NUMERICAL MODELING OF 1-DIMENSIONAL WAVE EQUATION USING FINITE MODAL SYNTHESIS

SYAHIDATUL SYAHIRA BINTI ABDUL RAHMAN

UNIVERSITI TEKNIKALMALAYSIA MELAKA

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"I hereby declare that I have read this thesis and in my opinion this report is sufficient in terms of scope and quality for the award of the degree of Bachelor of Mechanical Engineering (Structure and Material)"

Signature: ……………………

Supervisor: ……………………

Date: ……………………

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SYAHIDATUL SYAHIRA BINTI ABDUL RAHMAN

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Faculty of Mechanical Engineering

Universiti Teknikal Malaysia Melaka

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DECLARATION

“I hereby declare that the work in this report is my own except for summaries and quotations which have been duly acknowledgement.”

Signature: ……………………….

Author: ……………………….

Date: ……………………….

Special for Mom and Dad Loved

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ABSTRACT

This project is to perform numerical modelling of behavior of 1-dimensional wave equation and investigate the behavior through the variation of the system's parameters. The study of wave is useful nowadays to improve world achievement in order to settle down problems regarding physical phenomena. This is because some physical phenomena happened are based on the principles of wave motion. For information, the simplest example of physical phenomena is a vibrating string. From this project the 1-dimensional wave equation needed to be study using Finite Modal Synthesis method to model the wave equation in the form of numerical modeling. Finite Modal Synthesis is a real-time synthesis as the stimulus of the (virtual) objects before they occur. The vibrating object is modeled by a bank of damped harmonic oscillators which are excited by an external stimulus. The frequency and damping of the oscillators are determined by the geometry and material properties, while the coupling gains are determined by the location of the force applied to the object. This method is a linear partial differential equation for a vibrating system with its boundary conditions. The advantages of this method are the feasibility of analysis of each component separately and it simplifies the test and analysis of the models. Hence, all the parameters that needed to simulate the equation are identified using Matlab. All the parameters needed to be understood more in effort to relate with the method used. Then the selected parameters are manipulated to study and discuss the behavior of the wave.

ABSTRAK

Projek ini adalah untuk melaksanakan pemodelan berangka kelakuan persamaan gelombang 1-dimensi dan menyiasat kelakuan ini melalui perubahan parameter sistem. Kajian gelombang adalah berguna pada hari ini untuk meningkatkan pencapaian dunia untuk menyelesaikan masalah mengenai fenomena fizikal. Ini adalah kerana sebahagian fenomena fizikal berlaku berdasarkan prinsip gerakan gelombang. Untuk maklumat, contoh yang paling mudah fenomena fizikal adalah getaran tali. Dari projek ini persamaan gelombang 1-dimensi perlu kajian dengan menggunakan kaedah Sintesis Bermod Terhingga untuk model persamaan gelombang dalam permodelan berangka. Sintesis Bermod Terhingga adalah sintesis masa sebenar sebagai rangsangan objek (maya) sebelum ia berlaku. Objek bergetar dimodelkan oleh sebuah struktur pengayun harmonik teredam yang teruja dengan rangsangan luar. Kekerapan dan redaman pengayun ditentukan oleh geometri dan sifat bahan, manakala gandaan gandingan ditentukan oleh lokasi daya yang dikenakan kepada objek. Kaedah ini adalah persamaan pembezaan linear separa untuk sistem yang bergetar dengan syarat sempadan. Kelebihan kaedah ini adalah kebolehlaksanaan analisis setiap komponen secara berasingan dan ia mempermudahkan ujian dan analisis model. Oleh itu, semua parameter yang perlu untuk mensimulasikan persamaan yang dikenal pasti dengan mengekalkan latihan menggunakan Matlab. Semua parameter perlu difahami dengan lebih dalam usaha untuk mengaitkan dengan kaedah yang digunakan. Kemudian parameter yang terpilih dimanipulasikan untuk mengkaji dan membincangkan perilaku gelombang tersebut.

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CHAPTER 1

INTRODUCTION

1.1 Introduction to Project

Wave is a vibrating source that periodically disturbs the first particle of a medium. This produces a wave pattern which travels along the medium. The frequency of vibrating particles is equal to the frequency of source vibration. This forms a wave equation. The wave equation can vary to situations which one of it is a one-dimensional wave equation. The wave equation can be modeled as numerical modeling where it undergoes computer simulation. In this project behavior of the wave equation is determined by using finite modal synthesis. Finite modal synthesis is a method where it divides the equation into several substructures of a complex structure that reduced the modal bases will be grouped and synthesized as the given modal base of the original system. One of the useful advantages of this method is the feasibility of analysis of each component separately. It also simplifies the test and analysis of the models.

1.2 Problem Statement

The study of wave is important as many physical phenomena are based on the principles of wave motion. All forms of wave are associated with the transport of energy. The wave carries energy when travels from one point to another without transporting the material particles. The wave equation numerical simulation is needed to increase the understanding of wave equation. With this understanding, many useful applied actions can be achieved such as to act against tsunamis, assist warning system, assist building of harbors protection which to break waters, recognize critical coastal areas as need to move population, help to detect earthquake and hindcast historical tsunamis which assist geologies.

1.3 Objectives of Project

There are two main objectives goes to achieve by this project. The first target is to perform numerical modeling of the behavior of 1-dimensional wave equation using finite modal synthesis. Next is to investigate the behavior of 1-dimensional wave through the variation of the system's parameters.

1.4 Scope of Project

The there are four related scope that will be discuss in this project which are 1-dimensional wave equation focused on vibrating string. Vibrating string is one of the 1-

dimensional physical phenomena of wave. Besides, the numerical modeling is developed through Matlab. The development is based on program writing or coding. The Matlab software is function to analyze data, develop algorithm and create models. The method to simulate the equation is performed by using finite modal synthesis method which one of the physical models method.

CHAPTER 2

LITERATURE REVIEW

2.1 Numerical Modeling

Nowadays it is necessary to find, calculate and test scenarios mathematically, in order to predict what will happen in a given situation. Numerical modeling is an optimum method that can be used especially to visualize the dynamic behavior of physical systems. Numerical modeling can be explained as mathematical models that use some kind of numerical time step procedure to obtain the models behavior over time. There are some advantages of numerical solution over analytical solution those are the equations are much more intuitive and it is easier to understand the meaning of the equation.

Mathematical modeling computer simulations made shown as a useful part of it for many natural systems in physics. The simulation explains about the process of the model system. The mathematical modeling is good in develop new technology and estimation on the performance of complex systems for analytical solutions.

Computer simulations are great at reflect scenarios and comparing it theoretically. There are three consequences that need to be followed in order to produce simulation model which are calibration, verification and validation. For calibration it can be obtained through adjusting any available parameters to adjust the model's operation in the process. Next, to confirm that it can be done with the data output from the model and compare it with those projected from the input data. Then the last step required to confirm the model through comparing the results with of what prediction is based on the scope of study historical data [13]. It is will be a great successful if the model can produce similar results with the historical data

2.2 Wave

A wave is a disturbance or oscillation that travels through space-time, accompanied by a transfer of energy. Many physical phenomena are based on the principles of wave motion. All forms of waves are associated with the transport of energy. Waves transport energy without transporting matter. A wave carries energy when it travels from one point to another without transporting the material particles. Wave can be described by a wave equation which sets out how the disturbance proceeds over time.

A wave can be transverse or longitudinal. It can be differentiated depending on the direction of its oscillation. The shape of the wave is moving either forward or backward.

2.2.1 Wave Categories

Wave is produce in various shapes and forms. They can be distinguished based on certain characteristics. One characteristic that can categorize waves is based on the individual particles of wave movement direction in a medium where waves travel. Basically there are two categories of wave which are transverse and longitudinal wave.

2.2.1.1 Transverse Wave

Transverse wave is described as the direction of vibration of the particles in this wave is perpendicular to the direction of motion of the wave. There are some examples of phenomenon that can show how transverse wave occur in everyday life such as stretched string and waves on the surface of water. The displacement of particles results in the shape of the wave such as the sinusoidal shape [10]. When a transverse wave is moving, it will oscillate in up and down direction. Figure 2.1 below illustrate the transverse wave.

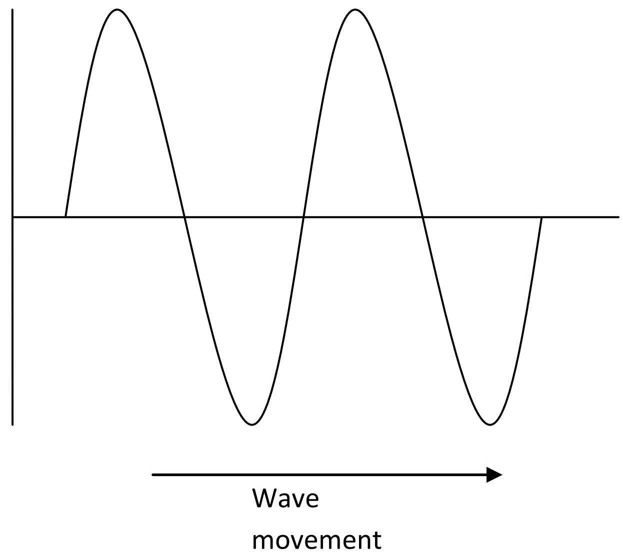

Figure 2.1 Transverse Wave

2.2.1.2 Longitudinal Wave

This category of wave, the direction of vibration of the particles is parallel to the direction of the motion of the wave. The particles of wave oscillate back and forth about the equilibrium positions. The example of this wave is sound waves in air. The displacement of particles shows in regions of high pressure (compression) and low pressure (rarefraction) [10]. Figure 2.2 below shows a longitudinal wave.

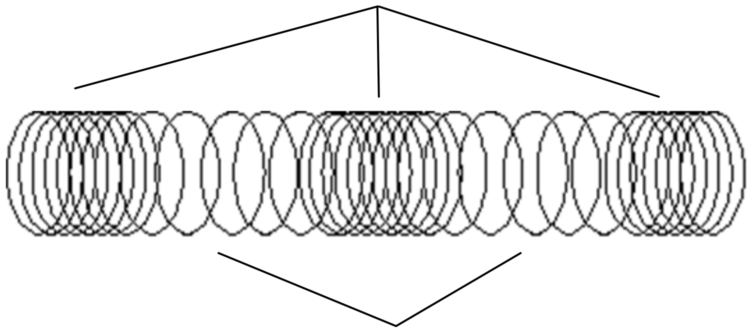

Figure 2.2: Longitudinal Wave

2.2.2 Properties of Waves

Wave motion can be studied using a mathematical approach. Characteristic of the mathematical method are as in Table 2.1 and the properties illustrate in Figure 2.3 below:

Table 2.1: Characteristic of Mathematical Model

No.	Characteristics	Descriptions
1	Amplitude (A)	• Magnitude of the maximum displacement of a particle along a wave
2	Frequency (f)	• Number of complete oscillations which come from each particle per second.
3	Period (t)	• Time taken for one complete oscillation of a particle.
4	Wavelength (λ)	• Distance between any two points with the same phase • Example: distance between two adjacent peaks
5	Wave number (k)	• Known as the spatial frequency of a wave. • The sum of 2π divided by wavelength or can be written as $k = \frac{2\pi}{\lambda}$.

6	Angular Frequency (ω)	• A scalar measure of rotation rate. • Angular frequency is define by $\omega = 2\pi f$
7	Phase angle (Φ)	• The phase angle tells whether two vibrating particles are moving together, in opposite directions or in any other relationship between one another. • If the two particles of the wave are at the same displacement and moving at the same speed in the same direction, it is called in phase. The phase difference $\Delta\Phi$ is equal to zero. • If the two particles are not at the same displacement and not moving at the same speed, it is call out-of-phase. The phase difference $\Delta\Phi$ is 180^{o} or π radian. • Phase difference is a fraction of a cycle by which one wave moves behind the other.

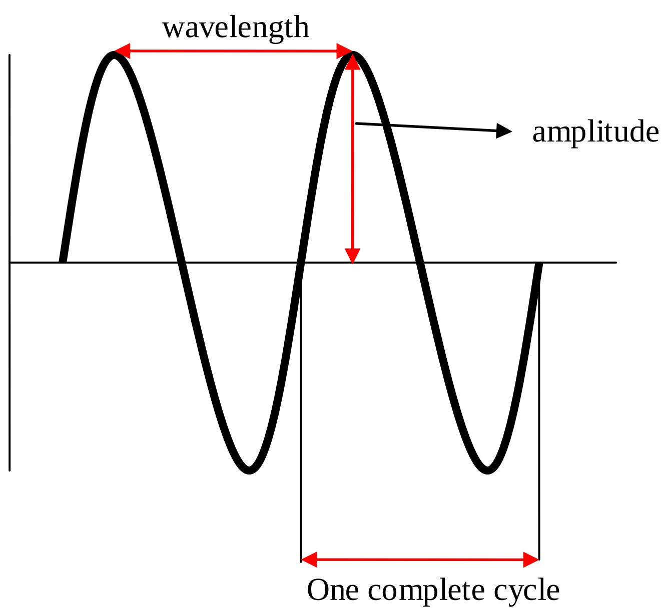

Figure 2.3 Properties of Wave