NUMERICAL MODELLING OF 1-DIMENSIONAL WAVE EQUATION USING FINITE DIFFERENCE SCHEME

NOR SYAZWANI BINTI MOHD RIDZUN

UNIVERSITI TEKNIKAL MALAYSIA MELAKA

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NOR SYAZWANI BINTI MOHD RIDZUN

This report is representing as a partial fulfilment of the requirement for the award of the degree of Bachelor of Mechanical Engineering (Structure and Material)

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DECLARATION

"I hereby declare that the work in this report is my own except for summaries and quotations which have been duly acknowledged."

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ACKNOWLEGDEMENT

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ABSTRACT

Finite difference scheme numerical technique was used to simulate the 1-dimensional wave equation. The 1-imensional wave equation was modelled in the MATLAB software. One-dimensional wave equation is a physical phenomenon that happens in vibrating string. From the complete numerical modelling in MATLAB, the wave behaviour from the graph was studied from the variation of parameters. In order to write a complete model using MATLAB, several steps were needed for examples function declaration, global parameters definition, stability condition setting, derived parameters declaration and graph plotting sections. The complete program of numerical simulation of 1-dimensional wave equation was analysed. As conclusion, the 1-dimensional wave equation can be derived by using Finite Difference Scheme. The center finite difference approximation is used to derive 1-dimensional wave equation. The study guides on the behaviour of 1-dimensional wave towards the prediction and assisting warning precaution on natural disasters like earthquakes, tsunamis, slides and also on design of string musical instrumentation.

ABSTRAK

Skim perbezaan terhingga ialah satu teknik berangka yang digunakan untuk menyelesaikan persamaan gelombang 1-dimensi. Persamaan gelombang 1-dimensi ini dimodelkan di dalam perisian MATLAB. Persamaan gelombang 1-dimensi adalah satu fenomena fizikal yang berlaku seperti getaran pada benang. Model yang telah lengkap dalam MATLAB dipelajari dengan melihat kelakuan gelombang dari graf dengan memperkenalkan variasi parameter. Dalam usaha untuk menulis satu model lengkap dengan menggunakan MATLAB, beberapa langkah penting perlu diambil seperti memperkenalkan pelbagai fungsi dan arahan pada parameter global, keadaan kestabilan, proses pengendalian parameter dan bahagian memplot graf. Daripada program yang lengkap itu juga akan dianalisa. Sebagai konklusi, persamaan gelombang 1-dimensi dapat diperolehi daripada skim perbezaan terhingga. Anggaran perbezaan terhingga ditengah telah digunakan untuk mengdapatkan persamaan gelombang 1-dimensi. Tujuan daripada pembelajaran ini membolehkan kita untuk membuat ramalan dan mengambil langkah berjaga-jaga dengan amaran mengenai bencana alam seperti gempa bumi, tsunami, tanah runtuh dan kajian ini juga digunakan dalam rekabentuk peralatan alat muzik yang menggunakan tali.

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LIST OF SYMBOLS

f0	=	Fundamental frequency, Hz
f_s	=	Sampling frequency, Hz
f(x)	=	Function of x
h,Δx	=	Step size of displacement, x
k,Δt	=	Step size of time, s
N	=	Sample rate, sample/s
T	=	Period, s
T	=	Tension, N
t	=	Time, s
u,x	=	Displacement, m
v	=	Velocity, m/s
x,y	=	Unknown
ρ	=	Mass density, kg/m^3
∇^2	=	Laplacian condition

CHAPTER I

INTRODUCTION

1.1 BACKGROUND

This research is to study about the wave and the wave equation to make prediction and assumption of physical phenomenon such as in vibrating string. The definition of wave is forward or backward propagating directions that transfer of energy in a system. In the mathematical sense, a wave is any function that moves.

The linear wave equation is widely used model in mathematical physics. However, explicit solutions for this equation when the wave speed varies spatially can only be obtained in certain special cases. These solutions are nevertheless useful to describe wave propagation and to test numerical algorithms. Based on the scope of this thesis; the focused is on 1-dimensional wave equation.

According to Andreas (2005) stated that the 1-dimensional wave equation can be defined where the wave is propagate in x-direction by using partial derivatives and just exist in plane wave. Means that, the derivation of 1-dimensional equation is from second partial derivatives and can be derived by using Maxwell and Newton's 2^{nd} Law equation. The finite difference scheme was used to derive the second partial derivatives and be model in type of numerical modelling in MATLAB.

1.2 PROBLEM STATEMENT

This research to perform numerical modeling of behavior of 1D wave equation by using finite difference scheme and to investigate the behavior of 1D wave through the variation of the system's parameters. The numerical modeling is designed to calculate the 1-dimensional wave equation by using finite difference schemes method. The finite difference approximation is a numerical technique based on the finite difference concept from Taylor series that is employed to solve Maxwell's equation and well suited to analyze problems with complex geometrical features like 1-dimensional wave equation.

1.3 OBJECTIVES

The main objective of this research is to perform numerical modeling of the behavior of 1D wave equation by using finite difference scheme. Other objective is to investigate the behavior of 1D wave through the variation of the system's parameters.

1.4 SCOPE OF PROJECT

The scope of this project is all the simulations, manipulations and implementations must be done by using MATLAB.

The method used in the numerical modeling is finite difference scheme. Finite difference scheme is suitable for analyzing problems with complex geometrical features that include parameters such as frequency, velocity and time.

The centered approximation of finite difference scheme is used in this study because it is widely used. Besides that, the 1-dimensional wave equation can be solve by using centered approximation of finite difference scheme by introducing the initial velocity condition according to Pierce (2012).

1.5 SUMMARY

This project was conducted by using MATLAB software to study the wave equation based on the variation of the system parameters. The 1-dimensional wave equation is going to be modeled in the software using finite difference scheme. To model a program, there are a lot of commands that we are need to study and apply. Then, after writing a complete program, we need to test the program either obey the 1-dimensional wave equation or not. From the 1-dimensional wave equation of finite difference scheme, different parameters indicate different behavior of the wave. From these variations of parameters will indicated different simulation and behavior of the wave equation. Apart from that, the analysis of the model program will be compared with manual calculation of 1-dimensional wave equation.

CHAPTER II

LITERATURE REVIEW

2.1 INTRODUCTION

The purpose of this project is to do a numerical modelling of 1-dimensional wave equation by using finite difference scheme.

According to Allaire (2007), numerical modelling is a combination of two distinct aspects of mathematical modelling and numerical simulation. First aspect is a combination of a set of number that has the ability to solve an assumptions, constraints and equation. Numerical modelling is the study of the actual arithmetic operations such as addition subtraction, multiplication and division that will perform by computer. A computer program needs in order to solve a larger problem. So, numerical modelling is combination of mathematics, computer science and engineering. Numerical simulation is of course, about the process which allows us to calculate the solutions of these models on a computer, and thus to stimulate physical reality

At present, many commercial software have been developed and used widely, these software contain numerical methods with some specific procedures to provide approximate solutions. Thus, it is very important for users to understand the computational procedures that are built inside the software prior to using them. In this project, all the computational procedure will be done by using MATLAB.

According to Goering (2004), the MATLAB (matrix laboratory) is a numerical computing environment and fourth-generation programming language. This programming was developed by MathWorks that allows matrix manipulations, plotting of functions and data, implementations of algorithms and creation of user interfaces.

The most important aspect for a student who needs to learn numerical modelling by using computer language is that the methods and their computational procedures must be understood clearly prior to developing any computer program.

2.2 WAVES

2.2.1 Definition of Wave

A wave has the same sense as a particle. Wave is a disturbance or oscillation that travels through space-time and also transferring energy. Wave is a disturbance that moves through a medium. A medium here is a substance or material that carries the disturbance from one point or place to another.

"Wave motion is an energy transfer from one point to another", as mentioned by Richard *et. al*. (2005). So, the study of wave is important where many physical phenomena are based on the principles of wave motion.

A wave is a transfer of energy from one point to another. There are two basic types of wave; a wave can be transverse or longitudinal depending on the direction of its oscillation.

The transverse wave is the wave that causes the medium to vibrate at a right angle to the direction of the wave. The transverse wave has crest (highest point) and trough (lowest point) of the wave. An example of transverse wave is light wave (electromagnetic wave).

The longitudinal wave (also known as l-wave) is the wave has the same direction of vibration as their direction of travel. According to Richard, (2005), the characteristic of longitudinal wave is same with compression and extension wave in spring.

But, any kinds of waves have same properties either its longitudinal or transverse wave such as amplitude, wavelength, period, frequency and speed.

2.2.2 Properties of Wave

The main properties of a wave are defined below and as illustrated in the Figure 2.1.

i. Amplitude: the height of the wave from equilibrium position line to crest, measured in meters.
ii. Wavelength: the distance between a crest to another crest, measured in meters.
iii. Period: the time it takes for one complete wave to pass a given point, measured in seconds means that wavelength over time where the unit is cycle/ second.

iv. Frequency: the number of crest that pass a point in one second, measured in inverse seconds or Hertz (Hz).

v. Speed: the horizontal speed of a point on a wave as it propagates or the number of crest passages per unit time, measured in meters / second.

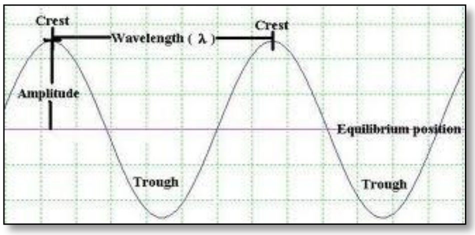

Figure 2.1: An example of Wave

(Adapted from butane.chem.uiuc.edu)

2.2.3 Dimensions of Waves

There are three types of wave that known as 1-dimension, 2-dimension and 3-dimension of waves. All the waves are differentiating by it point of source and path of the wave propagate.

i. 1-Dimensional Wave:

Examples of wave in one dimension are a wave that travels along string or sound waves going down a narrow tube. The energy of the wave motion of a 1-dimensional wave has only one dimension in which to travel. The waves that have only one dimension to travel are transverse and longitudinal wave.

According to Richard *et. al.* (2005), a transverse wave is the oscillations forming the wave are perpendicular to the direction of propagation for an example is rope waves which the wave is created by motion perpendicular along the rope. A transverse wave in a rope is created by sinusoidal oscillation of the end of the rope. An example is transverse wave that travels along a rope as shown in Figure 2.2.

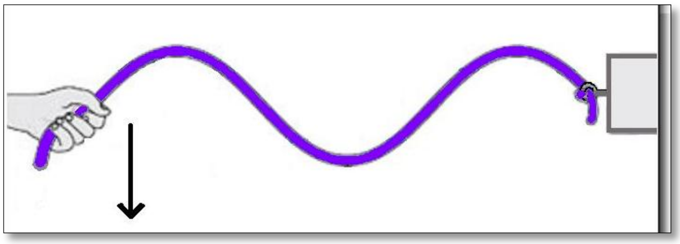

Figure 2.2: Transverse Wave
(Adapted from physics.tutorvista.com)

Then, the longitudinal wave is the motion of the points of the medium (forming the wave) is in the same direction as the direction of propagation of the wave pattern as shown in Figure 2.3. The examples of longitudinal waves are the spring coil, sound waves, ultrasonic waves, shock waves (sonic booms), light waves and so on. The longitudinal wave in slinky spring are contains in compression and extension of the spring as mentioned by Richard *et. al.* (2005) and as shown in Figure 2.3.

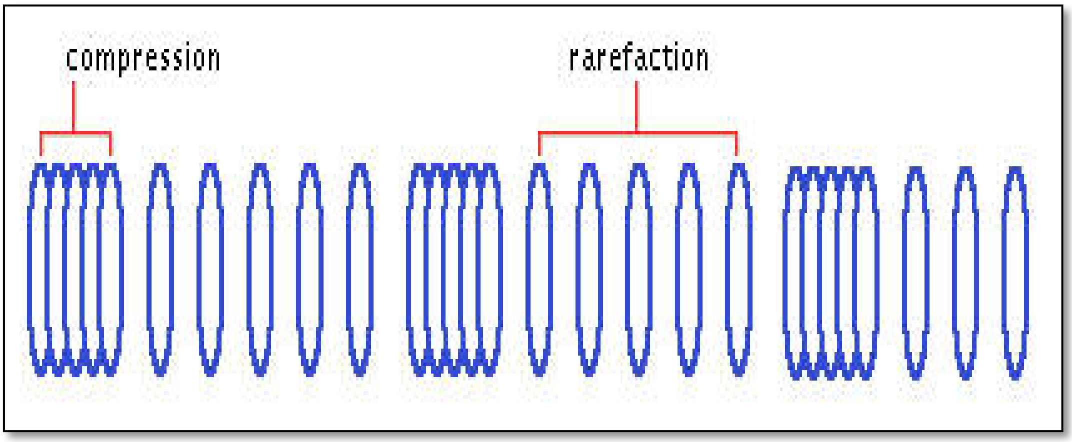

Figure 2.3: Longitudinal Wave
(Adapted from www.mundos-fantasticos.com)

These two types of wave are distinguished by the direction of the wave oscillation relative to the direction of propagation of the wave. That is, the direction of motion of vibration in the medium.

ii. 2-Dimensional Wave

By referring to Kneubühl (2010) stated that the energy of the wave motion of a 2-dimensional wave has the ability to travel around corners. The wave has a point of source and a path of the wave. An example is a ripple tank that contains water waves as illustrated in Figure 2.4.

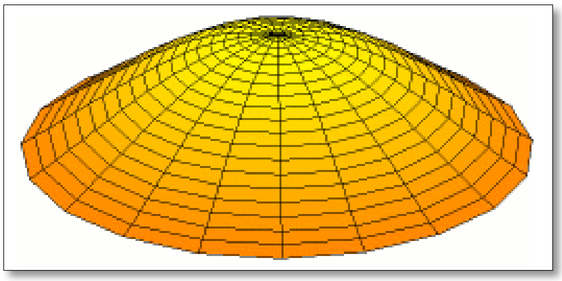

Figure 2.4: Ripple Tank of Water Wave
(Adapted from Drum_vibration_mode01.gif)

iii. 3-Dimensional Wave

The 3-dimensional wave is a wave that shows point sources, line sources and plane of wave such as the wave moving into different medium also called, interference between sources is the definition gave by Kneubühl (2010). For an example, a standing wave on a disk (as shown in Figure 2.5) with two nodal lines crossing at the centre (overtone)

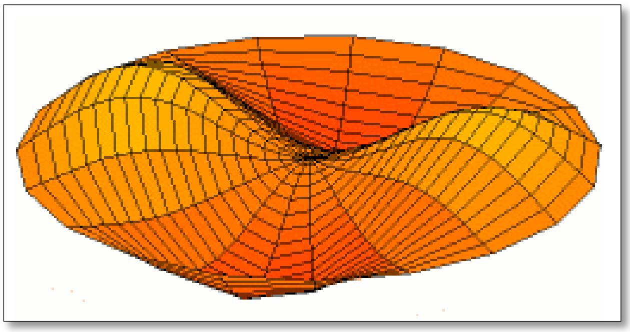

Figure 2.5: Standing Wave on A Disk (overtone)
(Adapted from Drum_vibration_mode21.gif)

2.2.4 Types of Mechanical Wave

The wave equation is a part of mechanical wave that can be derived in vibrating string. A mechanical wave is a wave that needs a medium to travel and cause a local oscillation of material. The mechanical wave is also known as material wave. The mechanical wave can be divided into four types.

The first type of mechanical wave is surface wave or also known as ground wave. The surface wave is a mechanical wave that propagates along the interface between differing media or two fluids that have different densities or refractive index gradient. The suitable example of surface wave is radio propagation that travels as ground wave as shown in Figure 2.6. When the wave travels in low frequency, the wavelength of the wave is high due to the diffracted around obstacles.

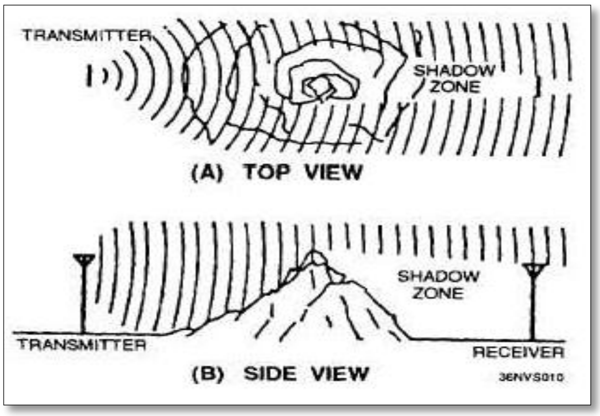

Figure 2.6: (a) Top view and (b) Side view of Ground Wave (radio transmitter propagation)
(Adapted from www.edinformatics.com)

The other type of wave is water wave (wind wave) where this wave moves in fluid dynamic because of wind-generated waves that commonly occur on the free surface of oceans, seas, lakes, rivers and canals. The factors that generate this water wave are wind speed, distance of open water (fetch), width of area affected by fetch, time duration over given area and water depth. This water wave can be divided into two types those are capillary and swell type as shown in Figures 2.7: