its parameters.

1.5 SCOPE OF PROJECT

All simulations are performed using MATLAB. The field of study is related to physics and mechanical vibrations. Most cases related to this study can be seen in daily life such as swing, spring system and grandfather's clock.

CHAPTER 2

LITERATURE REVIEW

2.1 OSCILLATION AND WAVES

Referring to deduction of Zamora (2006), oscillation is the repetitive variation, typically in time, of some measure about a central value or between two or more different states as shown in Figure 2.1. The term vibration is sometimes used more narrowly to mean a mechanical oscillation but sometimes is used to be synonymous with oscillation. Oscillation occurs not only in physical system but also in biological system and in human society. In mathematics, oscillation quantifies the amount which a sequence or function tends to move between extremes

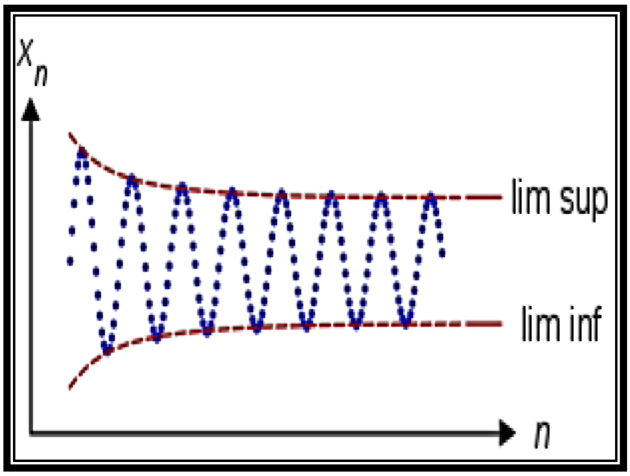

Figure 2.1: Oscillation of a Sequence; Difference between the Limit Superior and Limit Inferior of the Sequence.

There are several types of oscillation and these are:

1) Simple harmonic motion

The simplest mechanical oscillating system is a mass attached to a linear spring subject to no other forces. Such a system may be approximated on an air table or ice surface. The system is in an equilibrium state when the spring is static. If the system is displaced from the equilibrium, there is a net restoring force on the mass, tending to bring it back to equilibrium. If a constant force such as gravity is added to the system, the point of equilibrium is shifted. The time taken for an oscillation to occur is often referred to as the oscillatory period. In the spring-mass system, oscillations occur because, at the static equilibrium displacement, the mass has kinetic energy which is converted into potential energy stored in the spring at the extremes of its path.

2) Damped and driven oscillation

All real-world oscillator systems are thermodynamically irreversible. This means there are dissipative processes such as friction or electrical resistance which continually convert some of the energy stored in the oscillator into heat in the environment. This is called damping. For example of driven oscillation , the phenomenon of flutter in aerodynamics occurs when an arbitrarily small displacement of an aircraft wing (from its equilibrium) results in an increase in the angle of attack of the wing on the air flow and a consequential increase in lift coefficient, leading to a still greater displacement. Another example is a child plays a swing on a playground. To start swinging, the child will push herself backward using her leg and release it. After stay swinging for a few swings, the swing had become slow and the child will push herself again. Hence, the swing continues to swing. In this phenomenon, it can be said that the child's legs are some driven force applied (driven) to the system and the movements of swing become slower due to the damped phenomenon.

3) Coupled oscillation

The harmonic oscillator and the systems it models have a single degree of freedom. More complicated systems have more degrees of freedom, for example two masses and three springs. For example, two pendulum clocks (of identical frequency) mounted on a common wall will tend to synchronise. Well-known is the Wilberforce pendulum as shown in Figure 1.3, where the oscillation alternates between an elongation of a vertical spring and the rotation of an object at the end of that spring. Two pendulums with the same period fixed on a string act as pair of coupled oscillators. The oscillation alternates between the two.

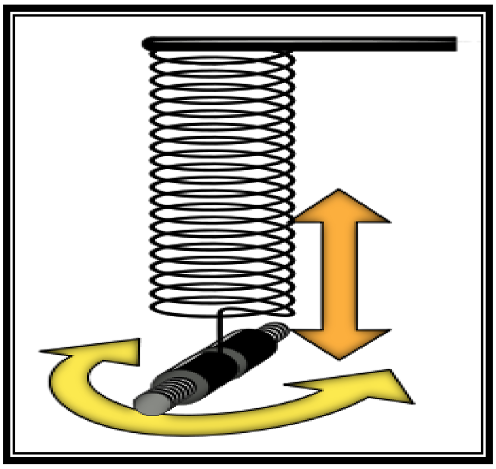

Figure 2.2: A Wilberforce Pendulum Alternates between Two Oscillation Modes

An oscillation is a disturbance in a physical that is repetitive in time. A wave is a disturbance in an extended physical system that is both repetitive in time and periodic in space. In general, an oscillation involves a continuous back and forth flows of energy between two different types of energy which are kinetic and potential energy, for example in the case of a pendulum(Fitzpatrick, 2000). A wave involves similar repetitive energy flows to an oscillation but in addition, is capable of transmitting energy and information from place to place.

One of the examples of simple harmonic oscillation is a mass on a spring (Figure 2.3). Consider a compact mass m which slides over a frictionless horizontal surface. Suppose that the mass is attached to one end of a light horizontal spring whose other end is anchored in an immovable wall. At time *t*, let *x (t)* be the extension of the spring, the difference between the spring's actual length and its unstretched length. Obviously, *x (t)* can also be used as a coordinate to determine the instantaneous horizontal displacement of the mass. The equilibrium state of the system corresponds to the situation in which the mass is at rest, and the spring is unextended. In this state, zero horizontal force acts on the mass, and so there is no reason for it to start to move. However, if the system is perturbed from its equilibrium state (if the mass is displaced, so that the spring becomes extended) then the mass experiences a horizontal restoring force given by Hooke's law:

$$f(x) = -kx$$

Here, k > 0 is the so-called force constant of the spring. The negative sign indicates that f(x) is indeed a restoring force. Hooke's law only holds for relatively small spring extensions. Hence, the displacement of the mass cannot be made too large. Incidentally, the motion of this particular dynamical system is representative of the motion of a wide variety of mechanical systems when they are slightly disturbed from a stable equilibrium state. Newton's second law of motion gives following time evolution equation for the system:

$$m\ddot{x} = -kx$$

Where $\ddot{x} = {d^2x}/{dt^2}$ .This differential equation is known as the simple harmonic oscillator equation and its solution has been known for centuries. The solution is:

$$x(t) = a\ cos\ (\omega t - \varphi)$$

$$\omega = \sqrt{\frac{k}{m}}$$

where; ω is angular frequency and φ is phase constant.

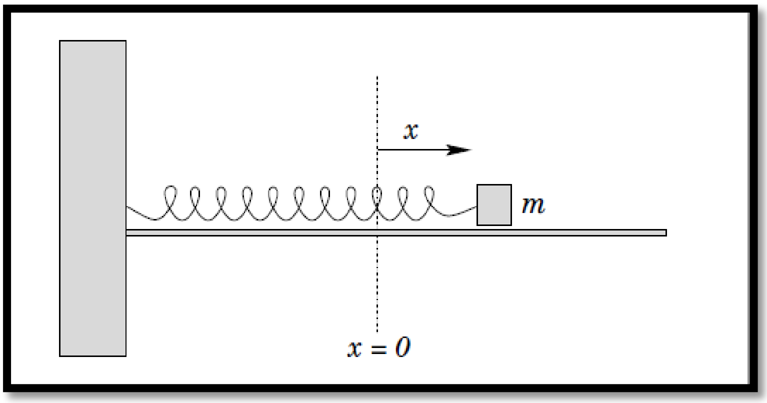

Figure 2.3: Mass on a Spring

The type of behaviour shown in Figure 2.4 is called simple harmonic oscillation. Fitzpatrick (2012) explained that it can be seen that the displacement x