

**MODELING, SIMULATION AND VALIDATION OF REDUCED SCALE
VEHICLE DYNAMICS MODEL**

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SUPERVISOR DECLARATION

“I hereby declare that I have read this thesis and in my opinion this report is sufficient in terms of scope and quality for the award of the degree of Bachelor of Mechanical Engineering (Automotive)”

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ABSTRAK

Tujuan projek ini adalah model, simulasi dan pengesahan model kenderaan dinamik skala dikurangkan. Penyelidikan yang berkaitan dengan dinamik kenderaan sering memerlukan model matematik untuk mewakili kelakuan dinamik kenderaan dengan bantuan simulasi komputer perisian. Tetapi, ramalan daripada simulasi model matematik yang dihasilkan mungkin berbeza daripada kelakuan dinamik kenderaan yang sebenar. Oleh itu, pengesahan keputusan simulasi dengan perisian simulasi kenderaan dinamik atau pengesahan dengan eksperimen. Walau bagaimanapun, kos dan masa pembangunan yang tinggi untuk mengesahkan keputusan simulasi dengan ujian untuk kenderaan berskala penuh adalah suatu halangan yang besar kepada para penyelidik. Untuk mengurangkan kos dan masa pembangunan, kenderaan skala dikurangkan diperkenalkan dalam projek ini. Kenderaan model dinamik yang mempunyai 8 darjah kebebasan telah dibangunkan untuk mewakili kelakuan dinamik kenderaan skala dikurangkan. Selain itu, kereta skala dikurangkan aksn dilengkapi dengan alat-alat penderia yang tanpa wayar dibangunkan dan digunakan sebagai katil ujian dinamik kenderaan untuk menyiasat pelbagai operasi penuh dikurangkan pengendalian kenderaan skala. Akhirnya, keputusan yang diperolehi dari eksperimen adalah mewakili tingkah laku sebenar kenderaan skala dikurangkan dan keputusan ini digunakan untuk mengesahkan model dinamik kenderaan dibangunkan. Model dinamik kenderaan yang dibangunkan boleh digunakan untuk mengkaji tingkah laku dinamik kenderaan, reka bentuk strategi pengawal kenderaan dan pelaksanaan tanpa menjalankan eksperimen.

ABSTRACT

This project focuses on modeling, simulation and validation of a reduced scale vehicle dynamics model. Research in the area of vehicle dynamics often requires the mathematical model to represent the vehicle dynamics behavior with the help of virtual simulations but the prediction by simulation might differ from the actual vehicle dynamics behavior. Thus, validation of the simulation results with vehicle dynamics simulation software or by experimental is required. However, the high development cost and time to validate the simulation results with full scale vehicle testing was an obstruction to researchers. In order to reduce the development cost and time, reduced scale vehicle was introduced in this project. A 8 degree of freedom vehicle dynamics model was developed to represent the dynamics behavior of reduced scale vehicle. Besides, a reduced scaled instrumented vehicle is developed and used as a vehicle dynamics testbed to investigate the full operational range of reduced scale vehicle handling. Wireless sensing system was used to capture vehicle response wirelessly. Finally, the results obtained from the experimental of a scale vehicle with J-turn test was represent the actual behavior of reduced scale vehicle and these results used to validate the vehicle dynamics model developed. The vehicle model developed could be used to study dynamic behavior of the vehicle and for controller strategies design and implementation without the need to conduct experiment.

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LIST OF SYMBOLS

| | |
|--------------|--|
| z_s | = Sprung mass vertical displacement |
| \dot{z}_s | = Sprung mass vertical velocity |
| \ddot{z}_s | = Sprung mass vertical acceleration |
| z_u | = Unsprung mass vertical displacement |
| \dot{z}_u | = Unsprung mass vertical velocity |
| \ddot{z}_u | = Unsprung mass vertical acceleration |
| k_s | = Suspension spring stiffness |
| k_t | = Tire stiffness |
| c_s | = Suspension damping |
| F_s | = Spring forces |
| F_d | = Damping forces |
| F_t | = Tire forces |
| L | = Wheelbase of vehicle |
| L_f | = Distance of vehicle C.G. from front axle |
| L_r | = Distance of vehicle C.G. from rear axle |
| V_x | = Longitudinal velocity |
| \dot{V}_x | = Longitudinal acceleration |
| V_y | = Lateral velocity |
| \dot{V}_y | = Lateral acceleration |
| w | = Track width |
| F_{xfl} | = Front left tire longitudinal force |

| | |
|--------------|---|
| F_{xfr} | = Front right tire longitudinal force |
| F_{xrl} | = Rear left tire longitudinal force |
| F_{xrr} | = Rear right tire longitudinal force |
| F_{yfl} | = Front left tire lateral force |
| F_{yfr} | = Front right tire lateral force |
| F_{yrl} | = Rear left tire lateral force |
| F_{yrr} | = Rear right tire lateral force |
| F_{zfl} | = Front left tire normal force |
| F_{zfr} | = Front right tire normal force |
| F_{zrl} | = Rear left tire normal force |
| F_{zrr} | = Rear right tire normal force |
| m_T | = Total mass of vehicle |
| m_s | = Sprung mass |
| m_{ufl} | = Front left unsprung mass |
| m_{ufr} | = Front right unsprung mass |
| m_{url} | = Rear left unsprung mass |
| m_{urr} | = Rear right unsprung mass |
| h_{cg} | = Height of vehicle C.G. from ground |
| h_{rc} | = Height of roll center below sprung mass |
| h_{rcf} | = Front roll center distance below sprung mass C.G. |
| h_{rcr} | = Rear roll center distance below sprung mass C.G. |
| $k_{\phi f}$ | = Front equivalent roll stiffness |
| $k_{\phi r}$ | = Rear equivalent roll stiffness |
| $c_{\phi f}$ | = Front equivalent roll damping |
| $c_{\phi r}$ | = Rear equivalent roll damping |
| I_x | = Roll moment of inertia |
| I_z | = Yaw moment of inertia |

| | |
|---------------------|--|
| I_w | = Rotational inertia of each wheel |
| T_{dfl} | = Front left wheel drive torque |
| T_{dfr} | = Front right wheel drive torque |
| T_{drl} | = Rear left wheel drive torque |
| T_{drr} | = Rear right wheel drive torque |
| T_{bfl} | = Front left wheel brake torque |
| T_{bfr} | = Front right wheel brake torque |
| T_{brl} | = Rear left wheel brake torque |
| T_{brr} | = Rear right wheel brake torque |
| R | = Tire effective radius |
| ω_{fl} | = Front left angular velocity of wheel rotation |
| ω_{fr} | = Front right angular velocity of wheel rotation |
| ω_{rl} | = Rear left angular velocity of wheel rotation |
| ω_{rr} | = Rear right angular velocity of wheel rotation |
| $\dot{\omega}_{fl}$ | = Front left angular acceleration of wheel rotation |
| $\dot{\omega}_{fr}$ | = Front right angular acceleration of wheel rotation |
| $\dot{\omega}_{rl}$ | = Rear left angular acceleration of wheel rotation |
| $\dot{\omega}_{rr}$ | = Rear right angular acceleration of wheel rotation |
| δ | = Front wheel steer angle |
| ϕ | = Roll angle |
| $\dot{\phi}$ | = Roll rate |
| $\ddot{\phi}$ | = Roll angular acceleration |
| ψ | = Yaw angle |
| $\dot{\psi}$ | = Yaw rate |
| $\ddot{\psi}$ | = Yaw angular acceleration |
| α_f | = Front tire side slip angle |
| α_r | = Rear tire side slip angle |
| σ_{xfl} | = Front left tire longitudinal slip ratio |

- σ_{xfr} = Front right tire longitudinal slip ratio
- σ_{xrl} = Rear left tire longitudinal slip ratio
- σ_{xrr} = Rear right tire longitudinal slip ratio
- g = Gravitational acceleration (9.81 ms^{-2})
- μ = Coefficient of friction of road surface
- C_{σ} = Longitudinal tire stiffness
- C_{α} = Tire cornering stiffness

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CHAPTER I

INTRODUCTION

1.1 OVERVIEW

In the automotive field, computer simulation is the useful tool to design, analyze and develop a vehicle dynamic model. These virtual dynamic simulations have been proving to be an effective and accurate method to assess or predict vehicle behavior on various operating conditions (Liburdi, 2010; Garott and Heydinger, 1992). Without computer simulation, the traditional full-scale vehicle testing is required to evaluate the vehicle dynamics behavior which is high costing and endangering the test driver. However, full-scale vehicle testing will not eliminate from the development processes. It just postpone to the final stage of evaluation to validate the model design (Longoria et al., 2004).

Recently, many studies on reduced scale vehicle dynamics model in order to reduce development costs and time. The dynamic behavior of reduced scale vehicle is proportionally similar to full scale vehicle (Witaya et al., 2009). Various researchers have been proved that reduced scale vehicle model is an alternative technology to test with full-scale application (William et al., 2004).

There are several factors or advantages of reduced scaled vehicle over full scale vehicle for vehicle dynamics testing. Firstly, the costing factor. Compared with full scale vehicle, the reduced scale vehicle is simpler and relative low cost. The initial purchase cost of reduced scale vehicle (for example radio control car, RC car) are relatively low compared to full scale vehicle. Besides that, the replacement parts are easily available in the market and cheaper than full scale vehicle. Since, reduced scale vehicle has simple construction, thus the modification will be more

convenience and less costly (Witayaet al., 2009; Brennam and Alleyne, 1999).

Next, the safety factor of the test driver during vehicle dynamics testing. Generally, the reduced scale vehicles are safe to use, easy to operate and less critical errors or mistakes. Since the operation of reduced scale vehicles are controlled by using controller, so driver can be excluded and no risk during various testing.

Lastly, the flexibility of the experimental equipment and apparatus by using reduced scale vehicle. To determine the vehicle parameter, a lot of equipments, apparatus and sensors will be needed. By using reduced scale vehicle (5 times smaller than full scale vehicle), the equipments and apparatus used are simpler and smaller compare to full scale vehicle. For example, to determine the weight of vehicle, a simple and small weighing machine is sufficient for reduced scale vehicle.

1.2 PROBLEM STATEMENT

Field testing using full scale vehicle to evaluate the vehicle dynamic behavior has some drawbacks especially endangering the test driver. It is very dangerous to the driver to test full scale vehicle especially when it is required to push the vehicle to its limits to observe what happens in non-linear regions of vehicle behavior (Yih, 2000). The cost to develop full scale vehicle and maintenance cost are relatively high. Furthermore, modification of vehicle parameters requires a lot of time and this limit the repeatability of the test.

A reduced scale vehicle dynamic model is a model that has a smaller size than the actual size with some ratio 1:5. A reduced scale model chosen in this project is to obtain results with a lower dimension but an accurate approximation of the full scale vehicle dynamics model. This way can drastically reduce the time required to measure the vehicle parameter and to validate the vehicle dynamic model developed (Witaya et al., 2009). Development of a full scale vehicle dynamic model is very costly. A higher initial cost is need to purchase the material and equipment required (Brennam and Alleyne, 1999). By using reduced scale vehicle model, the initial cost, purchasing cost can be greatly reduced. Besides, the development cost can be reduced such as the installation cost, modification cost, validation cost and others costing.

By using reduced scale model, the speed of vehicle dynamics model development process is increased. Furthermore, it can assist to identify any problem occur at the earlier development and requirements analysis. Various aspects can be tested by using reduced scale vehicle dynamics model and quicker feedback can be retained (Longoria et al., 2004).

In addition, a more accurate control testing environment can be done since the testing area requires for a reduced scale vehicle dynamic model is relatively smaller than a full-scale model (Brennam and Alleyne, 1999). Besides that, the reduced scale can be driven using controller, so driver can be excluded, driver safety problem will not occur. Furthermore, modification of vehicle parameter (if needed) will be easier than full scale vehicle dynamic model.

1.3 OBJECTIVE

The objectives of this project are:

1. To develop reduced scale vehicle dynamic model
2. To validate the vehicle dynamics model with experiment

1.4 SCOPE

The scope of this project is as follow:

1. Development of mathematical model and Matlab/SIMULINK model to represent dynamic behavior of a reduced scale vehicle
2. Fabrication of vehicle dynamics test bed
3. Validation of reduced scale vehicle dynamics model with experiment

1.5 THESIS OUTLINE

The remaining report is broken down into five further chapters as summarize follows. Chapter 2 present the literature review on concepts related to vehicle dynamics and vehicle modeling. Vehicle modeling can be classified as vehicle ride model, vehicle handling model and full vehicle model. Besides, this chapter also discuss about the available test maneuver for vehicle handling test and the method used for vehicle modeling validation. Chapter 3 presents the methodology used to complete this project, in which how to develop the mathematical model, Simulink model, vehicle parameter measurement, sensor selection and installation, design and develop wireless DAQ and functional test, vehicle dynamic model validation and performance evaluation. Besides, this chapter will also discuss detail about the experimental setup and sensors installation. Chapter 4 explain the results obtain from experimental and validation of vehicle dynamics model. Finally, Chapter 5 presents conclusions that were obtained from this research and some recommendations.

CHAPTER II

LITERATURE REVIEW

2.1 VEHICLE MODELING

Nowadays, the safety, performance, efficiency and comfort of modern vehicles have improved drastically. In order to make improvement continuously, manufacturers have relied on the vehicle modeling and simulations. There have many advanced software such as CarSim, Matlab/SIMULINK and Adams/Car are widely used to model and evaluate the vehicle dynamics behavior throughout various operating conditions. With the help of this advanced software, manufacturers are able to introduce new design vehicle to the market in shorter period (reducing development time) as well as reducing the development cost. Furthermore, the development cost and time saved can be used to optimize vehicle systems such as anti-braking system (ABS), electronic brake force distribution (EBD) and dynamics stability control in order to increase the quality and refinement that customer expected (Liburdi, 2010).

The effort to develop vehicle modeling is to represent the vehicle dynamic behavior as exactly as possible. Simulation of full vehicle model is complicated and time consuming. The vehicle model complexity can be reduce to a sufficient level for vehicle dynamics, it is depend on the which field to be assessed. Basically, vehicle model is classified into ride model and handling model. Ride model and handling model can be simplified to a lower degree of freedom (DOF) and it is depended on the finding to be obtained (Wang, 2001).

2.1.1 Vehicle Ride Model

Vehicle ride model is used to study the vehicle body displacement, rolling and pitching of the vehicle and the reaction of wheels (Fauzi et al., 2009). The vehicle body displacement, roll rate and pitch rate are affected by spring forces, damping forces and tire stiffness forces. The spring forces and damping forces are come from the suspension system; while the tire stiffness forces come from the stiffness of tires acting as spring behavior.

The ride model can be classified into three types:

- 1) 2 DOF vehicle ride model (Liburdi, 2010)
- 2) 4 DOF vehicle ride model (Alexandru and Alexandru, 2011)
- 3) 7 DOF vehicle ride model (Fauzi et al., 2009)

2.1.1.1 2 DOF Vehicle Ride Model

The quarter car model is a 2 DOF vehicle ride model which is the simplest vehicle ride model used to evaluate the vertical motion of vehicles causes by unwanted vibrations (Thite, 2012). Driving discomfort mostly is come from the unwanted vibrations, so suspension system playing important role to reduce or eliminate unwanted vibrations. Many researcher works in this field to design and develop a better active suspension system to improve driving comfort. Kumar et al. (2008) conducts a research on development of active suspension automobile using PID controller. In his research, a quarter car models with 2 DOF was developed to analyze and compare the performance of passive suspension system with active suspension system as shown in Figure 2.1. Besides, Yahaya (2006) also conducts a study on robust control of active suspension for a quarter car model. Quarter car model are the simplest model that can represent most features of full car model. Generally, researchers will develop the quarter car model to validate the basic properties or response before developing the complicated full car model.

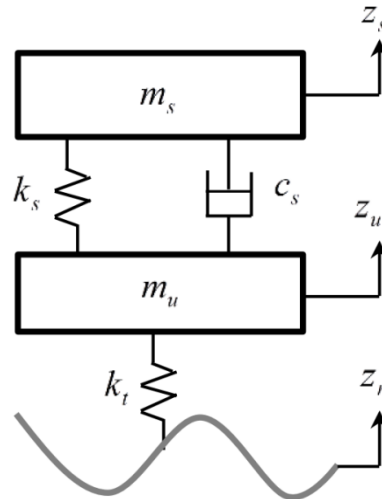


Figure 2.1: FBD of quarter car model

Equation of motions for quarter car model:

$$m_s \ddot{z}_s = c_s (\dot{z}_u - \dot{z}_s) + k_s (z_u - z_s) = 0 \quad (1)$$

$$m_u \ddot{z}_u = k_t (z_r - z_u) - k_s (z_u - z_s) - c_s (\dot{z}_u - \dot{z}_s) = 0 \quad (2)$$

2.1.1.2 4 DOF Vehicle Ride Model

The half car model is a 4 DOF vehicle ride model; it can be categorized into two types that are half car roll plane model and half car pitch plane model. The applications of half car model is used to investigate the dynamic behavior caused by vibration. Many researchers applied quarter car model or half car model in preliminary stages of their research before developing the full car model. The half car model is frequently used to assess the vertical motion of vehicle as well as roll dynamics and pitch dynamics (squat and dive) (Alexandru and Alexandru 2011). Thus, the half car model has advantages over the quarter car model. Alexandru and Alexandru (2011) conducted a research on dynamic analysis of active suspension with the half car model to improve the vehicle dynamics behavior in terms of comfort and stability. Wang (2001) also conducted a study on design and synthesis of active and passive vehicle suspension. He introduced a trailing arm model for anti-squat and anti-dive purposes to increase vehicle comfort and stability.