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Name of supervisor : ..Lee Yuk Choi....
Date : ....9/12/2005........

NUMERICAL STUDY OF TWO – DIMENSIONAL TRANSIENT HEAT CONDUCTION USING FINITE ELEMENT METHOD

WONG SAU KEONG

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Faculty of Mechanical Engineering
Kolej Universiti Teknikal Kebangsaan Malaysia

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"I hereby declared that this thesis is my own work except the idea and summaries which I have clarified their sources"

Signature : ....
Author : WONG SAU KEONG
Date : 9/12/2005

Specially dedicated to my family, friends and companion...

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ABSTRACT

This thesis presents a finite element method in numerical study of two-dimensional transient heat conduction to determine the temperature distribution as a function of time by using MATLAB. This study was mainly focus on time dependents heat transfer problem which was contributed by several factors such as unsteady state or transient problem, typically arises when the boundary conditions of a system are changed. The Galerkin method is the method of choice in the formulation which involves transient heat conduction. By using the bilinear rectangular element (quadrilateral element) in disretization and integration of weighted residual of the differential equation and also boundary condition was performed using finite element method. The formulation of the final matrix equation or global matrixes of two-dimensional transient heat conduction could then be determined. The temperature distribution as a function of time was performed using backward difference scheme, it was found that the temperature decline in early stage for Δt = 1 second. It can be concluded that it is better in the sense that it is computationally more efficient.

ABSTRAK

Tesis ini adalah mempersembahkan kajian simulasi berangka pada unsur dua dimensi konduksi haba dengan mengunakan kaedah unsur terhingga (*Finite Element Method)* bagi menentukan pengaliran haba dan pengagihan suhu yang bergantung pada masa dengan simulasi MATLAB untuk menyelesaikan masalah ini. Tujuan utama tesis ini adalah memfokus kepada permindahan haba yang bergantung kepada masa di mana ia disebabkan oleh beberapa faktor seperti ketidakstabilan fana dan masalah yang ditimbulkan semasa keadaan sempadan sesuatu sistem berubah. Dengan menyelesaikan masalah ini kaedah *Galerkin* digunakan untuk membabitkan konduksi haba pada keadaan sempadan sesuatu sistem berubah. Untuk *disretization* pada unsur segi empat dwilelurus digunakan dengan integrasi persamaan perbezaan dari baki berpemberat dan keadaan sempadan dilakukan pada semasa memilih unsur dua dimensi dengan kaedah unsur terhingga (FEM). Untuk menentukan unsur dua dimensi konduksi haba dengan mengunakan kaedah unsur terhingga (*Finite Element Method)*, formula bagi persamaan adalah susunan angka – angka atau matriks sejagat digunakan untuk menyelesaikan masalan tesis ini. Pengagihan suhu yang bergantung pada masa menggunakan kaedah pembezaan kebelakang *(Backward Difference Method)* membuktikan kadar kejatuhan suhu awal dalam perubahan suhu adalah 1 saat menjadi semakin stabil. Boleh disimpulkan bahawa ia adalah lebih baik dari segi pengiraan dimana ia lebih efisien.

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NOMENCLATURE

C_p	=	Specific heat capacity, J/kg K
ρ	=	Density of the material, kg/m^3
Γ	=	Boundary for computational domain
T	=	Temperature, K
t	=	Time, s
α	=	Is a know function usually is equal to $\frac{\rho C_p}{k}$, for heat conduction problem witħ constant material properties.
N	=	Shape function for node.
K	=	Conductance matrix.
M	=	Element capacitance matrix.
k	=	Global conductivity matrix and effect of conduction.
$\dot{T}^t$	=	Temperature time dependent.
T^t	=	Temperature vector.
R	=	Global heat rate vector.

CHAPTER I

INTRODUCTION

1.1 Overview

Many engineering applications rely on the dynamics of heat intensity and flow within objects. Two-dimensional heat transfer problem find significant importance in building because thermal bridge in walls, windows and other component can have significant effects on energy performance and occupant comfort. The insulating value of a material is not sufficient to determine the energy performance of a wall or other component in when the material is used because the entire area of the wall is not completely filled with insulating material. With theory of two-dimensional heat transfer and utilizing the finite element method, it becomes easy to analyze the heat flow. Therefore, knowledge of the temperature and its transient distribution is vital in design and implementation.

Time dependent problem of engineering interact involve bath space and time. The time variations start from some initial conditions and then propagate through time. It can efficiently solve with the finite element method to consider the temperature for each nodal point. In this study, finite element method is used to determine the two–dimensional transient heat conduction in a slab.

Since the late 1960s the mathematic literature on the finite element method has grown more than in any previous period. Today, the finite element method is strongly consider for simulating all modes of heat transfer process, rivaling performance standards associated with finite different method. The finite element method is a one of the major numerical solution techniques. The advantage by using FEM in transient heat conduction is that a general purpose be developed easily via computer program to analyze the problems.

In this study, the method used to formulate the finite element method is Galerkin's procedure. It is used to solve the two dimensional transient heat conduction problems. Galerkin's method uses a set of governing equations in the development of an integral form. Starting with a governing equation weighted residual integrals are evaluated at each element to form a system of linear algebra equation.

From the formulation of finite element method equation and be using the Matlab to solve the two dimensional transient heat conduction in a slab. Matlab is a high performance language for technical computing. It can be programmed in an easy to use environment where problem and solution are expressed in familiar mathematical notation. One attractive aspect of Matlab is that it is relatively easy to learn. The finite element method is a well – defined method for which Matlab can be very useful as a solution tool. Matrix and vector manipulation are essential parts in this method.

1.2 Objective Of The Project

The project was concerned with numerical study of the two-dimensional transient heat conduction using finite element method. The objectives are as follows:

1. Identify a better method to derive finite element equation on heat flows.
2. To determine the temperature distribution as a function of time.

1.3 Scopes Of The Project

To achieve the objectives, the scopes are carried out to achieve the above objective. The scopes of project are as follows:

1. Literature survey.
2. Study of heat flow with variation in time.
3. Development of a computer programs using matlab.
4. Comparison with other available works.

1.4 Gantt Chart

The progress of the project was shown in Chart 1.1 and Chart 1.2

Project Activities	Week														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Select title project PSM And confirmed title.	■	■													
Research, collect and study information		■	■	■	■	■	■	■							
Literature review				■	■	■	■	■	■	■	■	■			
Create the equation								■	■	■	■	■	■		
First draft													■	■	
Power point														■	■
First presentation															■

Chart 1.1: Progress of the 1st Project

Project Activities	Week														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Writer the program by using Matlab	■	■	■	■	■	■	■								
Collect data and rewriter						■	■	■							
Data analysis & Discussion									■	■	■	■			
Second draft												■	■	■	
Power point														■	■
Second presentation															■
Arrangement the draft 1& 2	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■

Chart 1.2: Progress of the 2nd Project.

CHAPTER II

LITERATURE REVIEW

A general literature review on transient heat conduction first is presented. This will be followed by a review of the studies on transient heat conduction using finite element.

G. Ibrahim, et al (2002) analyzes transient heat conduction of electronic packages by coupled boundary and finite element method. Electronic packages are presence of geometric and material discontinuities. The finite element method is highly efficient and commonly used method, its application with conventional elements suffers from poor accuracy of the result from the boundary element method formulation, which requires computationally intensive time integration schemes, is much higher than that of the finite element. In this type of coupled formulation avoids the fine discretization required by finite element method are avoids the fine discretion to achieve accurate results in regions with small length scales and geometric and material discontinuities. The combination of material and geometric discontinuities caused the heat flux to exhibit a singular behavior around these regions and the result is a higher level of thermal stresses in the package.

R. Raimund and T. Jan (2001) analyze 2D finite element formulation for 3D finite element method in temperature fields for hybrid and conventional composite structures. The three dimensional temperature fields very effectively than the two – dimensional finite element discretisation. However, it is desirable to have 2D finite elements on the model, which can calculate the full three dimensional temperature distribution reducing the modeling and numerical effort drastically. Compare with the 2D finite element a layerwise discretisation with 3D finite element is very costly and comparison of the 2D and 3D thermal analysis the 3D result a match excellently.

M.D.Mikhailov and M.N.Ozisik (1986) performed the transient heat conduction in a three dimensional composite slab. The eigenvalues problem is a solution to the transient heat conduction in two – dimensional composite slab to the layers subjected to a uniform temperature. The physical significance of the eigenvalues that mean when temperature across a fully insulated composite that is no temperature variation along it, it is the temperature variation along the slab without having temperature variation across it. The generalization to three dimensional situation of the two dimensional multi layer slab can readily be used to compute the eigenvalues associated with the composite layer problem.

B.B. Terry (2002) presented of transient conduction heat flux phenomena. Using the two experiments to measure the transient local heat flux and transient temperature which are then compare with predictions from conduction theory. The resulting temperature distribution for three different times corresponding to the early stages of the transient are compared. The temperature distribution effectively reaches the initial uniform slab temperature just a few centimeters beneath the surface of the slab. The solid line predictions of the local heat flux at several discrete locations within the slab, which demonstrate conduction phenomena, have largely been limited to simple one-dimensional steady-state conduction.

Q.T. Pham (1994) performed the comparison of general purpose finite element methods for this Stefan problem. A number of fixed grid finite element methods were tested on problems involving heat conduction with phase change. Only methods that can deal with arbitrary enthalpy temperature relationships were considered. Comparisons were made for temperature gradient versus enthalpy gradients formulations, lumped versus distributed capacitance, time average versus space-average apparent heat capacity and iterative versus non-iterative methods. The apparent heat capacity method which incorporates lumped capacitances and Pham's correction performed best, in terms of agreement with analytical solutions and speed of computation (as measured by the number of matrix solutions). The best iterative method allows marginally larger time intervals to be used and guarantees perfect heat balance, but for a given accuracy it is usually slower than the best non-iterative methods. A further advantage of the non-iterative methods is that the heat balance can serve as a useful check of convergence, a heat balance error of more than 1% generally indicating that convergence has not been reached.

A. Sutradhar, et al (2002) presented functionally graded materials in three-dimensional heat conduction. The Green's function for three-dimensional transient heat conduction (diffusion equation) for functionally graded materials (FGMs) is derived. The thermal conductivity and heat capacitance both vary exponentially in one coordinate. In the process of solving this diffusion problem numerically, a Laplace transform approach is used to eliminate the dependence on time. The fundamental solution in Laplace space is derived and the boundary integral equation formulation for the Laplace Transform Boundary Element Method (LTBEM) is obtained. The numerical implementation is performed using a Galerkin approximation, and the time-dependence is restored by numerical inversion of the Laplace transform using Stehfest's algorithm. A number of test problems have been examined, and the results are in excellent agreement with available analytical solutions.

C. Axnessa, J. Carrera and M. Bayer (2004) analyze the hydrodynamic flow equation under radial flow conditions solving by finite element formulation. The numerical solution of the flow equation in hydrological modeling has traditionally been accomplished through the use of finite-element or finite-difference models using Cartesian coordinates. Radial flow problems, however, are more easily posed in polar or cylindrical coordinates utilizing truncated ''sector'' type elements. The purpose of this work is to introduce a simple transformation of variables from (x; y) to (ln r; y) to facilitate using such elements. The transformation converts truncated ''sector'' elements in an annular domain to rectangular elements in a connected rectangular domain where the flow equation is then solved. The transformation makes mesh generation easy and allows a fine discretization about a well. The elements allow an exact representation of the well geometry. The method computes the head exactly at discrete points for homogeneous problems. It displays a reduced truncation error in head and flux computations for heterogeneous problems with racially dominant flow, when compared to methods using conventional elements. Numerical comparisons on a suite of heterogeneous media problems illustrate these advantages with respect to conventional methods.

B S Varaprasad Patnaik, et al (2002) presented the finite element simulation of internal flows with heat transfer using a velocity correction approach. The present numerical simulations employ a velocity correction algorithm, with a Galerkin weighted residual formulation. Two problems each in laminar and turbulent flow regimes are investigated, by solving full Navier–Stokes equations. Flow over a backward-facing step is studied with extensive validations. The robustness of the algorithm is demonstrated by solving a very complex problem viz. a disk and doughnut baffled heat exchanger, which has several obstructions in its flow path. The effect of wall conductivity in turbulent heat transfer is also studied by performing a conjugate analysis. Temporal evolution of flow in a channel due to circular, square and elliptic obstructions is investigated, to simulate the vortex dynamics. Flow past an in-line tube bank of a heat exchanger shell is numerically studied. Resulting heat and fluid flow patterns are analyzed. Important design parameters of interest such as the Nusselt number, Strouhal number, skin friction coefficient, pressure drop etc. are obtained. It is successfully demonstrated that the velocity correction approach with a Galerkin weighted residual formulation is able to effectively simulate a wide range of fluid flow features.

X. Lun and P. Tervola (2005) analyze the transient heat conduction in the composite slab subject to periodic temperature changes has been developed. Taking advantage of the periodic properties of boundary changes, the corresponding analytical solution is obtained and expressed explicitly. Unlike most of the traditional methods, it involves no residue evaluation and no iterative computation such as a numerical search for eigenvalues. Furthermore, comparison of the method with numerical calculations demonstrates the applicability and accuracy of the method.

D. Parshant (2002) presented the alternating direction implicit technique for two dimensional diffusion equation transient flow problem. In case of two dimensional heat conduction equations the FTCS (forward difference in time and central in space) results in conditionally stable solution which is stable only for a diffusion number less then 0.4. The implicit scheme can be used which results in a pentadiagonal system of matrix to solve the constants. However solving a pentadiagonal system is cumbersome and very time consuming process, so in order to avoid this hassle; the ADI (Alternating Direction Implicit) scheme is introduced. Results for both the Dirichlet and Neuman boundary conditions are plotted. The program for ADI is written in Fortran 90. The equation is very much restricted by stability so cannot be solved very accurately with FTCS method as well.

N. Mendes, et al (2001) analyze with using MATLAB/simulink in building thermal performance. This focused on a mathematical model applied to both building thermal analysis and control systems design. A lumped approach is used to model the room air temperature and a multi-layer model for the building envelope. The capacitance model allows studying the transient analysis of room air temperature when it is submitted to sinusoidal variation of external air temperature, representing a case study for a cold day in the south Brazil. To evaluate the building performance with thermal parameters, they will use MATLAB/SIMULINK. In the results section, it show the influences of thermal parameters on the building air temperature, heating system performance, energy consumption and the advantages of using MATLAB/SIMULINK in building thermal and energy analysis.

From the above literature review, there are different methods to solve the heat transfer problem such as finite element method, boundary element method and etc. All of the above presented mostly on transient heat conduction to the electronic packages, hybrid & conventional composite structures, Stefan problem, flux phenomena, graded materials, hydrodynamic flow and internal flow by using finite element method. It is to find out an efficient way to analyze the heat transfer phenomena in which can be used to predict the temperature and heat flow in and around a structure. The results of a heat transfer analysis are used in subsequent analysis to determine a structure's thermally induced response. Mean while, MATLAB is a method of choice in programming.

CHAPTER III

THEORY ON FINITE ELEMENT METHOD

3.1 Introduction

The finite element method is a numerical analysis technique solution to a range of engineering problems. It can be extended and applied to broad field of continuum mechanics. This is because due to the flexibility as an analysis tool. In this method of analysis, a còmplex region defining a continuum is discretized into simple geometric shapes called finite element[6].

Finite element method has three basic features that account for its superiority over other competing methods. The first basic features are the geometrically simple complex domain of the problem is represented as a collection of geometrically simple sub domains. The second is the approximation functions are derived using the basic idea that any continuous function can be represented by a linear combination of algebraic polynomials. The third is algebraic relation among the undetermined coefficients are obtained by satisfying the governing equations, often in weighted – integral sense, over each element. Nodes on the boundary and in the interior of the element represented the algebraic polynomials and undermined parameters represent the value of the solution.